Queuing_envelope_mg1

2024-04-24

Approximating Queuing Delay from M/GI/1 to M/M/1 Envelope

FUNCTIONS

Function for Calculating of M/M/1 envelope load for M/G/1

```
envelope_load_calc <- function(Capacity_Gbps, k_num, Load, PS_size, PS_weights)
{
  mg1_packets <- simmer_mg1(Capacity_Gbps, Load, PS_size, PS_weights, k_num = 1e4)
  cat("Load real M/G/1:", Load, "\n")

  # Step 1: Determine the average delay (E(T))
  E_T_real <- mean(mg1_packets)
  N = sum(PS_size*PS_weights)
  N
  var_N <- sum(PS_size^2*PS_weights) - N^2
  Cs2 <- var_N/(N^2)
  nodes_capacity_Bps = Capacity_Gbps*1e9
  Capacity_ps = nodes_capacity_Bps/(8*N)
  E_X = 1/Capacity_ps

  percentiles_seq <- seq(0.5, 0.99, 0.01)

  n = 1
  df_real <- sapply(percentiles_seq, function(x) quantile(mg1_packets, x))*1e6 #real
  rho_env = seq(from=0.01,to=0.99,by=0.01)
  for (rho in rho_env){
    df_env <- qexp(percentiles_seq,rate = ((1-rho)/E_X))*1e6 # mu s
    if (all(df_real < df_env)){
      cat("Envelope upper bound \n")
      cat("Load Envelope:", rho, "\n")
      cat("E(T) exponential envelope M/M/1:", E_X/(1-rho), "s \n")
      cat("E(T) Real M/GI/1:", E_T_real, "s \n")
      cat("\n")
      break
    }
    n = n + 1
  }
  return(rho)
}
```

INPUT DATA

```
Capacity_Gbps = 10
k_num = 1e4 #number of simulation packages
Load <- 0.7  # Load
# #V1
PS_size <- c(40, 576, 1500)  # Packet sizes in bytes
PS_weights <- c(7, 4, 1) / 12  # Packet weights
```

Part 1: simulation M/G/1 and envelope M/M/1 calculations

SIMULATION

```
mg1_packets <- simmer_mg1(Capacity_Gbps, Load, PS_size, PS_weights, k_num = 1e4)
```

M/G/1 calculations of parameters

```
real_cdf <- ecdf(mg1_packets)
cat("Load real M/G/1:", Load, "\n")
```

```
## Load real M/G/1: 0.7
```

```
# Step 1: Determine the average delay (E(T))
E_T_real <- mean(mg1_packets)
N = sum(PS_size*PS_weights)
N
```

```
## [1] 340.3333
```

```
var_N <- sum(PS_size^2*PS_weights) - N^2
Cs2 <- var_N/(N^2)
nodes_capacity_Bps = Capacity_Gbps*1e9
Capacity_ps = nodes_capacity_Bps/(8*N)
E_X = 1/Capacity_ps
```

Envelope M/M/1 fitting process

Envelope E(T) M/M/1 should be above real M/G/1 for percentiles from 50% - 99%

```
percentiles_seq <- seq(0.5, 0.99, 0.01)

n = 1
df_real <- sapply(percentiles_seq, function(x) quantile(mg1_packets, x))*1e6 #real
rho_env = seq(from=0.01,to=0.99,by=0.01)
for (rho in rho_env){
```

```
  df_env <- qexp(percentiles_seq,rate = ((1-rho)/E_X))*1e6 # mu s
  if (all(df_real < df_env)){
    cat("Envelope upper bound \n")
    cat("Load Envelope:", rho, "\n")
    cat("E(T) exponential envelope M/M/1:", E_X/(1-rho), "s \n")
    cat("E(T) Real M/GI/1:", E_T_real, "s \n")
    break
  }
  n = n + 1
}
```

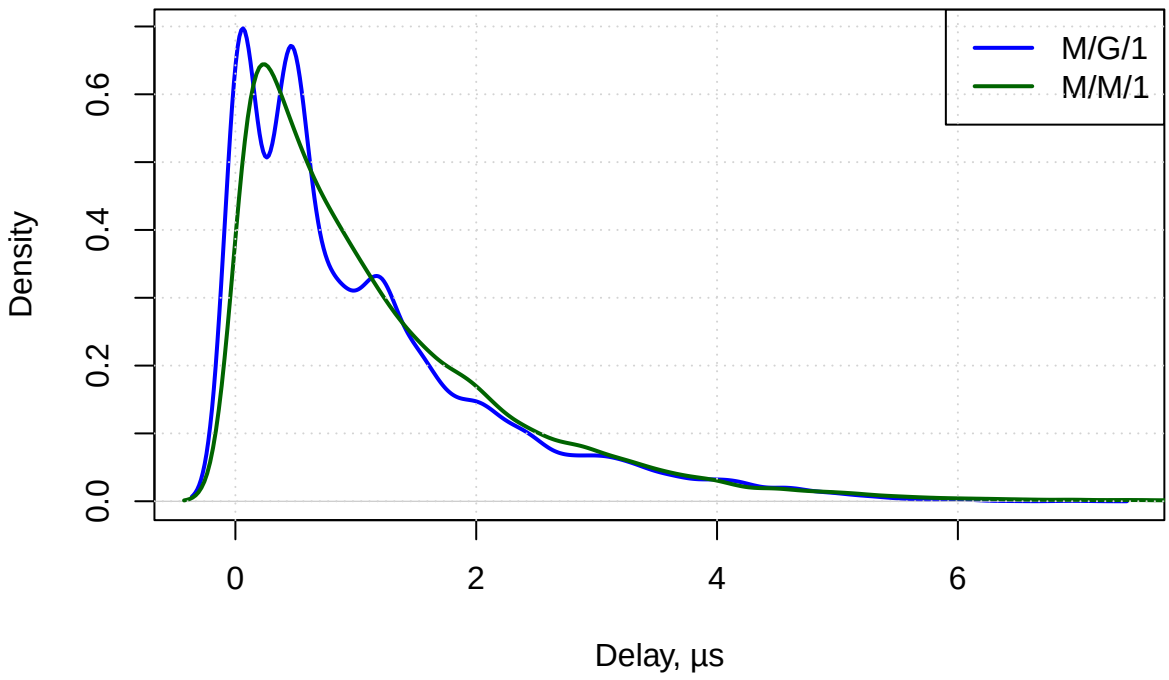
```
## Envelope upper bound
## Load Envelope: 0.77
## E(T) exponential envelope M/M/1: 1.183768e-06 s
## E(T) Real M/GI/1: 1.069956e-06 s
```

```
rho_env = rho
envelope_rate = (1-rho)/E_X
```

Plots

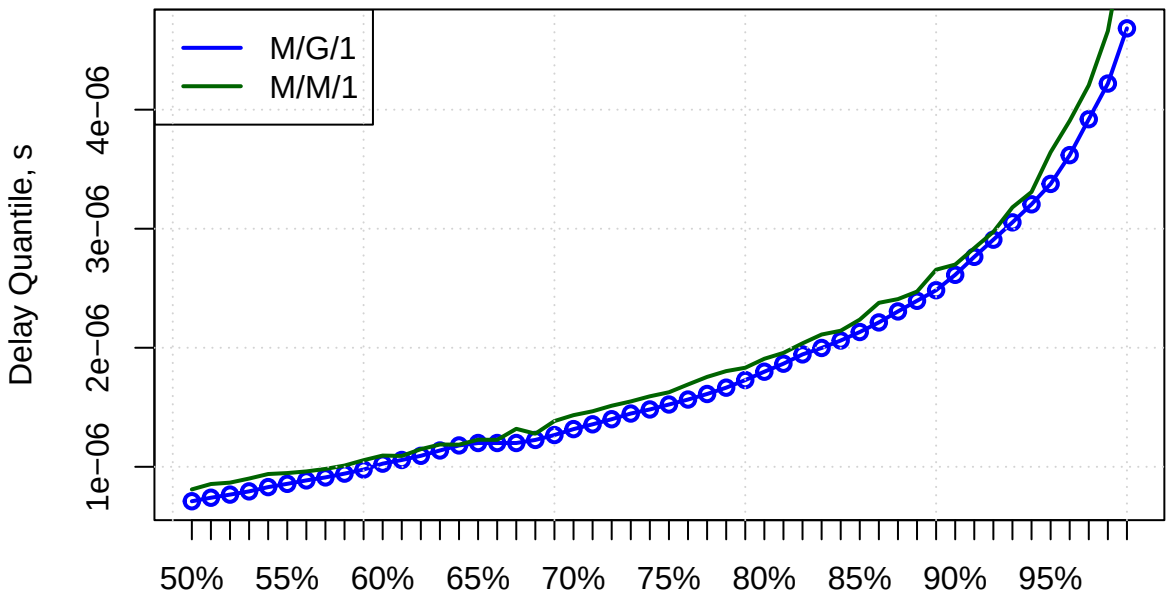
PDF of real and envelope

```
plot(density(mg1_packets*1e6), col = "blue", main = "", xlab = "Delay,  s", lwd = 2)
lines(density(rexp(1e4, rate = envelope_rate)*1e6), col = "darkgreen", lwd = 2)
legend("topright", legend = c("M/G/1", "M/M/1"), col = c("blue", "darkgreen"), lwd = 2)
grid()
```



