Simulating TB in the U.S. with an S.E.I. Model

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06-09-25

Load in necessary libraries, set up parameters and time values

```
# load in libraries

library("deSolve")
library(ggplot2)
library(tidyr)
```

```
# set up params

rho = 0.0179 # US birth rate: range 0.0179
q = 0.15 # Proportion of active cases that have potential to be infectious. Range (0, 1)
alpha = 0.00425 # Immigration rate. Range 0.00425
lambda = 8 # Effective Contact Rate. Range (0, 30)
gamma = 0.1 # Proportion H-resistance acquisition cases. Range (0, 1)
mu_0 = 0.013 # Mortality rate unrelated to TB. Range 0.013
mu = 0.15 # TB mortality rate. Range (0, 0.5)
v_L = 0.003 # Progression rate from latent to active infection. Range (0, 0.01)
l = 0.25 # Proportion of immigrants that have LTBI. Range (0, 0.3)
p = 0.15 # Proportion of exogenous infections that are acute. Range (0, 0.3)
trt = c(0.1, 0.1, 0.1, 0.1) # Proportion of treatment time when individuals are infectious (I 1-4). Ran
z = c(0.85, 0.7, 0.65, 0.4) # Proportion of treatment courses for TB (DS,H,R,MDR) that are successful.
phi = c(0.8,0.8,0.5,0.4) # Rate of end of treatment (DS, H, R, MDR). Ranges [(0.6, 0.9), (0.5, 0.9), (0
r = c(0.05,0.03,0.02) # Proportion of immigrant LTBI cases (H,R,MDR). Ranges [(0, 0.2), (0, 0.2), (0, 0
y = c(0.15,0.1) # Proportion of failed treatments for DS TB that result in H- or R-resistance Ranges [(
```

```
# time setup

years <- 50
dt = 0.25
time <- seq(0, years, dt)
```

Solve Differential Equations Numerically using Euler's Method

```
# variables and initial values setup

S = numeric(length(time))
E_1 = numeric(length(time))
I_1 = numeric(length(time))
E_2 = numeric(length(time))
I_2 = numeric(length(time))
E_3 = numeric(length(time))
```

```
I_3 = numeric(length(time))
E_4 = numeric(length(time))
I_4 = numeric(length(time))
D = numeric(length(time))
N = numeric(length(time))

S[1] = 950000
E_1[1] = 30000
I_1[1] = 2000
E_2[1] = 3000
I_2[1] = 500
E_3[1] = 2000
I_3[1] = 300
E_4[1] = 1000
I_4[1] = 200
D[1] = 0
N[1] = 1000000
```

```
# CDE Model

for (t in 1:(length(time) - 1)) {

  # differential equations

   # dS/dT
      dS <- (rho*N[t])
            - (q*trt[1]*lambda*(S[t]*I_2[t]/N[t])) - (q*trt[2]*lambda*(S[t]*I_2[t]/N[t])) - (q*trt[3]*la
            + (z[1]*phi[1]*I_1[t]) + (z[2]*phi[2]*I_2[t]) + (z[3]*phi[3]*I_3[t]) + (z[4]*phi[4]*I_4[t])
            + (1 - l)*(alpha*N[t]) - mu_0*S

      # dE_1/dT
      dE_1 <- (1 - p)*(q*trt[1]*lambda*(S[t]*I_1[t]/N[t])) - v_L*E_1[t] + (1-y[1])*(1-z[1])*phi[1]*I_1[

      # dI_1/dT
      dI_1 <- q*trt[1]*lambda*(S[t]*I_1[t]/N[t]) + q*v_L*E_1[t] - phi[1]*I_1[t] - mu_0*I_1[t] - mu*I_1[

      # dE_2/dT
      dE_2 <- (1 - p)*(q*trt[2]*lambda*(S[t]*I_2[t]/N[t])) - v_L*E_2[t] + (1-y[1])*(1-z[2])*phi[2]*I_2[

      # dI_2/dT
      dI_2 <- q*trt[2]*lambda*(S[t]*I_2[t]/N[t]) + v_L*E_2[t] - phi[2]*I_2[t] - mu_0*I_2[t] - mu*I_2[t]

      # dE_3/dT
      dE_3 <- (1 - p)*(q*trt[3]*lambda*(S[t]*I_3[t]/N[t])) - v_L*E_3[t] + (1-y[1])*(1-z[3])*phi[3]*I_3[

      # dI_3/dT
      dI_3 <- q*trt[3]*lambda*(S[t]*I_3[t]/N[t]) + v_L*E_3[t] - phi[3]*I_3[t] - mu_0*I_3[t] - mu*I_3[t]

      # dE_4/dT
      dE_4 <- (1 - p)*(q*trt[4]*lambda*(S[t]*I_4[t]/N[t])) - v_L*E_4[t]
      + (1-z[2])*y[1]*phi[2]*I_2[t] + (1-z[3])*y[1]*phi[3]*I_3[t] + (1-z[4])*phi[4]*I_4[t]
      + l*alpha*r[4]*N[t] - mu_0*E_4[t]

      # dI_4/dT
```

```
    dI_4 <- q*trt[4]*lambda*(S[t]*I_4[t]/N[t]) + v_L*E_4[t] - phi[4]*I_4[t] - mu_0*I_4[t] - mu*I_4[t]

    # dD/dT
    dD <- mu*(I_1[t] + I_2[t] + I_3[t] + I_4[t])

    # dN/dT
    dN <- rho*N[t] + alpha*N[t] - mu*(I_1[t] + I_2[t] + I_3[t] + I_4[t]) - mu_0*N[t]

  # Updated values

  S[t + 1] = S[t] + dS*dt
  E_1[t + 1] = E_1[t] + dE_1*dt
  I_1[t + 1] = I_1[t] + dI_1*dt
  E_2[t + 1] = E_2[t] + dE_2*dt
  I_2[t + 1] = I_2[t] + dI_2*dt
  E_3[t + 1] = E_3[t] + dE_3*dt
  I_3[t + 1] = I_3[t] + dI_3*dt
  E_4[t + 1] = E_4[t] + dE_4*dt
  I_4[t + 1] = I_4[t] + dI_4*dt
  D[t + 1] = D[t] + dD*dt
  N[t + 1] = N[t] + dN*dt

}

result1 = data.frame(time, S, E_1, I_1, E_2, I_2, E_3, I_3, E_4, I_4, D, N)
result1
```

```
##      time         S      E_1        I_1      E_2       I_2      E_3       I_3
## 1    0.00  950000.0 30000.00 2000.00000 3000.000 500.00000 2000.000 300.00000
## 2    0.25  954475.0 30218.51 1578.87500 3039.794 396.12500 2026.493 260.32500
## 3    0.50  958959.7 30415.83 1247.27221 3071.478 314.35588 2048.770 226.13403
## 4    0.75  963454.3 30596.45  986.13935 3096.778 249.98254 2067.545 196.66615
## 5    1.00  967958.9 30763.92  780.48448 3117.054 199.29984 2083.404 171.26648
## 6    1.25  972473.4 30921.04  618.50938 3133.377 159.39288 2096.830 149.37138
## 7    1.50  976998.1 31070.01  490.92820 3146.588 127.96816 2108.223 130.49562
## 8    1.75  981533.0 31212.58  390.43172 3157.354 103.22099 2117.914 114.22146
## 9    2.00  986078.1 31350.11  311.26603 3166.198  83.73112 2126.180 100.18918
## 10   2.25  990633.5 31483.69  248.90113 3173.532  68.38076 2133.248  88.08899
## 11   2.50  995199.2 31614.18  199.77024 3179.680  56.29007 2139.311  77.65409
## 12   2.75  999775.3 31742.23  161.06475 3184.901  46.76648 2144.526  68.65464
## 13   3.00 1004361.7 31868.40  130.57286 3189.394  39.26471 2149.028  60.89266
## 14   3.25 1008958.6 31993.08  106.55258 3193.320  33.35548 2152.929  54.19756
## 15   3.50 1013566.0 32116.62   87.63182 3196.803  28.70076 2156.324  48.42238
## 16   3.75 1018183.9 32239.28   72.72976 3199.943  25.03436 2159.290  43.44044
## 17   4.00 1022812.3 32361.26   60.99493 3202.818  22.14664 2161.895  39.14258
## 18   4.25 1027451.2 32482.73   51.75649 3205.488  19.87249 2164.196  35.43474
## 19   4.50 1032100.8 32603.81   44.48585 3208.004  18.08185 2166.239  32.23580
## 20   4.75 1036760.9 32724.61   38.76650 3210.402  16.67227 2168.067  29.47586
## 21   5.00 1041431.7 32845.20   34.27019 3212.712  15.56304 2169.711  27.09464
## 22   5.25 1046113.2 32965.66   30.73821 3214.960  14.69056 2171.203  25.04017
## 23   5.50 1050805.3 33086.02   27.96665 3217.162  14.00473 2172.566  23.26764
## 24   5.75 1055508.2 33206.34   25.79476 3219.335  13.46606 2173.822  21.73841
## 25   6.00 1060221.8 33326.64   24.09584 3221.490  13.04343 2174.989  20.41914
## 26   6.25 1064946.2 33446.95   22.76996 3223.635  12.71231 2176.082  19.28109
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```
## 27   6.50 1069681.3 33567.29   21.73837 3225.779  12.45337 2177.114  18.29946
## 28   6.75 1074427.3 33687.67   20.93894 3227.926  12.25137 2178.096  17.45287
## 29   7.00 1079184.2 33808.11   20.32268 3230.083  12.09430 2179.038  16.72285
## 30   7.25 1083951.9 33928.62   19.85097 3232.250  11.97268 2179.948  16.09349
## 31   7.50 1088730.5 34049.21   19.49333 3234.433  11.87904 2180.833  15.55104
## 32   7.75 1093520.0 34169.88   19.22573 3236.633  11.80750 2181.699  15.08365
## 33   8.00 1098320.5 34290.64   19.02920 3238.851  11.75341 2182.551  14.68109
## 34   8.25 1103131.9 34411.50   18.88878 3241.090  11.71310 2183.394  14.33452
## 35   8.50 1107954.4 34532.45   18.79266 3243.350  11.68370 2184.232  14.03633
## 36   8.75 1112787.8 34653.50   18.73155 3245.631  11.66291 2185.067  13.77993
## 37   9.00 1117632.3 34774.66   18.69808 3247.936  11.64894 2185.903  13.55964
## 38   9.25 1122487.9 34895.92   18.68645 3250.263  11.64037 2186.742  13.37057
## 39   9.50 1127354.6 35017.29   18.69210 3252.614  11.63609 2187.586  13.20846
## 40   9.75 1132232.4 35138.77   18.71139 3254.989  11.63520 2188.437  13.06968
## 41  10.00 1137121.4 35260.37   18.74149 3257.388  11.63702 2189.297  12.95106
## 42  10.25 1142021.5 35382.07   18.78014 3259.812  11.64099 