1 $\frac{\partial u}{\partial t} = u^q, \; t \in [0, 10]$

$u(t) = e^t$ para $q = 1$

$$\frac{du}{dt} = u \rightarrow \frac{dt}{du} = \frac{1}{u}$$

$$\int dt = \int \frac{du}{du}$$

$$c + t = \ln|u|$$

Si decimos que $u(0) = 1$ entonces $c = 0$.

$$\therefore \; t = \ln|u|$$

$$\boxed{e^t = u}$$

Ahora, si $q < 1$:

$$\frac{\partial u}{\partial t} \frac{1}{u^q} = 1$$

$$du \, \frac{1}{u^q} = dt$$

$$\int \frac{1}{u^q} du = \int dt$$

$$\frac{u^{-q+1}}{-q+1} = t + c$$

$$u^{-q+1} = t + c(-q+1)$$

Evaluando $u(0) = 1$:

$$1^{-q+1} = c(-q+1)$$

$$c = \frac{1^{-q+1}}{-q+1}$$ pero sabemos que $1^{a} = 1$ donde $a \in \mathbb{R}$.

$$c = \frac{1}{-q+1}$$

∴ nos queda: $$u^{-q+1} = \left(t + \frac{1}{-q+1}\right)(-q+1)$$

Dejando u sola nos queda:

$$u = \sqrt[-q+1]{(-q+1)t+1}$$

$$u = ((-q+1)t+1)^{1/-q+1}$$

Reorganizando:

$$\boxed{u = ((1-q)t+1)^{\frac{1}{1-q}}}$$

Por último, tenemos:

$$t(1-q)+1 > 0$$

Despejando t:

$$t > -\frac{1}{1-q}$$