Build and evaluate a regression model

In this notebook we build regression models using linear regression and knn algorithms and choose the best one.

Import the libraries we need

In [1]:

```
# Core libraries
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

# Sklearn functionality
from sklearn.preprocessing import MinMaxScaler
from sklearn.model_selection import train_test_split

from sklearn.linear_model import LinearRegression
from sklearn.neighbors import KNeighborsRegressor
from sklearn.tree import DecisionTreeRegressor
from sklearn.svm import SVR

from sklearn.metrics import mean_absolute_error

# Convenience functions.  This can be found on the course github
from functions import *
```

Define the task

"Make predictions about a country's life expectancy in years from a set of metrics for the country."

Acquire clean data

Load the data

We will load and clean our data.

In [2]:

```
# Load the data set
dataset = pd.read_csv("world_data.csv")

# Remove sparsely populated features
dataset = dataset.drop(["murder","urbanpopulation","unemployment"], axis=1)

# Impute all features with mean
means = dataset.mean().to_dict()
for m in means:
    dataset[m] = dataset[m].fillna(value=means[m])
```

Understand the data

Let's use a nice plot style

In [3]:

```
print(plt.style.available)
```

```
['bmh', 'classic', 'dark_background', 'fast', 'fivethirtyeight', 'ggplot', 'grayscale', 'seaborn-b
right', 'seaborn-colorblind', 'seaborn-dark-palette', 'seaborn-dark', 'seaborn-darkgrid',
'seaborn-deep', 'seaborn-muted', 'seaborn-notebook', 'seaborn-paper', 'seaborn-pastel', 'seaborn-p
```

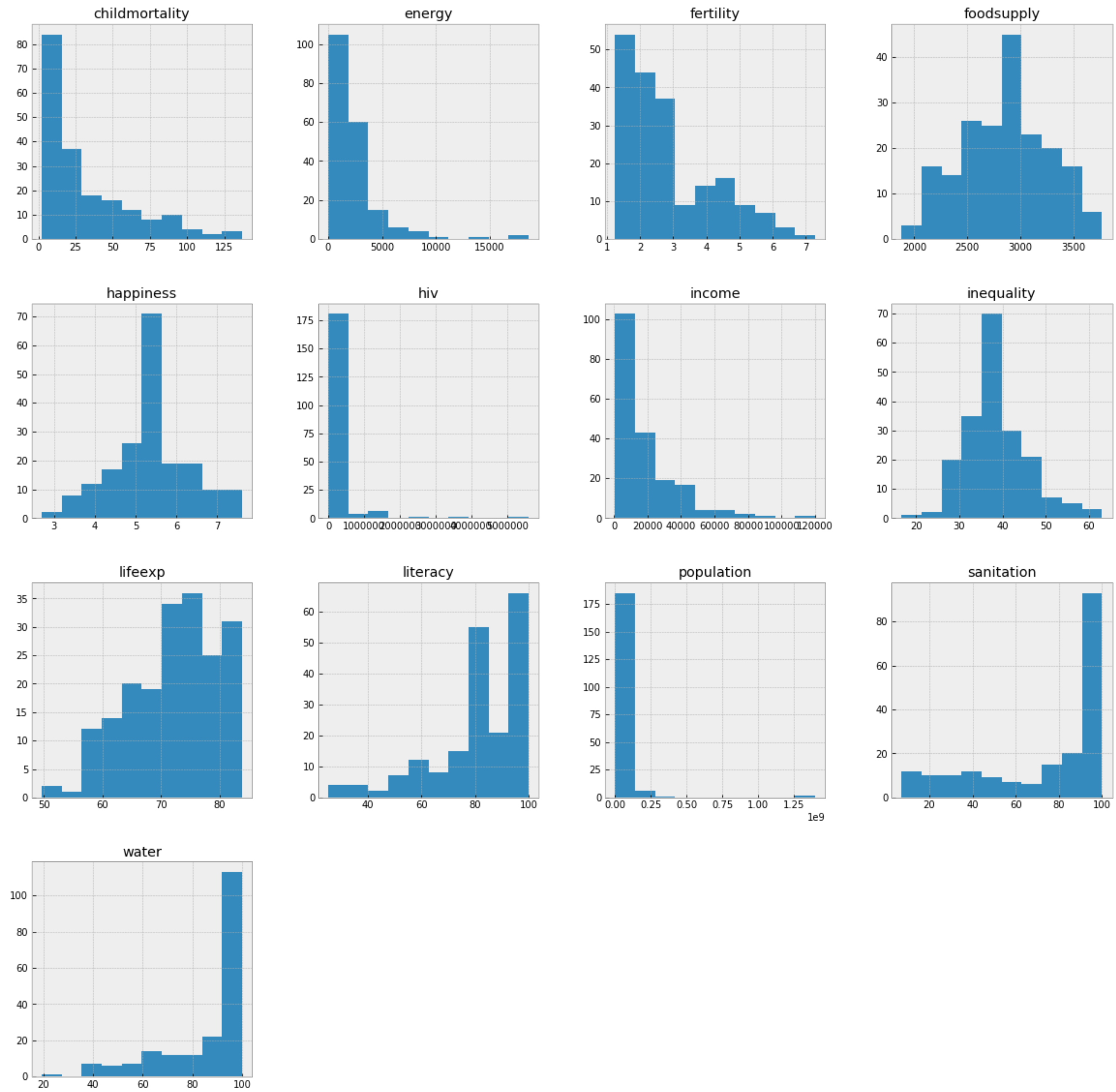
```
oster', 'seaborn-talk', 'seaborn-ticks', 'seaborn-white', 'seaborn-whitegrid', 'seaborn',
'Solarize_Light2', 'tableau-colorblind10', '_classic_test']
```