2190.166  12.84987
## 43  10.50 1146932.8 35503.89   18.82556 3262.260  11.64667 2191.046  12.76376
## 44  10.75 1151855.4 35625.83   18.87636 3264.732  11.65374 2191.937  12.69070
## 45  11.00 1156789.2 35747.89   18.93141 3267.230  11.66190 2192.842  12.62892
## 46  11.25 1161734.2 35870.06   18.98984 3269.752  11.67096 2193.760  12.57692
## 47  11.50 1166690.6 35992.35   19.05097 3272.299  11.68074 2194.692  12.53337
## 48  11.75 1171658.3 36114.76   19.11425 3274.870  11.69111 2195.638  12.49715
## 49  12.00 1176637.4 36237.29   19.17925 3277.467  11.70197 2196.600  12.46728
## 50  12.25 1181627.9 36359.95   19.24563 3280.088  11.71322 2197.577  12.44291
## 51  12.50 1186629.7 36482.72   19.31311 3282.735  11.72481 2198.569  12.42330
## 52  12.75 1191643.0 36605.63   19.38149 3285.406  11.73669 2199.578  12.40784
## 53  13.00 1196667.8 36728.65   19.45060 3288.102  11.74881 2200.603  12.39596
## 54  13.25 1201704.0 36851.81   19.52030 3290.823  11.76114 2201.645  12.38721
## 55  13.50 1206751.7 36975.09   19.59050 3293.569  11.77366 2202.703  12.38116
## 56  13.75 1211811.0 37098.50   19.66110 3296.340  11.78634 2203.778  12.37748
## 57  14.00 1216881.9 37222.03   19.73205 3299.135  11.79917 2204.871  12.37585
## 58  14.25 1221964.3 37345.70   19.80329 3301.956  11.81213 2205.980  12.37600
## 59  14.50 1227058.4 37469.50   19.87478 3304.801  11.82523 2207.106  12.37772
## 60  14.75 1232164.1 37593.43   19.94650 3307.672  11.83844 2208.250  12.38080
## 61  15.00 1237281.5 37717.49   20.01840 3310.567  11.85176 2209.411  12.38508
## 62  15.25 1242410.5 37841.69   20.09049 3313.487  11.86520 2210.589  12.39040
## 63  15.50 1247551.3 37966.02   20.16273 3316.431  11.87873 2211.785  12.39664
## 64  15.75 1252703.9 38090.48   20.23512 3319.401  11.89237 2212.998  12.40369
## 65  16.00 1257868.2 38215.08   20.30765 3322.395  11.90611 2214.229  12.41145
## 66  16.25 1263044.3 38339.82   20.38031 3325.414  11.91994 2215.478  12.41984
## 67  16.50 1268232.3 38464.70   20.45309 3328.458  11.93387 2216.743  12.42879
## 68  16.75 1273432.1 38589.71   20.52599 3331.526  11.94789 2218.027  12.43824
## 69  17.00 1278643.8 38714.87   20.59901 3334.620  11.96200 2219.328  12.44813
## 70  17.25 1283867.4 38840.17   20.67214 3337.738  11.97621 2220.646  12.45842
## 71  17.50 1289103.0 38965.60   20.74539 3340.881  11.99051 2221.983  12.46906
## 72  17.75 1294350.5 39091.18   20.81875 3344.048  12.00490 2223.336  12.48003
## 73  18.00 1299610.0 39216.91   20.89221 3347.240  12.01938 2224.708  12.49129
## 74  18.25 1304881.6 39342.78   20.96578 3350.457  12.03395 2226.096  12.50281
## 75  18.50 1310165.2 39468.79   21.03947 3353.699  12.04861 2227.503  12.51458
## 76  18.75 1315460.8 39594.95   21.11325 3356.965  12.06336 2228.927  12.52657
## 77  19.00 1320768.6 39721.26   21.18715 3360.256  12.07820 2230.369  12.53877
## 78  19.25 1326088.5 39847.71   21.26116 3363.571  12.09314 2231.828  12.55117
## 79  19.50 1331420.6 39974.31   21.33527 3366.911  12.10816 2233.304  12.56374
## 80  19.75 1336764.8 40101.07   21.40949 3370.276  12.12327 2234.799  12.57648
```

```
## 81  20.00 1342121.3 40227.97   21.48382 3373.665  12.13847 2236.310  12.58939
## 82  20.25 1347490.0 40355.02   21.55826 3377.080  12.15376 2237.840  12.60244
## 83  20.50 1352871.0 40482.23   21.63280 3380.518  12.16915 2239.386  12.61564
## 84  20.75 1358264.3 40609.59   21.70746 3383.981  12.18462 2240.951  12.62898
## 85  21.00 1363669.9 40737.11   21.78222 3387.469  12.20018 2242.533  12.64245
## 86  21.25 1369087.9 40864.78   21.85709 3390.982  12.21583 2244.132  12.65605
## 87  21.50 1374518.2 40992.61   21.93208 3394.519  12.23157 2245.749  12.66978
## 88  21.75 1379961.0 41120.59   22.00717 3398.080  12.24740 2247.383  12.68362
## 89  22.00 1385416.2 41248.73   22.08238 3401.666  12.26332 2249.034  12.69759
## 90  22.25 1390883.9 41377.03   22.15770 3405.277  12.27934 2250.703  12.71167
## 91  22.50 1396364.1 41505.49   22.23313 3408.912  12.29544 2252.390  12.72586
## 92  22.75 1401856.8 41634.11   22.30867 3412.572  12.31163 2254.094  12.74017
## 93  23.00 1407362.1 41762.89   22.38432 3416.256  12.32791 2255.815  12.75458
## 94  23.25 1412879.9 41891.84   22.46009 3419.965  12.34428 2257.554  12.76910
## 95  23.50 1418410.4 42020.94   22.53597 3423.699  12.36074 2259.310  12.78373
## 96  23.75 1423953.5 42150.21   22.61197 3427.457  12.37730 2261.083  12.79847
## 97  24.00 1429509.2 42279.65   22.68808 3431.239  12.39394 2262.874  12.81331
## 98  24.25 1435077.7 42409.25   22.76431 3435.046  12.41067 2264.682  12.82826
## 99  24.50 1440658.9 42539.02   22.84065 3438.877  12.42749 2266.508  12.84331
## 100 24.75 1446252.9 42668.96   22.91711 3442.733  12.44441 2268.351  12.85846
## 101 25.00 1451859.6 42799.06   22.99368 3446.614  12.46141 2270.211  12.87372
## 102 25.25 1457479.2 42929.34   23.07037 3450.519  12.47851 2272.088  12.88908
## 103 25.50 1463111.6 43059.78   23.14718 3454.448  12.49569 2273.983  12.90454
## 104 25.75 1468756.9 43190.40   23.22411 3458.402  12.51297 2275.895  12.92010
## 105 26.00 1474415.0 43321.19   23.30116 3462.381  12.53033 2277.825  12.93577
## 106 26.25 1480086.2 43452.15   23.37832 3466.384  12.54779 2279.771  12.95153
## 107 26.50 1485770.2 43583.29   23.45561 3470.411  12.56534 2281.735  12.96740
## 108 26.75 1491467.3 43714.60   23.53301 3474.463  12.58297 2283.716  12.98336
## 109 27.00 1497177.4 43846.09   23.61054 3478.539  12.60070 2285.714  12.99943
## 110 27.25 1502900.6 43977.75   23.68818 3482.640  12.61852 2287.730  13.01560
## 111 27.50 1508636.8 44109.59   23.76595 3486.766  12.63643 2289.763  13.03187
## 112 27.75 1514386.2 44241.61   23.84384 3490.915  12.65444 2291.813  13.04824
## 113 28.00 1520148.6 44373.81   23.92185 3495.090  12.67253 2293.880  13.06471
## 114 28.25 1525924.3 44506.19   23.99999 3499.289  12.69071 2295.965  13.08128
## 115 28.50 1531713.2 44638.75   24.07824 3503.512  12.70899 2298.066  13.09795
## 116 28.75 1537515.3 44771.49   24.15662 3507.759  12.72735 2300.185  13.11472
## 117 29.00 1543330.6 44904.42   24.23513 3512.032  12.74581 2302.321  13.13159
## 118 29.25 1549159.3 45037.53   24.31376 3516.328  12.76436 2304.474  13.14856
## 119 29.50 1555001.2 45170.82   24.39252 3520.649  12.78300 2306.645  13.16563
## 120 29.75 1560856.6 45304.30   24.47140 3524.995  12.80173 2308.832  13.18280
## 121 30.00 1566725.3 45437.97   24.55040 3529.365  12.82055 2311.037  13.20008
## 122 30.25 1572607.4 45571.82   24.62954 3533.760  12.83946 2313.259  13.21745
## 123 30.50 1578503.0 45705.86   24.70880 3538.179  12.85847 2315.498  13.23492
## 124 30.75 1584412.0 45840.10   24.78819 3542.622  12.87756 2317.754  13.25250
## 125 31.00 1590334.6 45974.52   24.86770 3547.090  12.89675 2320.027  13.27017
## 126 31.25 1596270.7 46109.13   24.94735 3551.582  12.91603 2322.317  13.28795
## 127 31.50 1602220.3 46243.94   25.02712 3556.099  12.93540 2324.625  13.30582
## 128 31.75 1608183.6 46378.93   25.10703 3560.641  12.95487 2326.949  13.32380
## 129 32.00 1614160.5 46514.13   25.18706 3565.207  12.97442 2329.291  13.34188
## 130 32.25 1620151.1 46649.51   25.26723 3569.797  12.99407 2331.650  13.36005
## 131 32.50 1626155.3 46785.09   25.34752 3574.412  13.01381 2334.026  13.37833
## 132 32.75 1632173.3 46920.87   25.42795 3579.051  13.03364 2336.419  13.39671
## 133 33.00 1638205.0 47056.85   25.50851 3583.715  13.05356 2338.829  13.41520
## 134 33.25 1644250.5 47193.02   25.58920 3588.404  13.07358 2341.256  13.43378
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## 135 33.50 1650309.9 47329.40   25.67003 3593.116  13.09368 2343.700  13.45246