Plot the percentiles of real and envelope

```r
df_real <- sapply(percentiles_seq, function(x) quantile(mg1_packets, x))
df_env <- sapply(percentiles_seq, function(x) quantile(rexp(1e4, rate = envelope_rate), x))

# Plot with no additional x-axis labels
plot(df_real, col = "blue", xlab = "", ylab = "Delay Quantile, s", main = "", xaxt = "n", lwd = 2)
lines(df_real, col = "blue", lwd = 2)
lines(df_env, col = "darkgreen", lwd = 2)

# Add x-axis labels for percentiles_seq
axis(1, at = seq_along(percentiles_seq), labels = paste0(percentiles_seq * 100, "%"))
legend("topleft", legend = c("M/G/1", "M/M/1"), col = c("blue", "darkgreen"), lwd = 2)
grid()
```



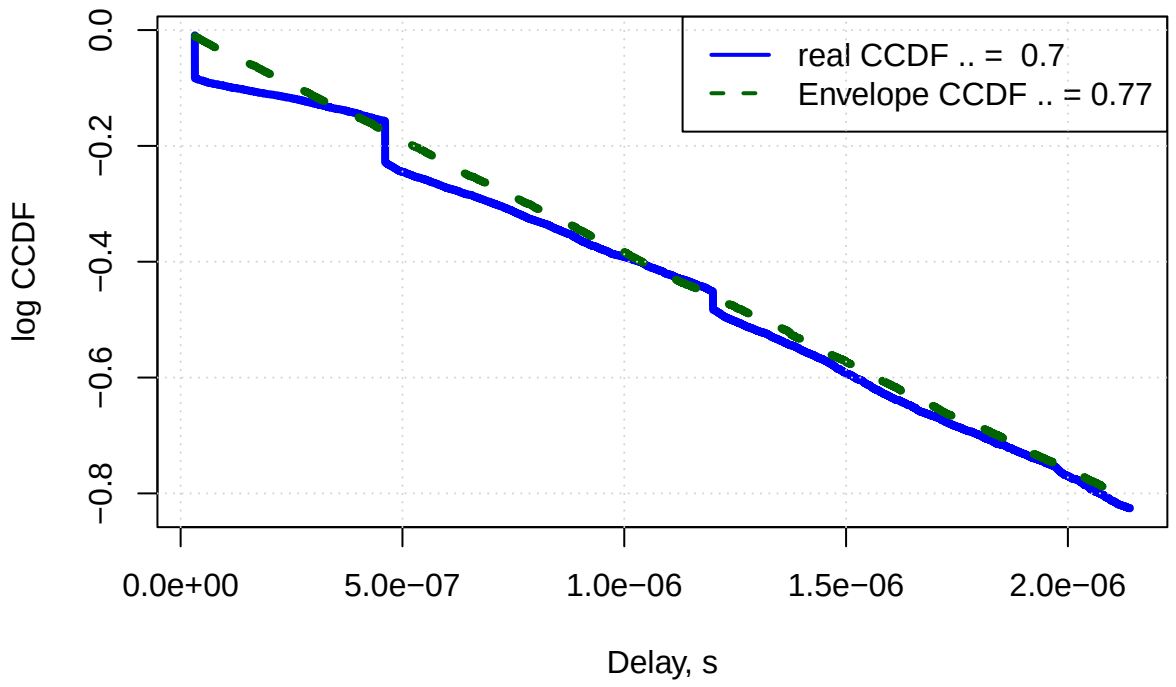
Plot the log CCDF

```
t_min = min(mg1_packets)
t_max = 2*E_T_real
# Compute the empirical CDF
ecdf_values <- ecdf(mg1_packets)
# Generate x values for the plot E(T)
x_values <- seq(t_min, t_max, length.out = 1e4)
# Compute the empirical CDF values
ecdf_y_values_sim <- ecdf_values(x_values)


envelope_ecdf_val <- envelope_cdf(envelope_rate)
ecdf_y_values_env_upper_bound <- envelope_ecdf_val(x_values)


lwd = 4
plot(x_values, log10(1 - ecdf_y_values_sim), type = "l", xlab = "Delay, s", ylab = "log CCDF", main = "
lines(x_values, log10(1 - ecdf_y_values_env_upper_bound), col = "darkgreen", lty = 2, lwd  = lwd )
legend("topright", legend = c(paste("real CCDF   = ", Load),
                              paste("Envelope CCDF   =", round(rho_env, 2))),
      col = c("blue", "darkgreen" ), lty = c(1, 2), lwd = 2)
```

```
## Warning in (function (s, units = "user", cex = NULL, font = NULL, vfont = NULL,
## : conversion failure on 'real CCDF   = 0.7' in 'mbcsToSbcs': dot substituted
## for <cf>
```

```
## Warning in (function (s, units = "user", cex = NULL, font = NULL, vfont = NULL,
## : conversion failure on 'real CCDF   = 0.7' in 'mbcsToSbcs': dot substituted
## for <81>

## Warning in (function (s, units = "user", cex = NULL, font = NULL, vfont = NULL,
## : conversion failure on 'Envelope CCDF   = 0.77' in 'mbcsToSbcs': dot
## substituted for <cf>

## Warning in (function (s, units = "user", cex = NULL, font = NULL, vfont = NULL,
## : conversion failure on 'Envelope CCDF   = 0.77' in 'mbcsToSbcs': dot
## substituted for <81>

## Warning in text.default(x, y, ...): conversion failure on 'real CCDF   = 0.7'
## in 'mbcsToSbcs': dot substituted for <cf>

## Warning in text.default(x, y, ...): conversion failure on 'real CCDF   = 0.7'
## in 'mbcsToSbcs': dot substituted for <81>

## Warning in text.default(x, y, ...): conversion failure on 'Envelope CCDF   =
## 0.77' in 'mbcsToSbcs': dot substituted for <cf>

## Warning in text.default(x, y, ...): conversion failure on 'Envelope CCDF   =
## 0.77' in 'mbcsToSbcs': dot substituted for <81>
```

```
grid()
```