In [4]:

```python
plt.style.use('bmh')
```

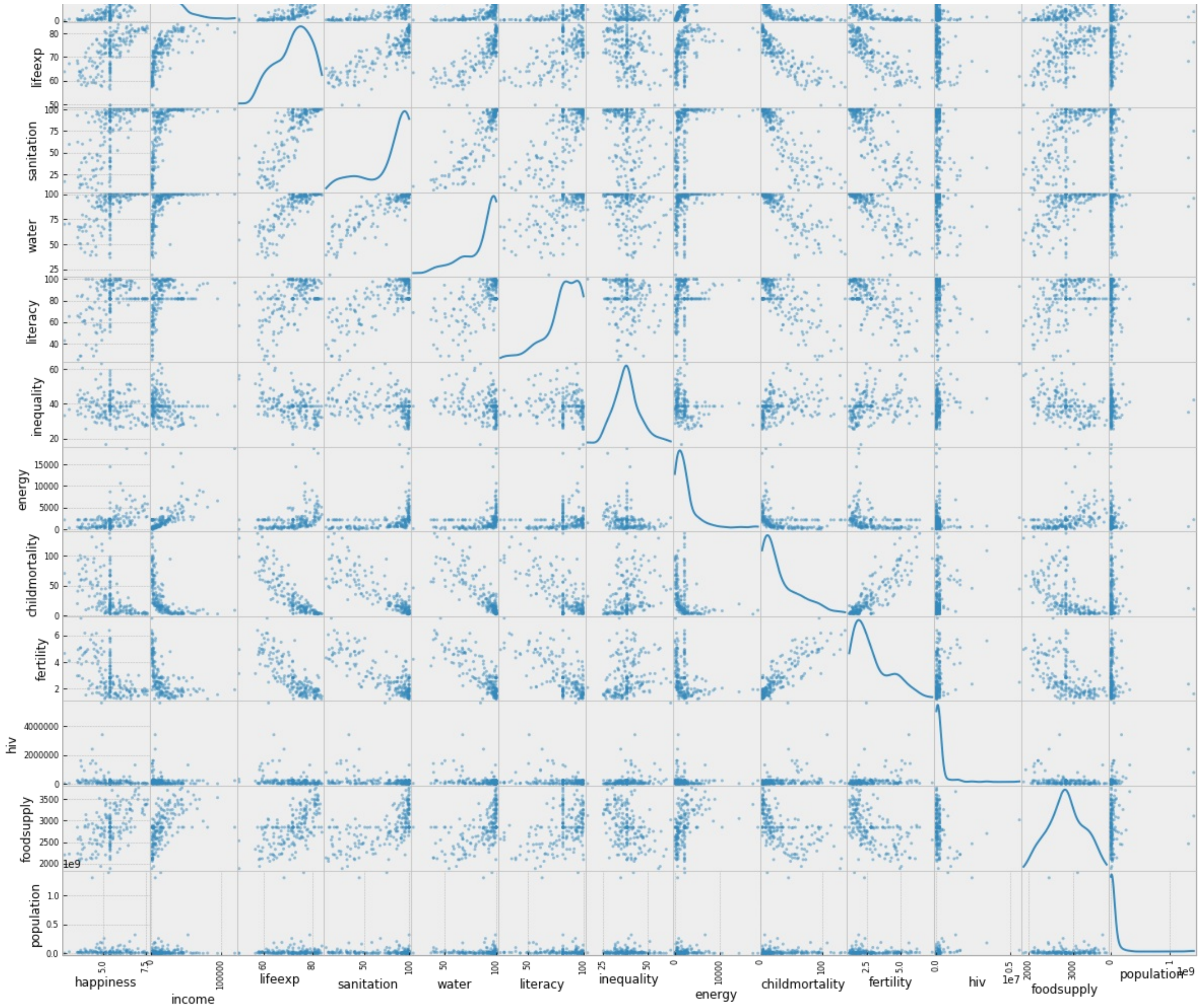
In [5]:

```python
# Histogram plot
dataset.hist(figsize=(20,20))
plt.show()
```