## 136 33.75 1656383.1 47465.97   25.75099 3597.854  13.11388 2346.161  13.47125
## 137 34.00 1662470.1 47602.75   25.83208 3602.616  13.13417 2348.640  13.49013
## 138 34.25 1668571.1 47739.72   25.91330 3607.402  13.15456 2351.135  13.50912
## 139 34.50 1674686.1 47876.90   25.99467 3612.213  13.17504 2353.647  13.52821
## 140 34.75 1680815.0 48014.28   26.07616 3617.048  13.19560 2356.177  13.54740
## 141 35.00 1686957.9 48151.87   26.15780 3621.908  13.21627 2358.724  13.56669
## 142 35.25 1693114.8 48289.67   26.23957 3626.793  13.23702 2361.287  13.58608
## 143 35.50 1699285.9 48427.67   26.32147 3631.702  13.25787 2363.868  13.60558
## 144 35.75 1705471.0 48565.87   26.40352 3636.635  13.27881 2366.465  13.62517
## 145 36.00 1711670.3 48704.29   26.48570 3641.594  13.29984 2369.080  13.64487
## 146 36.25 1717883.8 48842.91   26.56802 3646.576  13.32097 2371.712  13.66467
## 147 36.50 1724111.4 48981.75   26.65048 3651.583  13.34219 2374.361  13.68457
## 148 36.75 1730353.3 49120.79   26.73307 3656.615  13.36350 2377.027  13.70458
## 149 37.00 1736609.5 49260.05   26.81581 3661.672  13.38490 2379.709  13.72468
## 150 37.25 1742879.9 49399.52   26.89869 3666.753  13.40640 2382.409  13.74489
## 151 37.50 1749164.7 49539.20   26.98171 3671.858  13.42799 2385.126  13.76520
## 152 37.75 1755463.9 49679.10   27.06487 3676.988  13.44968 2387.860  13.78562
## 153 38.00 1761777.4 49819.22   27.14817 3682.143  13.47145 2390.611  13.80613
## 154 38.25 1768105.4 49959.55   27.23162 3687.322  13.49333 2393.379  13.82675
## 155 38.50 1774447.9 50100.10   27.31521 3692.526  13.51529 2396.164  13.84747
## 156 38.75 1780804.8 50240.86   27.39894 3697.755  13.53735 2398.966  13.86829
## 157 39.00 1787176.3 50381.85   27.48281 3703.008  13.55950 2401.785  13.88922
## 158 39.25 1793562.4 50523.05   27.56683 3708.286  13.58175 2404.621  13.91025
## 159 39.50 1799963.0 50664.48   27.65100 3713.589  13.60409 2407.474  13.93138
## 160 39.75 1806378.3 50806.13   27.73531 3718.916  13.62652 2410.344  13.95261
## 161 40.00 1812808.2 50948.00   27.81976 3724.268  13.64905 2413.231  13.97395
## 162 40.25 1819252.9 51090.09   27.90436 3729.644  13.67167 2416.135  13.99539
## 163 40.50 1825712.2 51232.41   27.98911 3735.046  13.69439 2419.056  14.01694
## 164 40.75 1832186.4 51374.96   28.07401 3740.472  13.71720 2421.994  14.03858
## 165 41.00 1838675.3 51517.73   28.15905 3745.922  13.74011 2424.950  14.06033
## 166 41.25 1845179.1 51660.73   28.24425 3751.398  13.76310 2427.922  14.08219
## 167 41.50 1851697.7 51803.95   28.32959 3756.898  13.78620 2430.911  14.10414
## 168 41.75 1858231.3 51947.41   28.41508 3762.423  13.80939 2433.917  14.12620
## 169 42.00 1864779.7 52091.09   28.50072 3767.972  13.83267 2436.940  14.14837
## 170 42.25 1871343.2 52235.01   28.58651 3773.547  13.85605 2439.980  14.17064
## 171 42.50 1877921.6 52379.16   28.67245 3779.146  13.87952 2443.038  14.19301
## 172 42.75 1884515.1 52523.54   28.75855 3784.770  13.90309 2446.112  14.21549
## 173 43.00 1891123.6 52668.16   28.84479 3790.418  13.92675 2449.203  14.23807
## 174 43.25 1897747.3 52813.01   28.93119 3796.092  13.95051 2452.311  14.26075
## 175 43.50 1904386.1 52958.09   29.01774 3801.790  13.97436 2455.437  14.28354
## 176 43.75 1911040.1 53103.42   29.10445 3807.513  13.99831 2458.579  14.30643
## 177 44.00 1917709.3 53248.98   29.19131 3813.261  14.02235 2461.739  14.32943
## 178 44.25 1924393.7 53394.78   29.27832 3819.034  14.04649 2464.915  14.35253
## 179 44.50 1931093.4 53540.81   29.36549 3824.832  14.07073 2468.108  14.37574
## 180 44.75 1937808.4 53687.09   29.45281 3830.655  14.09506 2471.319  14.39905
## 181 45.00 1944538.8 53833.61   29.54029 3836.502  14.11948 2474.546  14.42247
## 182 45.25 1951284.6 53980.37   29.62793 3842.375  14.14400 2477.791  14.44599
## 183 45.50 1958045.8 54127.38   29.71572 3848.272  14.16862 2481.053  14.46962
## 184 45.75 1964822.4 54274.62   29.80367 3854.194  14.19333 2484.331  14.49335
## 185 46.00 1971614.5 54422.12   29.89178 3860.141  14.21814 2487.627  14.51718
## 186 46.25 1978422.2 54569.85   29.98005 3866.114  14.24305 2490.940  14.54112
## 187 46.50 1985245.4 54717.84   30.06848 3872.111  14.26805 2494.269  14.56517
## 188 46.75 1992084.2 54866.07   30.15706 3878.133  14.29315 2497.616  14.58932
```

```
## 189 47.00 1998938.6 55014.55   30.24581 3884.180  14.31834 2500.980  14.61358
## 190 47.25 2005808.7 55163.29   30.33472 3890.252  14.34363 2504.361  14.63795
## 191 47.50 2012694.6 55312.27   30.42379 3896.349  14.36902 2507.759  14.66242
## 192 47.75 2019596.1 55461.50   30.51302 3902.472  14.39451 2511.174  14.68699
## 193 48.00 2026513.5 55610.98   30.60241 3908.619  14.42009 2514.607  14.71167
## 194 48.25 2033446.6 55760.72   30.69197 3914.791  14.44577 2518.056  14.73646
## 195 48.50 2040395.6 55910.71   30.78169 3920.989  14.47154 2521.523  14.76135
## 196 48.75 2047360.5 56060.96   30.87157 3927.212  14.49741 2525.006  14.78635
## 197 49.00 2054341.3 56211.47   30.96162 3933.459  14.52338 2528.507  14.81146
## 198 49.25 2061338.0 56362.23   31.05183 3939.732  14.54945 2532.024  14.83667
## 199 49.50 2068350.8 56513.24   31.14221 3946.030  14.57561 2535.559  14.86199
## 200 49.75 2075379.5 56664.52   31.23275 3952.354  14.60188 2539.111  14.88742
## 201 50.00 2082424.4 56816.06   31.32346 3958.702  14.62824 2542.680  14.91295
##          E_4        I_4        D       N
## 1   1000.0000 200.000000   0.0000 1000000
## 2   1004.0950 178.300000 112.5000 1002175
## 3   1007.6722 159.051753 203.0109 1004377
## 4   1010.7888 141.976759 276.0165 1006601
## 5   1013.4960 126.828353 335.0701 1008845
## 6   1015.8389 113.388033 382.9906 1011105
## 7   1017.8579 101.462221 422.0154 1013379
## 8   1019.5890  90.879398 453.9225 1015665
## 9   1021.0638  81.487575 480.1257 1017962
## 10  1022.3109  73.152055 501.7510 1020269
## 11  1023.3553  65.753453 519.6956 1022585
## 12  1024.2196  59.185946 534.6756 1024909
## 13  1024.9237  53.355725 547.2633 1027241
## 14  1025.4853  48.179620 557.9166 1029580
## 15  1025.9201  43.583891 567.0022 1031926
## 16  1026.2423  39.503152 574.8150 1034279
## 17  1026.4643  35.879418 581.5915 1036638
## 18  1026.5971  32.661264 587.5226 1039003
## 19  1026.6508  29.803083 592.7623 1041375
## 20  1026.6340  27.264417 597.4351 1043752
## 21  1026.5546  25.009379 601.6418 1046136
## 22  1026.4196  23.006133 605.4644 1048525
## 23  1026.2351  21.226431 608.9697 1050920
## 24  1026.0066  19.645210 612.2122 1053321
## 25  1025.7391  18.240232 615.2364 1055727
## 26  1025.4369  16.991760 618.0788 1058139
## 27  1025.1039  15.882280 620.7696 1060557
## 28  1024.7436  14.896244 623.3336 1062981
## 29  1024.3589  14.019855 625.7914 1065410
## 30  1023.9528  13.240863 628.1598 1067844
## 31  1023.5276  12.548394 630.4533 1070285
## 32  1023.0854  11.932793 632.6835 1072731
## 33  1022.6283  11.385488 634.8603 1075183
## 34  1022.1579  10.898867 636.9922 1077640
## 35  1021.6758  10.466170 639.0860 1080103
## 36  1021.1833  10.081395 641.1477 1082572
## 37  1020.6816   9.739210 643.1823 1085046
## 38  1020.1719   9.434878 645.1940 1087526
## 39  1019.6551   9.164194 647.1865 1090012
## 40  1019.1321   8.923419 649.1628 1092503
```

```
