

$$\textcircled{1} \quad \frac{\partial u}{\partial t} = u^q, \quad t \in [0, 10]$$

$$u(t) = e^t \quad \text{pour } q = 1$$

$$\frac{du}{dt} = u \rightarrow \frac{dt}{du} = \frac{1}{u}$$

$$\int dt = \int \frac{du}{u}$$

$$c + t = \ln|u|$$

Si choisit que  $u(0) = 1$  alors  $c = 0$ .

$$\therefore t = \ln|u|$$

$$\boxed{e^t = u}$$

Donc, si  $q < 1$ :

$$\frac{\partial u}{\partial t} \frac{1}{u^q} = 1$$

$$du \frac{1}{u^q} = dt$$

$$\int \frac{1}{u^q} du = \int dt$$

$$\frac{u^{-q+1}}{-q+1} = t + c$$

$$u^{-q+1} = t + c(-q+1)$$

Enfin, si  $u(0) = 1$  :

$$1^{q+1} = C(-q+1)$$

$$C = \frac{1^{-q+1}}{-q+1}$$

para obtener que  $1^q = 1$  donde  $a \in \mathbb{R}$ .

$$C = \frac{1}{-q+1}$$

∴ nos queda:  $u^{-q+1} = \left(t + \frac{1}{-q+1}\right)(-q+1)$

Despejando  $u$  nos queda:

$$u = \sqrt[-q+1]{(-q+1)t + 1}$$

$$u = ((-q+1)t + 1)^{1/-q+1}$$

Reorganizando:

$$u = ((1-q)t + 1)^{\frac{1}{1-q}}$$

Por último, tenemos:

$$t(1-q) + 1 > 0$$

Despejando  $t$ :

$$t > -\frac{1}{1-q}$$