In [6]:

```python
# Scatter plot
scatterMatrix(dataset)
```



Prepare data

Select features

In [9]:

```
dataset.columns
```

Out[9]:

```
Index(['country', 'happiness', 'income', 'lifeexp', 'sanitation', 'water',
       'literacy', 'inequality', 'energy', 'childmortality', 'fertility',
       'hiv', 'foodsupply', 'population'],
      dtype='object')
```

In [10]:

```
y = dataset["lifeexp"]
X = dataset[['happiness', 'income', 'sanitation', 'water', 'literacy', 'inequality', 'energy', 'chi
ldmortality', 'fertility',  'hiv', 'foodsupply', 'population']]
```

Scale features

In [12]:

```
# Rescale the data
scaler = MinMaxScaler(feature_range=(0,1))
rescaledX = scaler.fit_transform(X)
```

```
rescaledX = scaler.fit_transform(X)

# Convert X back to a Pandas DataFrame, for convenience
X = pd.DataFrame(rescaledX, index=X.index, columns=X.columns)
```

Build models

Split into test and training sets

In [13]:

```
test_size = 0.33
seed = 1
X_train, X_test, Y_train, Y_test =  train_test_split(X, y, test_size=test_size, random_state=seed)
```

Create multiple models, fit them and check them

In [16]:

```
# Create and check a number of models
models = [LinearRegression(), KNeighborsRegressor(), SVR(gamma='auto')]

for model in models:
    model.fit(X_train, Y_train)
    predictions = model.predict(X_train)
    print(type(model).__name__, mean_absolute_error(Y_train, predictions))
```

```
LinearRegression 2.2920035925091766
KNeighborsRegressor 2.1955055341375442
SVR 3.6117510998705655
```

Evaluate models

In [17]:

```
# Evaluate the models
for model in models:
    predictions = model.predict(X_test)
    print(type(model).__name__, mean_absolute_error(Y_test, predictions))
```

```
LinearRegression 2.4463956508110307
KNeighborsRegressor 2.5532340600575907
SVR 3.6854103854533866
```

In [18]:

```
# Choose best model
model = models[0]
```

In [19]:

```
# See predictions made
predictions = model.predict(X_test)
df = X_test.copy()
df['Prediction'] = predictions
df['Actual'] = Y_test
df["Error"] = Y_test - predictions
df
```