Part 2: finding polinomial regression/relationship between load_real and load_envelope

```
# Data
loads_real <- seq(0.01, 0.99, 0.01)
loads_envelope <- sapply(loads_real, function(load) envelope_load_calc(Capacity_Gbps, k_num, load, PS_s
```

```
## Load real M/G/1: 0.01
## Envelope upper bound
## Load Envelope: 0.49
## E(T) exponential envelope M/M/1: 5.338562e-07 s
## E(T) Real M/GI/1: 2.824983e-07 s
##
## Load real M/G/1: 0.02
## Envelope upper bound
## Load Envelope: 0.48
## E(T) exponential envelope M/M/1: 5.235897e-07 s
## E(T) Real M/GI/1: 2.785732e-07 s
##
## Load real M/G/1: 0.03
## Envelope upper bound
## Load Envelope: 0.49
## E(T) exponential envelope M/M/1: 5.338562e-07 s
## E(T) Real M/GI/1: 2.823481e-07 s
##
## Load real M/G/1: 0.04
## Envelope upper bound
## Load Envelope: 0.48
## E(T) exponential envelope M/M/1: 5.235897e-07 s
## E(T) Real M/GI/1: 2.850714e-07 s
##
## Load real M/G/1: 0.05
## Envelope upper bound
## Load Envelope: 0.49
## E(T) exponential envelope M/M/1: 5.338562e-07 s
## E(T) Real M/GI/1: 2.943641e-07 s
##
## Load real M/G/1: 0.06
## Envelope upper bound
## Load Envelope: 0.49
## E(T) exponential envelope M/M/1: 5.338562e-07 s
## E(T) Real M/GI/1: 2.932052e-07 s
##
## Load real M/G/1: 0.07
## Envelope upper bound
## Load Envelope: 0.52
## E(T) exponential envelope M/M/1: 5.672222e-07 s
## E(T) Real M/GI/1: 3.049047e-07 s
##
## Load real M/G/1: 0.08
## Envelope upper bound
## Load Envelope: 0.49
```

```
## E(T) exponential envelope M/M/1: 5.338562e-07 s
## E(T) Real M/GI/1: 3.020969e-07 s
##
## Load real M/G/1: 0.09
## Envelope upper bound
## Load Envelope: 0.51
## E(T) exponential envelope M/M/1: 5.556463e-07 s
## E(T) Real M/GI/1: 3.07033e-07 s
##
## Load real M/G/1: 0.1
## Envelope upper bound
## Load Envelope: 0.51
## E(T) exponential envelope M/M/1: 5.556463e-07 s
## E(T) Real M/GI/1: 3.083089e-07 s
##
## Load real M/G/1: 0.11
## Envelope upper bound
## Load Envelope: 0.52
## E(T) exponential envelope M/M/1: 5.672222e-07 s
## E(T) Real M/GI/1: 3.268701e-07 s
##
## Load real M/G/1: 0.12
## Envelope upper bound
## Load Envelope: 0.53
## E(T) exponential envelope M/M/1: 5.792908e-07 s
## E(T) Real M/GI/1: 3.345738e-07 s
##
## Load real M/G/1: 0.13
## Envelope upper bound
## Load Envelope: 0.53
## E(T) exponential envelope M/M/1: 5.792908e-07 s
## E(T) Real M/GI/1: 3.284778e-07 s
##
## Load real M/G/1: 0.14
## Envelope upper bound
## Load Envelope: 0.52
## E(T) exponential envelope M/M/1: 5.672222e-07 s
## E(T) Real M/GI/1: 3.229115e-07 s
##
## Load real M/G/1: 0.15
## Envelope upper bound
## Load Envelope: 0.53
## E(T) exponential envelope M/M/1: 5.792908e-07 s
## E(T) Real M/GI/1: 3.349947e-07 s
##
## Load real M/G/1: 0.16
## Envelope upper bound
## Load Envelope: 0.53
## E(T) exponential envelope M/M/1: 5.792908e-07 s
## E(T) Real M/GI/1: 3.394216e-07 s
##
## Load real M/G/1: 0.17
## Envelope upper bound
## Load Envelope: 0.52
```

```
## E(T) exponential envelope M/M/1: 5.672222e-07 s
## E(T) Real M/GI/1: 3.329178e-07 s
##
## Load real M/G/1: 0.18
## Envelope upper bound
## Load Envelope: 0.53
## E(T) exponential envelope M/M/1: 5.792908e-07 s
## E(T) Real M/GI/1: 3.457983e-07 s
##
## Load real M/G/1: 0.19
## Envelope upper bound
## Load Envelope: 0.53
## E(T) exponential envelope M/M/1: 5.792908e-07 s
## E(T) Real M/GI/1: 3.603053e-07 s
##
## Load real M/G/1: 0.2
## Envelope upper bound
## Load Envelope: 0.56
## E(T) exponential envelope M/M/1: 6.187879e-07 s
## E(T) Real M/GI/1: 3.711692e-07 s
##
## Load real M/G/1: 0.21
## Envelope upper bound
## Load Envelope: 0.53
## E(T) exponential envelope M/M/1: 5.792908e-07 s
## E(T) Real M/GI/1: 3.560406e-07 s
##
## Load real M/G/1: 0.22
## Envelope upper bound
## Load Envelope: 0.54
## E(T) exponential envelope M/M/1: 5.918841e-07 s
## E(T) Real M/GI/1: 3.608895e-07 s
##
## Load real M/G/1: 0.23
## Envelope upper bound
## Load Envelope: 0.53
## E(T) exponential envelope M/M/1: 5.792908e-07 s
## E(T) Real M/GI/1: 3.684414e-07 s
##
## Load real M/G/1: 0.24
## Envelope upper bound
## Load Envelope: 0.55
## E(T) exponential envelope M/M/1: 6.05037e-07 s
## E(T) Real M/GI/1: 3.690312e-07 s
##
## Load real M/G/1: 0.25
## Envelope upper bound
## Load Envelope: 0.56
## E(T) exponential envelope M/M/1: 6.187879e-07 s
## E(T) Real M/GI/1: 3.939746e-07 s
##
## Load real M/G/1: 0.26
## Envelope upper bound
## Load Envelope: 0.56
```

```
## E(T) exponential envelope M/M/1: 6.187879e-07 s
## E(T) Real M/GI/1: 3.975794e-07 s
##
## Load real M/G/1: 0.27
## Envelope upper bound
## Load Envelope: 0.57
## E(T) exponential envelope M/M/1: 6.331783e-07 s
## E(T) Real M/GI/1: 4.102433e-07 s
##
## Load real M/G/1: 0.28
## Envelope upper bound
## Load Envelope: 0.56
## E(T) exponential envelope M/M/1: 6.187879e-07 s
## E(T) Real M/GI/1: 4.015561e-07 s
##
## Load real M/G/1: 0.29
## Envelope upper bound
## Load Envelope: 0.57
## E(T) exponential envelope M/M/1: 6.331783e-07 s
## E(T) Real M/GI/1: 4.169287e-07 s
##
## Load real M/G/1: 0.3
## Envelope upper bound
## Load Envelope: 0.58
## E(T) exponential envelope M/M/1: 6.48254e-07 s
## E(T) Real M/GI/1: 4.240627e-07 s
##
## Load real M/G/1: 0.31
## Envelope upper bound
## Load Envelope: 0.57
## E(T) exponential envelope M/M/1: 6.331783e-07 s
## E(T) Real M/GI/1: 4.173521e-07 s
##
## Load real M/G/1: 0.32
## Envelope upper bound
## Load Envelope: 0.58
## E(T) exponential envelope M/M/1: 6.48254e-07 s
## E(T) Real M/GI/1: 4.278643e-07 s
##
## Load real M/G/1: 0.33
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 4.446404e-07 s
##
## Load real M/G/1: 0.34
## Envelope upper bound
## Load Envelope: 0.59
## E(T) exponential envelope M/M/1: 6.64065e-07 s
## E(T) Real M/GI/1: 4.406016e-07 s
##
## Load real M/G/1: 0.35
## Envelope upper bound
## Load Envelope: 0.58
```

```
## E(T) exponential envelope M/M/1: 6.48254e-07 s
## E(T) Real M/GI/1: 4.440241e-07 s
##
## Load real M/G/1: 0.36
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 4.534018e-07 s
##
## Load real M/G/1: 0.37
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 4.731424e-07 s
##
## Load real M/G/1: 0.38
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 4.599918e-07 s
##
## Load real M/G/1: 0.39
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 5.001241e-07 s
##
## Load real M/G/1: 0.4
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 4.961688e-07 s
##
## Load real M/G/1: 0.41
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 5.025757e-07 s
##
## Load real M/G/1: 0.42
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 5.059995e-07 s
##
## Load real M/G/1: 0.43
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 5.572867e-07 s
##
## Load real M/G/1: 0.44
## Envelope upper bound
## Load Envelope: 0.6
```

```
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 5.499791e-07 s
##
## Load real M/G/1: 0.45
## Envelope upper bound
## Load Envelope: 0.62
## E(T) exponential envelope M/M/1: 7.164912e-07 s
## E(T) Real M/GI/1: 5.711058e-07 s
##
## Load real M/G/1: 0.46
## Envelope upper bound
## Load Envelope: 0.62
## E(T) exponential envelope M/M/1: 7.164912e-07 s
## E(T) Real M/GI/1: 5.842438e-07 s
##
## Load real M/G/1: 0.47
## Envelope upper bound
## Load Envelope: 0.62
## E(T) exponential envelope M/M/1: 7.164912e-07 s
## E(T) Real M/GI/1: 5.871319e-07 s
##
## Load real M/G/1: 0.48
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 5.701652e-07 s
##
## Load real M/G/1: 0.49
## Envelope upper bound
## Load Envelope: 0.63
## E(T) exponential envelope M/M/1: 7.358559e-07 s
## E(T) Real M/GI/1: 6.118861e-07 s
##
## Load real M/G/1: 0.5
## Envelope upper bound
## Load Envelope: 0.64
## E(T) exponential envelope M/M/1: 7.562963e-07 s
## E(T) Real M/GI/1: 6.357918e-07 s
##
## Load real M/G/1: 0.51
## Envelope upper bound
## Load Envelope: 0.65
## E(T) exponential envelope M/M/1: 7.779048e-07 s
## E(T) Real M/GI/1: 6.501574e-07 s
##
## Load real M/G/1: 0.52
## Envelope upper bound
## Load Envelope: 0.66
## E(T) exponential envelope M/M/1: 8.007843e-07 s
## E(T) Real M/GI/1: 6.782378e-07 s
##
## Load real M/G/1: 0.53
## Envelope upper bound
## Load Envelope: 0.66
```

```
## E(T) exponential envelope M/M/1: 8.007843e-07 s
## E(T) Real M/GI/1: 6.685514e-07 s
##
## Load real M/G/1: 0.54
## Envelope upper bound
## Load Envelope: 0.67
## E(T) exponential envelope M/M/1: 8.250505e-07 s
## E(T) Real M/GI/1: 7.016087e-07 s
##
## Load real M/G/1: 0.55
## Envelope upper bound
## Load Envelope: 0.65
## E(T) exponential envelope M/M/1: 7.779048e-07 s
## E(T) Real M/GI/1: 6.54001e-07 s
##
## Load real M/G/1: 0.56
## Envelope upper bound
## Load Envelope: 0.69
## E(T) exponential envelope M/M/1: 8.782796e-07 s
## E(T) Real M/GI/1: 7.613589e-07 s
##
## Load real M/G/1: 0.57
## Envelope upper bound
## Load Envelope: 0.69
## E(T) exponential envelope M/M/1: 8.782796e-07 s
## E(T) Real M/GI/1: 7.502777e-07 s
##
## Load real M/G/1: 0.58
## Envelope upper bound
## Load Envelope: 0.71
## E(T) exponential envelope M/M/1: 9.388506e-07 s
## E(T) Real M/GI/1: 8.157551e-07 s
##
## Load real M/G/1: 0.59
## Envelope upper bound
## Load Envelope: 0.7
## E(T) exponential envelope M/M/1: 9.075556e-07 s
## E(T) Real M/GI/1: 7.879726e-07 s
##
## Load real M/G/1: 0.6
## Envelope upper bound
## Load Envelope: 0.72
## E(T) exponential envelope M/M/1: 9.72381e-07 s
## E(T) Real M/GI/1: 8.582597e-07 s
##
## Load real M/G/1: 0.61
## Envelope upper bound
## Load Envelope: 0.69
## E(T) exponential envelope M/M/1: 8.782796e-07 s
## E(T) Real M/GI/1: 7.574569e-07 s
##
## Load real M/G/1: 0.62
## Envelope upper bound
## Load Envelope: 0.71
```

```
## E(T) exponential envelope M/M/1: 9.388506e-07 s
## E(T) Real M/GI/1: 8.123792e-07 s
##
## Load real M/G/1: 0.63
## Envelope upper bound
## Load Envelope: 0.73
## E(T) exponential envelope M/M/1: 1.008395e-06 s
## E(T) Real M/GI/1: 8.652342e-07 s
##
## Load real M/G/1: 0.64
## Envelope upper bound
## Load Envelope: 0.73
## E(T) exponential envelope M/M/1: 1.008395e-06 s
## E(T) Real M/GI/1: 9.154199e-07 s
##
## Load real M/G/1: 0.65
## Envelope upper bound
## Load Envelope: 0.72
## E(T) exponential envelope M/M/1: 9.72381e-07 s
## E(T) Real M/GI/1: 8.633192e-07 s
##
## Load real M/G/1: 0.66
## Envelope upper bound
## Load Envelope: 0.73
## E(T) exponential envelope M/M/1: 1.008395e-06 s
## E(T) Real M/GI/1: 8.831219e-07 s
##
## Load real M/G/1: 0.67
## Envelope upper bound
## Load Envelope: 0.77
## E(T) exponential envelope M/M/1: 1.183768e-06 s
## E(T) Real M/GI/1: 1.008397e-06 s
##
## Load real M/G/1: 0.68
## Envelope upper bound
## Load Envelope: 0.75
## E(T) exponential envelope M/M/1: 1.089067e-06 s
## E(T) Real M/GI/1: 9.664975e-07 s
##
## Load real M/G/1: 0.69
## Envelope upper bound
## Load Envelope: 0.77
## E(T) exponential envelope M/M/1: 1.183768e-06 s
## E(T) Real M/GI/1: 1.03691e-06 s
##
## Load real M/G/1: 0.7
## Envelope upper bound
## Load Envelope: 0.77
## E(T) exponential envelope M/M/1: 1.183768e-06 s
## E(T) Real M/GI/1: 1.075047e-06 s
##
## Load real M/G/1: 0.71
## Envelope upper bound
## Load Envelope: 0.8
```