## 41  1018.6036   8.709235 651.1255 1095000
## 42  1018.0702   8.518690 653.0770 1097503
## 43  1017.5327   8.349164 655.0191 1100012
## 44  1016.9916   8.198328 656.9535 1102526
## 45  1016.4472   8.064111 658.8817 1105046
## 46  1015.9002   7.944674 660.8050 1107572
## 47  1015.3507   7.838382 662.7243 1110104
## 48  1014.7993   7.743781 664.6407 1112641
## 49  1014.2461   7.659579 666.5549 1115184
## 50  1013.6915   7.584628 668.4677 1117733
## 51  1013.1357   7.517906 670.3797 1120288
## 52  1012.5789   7.458506 672.2914 1122849
## 53  1012.0213   7.405619 674.2033 1125416
## 54  1011.4631   7.358529 676.1159 1127988
## 55  1010.9044   7.316596 678.0294 1130567
## 56  1010.3454   7.279252 679.9442 1133151
## 57  1009.7861   7.245993 681.8606 1135741
## 58  1009.2268   7.216369 683.7789 1138337
## 59  1008.6674   7.189981 685.6992 1140939
## 60  1008.1081   7.166472 687.6217 1143547
## 61  1007.5489   7.145527 689.5467 1146161
## 62  1006.9899   7.126864 691.4742 1148781
## 63  1006.4312   7.110232 693.4044 1151407
## 64  1005.8729   7.095409 695.3375 1154039
## 65  1005.3149   7.082196 697.2735 1156677
## 66  1004.7573   7.070416 699.2125 1159321
## 67  1004.2001   7.059913 701.1547 1161971
## 68  1003.6435   7.050547 703.1000 1164627
## 69  1003.0873   7.042192 705.0486 1167289
## 70  1002.5317   7.034739 707.0005 1169957
## 71  1001.9767   7.028088 708.9558 1172631
## 72  1001.4222   7.022152 710.9146 1175312
## 73  1000.8683   7.016852 712.8768 1177998
## 74  1000.3151   7.012118 714.8425 1180691
## 75   999.7625   7.007889 716.8118 1183390
## 76   999.2105   7.004110 718.7847 1186095
## 77   998.6592   7.000731 720.7612 1188806
## 78   998.1085   6.997707 722.7414 1191524
## 79   997.5585   6.995001 724.7253 1194247
## 80   997.0092   6.992577 726.7129 1196977
## 81   996.4606   6.990405 728.7042 1199713
## 82   995.9127   6.988456 730.6993 1202456
## 83   995.3654   6.986707 732.6981 1205204
## 84   994.8189   6.985134 734.7008 1207959
## 85   994.2731   6.983719 736.7073 1210720
## 86   993.7279   6.982445 738.7176 1213488
## 87   993.1835   6.981295 740.7318 1216262
## 88   992.6398   6.980256 742.7498 1219042
## 89   992.0968   6.979315 744.7718 1221828
## 90   991.5546   6.978462 746.7976 1224621
## 91   991.0130   6.977686 748.8274 1227421
## 92   990.4722   6.976980 750.8611 1230226
## 93   989.9321   6.976335 752.8987 1233038
## 94   989.3927   6.975743 754.9404 1235857
```

```
## 95   988.8540   6.975200 756.9860 1238682
## 96   988.3160   6.974700 759.0355 1241513
## 97   987.7788   6.974237 761.0891 1244351
## 98   987.2422   6.973807 763.1467 1247196
## 99   986.7064   6.973406 765.2084 1250046
## 100  986.1713   6.973031 767.2741 1252904
## 101  985.6369   6.972679 769.3438 1255768
## 102  985.1033   6.972347 771.4176 1258638
## 103  984.5703   6.972032 773.4955 1261515
## 104  984.0381   6.971732 775.5775 1264399
## 105  983.5066   6.971445 777.6636 1267289
## 106  982.9758   6.971170 779.7538 1270186
## 107  982.4457   6.970905 781.8481 1273090
## 108  981.9163   6.970648 783.9466 1276000
## 109  981.3877   6.970398 786.0492 1278916
## 110  980.8597   6.970154 788.1560 1281840
## 111  980.3324   6.969916 790.2669 1284770
## 112  979.8059   6.969682 792.3821 1287707
## 113  979.2801   6.969451 794.5015 1290650
## 114  978.7549   6.969223 796.6250 1293600
## 115  978.2305   6.968996 798.7528 1296557
## 116  977.7067   6.968772 800.8849 1299521
## 117  977.1837   6.968548 803.0211 1302492
## 118  976.6614   6.968325 805.1617 1305469
## 119  976.1397   6.968102 807.3065 1308453
## 120  975.6188   6.967879 809.4556 1311444
## 121  975.0986   6.967655 811.6090 1314442
## 122  974.5790   6.967431 813.7667 1317446
## 123  974.0602   6.967206 815.9287 1320458
## 124  973.5420   6.966979 818.0950 1323476
## 125  973.0245   6.966751 820.2657 1326502
## 126  972.5078   6.966521 822.4408 1329534
## 127  971.9917   6.966289 824.6202 1332573
## 128  971.4763   6.966055 826.8040 1335619
## 129  970.9615   6.965819 828.9922 1338672
## 130  970.4475   6.965581 831.1848 1341732
## 131  969.9341   6.965341 833.3818 1344799
## 132  969.4215   6.965098 835.5832 1347873
## 133  968.9095   6.964853 837.7891 1350954
## 134  968.3982   6.964606 839.9995 1354042
## 135  967.8875   6.964355 842.2142 1357137
## 136  967.3776   6.964102 844.4335 1360240
## 137  966.8683   6.963847 846.6573 1363349
## 138  966.3597   6.963588 848.8855 1366465
## 139  965.8517   6.963327 851.1183 1369589
## 140  965.3444   6.963063 853.3556 1372720
## 141  964.8378   6.962796 855.5974 1375858
## 142  964.3319   6.962526 857.8438 1379003
## 143  963.8267   6.962254 860.0948 1382155
## 144  963.3221   6.961978 862.3503 1385314
## 145  962.8181   6.961700 864.6104 1388481
## 146  962.3149   6.961418 866.8751 1391655
## 147  961.8123   6.961134 869.1444 1394836
## 148  961.3103   6.960846 871.4183 1398024
```

```
## 149  960.8090   6.960556 873.6969 1401220
## 150  960.3084   6.960262 875.9801 1404423
## 151  959.8084   6.959966 878.2680 1407633
## 152  959.3091   6.959667 880.5606 1410851
## 153  958.8104   6.959364 882.8578 1414076
## 154  958.3124   6.959059 885.1598 1417308
## 155  957.8151   6.958750 887.4664 1420548
## 156  957.3184   6.958439 889.7778 1423795
## 157  956.8223   6.958124 892.0939 1427050
## 158  956.3269   6.957807 894.4148 1430312
## 159  955.8321   6.957486 896.7404 1433582
## 160  955.3380   6.957163 899.0708 1436859
## 161  954.8446   6.956836 901.4060 1440143
## 162  954.3517   6.956506 903.7460 1443435
## 163  953.8595   6.956174 906.0908 1446735
## 164  953.3680   6.955838 908.4404 1450042
## 165  952.8771   6.955499 910.7949 1453356
## 166  952.3868   6.955158 913.1542 1456678
## 167  951.8972   6.954813 915.5183 1460008
## 168  951.4082   6.954465 917.8874 1463346
## 169  950.9198   6.954114 920.2613 1466691
## 170  950.4321   6.953761 922.6402 1470043
## 171  949.9450   6.953404 925.0239 1473404
## 172  949.4585   6.953044 927.4126 1476772
## 173  948.9727   6.952681 929.8063 1480147
## 174  948.4875   6.952316 932.2048 1483531
## 175  948.0029   6.951947 934.6084 1486922
## 176  947.5190   6.951575 937.0169 1490321
## 177  947.0356   6.951201 939.4305 1493728
## 178  946.5529   6.950823 941.8490 1497142
## 179  946.0708   6.950443 944.2726 1500564
## 180  945.5894   6.950059 946.7011 1503994
## 181  945.1085   6.949672 949.1348 1507432
## 182  944.6283   6.949283 951.5735 1510878
## 183  944.1487   6.948891 954.0172 1514332
## 184  943.6697   6.948495 956.4661 1517794
## 185  943.1913   6.948097 958.9201 1521263
## 186  942.7135   6.947696 961.3791 1524740
## 187  942.2364   6.947291 963.8433 1528226
## 188  941.7598   6.946884 966.3127 1531719
## 189  941.2839   6.946474 968.7872 1535221
## 190  940.8086   6.946061 971.2668 1538730
## 191  940.3339   6.945645 973.7517 1542247
## 192  939.8598   6.945227 976.2417 1545773
## 193  939.3863   6.944805 978.7369 1549306
## 194  938.9134   6.944380 981.2374 1552848
## 195  938.4411   6.943953 983.7431 1556397
## 196  937.9694   6.943522 986.2540 1559955
## 197  937.4983   6.943089 988.7702 1563521
## 198  937.0278   6.942653 991.2917 1567095
## 199  936.5579   6.942214 993.8185 1570677
## 200  936.0886   6.941772 996.3506 1574267
## 201  935.6199   6.941327 998.8880 1577866
```

Check Model using DeSolve

```
# set up params for deSolve()

params <- list(
  rho = 0.0179, # US birth rate: range 0.0179
  q = 0.15, # Proportion of active cases that have potential to be infectious. Range (0, 1)
  alpha = 0.00425, # Immigration rate. Range 0.00425
  lambda = 8, # Effective Contact Rate. Range (0, 30)
  gamma = 0.1, # Proportion H-resistance acquisition cases. Range (0, 1)
  mu_0 = 0.013, # Mortality rate unrelated to TB. Range 0.013
  mu = 0.15, # TB mortality rate. Range (0, 0.5)
  v_L = 0.003, # Progression rate from latent to active infection. Range (0, 0.01)
  l = 0.25, # Proportion of immigrants that have LTBI. Range (0, 0.3)
  p = 0.15, # Proportion of exogenous infections that are acute. Range (0, 0.3)
  trt = c(0.1, 0.1, 0.1, 0.1), # Proportion of treatment time when individuals are infectious (I 1-4). 
