1 (a) Evaluate the indefinite integral $\int \frac{3x - 5x^{1/3}}{\sqrt{x}} dx$

$$-6 \times \frac{5/6}{11} + 2 \times \frac{3/2}{11}$$

(b) Calculate the area between $y = \frac{1}{x^2+1}$ and the x-axis for $0 \le x \le 1$. 2%

$$\int_{0}^{1} \frac{dx}{x^{2}+1} = \cot^{-1}x \Big|_{0}^{1} = \frac{\pi}{4}$$

$$0.5\%$$

(c) Express as a definite integral and evaluate the limit of the Riemann sum $\lim_{n\to\infty}\sum_{i=1}^n \left(x_i^2 + \sin(\sin x_i)\right) \triangle x$, where P is the partition with $x_i = -1 + \frac{2i}{n}$ for $i = 0, 1, \dots, n$ and $\triangle x \equiv x_i - x_{i-1}$. 3%

$$\int \left(X^2 + \sin(\sin x)\right) dx = \frac{1}{2}$$

$$(wrong limits \Rightarrow 0.5\%)$$

$$= \frac{2}{3} + 0$$

$$1\%, \text{ since sin (AINX)}$$

$$1\text{ is odd}$$

2%

3%

3%

1%

(d) Evaluate
$$\frac{d}{dx} \int_{\sin x}^{1} e^{-(1-t^2)} dt.$$

$$= -exp\{-(1-sin^2x)\} \cdot (sin x)'$$

$$= - enp \left\{ -\cos^2 x \right\} \cos x \qquad /1.$$

(e) Find an upper bound for the error E_T in the Trapezoidal Rule approximation of the definite integral $\int_0^1 \sin(2x) \, dx$, using n subintervals, given that $M_2 \equiv \max_{x \in [0,1]} \left| \frac{d^2}{dx^2} \sin(2x) \right| = \max_{x \in [0,1]} \left| -4 \sin(2x) \right| = 4$.

Choose n such that $E_T \leq \frac{1}{3}10^{-6}$.

$$E_T \le \frac{1}{12} \frac{(\beta - \alpha)^3 M_2}{h^2} = \frac{1}{3h^2} 1/.$$

$$\frac{1}{3h^2} \le \frac{1}{3} 10^{-6}$$

$$h \ge 10^3$$
17.

2 Evaluate the indefinite integral $\int \frac{\cos(\ln(t+1))}{t+1} dt$.

$$u = ln(t+1)$$

$$= \int cos u \cdot du$$

$$= sin u = sin(ln(t+1))$$

$$= 2$$

5%

3 Find the average value of
$$\frac{x-2}{x^2+5x+4}$$
 on the interval [0, 2].

$$f = \frac{1}{2} \int_{0}^{2} \frac{x-2}{x^2+5x+4} dx$$
 /1/2

(a) pointial fr-ns:

$$\bar{f} = \frac{1}{2} \int_{0}^{2} \left(\frac{2}{x+4} - \frac{1}{x+1} \right) dx + 2\%$$

$$= \frac{1}{2} \left(2 \ln (k+4) - \ln (x+1) \right) \Big|_{0}^{2} + 1\%$$

$$= \frac{1}{2} \left(\ln 3 - 2 \ln 2 \right) + 1\%$$

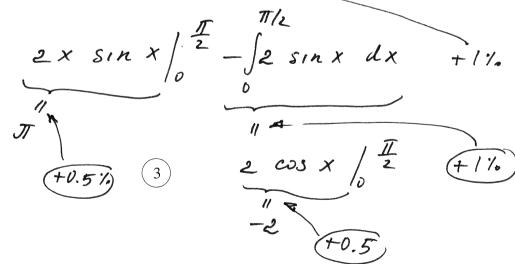
4 Evaluate the definite integral
$$\int_0^{\pi/2} x^2 \sin(x) dx$$
. 5%

$$= -x^{2} \cos x \Big|_{0}^{\pi/2} + \int_{0}^{\pi/2} 2x \cos x \, dx \qquad (\pm 1\%)$$

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5 Perform a partial fraction expansion of $\frac{2x-1}{(x+1)(x^2-3x+2)}$. (but do not integrate this function.)

5%

$$= \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x-1}$$

+2%

$$= \frac{-1/2}{X+1} + \frac{1}{X-2} + \frac{-1/2}{X-1} + \frac{3\%}{X-1}$$

$$= \frac{-1/2}{X+1} + \frac{1}{X-2} + \frac{-1/2}{X-1} + \frac{3\%}{X-1}$$