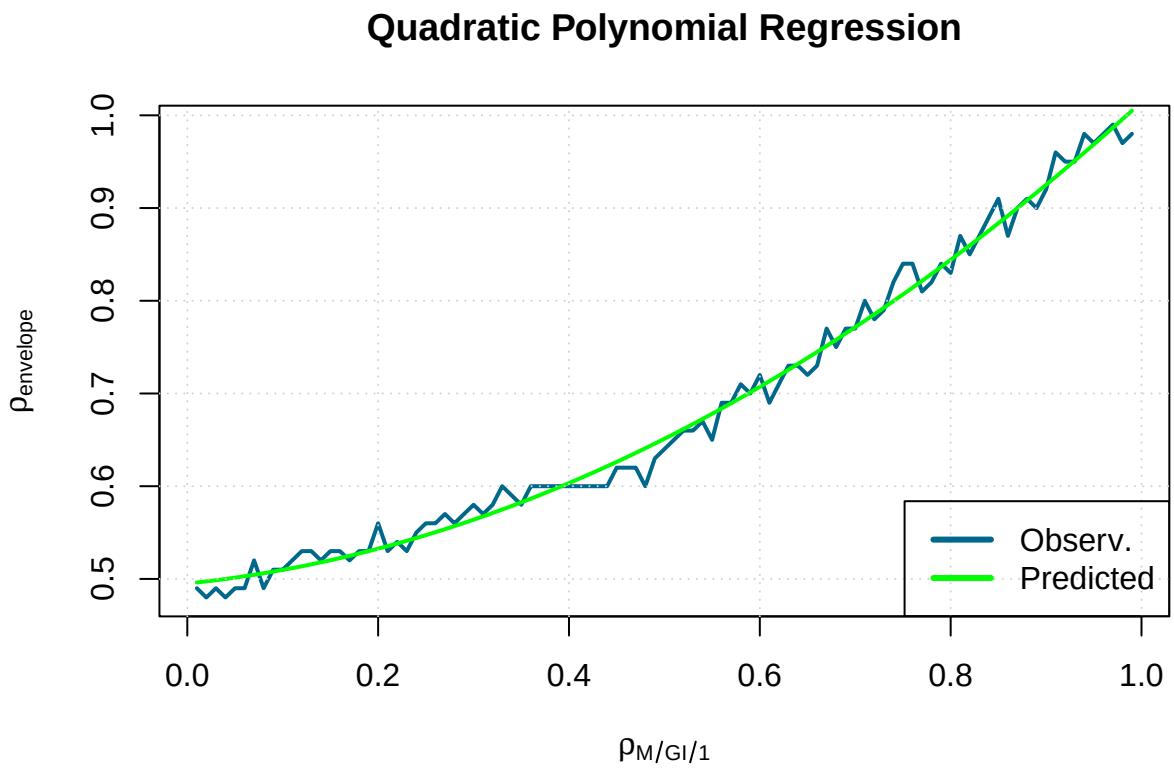
```
## E(T) exponential envelope M/M/1: 1.361333e-06 s
## E(T) Real M/GI/1: 1.185101e-06 s
##
## Load real M/G/1: 0.72
## Envelope upper bound
## Load Envelope: 0.78
## E(T) exponential envelope M/M/1: 1.237576e-06 s
## E(T) Real M/GI/1: 1.113367e-06 s
##
## Load real M/G/1: 0.73
## Envelope upper bound
## Load Envelope: 0.79
## E(T) exponential envelope M/M/1: 1.296508e-06 s
## E(T) Real M/GI/1: 1.160287e-06 s
##
## Load real M/G/1: 0.74
## Envelope upper bound
## Load Envelope: 0.82
## E(T) exponential envelope M/M/1: 1.512593e-06 s
## E(T) Real M/GI/1: 1.335504e-06 s
##
## Load real M/G/1: 0.75
## Envelope upper bound
## Load Envelope: 0.84
## E(T) exponential envelope M/M/1: 1.701667e-06 s
## E(T) Real M/GI/1: 1.468535e-06 s
##
## Load real M/G/1: 0.76
## Envelope upper bound
## Load Envelope: 0.84
## E(T) exponential envelope M/M/1: 1.701667e-06 s
## E(T) Real M/GI/1: 1.460801e-06 s
##
## Load real M/G/1: 0.77
## Envelope upper bound
## Load Envelope: 0.81
## E(T) exponential envelope M/M/1: 1.432982e-06 s
## E(T) Real M/GI/1: 1.288579e-06 s
##
## Load real M/G/1: 0.78
## Envelope upper bound
## Load Envelope: 0.82
## E(T) exponential envelope M/M/1: 1.512593e-06 s
## E(T) Real M/GI/1: 1.372981e-06 s
##
## Load real M/G/1: 0.79
## Envelope upper bound
## Load Envelope: 0.84
## E(T) exponential envelope M/M/1: 1.701667e-06 s
## E(T) Real M/GI/1: 1.433216e-06 s
##
## Load real M/G/1: 0.8
## Envelope upper bound
## Load Envelope: 0.83
```

```
## E(T) exponential envelope M/M/1: 1.601569e-06 s
## E(T) Real M/GI/1: 1.448781e-06 s
##
## Load real M/G/1: 0.81
## Envelope upper bound
## Load Envelope: 0.87
## E(T) exponential envelope M/M/1: 2.094359e-06 s
## E(T) Real M/GI/1: 1.928735e-06 s
##
## Load real M/G/1: 0.82
## Envelope upper bound
## Load Envelope: 0.85
## E(T) exponential envelope M/M/1: 1.815111e-06 s
## E(T) Real M/GI/1: 1.676457e-06 s
##
## Load real M/G/1: 0.83
## Envelope upper bound
## Load Envelope: 0.87
## E(T) exponential envelope M/M/1: 2.094359e-06 s
## E(T) Real M/GI/1: 2.008879e-06 s
##
## Load real M/G/1: 0.84
## Envelope upper bound
## Load Envelope: 0.89
## E(T) exponential envelope M/M/1: 2.475152e-06 s
## E(T) Real M/GI/1: 2.105306e-06 s
##
## Load real M/G/1: 0.85
## Envelope upper bound
## Load Envelope: 0.91
## E(T) exponential envelope M/M/1: 3.025185e-06 s
## E(T) Real M/GI/1: 2.491804e-06 s
##
## Load real M/G/1: 0.86
## Envelope upper bound
## Load Envelope: 0.87
## E(T) exponential envelope M/M/1: 2.094359e-06 s
## E(T) Real M/GI/1: 1.793664e-06 s
##
## Load real M/G/1: 0.87
## Envelope upper bound
## Load Envelope: 0.9
## E(T) exponential envelope M/M/1: 2.722667e-06 s
## E(T) Real M/GI/1: 2.395325e-06 s
##
## Load real M/G/1: 0.88
## Envelope upper bound
## Load Envelope: 0.91
## E(T) exponential envelope M/M/1: 3.025185e-06 s
## E(T) Real M/GI/1: 2.550739e-06 s
##
## Load real M/G/1: 0.89
## Envelope upper bound
## Load Envelope: 0.9
```

```
## E(T) exponential envelope M/M/1: 2.722667e-06 s
## E(T) Real M/GI/1: 2.435541e-06 s
##
## Load real M/G/1: 0.9
## Envelope upper bound
## Load Envelope: 0.92
## E(T) exponential envelope M/M/1: 3.403333e-06 s
## E(T) Real M/GI/1: 2.851503e-06 s
##
## Load real M/G/1: 0.91
## Envelope upper bound
## Load Envelope: 0.96
## E(T) exponential envelope M/M/1: 6.806667e-06 s
## E(T) Real M/GI/1: 4.958001e-06 s
##
## Load real M/G/1: 0.92
## Envelope upper bound
## Load Envelope: 0.95
## E(T) exponential envelope M/M/1: 5.445333e-06 s
## E(T) Real M/GI/1: 4.182841e-06 s
##
## Load real M/G/1: 0.93
## Envelope upper bound
## Load Envelope: 0.95
## E(T) exponential envelope M/M/1: 5.445333e-06 s
## E(T) Real M/GI/1: 4.491813e-06 s
##
## Load real M/G/1: 0.94
## Envelope upper bound
## Load Envelope: 0.98
## E(T) exponential envelope M/M/1: 1.361333e-05 s
## E(T) Real M/GI/1: 7.559581e-06 s
##
## Load real M/G/1: 0.95
## Envelope upper bound
## Load Envelope: 0.97
## E(T) exponential envelope M/M/1: 9.075556e-06 s
## E(T) Real M/GI/1: 6.197234e-06 s
##
## Load real M/G/1: 0.96
## Envelope upper bound
## Load Envelope: 0.98
## E(T) exponential envelope M/M/1: 1.361333e-05 s
## E(T) Real M/GI/1: 9.627801e-06 s
##
## Load real M/G/1: 0.97
## Envelope upper bound
## Load Envelope: 0.99
## E(T) exponential envelope M/M/1: 2.722667e-05 s
## E(T) Real M/GI/1: 1.706911e-05 s
##
## Load real M/G/1: 0.98
## Envelope upper bound
## Load Envelope: 0.97
```

```
## E(T) exponential envelope M/M/1: 9.075556e-06 s
## E(T) Real M/GI/1: 6.848687e-06 s
##
## Load real M/G/1: 0.99
## Envelope upper bound
## Load Envelope: 0.98
## E(T) exponential envelope M/M/1: 1.361333e-05 s
## E(T) Real M/GI/1: 8.242209e-06 s
```

```r
# Fitting a quadratic polynomial regression model
model <- lm(loads_envelope ~ poly(loads_real, 2, raw = TRUE))

# Generating sequence of loads_real values for prediction
load_real_pred <- seq(min(loads_real), max(loads_real), length.out = 100)

# Predicting loads_envelope values for the generated loads_real values
load_envelope_pred <- predict(model, newdata = data.frame(loads_real = load_real_pred))

# Plotting the observed data and the predicted values
plot(loads_real, loads_envelope, type = 'l', col = 'deepskyblue4', xlab = expression(rho[M/GI/1]), ylab
lines(load_real_pred, load_envelope_pred, col = 'green', lwd = 2)
grid()
legend("bottomright",c("Observ.","Predicted"),
       col=c("deepskyblue4","green"), lwd=3)
```