  z = c(0.85, 0.7, 0.65, 0.4), # Proportion of treatment courses for TB (DS,H,R,MDR) that are successfu
  phi = c(0.8,0.8,0.5,0.4), # Rate of end of treatment (DS, H, R, MDR). Ranges [(0.6, 0.9), (0.5, 0.9),
  r = c(0.05,0.03,0.02), # Proportion of immigrant LTBI cases (H,R,MDR). Ranges [(0, 0.2), (0, 0.2), (0
  y = c(0.15,0.1) # Proportion of failed treatments for DS TB that result in H- or R-resistance Ranges
)
```

```
# set up initial values for DeSolve method

initial <- c(
  S = 950000,
  E_1 = 30000,
  I_1 = 2000,
  E_2 = 3000,
  I_2 = 500,
  E_3 = 2000,
  I_3 = 300,
  E_4 = 1000,
  I_4 = 200,
  D = 0,
  N = 1000000
)
```

```
# Set up differential equations for DeSolve method

diffeqs <- function(t, y, params) {
  with(as.list(c(y, params)), {
      # dS/dT
      dS <- (rho*N)
            - (q*trt[1]*lambda*(S*I_2/N)) - (q*trt[2]*lambda*(S*I_2/N)) - (q*trt[3]*lambda*(S*I_3/N)) -
            + (z[1]*phi[1]*I_1) + (z[2]*phi[2]*I_2) + (z[3]*phi[3]*I_3) + (z[4]*phi[4]*I_4)
            + (1 - l)*(alpha*N) - mu_0*S

      # dE_1/dT
      dE_1 <- (1 - p)*(q*trt[1]*lambda*(S*I_1/N)) - v_L*E_1 + (1-y[1])*(1-z[1])*phi[1]*I_1 + l*alpha*(1

      # dI_1/dT
      dI_1 <- q*trt[1]*lambda*(S*I_1/N) + q*v_L*E_1 - phi[1]*I_1 - mu_0*I_1 - mu*I_1

      # dE_2/dT
```

```
      dE_2 <- (1 - p)*(q*trt[2]*lambda*(S*I_2/N)) - v_L*E_2 + (1-y[1])*(1-z[2])*phi[2]*I_2 + gamma*(1-z

      # dI_2/dT
      dI_2 <- q*trt[2]*lambda*(S*I_2/N) + v_L*E_2 - phi[2]*I_2 - mu_0*I_2 - mu*I_2

      # dE_3/dT
      dE_3 <- (1 - p)*(q*trt[3]*lambda*(S*I_3/N)) - v_L*E_3 + (1-y[1])*(1-z[3])*phi[3]*I_3 + (1-gamma)*

      # dI_3/dT
      dI_3 <- q*trt[3]*lambda*(S*I_3/N) + v_L*E_3 - phi[3]*I_3 - mu_0*I_3 - mu*I_3

      # dE_4/dT
      dE_4 <- (1 - p)*(q*trt[4]*lambda*(S*I_4/N)) - v_L*E_4
      + (1-z[2])*y[1]*phi[2]*I_2 + (1-z[3])*y[1]*phi[3]*I_3 + (1-z[4])*phi[4]*I_4
      + l*alpha*r[4]*N - mu_0*E_4

      # dI_4/dT
      dI_4 <- q*trt[4]*lambda*(S*I_4/N) + v_L*E_4 - phi[4]*I_4 - mu_0*I_4 - mu*I_4

      # dD/dT
      dD <- mu*(I_1 + I_2 + I_3 + I_4)

      # dN/dT
      dN <- rho*N + alpha*N - mu*(I_1 + I_2 + I_3 + I_4) - mu_0*N

      return(list(c(dS, dE_1, dI_1, dE_2, dI_2, dE_3, dI_3, dE_4, dI_4, dD, dN)))
    })
   }
```

```
# Solve Differential Equations Using DeSolve()

result2 <- ode(
  y = initial,
  times = time,
  func = diffeqs,
  parms = params,
  rtol = 1e-6,
  atol = 1e-6
)

result2 <- as.data.frame(result2)
result2
```

```
##      time         S      E_1        I_1      E_2       I_2      E_3       I_3
## 1    0.00  950000.0 30000.00 2000.00000 3000.000 500.00000 2000.000 300.00000
## 2    0.25  954479.9 30208.64 1620.63520 3036.015 406.43610 2024.500 262.94575
## 3    0.50  958969.6 30400.09 1313.92200 3065.455 330.80788 2045.431 230.67476
## 4    0.75  963469.3 30577.64 1065.93211 3089.580 269.67326 2063.343 202.56731
## 5    1.00  967979.0 30743.95  865.40922 3109.411 220.25113 2078.696 178.08429
## 6    1.25  972498.8 30901.18  703.25884 3125.772 180.29509 2091.880 156.75672
## 7    1.50  977028.7 31051.07  572.13137 3139.329 147.99016 2103.221 138.17655
## 8    1.75  981568.8 31195.04  466.08537 3150.623 121.86948 2112.995 121.98870
## 9    2.00  986119.3 31334.23  380.32089 3160.091 100.74833 2121.436 107.88418
## 10   2.25  990680.0 31469.56  310.95625 3168.085  83.66882 2128.743  95.59405
```

```
## 11   2.50  995251.0 31601.79  254.85438 3174.891  69.85706 2135.081  84.88425
## 12   2.75  999832.5 31731.52  209.47855 3180.740  58.68737 2140.594  75.55098
## 13   3.00 1004424.4 31859.25  172.77812 3185.820  49.65411 2145.403  67.41685
## 14   3.25 1009026.7 31985.37  143.09485 3190.283  42.34850 2149.611  60.32742
## 15   3.50 1013639.5 32110.20  119.08794 3194.249  36.44008 2153.304  54.14819
## 16   3.75 1018262.9 32234.01   99.67314 3197.820  31.66171 2156.559  48.76204
## 17   4.00 1022896.8 32357.02   83.97346 3201.074  27.79736 2159.438  44.06701
## 18   4.25 1027541.3 32479.38   71.27968 3204.077  24.67236 2161.997  39.97424
## 19   4.50 1032196.3 32601.25   61.01811 3206.882  22.14547 2164.283  36.40638
## 20   4.75 1036862.1 32722.74   52.72470 3209.531  20.10246 2166.336  33.29603
## 21   5.00 1041538.4 32843.93   46.02403 3212.058  18.45095 2168.189  30.58447
## 22   5.25 1046225.5 32964.90   40.61240 3214.493  17.11622 2169.873  28.22057
## 23   5.50 1050923.3 33085.72   36.24408 3216.856  16.03783 2171.414  26.15974
## 24   5.75 1055631.8 33206.43   32.72026 3219.166  15.16689 2172.832  24.36316
## 25   6.00 1060351.1 33327.07   29.88004 3221.439  14.46385 2174.147  22.79699
## 26   6.25 1065081.2 33447.68   27.59323 3223.686  13.89672 2175.376  21.43174
## 27   6.50 1069822.1 33568.28   25.75447 3225.917  13.43959 2176.531  20.24170
## 28   6.75 1074573.8 33688.89   24.27848 3228.139  13.07153 2177.626  19.20448
## 29   7.00 1079336.5 33809.54   23.09624 3230.360  12.77558 2178.671  18.30054
## 30   7.25 1084110.0 33930.23   22.15187 3232.584  12.53803 2179.675  17.51287
## 31   7.50 1088894.4 34050.98   21.40015 3234.815  12.34776 2180.645  16.82662
## 32   7.75 1093689.8 34171.80   20.80446 3237.058  12.19580 2181.589  16.22886
## 33   8.00 1098496.1 34292.69   20.33515 3239.313  12.07488 2182.511  15.70830
## 34   8.25 1103313.5 34413.67   19.96822 3241.585  11.97910 2183.419  15.25512
## 35   8.50 1108141.9 34534.74   19.68424 3243.874  11.90370 2184.315  14.86073
## 36   8.75 1112981.3 34655.90   19.46747 3246.182  11.84482 2185.203  14.51764
## 37   9.00 1117831.8 34777.16   19.30517 3248.510  11.79934 2186.088  14.21935
## 38   9.25 1122693.4 34898.52   19.18701 3250.859  11.76473 2186.971  13.96015
## 39   9.50 1127566.1 35019.98   19.10462 3253.230  11.73894 2187.855  13.73508
## 40   9.75 1132450.0 35141.55   19.05125 3255.623  11.72031 2188.743  13.53982
## 41  10.00 1137345.0 35263.23   19.02141 3258.040  11.70750 2189.637  13.37058
## 42  10.25 1142251.2 35385.03   19.01064 3260.480  11.69942 2190.537  13.22407
## 43  10.50 1147168.7 35506.93   19.01536 3262.943  11.69519 2191.445  13.09742
## 44  10.75 1152097.4 35628.95   19.03264 3265.430  11.69410 2192.364  12.98811
## 45  11.00 1157037.4 35751.09   19.06012 3267.942  11.69557 2193.293  12.89396
## 46  11.25 1161988.7 35873.35   19.09589 3270.478  11.69912 2194.233  12.81305
## 47  11.50 1166951.4 35995.72   19.13839 3273.038  11.70439 2195.186  12.74372
## 48  11.75 1171925.4 36118.21   19.18638 3275.622  11.71106 2196.152  12.68452
## 49  12.00 1176910.7 36240.83   19.23882 3278.232  11.71889 2197.132  12.63416
## 50  12.25 1181907.5 36363.57   19.29489 3280.866  11.72766 2198.126  12.59155
## 51  12.50 1186915.7 36486.43   19.35394 3283.524  11.73723 2199.135  12.55572
## 52  12.75 1191935.4 36609.42   19.41540 3286.208  11.74745 2200.159  12.52582
## 53  13.00 1196966.6 36732.53   19.47884 3288.916  11.75823 2201.199  12.50110
## 54  13.25 1202009.3 36855.77   19.54391 3291.649  11.76946 2202.254  12.48094
## 55  13.50 1207063.5 36979.14   19.61032 3294.407  11.78109 2203.325  12.46477
## 56  13.75 1212129.3 37102.63   19.67784 3297.189  11.79305 2204.413  12.45209
## 57  14.00 1217206.7 37226.26   19.74626 3299.997  11.80530 2205.517  12.44247
## 58  14.25 1222295.7 37350.02   19.81545 3302.829  11.81780 2206.638  12.43555
## 59  14.50 1227396.3 37473.90   19.88527 3305.686  11.83051 2207.775  12.43099
## 60  14.75 1232508.6 37597.92   19.95562 3308.568  11.84343 2208.930  12.42851
## 61  15.00 1237632.6 37722.08   20.02643 3311.474  11.85652 2210.101  12.42785
## 62  15.25 1242768.4 37846.37   20.09762 3314.406  11.86977 2211.289  12.42881
## 63  15.50 1247915.9 37970.79   20.16915 3317.362  11.88316 2212.495  12.43119
## 64  15.75 1253075.1 38095.35   20.24096 3320.343  11.89669 2213.718  12.43482
```

