Question 1(a)

$$\int \frac{1}{x \cdot \ln x} \, dx = \int \frac{1}{u} \, du \quad \text{0.5\%}$$

$$u = \ln x$$
$$du = \frac{1}{x} \, dx \Big\} \text{0.5\%}$$

$$= \ln u + C \quad \text{0.5\%}$$

$$= \ln(\ln x) + C \quad \text{0.5\%}$$

2%

Question 1(b)

$$A = \int_0^2 \left(3^x - x^3\right) dx = \quad \text{0.25\%}$$

$$= \left(\frac{3^x}{\ln 3} - \frac{x^4}{4}\right)\Big|_0^2$$
$$\underbrace{\text{0.25\%}} \quad \underbrace{\text{0.25\%}}$$

$$= \frac{9-1}{\ln 3} - \frac{16}{4} = \frac{8}{\ln 3} - 4 \Big\} \text{0.25\%}$$

1%

Question 1(c)

2%

(A)
$$\lim_{n \to \infty} = \lim_{n \to \infty} \sum_{i=1}^{n} \cos(1 + x_i) \cdot \frac{\Delta x}{3}$$
$$\left(\text{using } x_i = \frac{3i}{n}\right)$$

$$= \frac{1}{3} \int_0^3 \cos(1+x) \, dx \quad 1\% \left(\begin{array}{l}\text{0.5\% if}\\\text{close to this}\end{array}\right)$$

$$= \frac{1}{3} \sin(1+x)\Big|_0^3 = \boxed{\frac{1}{3}(\sin 4 - \sin 1)}$$
$$\text{0.5\%} \qquad\qquad\qquad \text{0.5\%}$$

(B)
$$\left\{x_i = \frac{i}{n}\right\} \Rightarrow \int_0^1 \cos(1+3x) \, dx \quad 1\% \left(\begin{array}{l}\text{0.5\% if}\\\text{close to}\\\text{this}\end{array}\right)$$

$$= \frac{1}{3}\sin(1+3x)\Big|_0^1 = \boxed{\frac{1}{3}(\sin 4 - \sin 1)}$$
$$\text{0.5\%} \qquad\qquad\qquad \text{0.5\%}$$

①

Question 1(d)

(circled in red: 1%)

$$\frac{d}{dx}\left(\int_{x^2}^{x^4+1} \sqrt{t \cdot \sin t}\ dt \right)$$

$$= \underbrace{4x^3}_{0.25\%} \cdot \underbrace{\sqrt{(x^4+1)\cdot \sin(x^4+1)}}_{0.25\%}$$

$$\underbrace{- 2x}_{\substack{0.25\% \\ (\text{including } "-")}} \cdot \underbrace{\sqrt{x^2 \cdot \sin(x^2)}}_{0.25\%}$$

....

Question 1(e)

(circled in red: 2%)

odd: $\sin x$, $\sin(x^3)$ $\}$ 0.25% (both)

even: $\cos x$, $\cos(x^3)$ 0.25% + 0.25%

$$I = \int_{-\frac{\pi}{2}}^{\pi/2} \cos x \cdot dx + \underbrace{\int_{-\pi/2}^{\pi/2} \sin(x^3) \cdot dx}_{\underset{0}{"} \leftarrow 0.5\%}$$

$$= \underbrace{\sin x \Big|_{-\frac{\pi}{2}}^{\pi/2}}_{0.5\%} = \boxed{2} \longrightarrow +0.25\% \text{ if the answer is also correct.}$$

....