Extract the coefficients from the fitted quadratic polynomial regression model:

```r
# Extract the coefficients of the fitted model
coef_model <- coef(model)

# Construct the formula
formula <- paste0("y(x) = ", coef_model[1], " + ", coef_model[2], "x",
                  " + ", coef_model[3], "x^2")

# Print the formula
print(formula)
```

```
## [1] "y(x) = 0.495076347314934 + 0.105061982739405x + 0.414096953562388x^2"
```

Part 3: Checking the polinomial prediction

```r
loads_real = 0.57
cat("Load real M/G/1:", loads_real, "\n")
```

```
## Load real M/G/1: 0.57
```

```r
mg1_packets <- simmer_mg1(Capacity_Gbps, loads_real, PS_size, PS_weights, k_num = 1e4)


load_envelope_predicted = predict(model, newdata = data.frame(loads_real))


n = 1
df_real <- sapply(percentiles_seq, function(x) quantile(mg1_packets, x))*1e6 #real
rho_env = seq(from=0.01,to=0.99,by=0.01)

df_env <- qexp(percentiles_seq,rate = ((1-load_envelope_predicted)/E_X))*1e6 # mu s
cat("All quantiles df_real are below envelope:", all(df_real < df_env), "\n")
```

```
## All quantiles df_real are below envelope: TRUE
```

```r
cat("Envelope upper bound \n")
```

```
## Envelope upper bound
```

```r
cat("Load envelope M/M/1:", load_envelope_predicted, "\n")
```

```
## Load envelope M/M/1: 0.6895018
```

```r
cat("E(T) exponential envelope M/M/1:", E_X/(1-load_envelope_predicted), "s \n")
```

```
## E(T) exponential envelope M/M/1: 8.768703e-07 s
```

```r
cat("E(T) Real M/GI/1:", mean(mg1_packets), "s \n")
```

```
## E(T) Real M/GI/1: 7.412351e-07 s
```

```r
print(paste0("Envelope load obtained from polynomial: ", formula,"=",coef_model[1] +coef_model[2]*loads
```

```
## [1] "Envelope load obtained from polynomial: y(x) = 0.495076347314934 + 0.105061982739405x + 0.41409
```