Out[19]:

	happiness	income	sanitation	water	literacy	inequality	energy	childmortality	fertility	hiv	foodsupply	populati
44	0.262009	0.027702	0.077701	0.724907	0.618474	0.556034	0.017505	0.436202	0.462810	0.041061	0.603175	1.970658

	happiness	income	sanitation	water	literacy	inequality	energy	childmortality	fertility	hiv	foodsupply	populati
172	0.549625	0.097816	0.629789	1.000000	0.753331	0.473693	0.116950	0.247774	0.263789	0.038917	0.511086	2.142874
163	0.549625	0.010446	0.352131	0.558860	0.753331	0.439655	0.005617	0.398368	0.404959	0.038917	0.613757	7.214342
35	0.420443	0.079387	0.926819	0.980173	0.638554	0.327586	0.043324	0.159496	0.342149	0.001686	0.867725	6.699267
136	0.484455	0.042278	0.765390	0.890954	0.911647	0.392241	0.034718	0.146884	0.119008	0.044633	0.460317	6.684981
11	0.188311	0.011451	0.073396	0.591078	0.045515	0.672414	0.021916	0.725519	0.629752	0.011418	0.391534	7.563631
123	0.714052	0.122109	0.946190	0.977695	0.912985	0.418103	0.105453	0.077151	0.042975	0.087490	0.476190	4.906396
82	0.494371	0.122947	0.955876	0.970260	0.979920	0.329741	0.082323	0.014095	0.072727	0.038917	0.851852	4.407177
175	0.549625	0.115407	1.000000	0.995043	0.753331	0.473693	0.261448	0.105341	0.263789	0.038917	0.511086	7.357201
102	0.627499	0.166506	0.804133	1.000000	0.969210	0.415948	0.085013	0.051929	0.044628	0.002846	0.783069	1.420654
174	0.549625	0.330692	0.992467	0.887237	0.824632	0.473693	0.329762	0.064540	0.247934	0.038917	0.666667	2.992166
180	0.549625	0.208390	1.000000	0.954151	0.890228	0.650862	0.129121	0.091246	0.180165	0.038917	0.511086	5.907189
29	0.510797	0.167344	0.973095	0.995043	0.985274	0.312500	0.101688	0.019288	0.038017	0.000204	0.624339	3.020738
16	0.784461	0.117921	0.850409	0.969021	0.871486	0.747845	0.079096	0.100148	0.082645	0.087490	0.730159	1.471362
143	0.549625	0.163155	0.865476	0.959108	0.986613	0.473693	0.085551	0.048220	0.135537	0.038917	0.285714	6.350050
166	0.549625	0.095303	0.955876	0.973978	0.978581	0.469828	0.047359	0.050445	0.147107	0.000000	0.449735	2.907166
28	0.847113	0.119596	0.968790	0.996283	0.950469	0.685345	0.054890	0.051187	0.092562	0.001561	0.513228	3.427884
51	0.978505	0.352472	0.987086	1.000000	0.753331	0.241379	0.940830	0.000000	0.117355	0.000096	0.793651	2.278589
31	0.796930	0.249437	0.990314	0.998761	0.753331	0.200431	0.207119	0.007418	0.046281	0.000364	0.730159	7.563631
122	0.195642	0.015640	0.176711	0.381660	0.568942	0.456897	0.025035	0.419881	0.634711	0.285707	0.174603	3.849245
47	0.163837	0.004666	0.160568	0.596035	0.000000	0.368534	0.116950	0.666172	0.609917	0.015168	0.365079	8.635068
4	0.939946	0.361686	1.000000	1.000000	0.753331	0.390086	0.294260	0.011869	0.102479	0.003918	0.740741	1.699228
98	0.580577	0.052414	0.730951	0.882280	0.938420	0.473693	0.025089	0.191395	0.284298	0.003382	0.365079	7.284986
56	0.842241	0.504930	0.916057	0.986369	0.753331	0.327586	0.151176	0.011128	0.123967	0.001382	0.910053	3.349312
78	0.249105	0.024938	0.403788	0.623296	0.445783	0.344828	0.116950	0.606083	0.578512	0.004275	0.529101	2.977881
58	0.641818	0.281269	0.992467	1.000000	0.986613	0.405172	0.132887	0.008902	0.036364	0.026775	0.899471	4.249248
132	0.849141	0.437077	1.000000	0.990087	0.753331	0.525862	0.365264	0.032641	0.114050	0.232135	0.952381	2.285654
124	0.217614	0.006090	0.073396	0.539033	0.469880	0.571121	0.024067	0.562315	0.542149	0.026775	0.301587	5.292184
53	0.477618	0.081900	0.654542	0.869888	0.903614	0.493534	0.047036	0.186202	0.190083	0.067847	0.476190	1.842793
127	0.630348	0.120434	0.963409	0.931846	0.994645	0.521552	0.262523	0.373887	0.279339	0.038917	0.507937	3.970745
...	...	...	...	...	...	...	...	...	...	...	...	
14	0.492792	0.086089	0.944038	0.971499	0.973226	0.346983	0.117287	0.028932	0.018182	0.038917	0.671958	2.520734
40	0.745849	0.311425	0.986009	1.000000	0.753331	0.346983	0.197974	0.012611	0.122314	0.028561	0.846561	4.606393
19	0.297964	0.022341	0.448988	0.690211	0.650602	0.473693	0.021916	0.221068	0.223140	0.011418	0.317460	1.106366