```
## 65  16.00 1258246.2 38220.04   20.31303 3323.349  11.91035 2214.958  12.43956
## 66  16.25 1263429.1 38344.88   20.38532 3326.379  11.92412 2216.216  12.44528
## 67  16.50 1268623.9 38469.85   20.45782 3329.435  11.93801 2217.491  12.45187
## 68  16.75 1273830.6 38594.97   20.53050 3332.515  11.95201 2218.783  12.45924
## 69  17.00 1279049.1 38720.22   20.60335 3335.620  11.96612 2220.093  12.46729
## 70  17.25 1284279.6 38845.62   20.67635 3338.749  11.98032 2221.420  12.47596
## 71  17.50 1289522.1 38971.15   20.74950 3341.903  11.99463 2222.764  12.48518
## 72  17.75 1294776.6 39096.83   20.82279 3345.082  12.00904 2224.127  12.49489
## 73  18.00 1300043.1 39222.66   20.89622 3348.286  12.02354 2225.506  12.50504
## 74  18.25 1305321.7 39348.63   20.96977 3351.514  12.03814 2226.903  12.51559
## 75  18.50 1310612.3 39474.75   21.04345 3354.767  12.05283 2228.318  12.52651
## 76  18.75 1315915.1 39601.01   21.11724 3358.045  12.06761 2229.750  12.53775
## 77  19.00 1321230.0 39727.42   21.19116 3361.347  12.08249 2231.200  12.54930
## 78  19.25 1326557.0 39853.99   21.26519 3364.674  12.09746 2232.667  12.56112
## 79  19.50 1331896.3 39980.70   21.33934 3368.026  12.11252 2234.152  12.57319
## 80  19.75 1337247.7 40107.56   21.41360 3371.402  12.12767 2235.654  12.58550
## 81  20.00 1342611.4 40234.57   21.48798 3374.803  12.14291 2237.174  12.59802
## 82  20.25 1347987.4 40361.74   21.56246 3378.229  12.15825 2238.711  12.61075
## 83  20.50 1353375.7 40489.05   21.63706 3381.679  12.17367 2240.266  12.62366
## 84  20.75 1358776.3 40616.53   21.71177 3385.154  12.18919 2241.838  12.63676
## 85  21.00 1364189.3 40744.16   21.78660 3388.653  12.20479 2243.428  12.65002
## 86  21.25 1369614.6 40871.94   21.86153 3392.177  12.22049 2245.035  12.66344
## 87  21.50 1375052.4 40999.88   21.93658 3395.725  12.23628 2246.659  12.67701
## 88  21.75 1380502.6 41127.98   22.01174 3399.298  12.25215 2248.301  12.69073
## 89  22.00 1385965.3 41256.24   22.08702 3402.896  12.26812 2249.961  12.70459
## 90  22.25 1391440.5 41384.65   22.16240 3406.518  12.28418 2251.638  12.71858
## 91  22.50 1396928.2 41513.23   22.23790 3410.165  12.30032 2253.332  12.73270
## 92  22.75 1402428.5 41641.97   22.31351 3413.836  12.31656 2255.044  12.74695
## 93  23.00 1407941.4 41770.87   22.38924 3417.532  12.33289 2256.773  12.76132
## 94  23.25 1413466.8 41899.94   22.46508 3421.253  12.34931 2258.519  12.77581
## 95  23.50 1419005.0 42029.17   22.54103 3424.998  12.36582 2260.283  12.79041
## 96  23.75 1424555.8 42158.56   22.61710 3428.767  12.38241 2262.064  12.80514
## 97  24.00 1430119.3 42288.12   22.69329 3432.561  12.39910 2263.863  12.81997
## 98  24.25 1435695.5 42417.85   22.76959 3436.380  12.41588 2265.679  12.83492
## 99  24.50 1441284.5 42547.74   22.84601 3440.223  12.43275 2267.512  12.84998
## 100 24.75 1446886.3 42677.81   22.92254 3444.090  12.44971 2269.362  12.86514
## 101 25.00 1452500.9 42808.04   22.99919 3447.982  12.46676 2271.230  12.88041
## 102 25.25 1458128.4 42938.44   23.07596 3451.899  12.48391 2273.115  12.89579
## 103 25.50 1463768.7 43069.02   23.15285 3455.840  12.50114 2275.018  12.91128
## 104 25.75 1469421.9 43199.76   23.22986 3459.806  12.51846 2276.938  12.92687
## 105 26.00 1475088.1 43330.68   23.30698 3463.796  12.53587 2278.875  12.94256
## 106 26.25 1480767.3 43461.78   23.38423 3467.811  12.55338 2280.829  12.95836
## 107 26.50 1486459.4 43593.05   23.46159 3471.850  12.57097 2282.801  12.97426
## 108 26.75 1492164.6 43724.49   23.53908 3475.913  12.58866 2284.789  12.99027
## 109 27.00 1497882.8 43856.11   23.61668 3480.001  12.60643 2286.796  13.00638
## 110 27.25 1503614.1 43987.91   23.69441 3484.114  12.62430 2288.819  13.02259
## 111 27.50 1509358.5 44119.89   23.77226 3488.251  12.64226 2290.859  13.03890
## 112 27.75 1515116.1 44252.04   23.85023 3492.413  12.66031 2292.917  13.05531
## 113 28.00 1520886.8 44384.38   23.92833 3496.599  12.67845 2294.992  13.07182
## 114 28.25 1526670.8 44516.90   24.00655 3500.809  12.69668 2297.084  13.08844
## 115 28.50 1532468.0 44649.60   24.08489 3505.044  12.71500 2299.194  13.10516
## 116 28.75 1538278.4 44782.48   24.16336 3509.304  12.73342 2301.320  13.12197
## 117 29.00 1544102.2 44915.55   24.24195 3513.588  12.75192 2303.464  13.13889
## 118 29.25 1549939.3 45048.80   24.32066 3517.896  12.77052 2305.625  13.15591
```

```
## 119 29.50 1555789.7 45182.24   24.39951 3522.229  12.78920 2307.803  13.17303
## 120 29.75 1561653.6 45315.87   24.47848 3526.587  12.80798 2309.999  13.19026
## 121 30.00 1567530.8 45449.68   24.55757 3530.968  12.82685 2312.211  13.20758
## 122 30.25 1573421.5 45583.68   24.63679 3535.375  12.84581 2314.441  13.22500
## 123 30.50 1579325.7 45717.87   24.71614 3539.806  12.86487 2316.687  13.24253
## 124 30.75 1585243.4 45852.25   24.79562 3544.261  12.88401 2318.951  13.26015
## 125 31.00 1591174.6 45986.82   24.87523 3548.741  12.90325 2321.232  13.27788
## 126 31.25 1597119.4 46121.58   24.95497 3553.245  12.92258 2323.530  13.29571
## 127 31.50 1603077.8 46256.54   25.03483 3557.774  12.94200 2325.846  13.31363
## 128 31.75 1609049.9 46391.69   25.11483 3562.328  12.96151 2328.178  13.33166
## 129 32.00 1615035.6 46527.03   25.19495 3566.906  12.98111 2330.527  13.34979
## 130 32.25 1621035.0 46662.57   25.27521 3571.508  13.00081 2332.894  13.36802
## 131 32.50 1627048.1 46798.31   25.35560 3576.135  13.02060 2335.278  13.38636
## 132 32.75 1633075.0 46934.25   25.43612 3580.786  13.04048 2337.679  13.40479
## 133 33.00 1639115.7 47070.38   25.51678 3585.462  13.06045 2340.096  13.42332
## 134 33.25 1645170.3 47206.71   25.59757 3590.163  13.08051 2342.531  13.44196
## 135 33.50 1651238.6 47343.24   25.67849 3594.888  13.10067 2344.983  13.46070
## 136 33.75 1657320.9 47479.97   25.75954 3599.637  13.12092 2347.453  13.47953
## 137 34.00 1663417.1 47616.91   25.84073 3604.411  13.14126 2349.939  13.49847
## 138 34.25 1669527.2 47754.05   25.92205 3609.210  13.16169 2352.442  13.51751
## 139 34.50 1675651.3 47891.39   26.00351 3614.033  13.18222 2354.962  13.53666
## 140 34.75 1681789.5 48028.94   26.08511 3618.881  13.20284 2357.500  13.55590
## 141 35.00 1687941.6 48166.69   26.16684 3623.753  13.22355 2360.054  13.57525
## 142 35.25 1694107.9 48304.65   26.24871 3628.650  13.24436 2362.626  13.59469
## 143 35.50 1700288.3 48442.81   26.33072 3633.571  13.26525 2365.214  13.61424
## 144 35.75 1706482.8 48581.18   26.41286 3638.517  13.28624 2367.820  13.63389
## 145 36.00 1712691.5 48719.77   26.49515 3643.487  13.30733 2370.443  13.65365
## 146 36.25 1718914.4 48858.56   26.57757 3648.482  13.32850 2373.082  13.67350
## 147 36.50 1725151.5 48997.57   26.66013 3653.502  13.34977 2375.739  13.69346
## 148 36.75 1731402.9 49136.78   26.74283 3658.546  13.37113 2378.413  13.71352
## 149 37.00 1737668.6 49276.21   26.82567 3663.615  