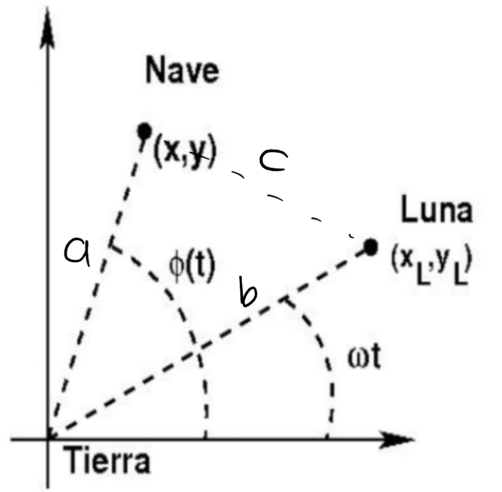
c)

$$x = r\cos\phi \qquad x(t) = r(t)\cos\phi$$
$$y = r\cos\phi \qquad y(t) = r(t)\cos\phi$$
$$r = \sqrt{x^2+y^2} \qquad r(t) = \sqrt{x(t)+y(t)^2}$$



Observando la imagen podemos utilizar la ley del Coseno.

$$a^2 + b^2 - 2ab\cos(\gamma) = c^2$$

Tenemos que el Lagrangiano: $L = k - u$

$$L = \left(\frac{mv^2}{2} + \frac{I\omega^2}{2}\right) - \left(-\frac{G m_L m}{r_L} - \frac{G m_T m}{r}\right)$$

$$L = \left(\frac{mr^2\phi^2}{2} + \frac{mr^2}{2} + \frac{G m_L m}{r_L} + \frac{G m_T m}{r}\right)$$

$$p_r = \frac{\partial L}{\partial \dot{r}} = m\dot{r} \qquad p_\phi = \frac{\partial L}{\partial \phi} = mr^2\dot{\phi}$$

$$\dot{r} = \frac{p_r}{m} \qquad \dot{\phi} = \frac{p_\phi}{mr^2}$$

Quedando el Lagrangiano como:

$$\mathcal{L} = \frac{P_\phi^2}{2mr^2} + \frac{P_r^2}{2m} + \frac{Gm_L m}{r_L} + \frac{Gm_T m}{r}$$

d) Ahora, el Hamiltoniano:

$$H = \dot{\phi} P_\phi + \dot{r} P_r - \mathcal{L}$$

$$H = \frac{P_\phi^2}{mr^2} + \frac{P_r^2}{m} - \mathcal{L}$$

$$H = \frac{P_\phi^2}{2mr^2} + \frac{P_r^2}{2m} - \frac{Gm_L m}{r_L} - \frac{Gm_T m}{r}$$

e) Tenemos:

$$\dot{P_r} = -\frac{\partial H}{\partial r} = \frac{P_\phi^2}{mr^3} - \frac{Gm_T m}{r^2} - \frac{G m_L m}{r_L^2}\left(\frac{\partial r_L}{\partial r}\right)$$

$$\dot{P_\phi} = -\frac{\partial H}{\partial \phi} = -\frac{G m_L m}{r_L^2}\left(\frac{\partial r_L}{\partial \phi}\right)$$

Reemplazamos r_L:

$$\frac{\partial r_L}{\partial r} = \frac{r + d\cos(\phi - \omega t)}{r_L}$$

$$\frac{\partial r_L}{\partial \phi} = \frac{rd\,\mathrm{Sen}(\phi - \omega t)}{r_L}$$

$\therefore$ $\dot{p}_r$ y $\dot{p}_\phi$:

$$\dot{p}_\phi = -\frac{G m_L m}{r_L^3}\left(r\, d\, \mathrm{Sen}\,(\phi - wt)\right)$$

$$\dot{p}_r = \frac{p_\phi^2}{m r^3} - \frac{G m_T m}{r^2} - \frac{G m_L m}{r_L^3}\left(r + d\,\mathrm{Cos}\,(\phi - wt)\right)$$

f)

$$\tilde{r} = \frac{r}{d} \qquad \tilde{p_r} = \frac{p_r}{md} \qquad \tilde{p_\phi} = \frac{p_\phi}{md^2}$$

$$\dot{\tilde{r}} = \frac{\dot{r}}{d} = \frac{\dot{p}_r}{md} = \tilde{p}_r$$

$$\dot{\phi} = \frac{p_\phi}{mr^2} = \frac{\tilde{p_\phi}}{\tilde{r}^2}$$

$$\tilde{r}_L = \frac{r_L}{d}\sqrt{\frac{r^2}{d^2} - \frac{2d^2}{d^2} + \frac{rd}{d^2}\,\mathrm{Cos}\,(\theta_L)}$$