Part 4:

Polynomial prediction for different packet size distributions

```r
# Data
#V1
PS_size <- c(40, 576, 1500)  # Packet sizes in bytes
PS_weights <- c(7, 4, 1) / 12  # Packet weights
loads_real <- seq(0.01, 0.99, 0.01)

loads_envelope_v1 <- sapply(loads_real, function(load) envelope_load_calc(Capacity_Gbps, k_num, load, PS
```

```
## Load real M/G/1: 0.01
## Envelope upper bound
## Load Envelope: 0.49
## E(T) exponential envelope M/M/1: 5.338562e-07 s
## E(T) Real M/GI/1: 2.791413e-07 s
##
## Load real M/G/1: 0.02
## Envelope upper bound
## Load Envelope: 0.48
## E(T) exponential envelope M/M/1: 5.235897e-07 s
## E(T) Real M/GI/1: 2.745303e-07 s
##
## Load real M/G/1: 0.03
## Envelope upper bound
## Load Envelope: 0.48
## E(T) exponential envelope M/M/1: 5.235897e-07 s
## E(T) Real M/GI/1: 2.843341e-07 s
##
## Load real M/G/1: 0.04
## Envelope upper bound
## Load Envelope: 0.49
## E(T) exponential envelope M/M/1: 5.338562e-07 s
## E(T) Real M/GI/1: 2.860208e-07 s
##
## Load real M/G/1: 0.05
## Envelope upper bound
## Load Envelope: 0.49
## E(T) exponential envelope M/M/1: 5.338562e-07 s
## E(T) Real M/GI/1: 2.834616e-07 s
```

```
##
## Load real M/G/1: 0.06
## Envelope upper bound
## Load Envelope: 0.49
## E(T) exponential envelope M/M/1: 5.338562e-07 s
## E(T) Real M/GI/1: 2.946321e-07 s
##
## Load real M/G/1: 0.07
## Envelope upper bound
## Load Envelope: 0.51
## E(T) exponential envelope M/M/1: 5.556463e-07 s
## E(T) Real M/GI/1: 3.038497e-07 s
##
## Load real M/G/1: 0.08
## Envelope upper bound
## Load Envelope: 0.49
## E(T) exponential envelope M/M/1: 5.338562e-07 s
## E(T) Real M/GI/1: 3.010863e-07 s
##
## Load real M/G/1: 0.09
## Envelope upper bound
## Load Envelope: 0.51
## E(T) exponential envelope M/M/1: 5.556463e-07 s
## E(T) Real M/GI/1: 3.055893e-07 s
##
## Load real M/G/1: 0.1
## Envelope upper bound
## Load Envelope: 0.5
## E(T) exponential envelope M/M/1: 5.445333e-07 s
## E(T) Real M/GI/1: 3.049412e-07 s
##
## Load real M/G/1: 0.11
## Envelope upper bound
## Load Envelope: 0.52
## E(T) exponential envelope M/M/1: 5.672222e-07 s
## E(T) Real M/GI/1: 3.212942e-07 s
##
## Load real M/G/1: 0.12
## Envelope upper bound
## Load Envelope: 0.52
## E(T) exponential envelope M/M/1: 5.672222e-07 s
## E(T) Real M/GI/1: 3.250589e-07 s
##
## Load real M/G/1: 0.13
## Envelope upper bound
## Load Envelope: 0.52
## E(T) exponential envelope M/M/1: 5.672222e-07 s
## E(T) Real M/GI/1: 3.167539e-07 s
##
## Load real M/G/1: 0.14
## Envelope upper bound
## Load Envelope: 0.53
## E(T) exponential envelope M/M/1: 5.792908e-07 s
## E(T) Real M/GI/1: 3.337236e-07 s
```

```
##
## Load real M/G/1: 0.15
## Envelope upper bound
## Load Envelope: 0.52
## E(T) exponential envelope M/M/1: 5.672222e-07 s
## E(T) Real M/GI/1: 3.298848e-07 s
##
## Load real M/G/1: 0.16
## Envelope upper bound
## Load Envelope: 0.54
## E(T) exponential envelope M/M/1: 5.918841e-07 s
## E(T) Real M/GI/1: 3.532612e-07 s
##
## Load real M/G/1: 0.17
## Envelope upper bound
## Load Envelope: 0.53
## E(T) exponential envelope M/M/1: 5.792908e-07 s
## E(T) Real M/GI/1: 3.491891e-07 s
##
## Load real M/G/1: 0.18
## Envelope upper bound
## Load Envelope: 0.53
## E(T) exponential envelope M/M/1: 5.792908e-07 s
## E(T) Real M/GI/1: 3.509556e-07 s
##
## Load real M/G/1: 0.19
## Envelope upper bound
## Load Envelope: 0.55
## E(T) exponential envelope M/M/1: 6.05037e-07 s
## E(T) Real M/GI/1: 3.68752e-07 s
##
## Load real M/G/1: 0.2
## Envelope upper bound
## Load Envelope: 0.54
## E(T) exponential envelope M/M/1: 5.918841e-07 s
## E(T) Real M/GI/1: 3.58088e-07 s
##
## Load real M/G/1: 0.21
## Envelope upper bound
## Load Envelope: 0.53
## E(T) exponential envelope M/M/1: 5.792908e-07 s
## E(T) Real M/GI/1: 3.639794e-07 s
##
## Load real M/G/1: 0.22
## Envelope upper bound
## Load Envelope: 0.55
## E(T) exponential envelope M/M/1: 6.05037e-07 s
## E(T) Real M/GI/1: 3.75545e-07 s
##
## Load real M/G/1: 0.23
## Envelope upper bound
## Load Envelope: 0.55
## E(T) exponential envelope M/M/1: 6.05037e-07 s
## E(T) Real M/GI/1: 3.82191e-07 s
```

```
##
## Load real M/G/1: 0.24
## Envelope upper bound
## Load Envelope: 0.56
## E(T) exponential envelope M/M/1: 6.187879e-07 s
## E(T) Real M/GI/1: 3.897571e-07 s
##
## Load real M/G/1: 0.25
## Envelope upper bound
## Load Envelope: 0.56
## E(T) exponential envelope M/M/1: 6.187879e-07 s
## E(T) Real M/GI/1: 3.939024e-07 s
##
## Load real M/G/1: 0.26
## Envelope upper bound
## Load Envelope: 0.56
## E(T) exponential envelope M/M/1: 6.187879e-07 s
## E(T) Real M/GI/1: 3.95638e-07 s
##
## Load real M/G/1: 0.27
## Envelope upper bound
## Load Envelope: 0.57
## E(T) exponential envelope M/M/1: 6.331783e-07 s
## E(T) Real M/GI/1: 3.945679e-07 s
##
## Load real M/G/1: 0.28
## Envelope upper bound
## Load Envelope: 0.58
## E(T) exponential envelope M/M/1: 6.48254e-07 s
## E(T) Real M/GI/1: 4.186771e-07 s
##
## Load real M/G/1: 0.29
## Envelope upper bound
## Load Envelope: 0.58
## E(T) exponential envelope M/M/1: 6.48254e-07 s
## E(T) Real M/GI/1: 4.182001e-07 s
##
## Load real M/G/1: 0.3
## Envelope upper bound
## Load Envelope: 0.58
## E(T) exponential envelope M/M/1: 6.48254e-07 s
## E(T) Real M/GI/1: 4.239363e-07 s
##
## Load real M/G/1: 0.31
## Envelope upper bound
## Load Envelope: 0.58
## E(T) exponential envelope M/M/1: 6.48254e-07 s
## E(T) Real M/GI/1: 4.294405e-07 s
##
## Load real M/G/1: 0.32
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 4.470032e-07 s
```

```
##
## Load real M/G/1: 0.33
## Envelope upper bound
## Load Envelope: 0.58
## E(T) exponential envelope M/M/1: 6.48254e-07 s
## E(T) Real M/GI/1: 4.38301e-07 s
##
## Load real M/G/1: 0.34
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 4.569963e-07 s
##
## Load real M/G/1: 0.35
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 4.494078e-07 s
##
## Load real M/G/1: 0.36
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 4.576677e-07 s
##
## Load real M/G/1: 0.37
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 4.980637e-07 s
##
## Load real M/G/1: 0.38
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 4.935585e-07 s