	happiness	income	sanitation	water	literacy	inequality	energy	childmortality	fertility	hiv	foodsupply	populati
110	0.705941	0.231845	0.988162	0.973978	0.753331	0.213362	0.160858	0.028190	0.031405	0.000052	0.560847	5.87788
112	0.541032	0.000000	0.098149	0.256506	0.753331	0.473693	0.116950	1.000000	0.847934	0.006239	0.511086	9.920792
114	0.628054	0.281269	0.998924	0.995043	0.753331	0.323276	0.290495	0.009644	0.006612	0.002668	0.767196	3.613528
167	0.549625	0.025524	0.859019	0.729864	0.753331	0.473693	0.032512	0.253709	0.263789	0.038917	0.511086	3.000024
69	0.594432	0.107868	0.996771	0.960347	0.859438	0.473693	0.154403	0.083086	0.176860	0.038917	0.511086	4.442178
152	0.549625	0.162318	0.900990	0.940520	0.997323	0.473693	0.054352	0.025223	0.079339	0.002489	0.809524	8.206493
33	0.481507	0.107031	0.813818	0.931846	0.867470	0.605603	0.038967	0.217359	0.200000	0.007847	0.386243	7.492202
89	0.657410	0.036330	0.744942	0.780669	0.705489	0.637931	0.032243	0.134273	0.163636	0.001346	0.402116	4.335034
154	0.549625	0.079387	0.762161	0.956629	0.753331	0.473693	0.032082	0.228487	0.263789	0.038917	0.555556	4.442892
188	0.549625	0.012791	0.397331	0.630731	0.441767	0.260776	0.002567	0.366469	0.723967	0.038917	0.132275	8.778640
105	0.743374	0.419486	1.000000	1.000000	0.828648	0.473693	0.372795	0.082344	0.221488	0.038917	0.730159	2.256375
107	0.533696	0.106193	0.941885	0.890954	0.973226	0.473693	0.099536	0.029674	0.059504	0.000614	0.449735	6.313621
42	0.289758	0.070424	0.837495	0.916976	0.995984	0.426724	0.062958	0.068249	0.125620	0.000864	0.544974	2.813594
17	0.441428	0.137187	0.849333	0.991326	0.977242	0.448276	0.132887	0.044510	0.051240	0.000686	0.502646	5.120755
5	0.892492	0.364199	1.000000	1.000000	0.753331	0.299569	0.203891	0.010386	0.041322	0.003204	1.000000	6.192192
117	0.389653	0.087764	0.937581	0.904585	0.882195	0.487069	0.027241	0.054896	0.135537	0.000739	0.349206	1.477797
182	0.549625	0.196663	0.909600	0.987608	0.753331	0.473693	0.089316	0.055638	0.263789	0.038917	0.322751	3.092881
48	0.177111	0.008603	0.252045	0.556382	0.313253	0.528017	0.020625	0.494807	0.285950	0.021418	0.111111	7.635060
39	0.968276	0.321477	0.993543	1.000000	0.753331	0.226293	0.317928	0.001484	0.092562	0.000507	0.788360	3.906459
116	0.750549	0.264515	0.998924	0.998761	0.970549	0.422414	0.137728	0.008902	0.018182	0.026775	0.682540	3.313526
66	0.648533	0.188286	0.923590	0.982652	0.997323	0.379310	0.116749	0.020772	0.047934	0.001614	0.682540	1.413583
99	0.674324	0.206715	0.979552	0.973978	0.995984	0.473693	0.133425	0.020030	0.011570	0.006239	0.830688	2.734950
189	0.549625	0.038257	0.930047	0.998761	0.986613	0.450431	0.029984	0.108309	0.403306	0.038917	0.511086	6.785768
18	0.350346	0.007765	0.165949	0.428748	0.045515	0.403017	0.116950	0.640208	0.694215	0.021418	0.444444	1.292082
159	0.549625	0.101167	0.766466	0.945477	0.753331	0.473693	0.041549	0.100890	0.147107	0.038917	0.301587	6.857197
113	0.445901	0.098654	0.710504	0.810409	0.906292	1.000000	0.144721	0.310831	0.206612	1.000000	0.603175	3.949245
54	0.417876	0.128810	0.874085	0.936803	0.799197	0.478448	0.161934	0.100148	0.074380	0.017132	0.640212	5.670687