Question 2

(circled in red: 3%)

$$I = \int \sin x \cdot (1 - \cos^2 x) \cdot \cos^2 x \cdot dx \ \Big\} 0.5\%$$

$$\left.\begin{array}{l} u = \cos x \\ du = -\sin x \cdot dx \end{array}\right\} 0.5\% \longrightarrow I = -\int (1 - u^2)\cdot u^2 \cdot du \ \Big\} 0.5\%$$

$$\text{②} \qquad = \int (u^4 - u^2)\, du \ \Big\} 0.5\%$$

$$= \frac{u^5}{5} - \frac{u^3}{3} + C \qquad 0.5\%$$

$$= \boxed{\frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + C} \qquad 0.5\%$$

Question 3

④ 4%.

$$\bar{f} = \frac{1}{3} \int_{-1}^{2} \frac{x+5}{x^2+6x+9} \cdot dx \qquad \bigg\} \begin{array}{l} 0.5\% \\ (\text{"}\frac{1}{3}\text{" should} \\ \text{be correct}) \end{array}$$

$$x^2 + 6x + 9 = (x+3)^2 \implies u = x+3$$

$\underbrace{\qquad}_{0.5\%}$

$$\bar{f} = \frac{1}{3} \int_{-1}^{2} \frac{x+5}{(x+3)^2} dx$$

Ⓐ $$\bar{f} = \frac{1}{3} \int_{u=2}^{u=5} \frac{u+2}{u^2} du \bigg\} 1\%$$

$$= \frac{1}{3} \left(\ln|u| - \frac{2}{u} \right) \bigg|_{u=2}^{u=5} \bigg] \begin{array}{l} 0.5\% \\ \text{each} \\ \text{term} \end{array}$$

$$= \boxed{\frac{1}{3} \left(\ln 5 - \ln 2 + \frac{3}{5} \right)}$$
$\underbrace{\qquad}_{0.5\%} \qquad \underbrace{}_{0.5\%}$

Ⓑ $$\bar{f} = \frac{1}{3} \int_{-1}^{2} \left(\frac{1}{x+3} + \frac{2}{(x+3)^2} \right) dx$$
$\underbrace{\qquad\qquad\qquad}_{1\%}$

$$= \frac{1}{3} \left(\ln|x+3| - \frac{2}{x+3} \right) \bigg|_{-1}^{2}$$

$$= \boxed{\frac{1}{3} \left(\ln 5 - \ln 2 + \frac{3}{5} \right)}$$
$\underbrace{\qquad}_{0.5\%} \qquad \underbrace{}_{0.5\%}$

Question 4

⑤ 5%.

$$I = \int \sin^{-1}x \cdot dx = x \cdot \sin^{-1}x \qquad \bigg\} 2\%$$
$$- \int x \cdot \frac{dx}{\sqrt{1-x^2}}$$

$1\% \begin{cases} u = \sin^{-1}x \\ du = \dfrac{dx}{\sqrt{1-x^2}} \, , \; dv = x \end{cases}$

$$\text{Next,} \quad w = 1-x^2, \quad dw = -2x \cdot dx \bigg\} 0.5\%$$

$$I = x \cdot \sin^{-1}x - \int \frac{\left(-\frac{1}{2} dw\right)}{\sqrt{w}} \bigg\} 1\%$$

③

$$= x \cdot \sin^{-1}x + \sqrt{w} + C$$

$$= \boxed{x \cdot \sin^{-1}x + \sqrt{1-x^2} + C} \bigg\} 0.5\%$$

Question 5

5 %

$$\frac{3x-1}{(x^2-2x+1)(x+1)} = \frac{3x-1}{(x-1)^2(x+1)}$$

$$= \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} \Big\} \; 2\%$$

$$3x-1 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$

$$x = -1 \implies -4 = A\cdot 0 + B\cdot 0 + C\cdot 4$$

$$\implies \boxed{C = -1}$$

$$x = 1 \implies 2 = A\cdot 0 + B\cdot 2 + C\cdot 0$$

$$\implies \boxed{B = 1}$$

$$x = 0 \implies -1 = A\cdot(-1) + B\cdot 1 + C\cdot 1$$

$$\underset{\underset{n}{0}}{\underbrace{\overset{=1}{} \qquad \qquad \overset{n}{\underset{-1}{}}}}$$

$$\implies \boxed{A = 1}$$

$$\implies \boxed{\frac{1}{x-1} + \frac{1}{(x-1)^2} - \frac{1}{x+1}}$$

0.5 % for each term

$$\boxed{I = \ln|x-1| - \frac{1}{x-1}} \; \text{(4)}$$

$$\xi \qquad - \ln|x+1| + C$$

0.5 % for each term