##
## Load real M/G/1: 0.39
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 5.051806e-07 s
##
## Load real M/G/1: 0.4
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 4.890065e-07 s
##
## Load real M/G/1: 0.41
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 4.868666e-07 s
```

```
##
## Load real M/G/1: 0.42
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 5.159231e-07 s
##
## Load real M/G/1: 0.43
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 5.297987e-07 s
##
## Load real M/G/1: 0.44
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 5.493948e-07 s
##
## Load real M/G/1: 0.45
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 6.806667e-07 s
## E(T) Real M/GI/1: 5.63198e-07 s
##
## Load real M/G/1: 0.46
## Envelope upper bound
## Load Envelope: 0.62
## E(T) exponential envelope M/M/1: 7.164912e-07 s
## E(T) Real M/GI/1: 5.774239e-07 s
##
## Load real M/G/1: 0.47
## Envelope upper bound
## Load Envelope: 0.62
## E(T) exponential envelope M/M/1: 7.164912e-07 s
## E(T) Real M/GI/1: 5.841624e-07 s
##
## Load real M/G/1: 0.48
## Envelope upper bound
## Load Envelope: 0.63
## E(T) exponential envelope M/M/1: 7.358559e-07 s
## E(T) Real M/GI/1: 6.15821e-07 s
##
## Load real M/G/1: 0.49
## Envelope upper bound
## Load Envelope: 0.64
## E(T) exponential envelope M/M/1: 7.562963e-07 s
## E(T) Real M/GI/1: 6.15408e-07 s
##
## Load real M/G/1: 0.5
## Envelope upper bound
## Load Envelope: 0.63
## E(T) exponential envelope M/M/1: 7.358559e-07 s
## E(T) Real M/GI/1: 6.072421e-07 s
```

```
##
## Load real M/G/1: 0.51
## Envelope upper bound
## Load Envelope: 0.63
## E(T) exponential envelope M/M/1: 7.358559e-07 s
## E(T) Real M/GI/1: 6.189114e-07 s
##
## Load real M/G/1: 0.52
## Envelope upper bound
## Load Envelope: 0.66
## E(T) exponential envelope M/M/1: 8.007843e-07 s
## E(T) Real M/GI/1: 6.783377e-07 s
##
## Load real M/G/1: 0.53
## Envelope upper bound
## Load Envelope: 0.65
## E(T) exponential envelope M/M/1: 7.779048e-07 s
## E(T) Real M/GI/1: 6.57582e-07 s
##
## Load real M/G/1: 0.54
## Envelope upper bound
## Load Envelope: 0.67
## E(T) exponential envelope M/M/1: 8.250505e-07 s
## E(T) Real M/GI/1: 6.95708e-07 s
##
## Load real M/G/1: 0.55
## Envelope upper bound
## Load Envelope: 0.67
## E(T) exponential envelope M/M/1: 8.250505e-07 s
## E(T) Real M/GI/1: 6.864939e-07 s
##
## Load real M/G/1: 0.56
## Envelope upper bound
## Load Envelope: 0.66
## E(T) exponential envelope M/M/1: 8.007843e-07 s
## E(T) Real M/GI/1: 6.795725e-07 s
##
## Load real M/G/1: 0.57
## Envelope upper bound
## Load Envelope: 0.7
## E(T) exponential envelope M/M/1: 9.075556e-07 s
## E(T) Real M/GI/1: 7.748786e-07 s
##
## Load real M/G/1: 0.58
## Envelope upper bound
## Load Envelope: 0.69
## E(T) exponential envelope M/M/1: 8.782796e-07 s
## E(T) Real M/GI/1: 7.683792e-07 s
##
## Load real M/G/1: 0.59
## Envelope upper bound
## Load Envelope: 0.7
## E(T) exponential envelope M/M/1: 9.075556e-07 s
## E(T) Real M/GI/1: 7.622807e-07 s
```

```
##
## Load real M/G/1: 0.6
## Envelope upper bound
## Load Envelope: 0.71
## E(T) exponential envelope M/M/1: 9.388506e-07 s
## E(T) Real M/GI/1: 8.195599e-07 s
##
## Load real M/G/1: 0.61
## Envelope upper bound
## Load Envelope: 0.72
## E(T) exponential envelope M/M/1: 9.72381e-07 s
## E(T) Real M/GI/1: 8.343262e-07 s
##
## Load real M/G/1: 0.62
## Envelope upper bound
## Load Envelope: 0.7
## E(T) exponential envelope M/M/1: 9.075556e-07 s
## E(T) Real M/GI/1: 7.781289e-07 s
##
## Load real M/G/1: 0.63
## Envelope upper bound
## Load Envelope: 0.71
## E(T) exponential envelope M/M/1: 9.388506e-07 s
## E(T) Real M/GI/1: 8.054655e-07 s
##
## Load real M/G/1: 0.64
## Envelope upper bound
## Load Envelope: 0.71
## E(T) exponential envelope M/M/1: 9.388506e-07 s
## E(T) Real M/GI/1: 8.38714e-07 s
##
## Load real M/G/1: 0.65
## Envelope upper bound
## Load Envelope: 0.75
## E(T) exponential envelope M/M/1: 1.089067e-06 s
## E(T) Real M/GI/1: 9.256267e-07 s
##
## Load real M/G/1: 0.66
## Envelope upper bound
## Load Envelope: 0.79
## E(T) exponential envelope M/M/1: 1.296508e-06 s
## E(T) Real M/GI/1: 1.031465e-06 s
##
## Load real M/G/1: 0.67
## Envelope upper bound
## Load Envelope: 0.76
## E(T) exponential envelope M/M/1: 1.134444e-06 s
## E(T) Real M/GI/1: 9.646786e-07 s
##
## Load real M/G/1: 0.68
## Envelope upper bound
## Load Envelope: 0.77
## E(T) exponential envelope M/M/1: 1.183768e-06 s
## E(T) Real M/GI/1: 1.019976e-06 s
```

```
##
## Load real M/G/1: 0.69
## Envelope upper bound
## Load Envelope: 0.75
## E(T) exponential envelope M/M/1: 1.089067e-06 s
## E(T) Real M/GI/1: 9.660959e-07 s
##
## Load real M/G/1: 0.7
## Envelope upper bound
## Load Envelope: 0.77
## E(T) exponential envelope M/M/1: 1.183768e-06 s
## E(T) Real M/GI/1: 1.011039e-06 s
##
## Load real M/G/1: 0.71
## Envelope upper bound
## Load Envelope: 0.79
## E(T) exponential envelope M/M/1: 1.296508e-06 s
## E(T) Real M/GI/1: 1.199557e-06 s
##
## Load real M/G/1: 0.72
## Envelope upper bound
## Load Envelope: 0.79
## E(T) exponential envelope M/M/1: 1.296508e-06 s
## E(T) Real M/GI/1: 1.140977e-06 s
##
## Load real M/G/1: 0.73
## Envelope upper bound
## Load Envelope: 0.81
## E(T) exponential envelope M/M/1: 1.432982e-06 s
## E(T) Real M/GI/1: 1.233423e-06 s
##
## Load real M/G/1: 0.74
## Envelope upper bound
## Load Envelope: 0.8
## E(T) exponential envelope M/M/1: 1.361333e-06 s
## E(T) Real M/GI/1: 1.228797e-06 s
##
## Load real M/G/1: 0.75
## Envelope upper bound
## Load Envelope: 0.8
## E(T) exponential envelope M/M/1: 1.361333e-06 s
## E(T) Real M/GI/1: 1.200609e-06 s
##
## Load real M/G/1: 0.76
## Envelope upper bound
## Load Envelope: 0.86
## E(T) exponential envelope M/M/1: 1.944762e-06 s
## E(T) Real M/GI/1: 1.641932e-06 s
##
## Load real M/G/1: 0.77
## Envelope upper bound
## Load Envelope: 0.84
## E(T) exponential envelope M/M/1: 1.701667e-06 s
## E(T) Real M/GI/1: 1.485687e-06 s
```

```