65 rows × 15 columns

Interpret model

Intepret linear regression model

In [20]:

```
models[0].coef_
```

Out[20]:

```
array([  5.00912637,   9.2096769 ,   5.41605897,   2.26122297,
        -4.13876626,  -3.31059309,  -5.53211396, -14.32217202,
        -1.63521978,  -2.01476135,   2.22581396,   3.20586791])
```

In [21]:

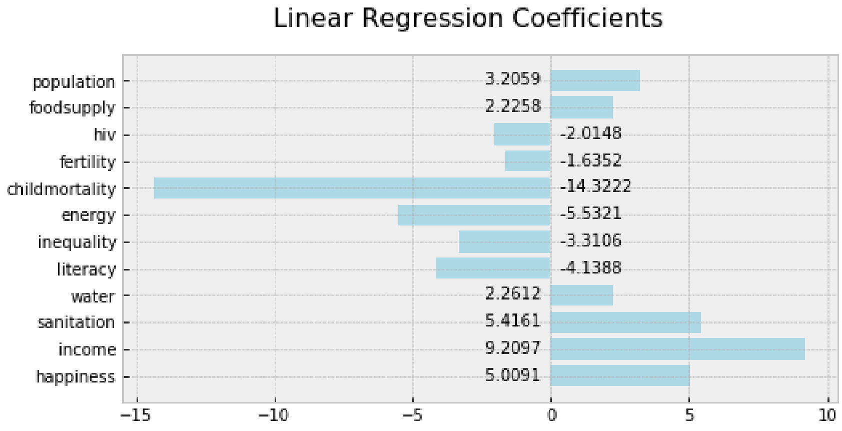
```
models[0].intercept_
```

Out[21]:

```
69.60409836404614
```

In [22]:

```
linearRegressionSummary(models[0], X.columns)
```



Interpret knn model

In [23]:

```
# Distances for k nearest neighbours to each point
models[1].kneighbors(X)
```