$$\mu = \frac{m_L}{m_T} \qquad \Delta = \frac{G m_T}{d^3}$$

$\tilde{p}_r$ queda :
$$\dot{\tilde{p}}_r = \frac{p_\phi^2}{r^3 d m^2} - \frac{G m_T}{r^2 d} - \frac{G m}{r_L^2 d}\left(r + d\,\mathrm{Cos}\,(\theta_L)\right)$$

$$\dot{\tilde{p}}_r = \frac{p_\phi^2}{r^3 d m^2}\left(\frac{d^3}{d^3}\right) - \frac{G m_T}{r^2 d}\left(\frac{d^2}{d^2}\right) - \frac{G m_L}{r_L^3}\left(\frac{\mu_T}{\mu_T}\right)\left(r + d\,\mathrm{Cos}\,(\theta_L)\right)\left(\frac{d^3}{d^3}\right)$$

$$\dot{\tilde{p}}_r = \frac{p_\phi^2}{d^4 m^2}\,\frac{1}{\tilde{r}^3} - \frac{G m_T}{d^3}\,\frac{1}{\tilde{r}^2} - \frac{G m_T}{d^3}\,\mu\,\frac{1}{\tilde{r}_L^3}\left(\tilde{r} + \mathrm{Cos}\,(\theta_L)\right)$$

$$\tilde{p}_r = \frac{\tilde{p}_\phi^2}{\tilde{r}^3} - \frac{\Delta}{\tilde{r}^2} - \frac{\Delta\mu}{r_L^3}(\tilde{r} - \cos(\theta_L))$$

$$\tilde{p}_r = \frac{\tilde{p}_\phi^2}{\tilde{r}^3} - \Delta\left(\frac{1}{\tilde{r}^2} + \frac{\mu}{\tilde{r}^3}(\tilde{r} - \cos(\theta - wt)\right)$$

Ahora $\tilde{p}_\phi$:

$$\tilde{p}_\phi = \frac{\dot{p}_\phi}{m\partial^2} = \frac{-G m_L \operatorname{Sen}(\theta_L)}{r_L^3}\left(\frac{r\partial}{\partial^2}\right)\left(\frac{\partial^3}{\partial^3}\right)\left(\frac{\mu_T}{\mu_T}\right)$$

$$\dot{p}_\phi = \frac{-\Delta \cdot \tilde{r}\mu \operatorname{Sen}(\theta_L)}{\tilde{r}_L^3}$$

$\tilde{p}_r^0 = \frac{p_r^0}{m\partial}$ en condiciones iniciales.

g)

$$p_r^0 = \frac{\dot{r}m}{\partial m} = \frac{1}{\partial}\left(\frac{\partial r}{\partial t}\right)^0 = \frac{1}{\partial}\left(\frac{\partial\sqrt{x^2+y^2}}{\partial t}\right)^0$$

$$\tilde{p}_r = \frac{1}{\partial}\left(\frac{x\dot{x} + y\dot{y}}{\sqrt{x^2+y^2}}\right)^0$$

$$\tilde{p}_r = \frac{1}{\partial}\left(\frac{\vec{r}\cdot\vec{V}_0}{r}\right)$$

$$\tilde{p}_r^0 = \tilde{r} \cdot \tilde{V}_0$$

$$\tilde{p}_r^0 \tilde{V}_0 \cos(\theta \cdot \phi)$$

$$\tilde{p}_\phi^0 = \frac{\dot{p}\partial}{\partial^2 m} = \frac{r^2 m}{\partial^2 m}\dot{\phi}^0$$

Reemplazamos:

$$\tilde{\rho}_\phi^{\circ} = \tilde{r}^2 \frac{\partial}{\partial t} \arctan\left(\frac{1}{x^\circ}\right)$$

$$\tilde{\rho}_\phi^{\circ} = \frac{\tilde{r}^2 \frac{\partial}{\partial t}\left(\frac{x}{y}\right)^\circ}{\frac{y^2}{x^2} + 1}$$

$$\Rightarrow \tilde{\rho}_\phi^{\circ} = \frac{\tilde{r}^2}{r^2}(x\dot{y} - y\dot{x})^\circ$$

$$\tilde{\rho}_\phi^{\circ} = \frac{\vec{r}_0 \vec{V}_0}{\partial^2} \Rightarrow \tilde{\rho}_\phi = \frac{r_0 v_0 \operatorname{Sen}(\theta - \phi)}{\partial^2}$$

$$\therefore \tilde{\rho}_\phi = \tilde{r}_0 \bar{v}_0 \operatorname{Sen}(\theta - \phi)$$