13.39259 2381.104  13.73368
## 150 37.25 1743948.7 49415.85   26.90866 3668.708  13.41414 2383.812  13.75394
## 151 37.50 1750243.1 49555.71   26.99178 3673.826  13.43578 2386.537  13.77431
## 152 37.75 1756551.9 49695.78   27.07505 3678.969  13.45752 2389.279  13.79477
## 153 38.00 1762875.2 49836.07   27.15846 3684.136  13.47934 2392.037  13.81535
## 154 38.25 1769212.9 49976.58   27.24201 3689.328  13.50127 2394.813  13.83602
## 155 38.50 1775565.2 50117.30   27.32570 3694.545  13.52328 2397.607  13.85679
## 156 38.75 1781932.0 50258.25   27.40954 3699.786  13.54539 2400.417  13.87767
## 157 39.00 1788313.3 50399.41   27.49353 3705.052  13.56760 2403.244  13.89865
## 158 39.25 1794709.3 50540.80   27.57765 3710.342  13.58990 2406.088  13.91974
## 159 39.50 1801119.9 50682.41   27.66193 3715.658  13.61229 2408.949  13.94093
## 160 39.75 1807545.1 50824.24   27.74635 3720.998  13.63477 2411.827  13.96222
## 161 40.00 1813985.1 50966.29   27.83091 3726.362  13.65735 2414.722  13.98361
## 162 40.25 1820439.8 51108.57   27.91563 3731.752  13.68003 2417.635  14.00511
## 163 40.50 1826909.3 51251.07   28.00049 3737.166  13.70280 2420.564  14.02671
## 164 40.75 1833393.6 51393.80   28.08550 3742.604  13.72566 2423.510  14.04841
## 165 41.00 1839892.7 51536.76   28.17065 3748.068  13.74862 2426.474  14.07022
## 166 41.25 1846406.7 51679.95   28.25596 3753.556  13.77167 2429.454  14.09213
## 167 41.50 1852935.6 51823.36   28.34142 3759.069  13.79482 2432.451  14.11414
## 168 41.75 1859479.4 51967.01   28.42702 3764.607  13.81806 2435.466  14.13626
## 169 42.00 1866038.2 52110.88   28.51278 3770.169  13.84139 2438.497  14.15848
## 170 42.25 1872612.1 52254.99   28.59869 3775.757  13.86482 2441.546  14.18081
## 171 42.50 1879200.9 52399.33   28.68475 3781.369  13.88835 2444.611  14.20324
## 172 42.75 1885804.9 52543.91   28.77096 3787.006  13.91197 2447.694  14.22577
```

```
## 173 43.00 1892424.0 52688.72   28.85732 3792.668  13.93569 2450.793  14.24841
## 174 43.25 1899058.2 52833.77   28.94384 3798.354  13.95950 2453.910  14.27115
## 175 43.50 1905707.6 52979.05   29.03051 3804.066  13.98340 2457.044  14.29400
## 176 43.75 1912372.2 53124.57   29.11733 3809.802  14.00741 2460.194  14.31695
## 177 44.00 1919052.0 53270.33   29.20431 3815.563  14.03150 2463.362  14.34000
## 178 44.25 1925747.2 53416.32   29.29144 3821.349  14.05570 2466.547  14.36316
## 179 44.50 1932457.6 53562.56   29.37873 3827.160  14.07998 2469.749  14.38643
## 180 44.75 1939183.5 53709.04   29.46618 3832.996  14.10437 2472.968  14.40980
## 181 45.00 1945924.7 53855.76   29.55378 3838.857  14.12885 2476.204  14.43327
## 182 45.25 1952681.3 54002.73   29.64154 3844.743  14.15342 2479.457  14.45685
## 183 45.50 1959453.5 54149.94   29.72946 3850.654  14.17810 2482.727  14.48054
## 184 45.75 1966241.1 54297.39   29.81753 3856.589  14.20287 2486.014  14.50433
## 185 46.00 1973044.2 54445.09   29.90577 3862.550  14.22773 2489.318  14.52822
## 186 46.25 1979862.9 54593.03   29.99416 3868.536  14.25269 2492.639  14.55222
## 187 46.50 1986697.2 54741.23   30.08272 3874.546  14.27775 2495.978  14.57633
## 188 46.75 1993547.2 54889.67   30.17143 3880.582  14.30290 2499.333  14.60054
## 189 47.00 2000412.8 55038.36   30.26030 3886.643  14.32815 2502.706  14.62486
## 190 47.25 2007294.2 55187.30   30.34934 3892.729  14.35350 2506.095  14.64928
## 191 47.50 2014191.2 55336.50   30.43854 3898.839  14.37894 2509.502  14.67381
## 192 47.75 2021104.1 55485.94   30.52790 3904.975  14.40448 2512.926  14.69845
## 193 48.00 2028032.8 55635.64   30.61742 3911.136  14.43012 2516.367  14.72319
## 194 48.25 2034977.4 55785.60   30.70711 3917.323  14.45585 2519.825  14.74803
## 195 48.50 2041937.8 55935.80   30.79696 3923.534  14.48168 2523.300  14.77299
## 196 48.75 2048914.2 56086.27   30.88697 3929.770  14.50761 2526.792  14.79805
## 197 49.00 2055906.5 56236.99   30.97715 3936.032  14.53364 2530.302  14.82322
## 198 49.25 2062914.9 56387.97   31.06750 3942.319  14.55976 2533.828  14.84849
## 199 49.50 2069939.2 56539.21   31.15801 3948.631  14.58598 2537.372  14.87387
## 200 49.75 2076979.7 56690.71   31.24869 3954.968  14.61230 2540.933  14.89936
## 201 50.00 2084036.3 56842.47   31.33953 3961.331  14.63872 2544.510  14.92495
##           E_4        I_4        D       N
## 1   1000.0000 200.000000   0.0000 1000000
## 2   1003.8460 179.481465 102.2158 1002188
## 3   1007.2291 161.154971 186.4302 1004399
## 4   1010.1976 144.785100 255.9480 1006629
## 5   1012.7949 130.161861 313.4576 1008877
## 6   1015.0598 117.097932 361.1469 1011140
## 7   1017.0270 105.426199 400.7976 1013416
## 8   1018.7277  94.997546 433.8619 1015703
## 9   1020.1898  85.678924 461.5237 1018002
## 10  1021.4381  77.351594 484.7495 1020310
## 11  1022.4950  69.909586 504.3287 1022627
## 12  1023.3806  63.258316 520.9064 1024952
## 13  1024.1127  57.313355 535.0106 1027285
## 14  1024.7074  51.999336 547.0736 1029626
## 15  1025.1791  47.248972 557.4500 1031973
## 16  1025.5406  43.002188 566.4307 1034328
## 17  1025.8034  39.205345 574.2545 1036689
## 18  1025.9780  35.810541 581.1182 1039056
## 19  1026.0735  32.775004 587.1836 1041429
## 20  1026.0982  30.060531 592.5844 1043809
## 21  1026.0596  27.633001 597.4308 1046195
## 22  1025.9642  25.461939 601.8139 1048586
## 23  1025.8181  23.520120 605.8095 1050983
## 24  1025.6266  21.783224 609.4800 1053387
```

```
## 25  1025.3945  20.229522 612.8778 1055796
## 26  1025.1261  18.839602 616.0460 1058210
## 27  1024.8251  17.596118 619.0209 1060631
## 28  1024.4951  16.483569 621.8326 1063057
## 29  1024.1391  15.488104 624.5064 1065489
## 30  1023.7599  14.597343 627.0636 1067926
## 31  1023.3600  13.800222 629.5218 1070370
## 32  1022.9416  13.086850 631.8962 1072818
## 33  1022.5066  12.448389 634.1994 1075273
## 34  1022.0569  11.876935 636.4420 1077733
## 35  1021.5941  11.365423 638.6331 1080199
## 36  1021.1196  10.907536 640.7804 1082671
## 37  1020.6347  10.497626 642.8905 1085148
## 38  1020.1405  10.130644 644.9688 1087631
## 39  1019.6382   9.802072 647.0201 1090120
## 40  1019.1285   9.507872 649.0486 1092614
## 41  1018.6123   9.244432 651.0576 1095115
## 42  1018.0904   9.008521 653.0503 1097621
## 43  1017.5635   8.797248 655.0291 1100132
## 44  1017.0321   8.608029 656.9964 1102650
## 45  1016.4968   8.438551 658.9539 1105173
## 46  1015.9581   8.286744 660.9035 1107702
## 47  1015.4163   8.150758 662.8465 1110237
## 48  1014.8720   8.028936 664.7842 1112778
## 49  1014.3254   7.919796 666.7176 1115324
## 50  1013.7768   7.822011 668.6478 1117876
## 51  1013.2266   7.734394 670.5756 1120434
## 52  1012.6749   7.655883 672.5017 1122998
## 53  1012.1221   7.585527 674.4267 1125568
## 54  1011.5682   7.522474 676.3511 1128144
## 55  1011.0135   7.465963 678.2756 1130726
## 56  1010.4582   7.415311 680.2004 1133313
## 57  1009.9024   7.369908 682.1261 1135907
## 58  1009.3462   7.329207 684.0528 1138506
## 59  1008.7897   7.292718 685.9810 1141112
## 60  1008.2331   7.260003 687.9109 1143723
## 61  1007.6764   7.230668 689.8426 1146340
## 62  1007.1197   7.204363 691.7765 1148964
## 63  1006.5631   7.180773 693.7127 1151593
## 64  1006.0067   7.159615 695.6514 1154228
## 65  1005.4506   7.140637 697.5927 1156870
## 66  1004.8947   7.123613 699.5367 1159517
## 67  1004.3391   7.108339 701.4836 1162171
## 68  1003.7839   7.094634 703.4335 1164830
## 69  1003.2291   7.082335 705.3864 1167496
## 70  1002.6748   7.071297 707.3424 1170167
## 71  1002.1209   7.061389 709.3017 1172845
## 72  1001.5676   7.052493 711.2642 1175529
## 73  1001.0147   7.044505 713.2301 1178219
## 74  1000.4625   7.037330 715.1995 1180916
## 75   999.9108   7.030885 717.1722 1183618
## 76   999.3596   7.025093 719.1485 1186327
## 77   998.8091   7.019888 721.1283 1189042
## 78   998.2593   7.015208 723.1117 1191763
```