Out[23]:

```
(array([[0.        , 0.38911668, 0.3929857 , 0.4201251 , 0.51618381],
        [0.        , 0.20505552, 0.20891422, 0.23226123, 0.26827204],
        [0.        , 0.12274953, 0.1953064 , 0.22064946, 0.22207317],
        [0.        , 0.16349608, 0.20891422, 0.22826679, 0.28847733],
        [0.13327753, 0.15176936, 0.16216559, 0.16956607, 0.18772587],
        [0.10256832, 0.15520456, 0.24629633, 0.24943538, 0.25508778],
        [0.        , 0.3593436 , 0.38793799, 0.38953126, 0.39317685],
        [0.        , 0.31469369, 0.32420127, 0.35698837, 0.41441586],
        [0.        , 0.1962574 , 0.34952635, 0.44393928, 0.51551159],
        [0.        , 0.12677335, 0.18037416, 0.18620954, 0.22384403],
        [0.        , 0.1768139 , 0.24628597, 0.25110484, 0.26795007],
        [0.50647823, 0.51036495, 0.51934624, 0.53394104, 0.5422487 ],
        [0.        , 0.28806232, 0.34952635, 0.46122169, 0.46576997],
        [0.        , 0.31134313, 0.31241819, 0.3828958 , 0.39244964],
        [0.15281102, 0.15938391, 0.19360514, 0.1973218 , 0.21007098],
        [0.        , 0.48004454, 0.49459404, 0.51470302, 0.54082671],
        [0.20985158, 0.28530332, 0.30840304, 0.31097787, 0.35484273],
        [0.13328827, 0.20812534, 0.24324601, 0.25921397, 0.25998832],
        [0.37155811, 0.40041014, 0.40058674, 0.46115617, 0.52932556],
        [0.35179564, 0.36415699, 0.40205587, 0.40813321, 0.41305967],
        [0.        , 0.2978747 , 0.35416281, 0.37108084, 0.38199833],
        [0.        , 0.22731765, 0.24609031, 0.2514421 , 0.26426626],
        [0.        , 0.44593904, 0.49654252, 0.55091864, 0.6034237 ],
        [0.        , 0.18248721, 0.18982473, 0.20722744, 0.22207317],
        [0.        , 0.81669763, 0.95775295, 0.95925478, 1.03565196],
        [0.        , 0.16950802, 0.21630199, 0.25427943, 0.26096556],
```

```
[0.        , 0.16950802, 0.21630199, 0.25427943, 0.26096556],
[0.        , 0.4236558 , 0.42931981, 0.4328422 , 0.43998536],
[0.        , 0.38717937, 0.4552416 , 0.53517022, 0.53784733],
[0.13476736, 0.19861057, 0.22056109, 0.23980557, 0.25151364],
[0.06012713, 0.17754024, 0.19235813, 0.19558804, 0.19833983],
[0.        , 0.23413216, 0.27100383, 0.2924633 , 0.29424509],
[0.21016054, 0.21317651, 0.21401717, 0.22530531, 0.23435293],
[0.        , 0.11551925, 0.13540946, 0.16807396, 0.17109652],
[0.14275493, 0.18337059, 0.18423998, 0.1911442 , 0.23841273],
[0.        , 0.18976031, 0.22171421, 0.23478481, 0.27052351],
[0.23796152, 0.26788789, 0.33464266, 0.40026901, 0.41369269],
[0.        , 0.19144591, 0.2018412 , 0.22074537, 0.22171421],
[0.        , 0.16560299, 0.17315359, 0.17392261, 0.18037416],
[0.        , 0.47312354, 0.47673224, 0.51618381, 0.55412114],
[0.13591545, 0.13928121, 0.17811116, 0.1949138 , 0.20628951],
[0.07170979, 0.17453854, 0.22052985, 0.23348755, 0.27410883],
[0.        , 0.32115285, 0.35666572, 0.38319009, 0.38575948],
[0.15280169, 0.21645216, 0.28807777, 0.31737569, 0.3383806 ],
[0.        , 0.15685008, 0.17109652, 0.1768139 , 0.179002  ],
[0.41053844, 0.46547848, 0.47721835, 0.48298201, 0.52123981],
[0.        , 0.10306423, 0.12386973, 0.16560299, 0.23479227],
[0.        , 0.31134313, 0.31206216, 0.3336656 , 0.35914784],
[0.39173788, 0.41672897, 0.41990415, 0.46036325, 0.49940464],
[0.37199265, 0.42779708, 0.44233556, 0.44901508, 0.47717194],
[0.        , 0.19130322, 0.21373038, 0.21652458, 0.24722352],
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In [24]:

```
# Actual nearest neighbour points
g = models[1].kneighbors_graph(X).toarray()
```

In [25]:

```
# First data point's nearest neighbours
g[0]
```

Out[25]:

```
array([0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 1., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 1., 0., 0., 0., 0., 0., 0., 0., 0., 0., 1., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 1., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 1., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
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       0., 0., 0., 0., 0., 0., 0., 0., 0., 0.])
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