Proving something is a domain

July 14, 2016

Given a set X, an element $\perp \in X$ and a relation $\sqsubseteq\ \subseteq X \times X$ we must prove

- $\forall x \in X.\ \perp \sqsubseteq X$
- That $\sqsubseteq$ is a partial order. For this we must prove that it is reflexive, antisymmetric and transitive, which are the following properties:
  - $\forall x \in X.\ x \sqsubseteq x$
  - $\forall x, y \in X.\ x \sqsubseteq y \wedge y \sqsubseteq x \Rightarrow x = y$
  - $\forall x, y, z \in X.\ x \sqsubseteq y \wedge y \sqsubseteq z \Rightarrow x \sqsubseteq z$
- For all chains $\underset{x}{\rightharpoonup}$, $\underset{x}{\rightharpoonup}$ has a limit. To prove this two properties we must prove that $\exists z \in X$ such that:
  - $\forall i.x_i \sqsubseteq z$ *(upper bound)*
  - $\forall y.(\forall i.x_i \sqsubseteq y) \Rightarrow\ z \sqsubseteq y$ *(least upper bound)*