```
## 79   997.7100   7.010998 725.0987 1194490
## 80   997.1614   7.007211 727.0894 1197224
## 81   996.6134   7.003802 729.0838 1199963
## 82   996.0661   7.000732 731.0819 1202709
## 83   995.5195   6.997966 733.0837 1205462
## 84   994.9735   6.995472 735.0893 1208220
## 85   994.4283   6.993223 737.0987 1210985
## 86   993.8837   6.991191 739.1119 1213757
## 87   993.3398   6.989356 741.1290 1216534
## 88   992.7966   6.987696 743.1499 1219318
## 89   992.2541   6.986194 745.1747 1222109
## 90   991.7123   6.984832 747.2033 1224905
## 91   991.1711   6.983596 749.2359 1227708
## 92   990.6308   6.982472 751.2724 1230518
## 93   990.0911   6.981450 753.3129 1233334
## 94   989.5521   6.980518 755.3573 1236156
## 95   989.0138   6.979666 757.4057 1238985
## 96   988.4762   6.978887 759.4581 1241821
## 97   987.9394   6.978172 761.5145 1244663
## 98   987.4032   6.977514 763.5749 1247511
## 99   986.8678   6.976908 765.6393 1250366
## 100  986.3331   6.976347 767.7078 1253227
## 101  985.7991   6.975828 769.7803 1256095
## 102  985.2658   6.975344 771.8570 1258970
## 103  984.7332   6.974893 773.9377 1261851
## 104  984.2013   6.974470 776.0225 1264739
## 105  983.6702   6.974073 778.1114 1267633
## 106  983.1397   6.973698 780.2044 1270534
## 107  982.6100   6.973343 782.3016 1273441
## 108  982.0809   6.973006 784.4030 1276356
## 109  981.5526   6.972684 786.5085 1279276
## 110  981.0250   6.972375 788.6181 1282204
## 111  980.4981   6.972079 790.7320 1285138
## 112  979.9718   6.971792 792.8500 1288079
## 113  979.4463   6.971515 794.9723 1291027
## 114  978.9215   6.971245 797.0988 1293982
## 115  978.3974   6.970982 799.2295 1296943
## 116  977.8740   6.970724 801.3645 1299911
## 117  977.3513   6.970471 803.5038 1302886
## 118  976.8293   6.970223 805.6473 1305867
## 119  976.3080   6.969977 807.7951 1308856
## 120  975.7874   6.969733 809.9471 1311851
## 121  975.2674   6.969492 812.1035 1314853
## 122  974.7482   6.969252 814.2643 1317862
## 123  974.2297   6.969014 816.4293 1320878
## 124  973.7118   6.968775 818.5987 1323901
## 125  973.1947   6.968537 820.7725 1326930
## 126  972.6782   6.968299 822.9506 1329967
## 127  972.1624   6.968060 825.1331 1333011
## 128  971.6473   6.967820 827.3200 1336061
## 129  971.1329   6.967580 829.5113 1339119
## 130  970.6192   6.967338 831.7070 1342183
## 131  970.1061   6.967095 833.9071 1345255
## 132  969.5938   6.966850 836.1117 1348334
```

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## 133  969.0821   6.966604 838.3208 1351419
## 134  968.5711   6.966356 840.5343 1354512
## 135  968.0608   6.966106 842.7522 1357612
## 136  967.5511   6.965853 844.9747 1360719
## 137  967.0421   6.965599 847.2017 1363832
## 138  966.5338   6.965342 849.4332 1366954
## 139  966.0262   6.965082 851.6692 1370082
## 140  965.5192   6.964821 853.9097 1373217
## 141  965.0129   6.964556 856.1548 1376360
## 142  964.5073   6.964289 858.4045 1379510
## 143  964.0024   6.964020 860.6588 1382667
## 144  963.4981   6.963748 862.9176 1385831
## 145  962.9944   6.963473 865.1810 1389002
## 146  962.4915   6.963195 867.4491 1392181
## 147  961.9892   6.962914 869.7217 1395367
## 148  961.4875   6.962631 871.9990 1398560
## 149  960.9865   6.962345 874.2810 1401761
## 150  960.4862   6.962056 876.5676 1404969
## 151  959.9865   6.961764 878.8589 1408184
## 152  959.4875   6.961469 881.1549 1411407
## 153  958.9892   6.961171 883.4556 1414637
## 154  958.4915   6.960871 885.7610 1417874
## 155  957.9944   6.960567 888.0711 1421119
## 156  957.4980   6.960260 890.3859 1424371
## 157  957.0022   6.959951 892.7056 1427631
## 158  956.5071   6.959638 895.0299 1430898
## 159  956.0127   6.959322 897.3591 1434172
## 160  955.5188   6.959004 899.6930 1437454
## 161  955.0257   6.958682 902.0318 1440744
## 162  954.5331   6.958358 904.3753 1444041
## 163  954.0412   6.958030 906.7237 1447346
## 164  953.5500   6.957700 909.0769 1450658
## 165  953.0594   6.957366 911.4350 1453978
## 166  952.5694   6.957030 913.7979 1457305
## 167  952.0801   6.956690 916.1658 1460640
## 168  951.5914   6.956348 918.5385 1463983
## 169  951.1033   6.956002 920.9161 1467333
## 170  950.6159   6.955653 923.2986 1470691
## 171  950.1291   6.955302 925.6861 1474057
## 172  949.6429   6.954947 928.0785 1477430
## 173  949.1573   6.954590 930.4759 1480811
## 174  948.6724   6.954229 932.8782 1484200
## 175  948.1881   6.953866 935.2855 1487597
## 176  947.7045   6.953499 937.6979 1491001
## 177  947.2214   6.953130 940.1152 1494413
## 178  946.7390   6.952757 942.5375 1497833
## 179  946.2572   6.952382 944.9649 1501261
## 180  945.7760   6.952004 947.3973 1504697
## 181  945.2955   6.951622 949.8348 1508140
## 182  944.8155   6.951238 952.2774 1511592
## 183  944.3362   6.950851 954.7251 1515051
## 184  943.8575   6.950460 957.1778 1518518
## 185  943.3794   6.950067 959.6357 1521993
## 186  942.9019   6.949671 962.0987 1525476
```

```
## 187  942.4251   6.949272 964.5669 1528967
## 188  941.9488   6.948870 967.0402 1532466
## 189  941.4732   6.948465 969.5187 1535973
## 190  940.9981   6.948057 972.0023 1539488
## 191  940.5237   6.947646 974.4912 1543012
## 192  940.0499   6.947233 976.9852 1546543
## 193  939.5766   6.946816 979.4845 1550082
## 194  939.1040   6.946396 981.9891 1553629
## 195  938.6320   6.945974 984.4988 1557185
## 196  938.1606   6.945548 987.0139 1560748
## 197  937.6898   6.945120 989.5342 1564320
## 198  937.2196   6.944689 992.0598 1567900
## 199  936.7500   6.944255 994.5908 1571488
## 200  936.2809   6.943818 997.1270 1575085
## 201  935.8125   6.943378 999.6686 1578689
```

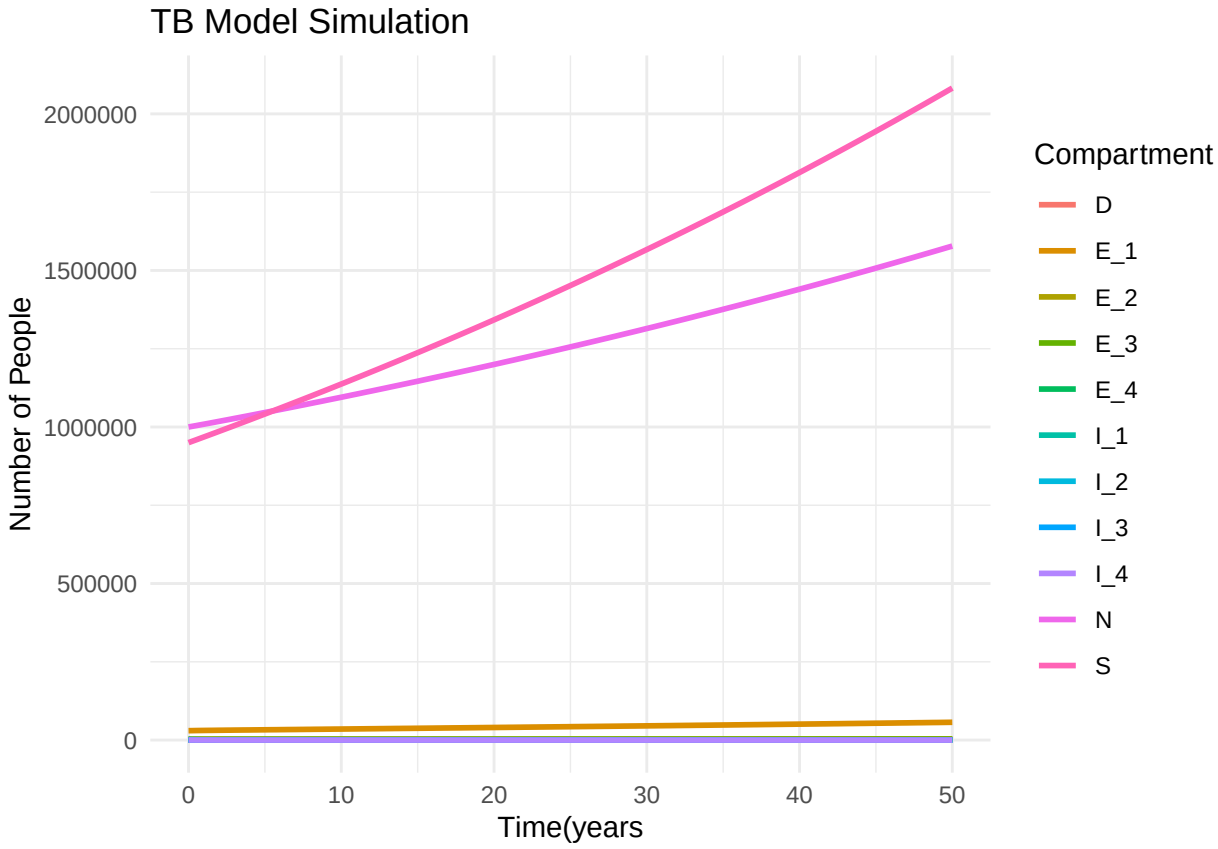
Compare Results

First, we'll plot the result of coding a solution by hand:

```
# plot the solution from the code-by-hand method

result1_long <- pivot_longer(result1, cols = -time, names_to = "compartment", values_to = "Count")

ggplot(result1_long[result1_long$compartment %in% c("S", "E_1", "I_1","E_2", "I_2", "E_3", "I_3", "E_4"
  geom_line(linewidth = 1) +
  labs(title = "TB Model Simulation", x = "Time(years", y = "Number of People", color = "Compartment") +
  theme_minimal()
```

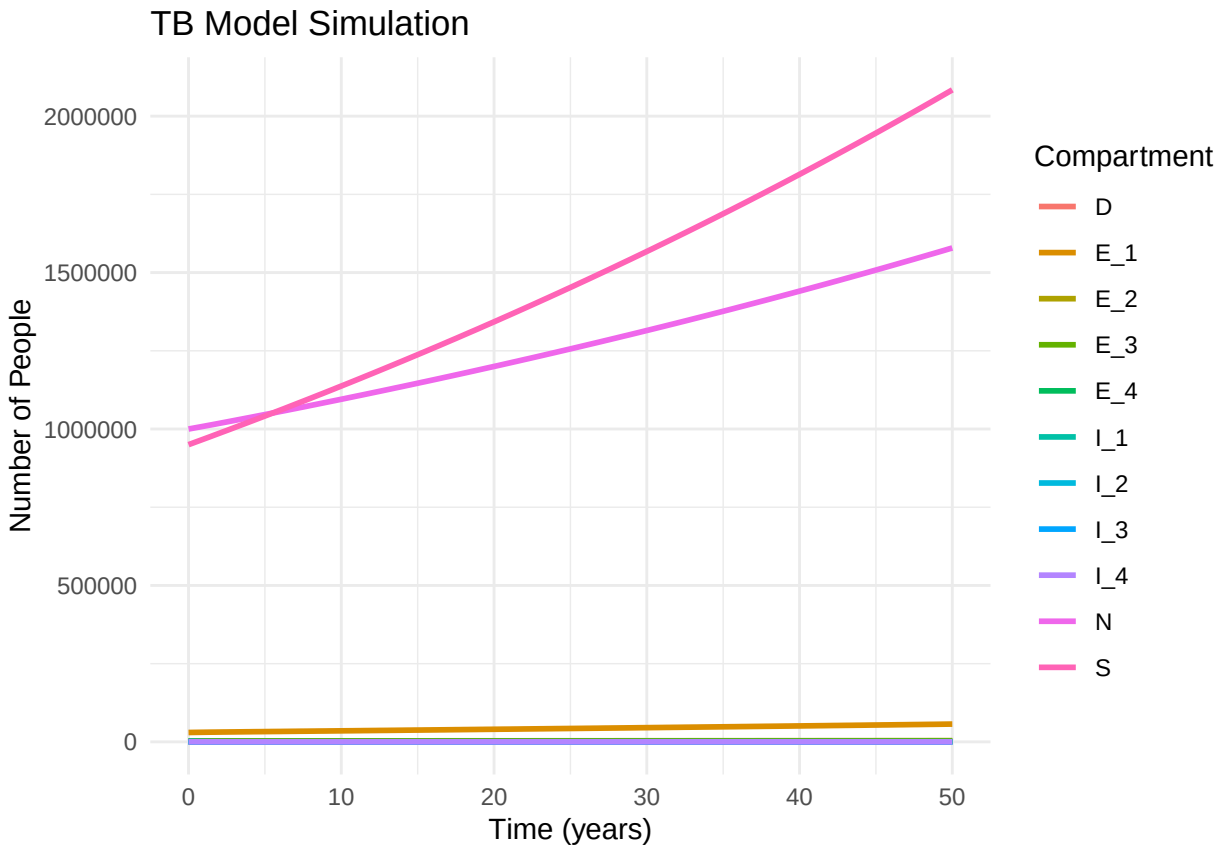


Now, we'll plot the result of solving using DeSolve to see if it yields the same result

```
# plot the solution from the DeSolve method

result2_long <- pivot_longer(result2,cols = -time, names_to = "compartment", values_to = "Count")

ggplot(result2_long[result2_long$compartment %in% c("S", "E_1", "I_1","E_2", "I_2", "E_3", "I_3", "E_4"
       aes(x = time, y = Count, color = compartment)) +
  geom_line(linewidth = 1) +
  labs(title = "TB Model Simulation",
       x = "Time (years)",
       y = "Number of People",
       color = "Compartment") +
  theme_minimal()
```



Same output!