Proving something is a domain

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Given a set X, an element $\bot \in X$ and a relation $\sqsubseteq \subseteq X \times X$ we must prove

- $\forall x \in X.\ \bot \sqsubseteq X$
- That $\sqsubseteq$ is a partial order. For this we must prove that it is reflexive, antisymmetric and transitive, which are the following properties:
  - $\forall x \in X.\ x \sqsubseteq x$
  - $\forall x, y \in X.\ x \sqsubseteq y \wedge y \sqsubseteq x \Rightarrow x = y$
  - $\forall x, y, z \in X.\ x \sqsubseteq y \wedge y \sqsubseteq z \Rightarrow x \sqsubseteq z$
- For all chains $\underset{x}{\rightharpoonup}$, $\underset{x}{\rightharpoonup}$ has a limit. To prove this there are two properties we must prove
  - $\exists z \in X.\ \forall i.x_i \sqsubseteq z$
  - $\exists z \in X.\ \forall y.(\forall i.x_i \sqsubseteq y) \Rightarrow\ z \sqsubseteq y$