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Marzo 19 de 2022

Métodos Computacionales 2 (202210_FISI2528)

Semana 8 – Punto # 2

Inicialmente, debe anotarse que las derivadas parciales de una función *f*, dependiente de dos variables *x* y *y* están dadas por:

$$\frac{\partial f}{\partial x} = \frac{f(x+h,y) - f(x,y)}{h} = \frac{f_{i+1,j} - f_{i,j}}{h} = \frac{f_{i+1,j} - f_{i-1,j}}{2h}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{f(x-h,y) - 2f(x,y) + f(x+h,y)}{h^2} = \frac{f_{i-1,j} - 2f_{i,j} + f_{i+1,j}}{h^2}$$

$$\frac{\partial f}{\partial y} = \frac{f(x,y+h) - f(x,y)}{h} = \frac{f_{i,j+1} - f_{i,j}}{h} = \frac{f_{i,j+1} - f_{i,j-1}}{2h}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{f(x,y-h) - 2f(x,y) + f(x,y+h)}{h^2} = \frac{f_{i,j-1} - 2f_{i,j} + f_{i,j+1}}{h^2}$$

De esta forma, para los casos particulares:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -w$$

$$v\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = \frac{\partial u}{\partial y}\frac{\partial w}{\partial x} - \frac{\partial u}{\partial x}\frac{\partial w}{\partial y}$$

Se tendría que:

$$\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2} + \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{h^2} = -w_{ij}$$

$$\frac{u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1} - 4u_{i,j}}{h^2} = -w_{ij}$$

Es decir,

$$u_{i,j} = \frac{1}{4}(u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1} + h^2 w_{ij}$$

Y, de manera semejante, se tendría que:

$$v\left(\frac{w_{i-1,j}-2w_{i,j}+w_{i+1,j}}{h^2}+\frac{w_{i,j-1}-2w_{i,j}+w_{i,j+1}}{h^2}\right)=\frac{u_{i,j+1}-u_{i,j-1}}{2h}\cdot\frac{w_{i+1,j}-w_{i-1,j}}{2h}-\frac{u_{i+1,j}-u_{i-1,j}}{2h}\cdot\frac{w_{i,j+1}-w_{i,j-1}}{2h}$$

Es decir,

$$w_{i,j}=\frac{1}{4}\left(w_{i-1,j}+w_{i+1,j}+w_{i,j-1}+w_{i,j+1}\right)-\frac{1}{16v}\left(u_{i,j+1}-u_{i,j-1}\right)\left(w_{i+1,j}-w_{i-1,j}\right)+\frac{1}{16v}(u_{i+1,j}-u_{i-1,j})(w_{i,j+1}-w_{i,j-1})$$

Lo cual, teniendo en cuenta que $R=\frac{1}{v}$, completa la demostración.