##
## Load real M/G/1: 0.78
## Envelope upper bound
## Load Envelope: 0.84
## E(T) exponential envelope M/M/1: 1.701667e-06 s
## E(T) Real M/GI/1: 1.586134e-06 s
##
## Load real M/G/1: 0.79
## Envelope upper bound
## Load Envelope: 0.84
## E(T) exponential envelope M/M/1: 1.701667e-06 s
## E(T) Real M/GI/1: 1.586178e-06 s
##
## Load real M/G/1: 0.8
## Envelope upper bound
## Load Envelope: 0.85
## E(T) exponential envelope M/M/1: 1.815111e-06 s
## E(T) Real M/GI/1: 1.646433e-06 s
##
## Load real M/G/1: 0.81
## Envelope upper bound
## Load Envelope: 0.88
## E(T) exponential envelope M/M/1: 2.268889e-06 s
## E(T) Real M/GI/1: 1.896498e-06 s
##
## Load real M/G/1: 0.82
## Envelope upper bound
## Load Envelope: 0.86
## E(T) exponential envelope M/M/1: 1.944762e-06 s
## E(T) Real M/GI/1: 1.721625e-06 s
##
## Load real M/G/1: 0.83
## Envelope upper bound
## Load Envelope: 0.87
## E(T) exponential envelope M/M/1: 2.094359e-06 s
## E(T) Real M/GI/1: 1.936145e-06 s
##
## Load real M/G/1: 0.84
## Envelope upper bound
## Load Envelope: 0.86
## E(T) exponential envelope M/M/1: 1.944762e-06 s
## E(T) Real M/GI/1: 1.876128e-06 s
##
## Load real M/G/1: 0.85
## Envelope upper bound
## Load Envelope: 0.91
## E(T) exponential envelope M/M/1: 3.025185e-06 s
## E(T) Real M/GI/1: 2.536904e-06 s
##
## Load real M/G/1: 0.86
## Envelope upper bound
## Load Envelope: 0.93
## E(T) exponential envelope M/M/1: 3.889524e-06 s
## E(T) Real M/GI/1: 2.483597e-06 s
```

```
##
## Load real M/G/1: 0.87
## Envelope upper bound
## Load Envelope: 0.93
## E(T) exponential envelope M/M/1: 3.889524e-06 s
## E(T) Real M/GI/1: 3.077548e-06 s
##
## Load real M/G/1: 0.88
## Envelope upper bound
## Load Envelope: 0.88
## E(T) exponential envelope M/M/1: 2.268889e-06 s
## E(T) Real M/GI/1: 2.055573e-06 s
##
## Load real M/G/1: 0.89
## Envelope upper bound
## Load Envelope: 0.91
## E(T) exponential envelope M/M/1: 3.025185e-06 s
## E(T) Real M/GI/1: 2.560554e-06 s
##
## Load real M/G/1: 0.9
## Envelope upper bound
## Load Envelope: 0.92
## E(T) exponential envelope M/M/1: 3.403333e-06 s
## E(T) Real M/GI/1: 3.029406e-06 s
##
## Load real M/G/1: 0.91
## Envelope upper bound
## Load Envelope: 0.94
## E(T) exponential envelope M/M/1: 4.537778e-06 s
## E(T) Real M/GI/1: 3.998816e-06 s
##
## Load real M/G/1: 0.92
## Envelope upper bound
## Load Envelope: 0.91
## E(T) exponential envelope M/M/1: 3.025185e-06 s
## E(T) Real M/GI/1: 2.418591e-06 s
##
## Load real M/G/1: 0.93
## Envelope upper bound
## Load Envelope: 0.93
## E(T) exponential envelope M/M/1: 3.889524e-06 s
## E(T) Real M/GI/1: 3.319849e-06 s
##
## Load real M/G/1: 0.94
## Envelope upper bound
## Load Envelope: 0.98
## E(T) exponential envelope M/M/1: 1.361333e-05 s
## E(T) Real M/GI/1: 8.459179e-06 s
##
## Load real M/G/1: 0.95
## Envelope upper bound
## Load Envelope: 0.95
## E(T) exponential envelope M/M/1: 5.445333e-06 s
## E(T) Real M/GI/1: 4.009715e-06 s
```

```
##
## Load real M/G/1: 0.96
## Envelope upper bound
## Load Envelope: 0.97
## E(T) exponential envelope M/M/1: 9.075556e-06 s
## E(T) Real M/GI/1: 6.165116e-06 s
##
## Load real M/G/1: 0.97
## Envelope upper bound
## Load Envelope: 0.97
## E(T) exponential envelope M/M/1: 9.075556e-06 s
## E(T) Real M/GI/1: 6.667481e-06 s
##
## Load real M/G/1: 0.98
## Envelope upper bound
## Load Envelope: 0.97
## E(T) exponential envelope M/M/1: 9.075556e-06 s
## E(T) Real M/GI/1: 7.719119e-06 s
##
## Load real M/G/1: 0.99
## Envelope upper bound
## Load Envelope: 0.99
## E(T) exponential envelope M/M/1: 2.722667e-05 s
## E(T) Real M/GI/1: 1.950163e-05 s
```

```r
# Fitting a quadratic polynomial regression model
model_v1 <- lm(loads_envelope_v1 ~ poly(loads_real, 2, raw = TRUE))

# Generating sequence of loads_real values for prediction
load_real_pred <- seq(min(loads_real), max(loads_real), length.out = 100)

# Predicting loads_envelope values for the generated loads_real values
load_envelope_pred_v1 <- predict(model_v1, newdata = data.frame(loads_real = load_real_pred))

#V2 https://www.ams-ix.net/ams
PS_size=c((64+127)/2,(128+255)/2,(256+511)/2, (512+1023)/2, (1024+1513)/2, 1514, (1515+9100)/2)
PS_weights=c(33.2/100, 5.4/100, 3.3/100, 3.7/100, 34.6/100, 14.6/100, 5.2/100)
loads_envelope_v2 <- sapply(loads_real, function(load) envelope_load_calc(Capacity_Gbps, k_num, load, PS
```

```
## Load real M/G/1: 0.01
## Envelope upper bound
## Load Envelope: 0.45
## E(T) exponential envelope M/M/1: 1.482233e-06 s
## E(T) Real M/GI/1: 8.205311e-07 s
##
## Load real M/G/1: 0.02
## Envelope upper bound
## Load Envelope: 0.45
## E(T) exponential envelope M/M/1: 1.482233e-06 s
## E(T) Real M/GI/1: 8.574681e-07 s
##
## Load real M/G/1: 0.03
## Envelope upper bound
## Load Envelope: 0.45
```

```
## E(T) exponential envelope M/M/1: 1.482233e-06 s
## E(T) Real M/GI/1: 8.411308e-07 s
##
## Load real M/G/1: 0.04
## Envelope upper bound
## Load Envelope: 0.45
## E(T) exponential envelope M/M/1: 1.482233e-06 s
## E(T) Real M/GI/1: 8.698631e-07 s
##
## Load real M/G/1: 0.05
## Envelope upper bound
## Load Envelope: 0.45
## E(T) exponential envelope M/M/1: 1.482233e-06 s
## E(T) Real M/GI/1: 8.607107e-07 s
##
## Load real M/G/1: 0.06
## Envelope upper bound
## Load Envelope: 0.45
## E(T) exponential envelope M/M/1: 1.482233e-06 s
## E(T) Real M/GI/1: 8.897541e-07 s
##
## Load real M/G/1: 0.07
## Envelope upper bound
## Load Envelope: 0.45
## E(T) exponential envelope M/M/1: 1.482233e-06 s
## E(T) Real M/GI/1: 8.788117e-07 s
##
## Load real M/G/1: 0.08
## Envelope upper bound
## Load Envelope: 0.45
## E(T) exponential envelope M/M/1: 1.482233e-06 s
## E(T) Real M/GI/1: 8.813221e-07 s
##
## Load real M/G/1: 0.09
## Envelope upper bound
## Load Envelope: 0.46
## E(T) exponential envelope M/M/1: 1.509681e-06 s
## E(T) Real M/GI/1: 9.174189e-07 s
##
## Load real M/G/1: 0.1
## Envelope upper bound
## Load Envelope: 0.45
## E(T) exponential envelope M/M/1: 1.482233e-06 s
## E(T) Real M/GI/1: 9.104107e-07 s
##
## Load real M/G/1: 0.11
## Envelope upper bound
## Load Envelope: 0.45
## E(T) exponential envelope M/M/1: 1.482233e-06 s
## E(T) Real M/GI/1: 9.339207e-07 s
##
## Load real M/G/1: 0.12
## Envelope upper bound
## Load Envelope: 0.45
```

```
## E(T) exponential envelope M/M/1: 1.482233e-06 s
## E(T) Real M/GI/1: 9.256928e-07 s
##
## Load real M/G/1: 0.13
## Envelope upper bound
## Load Envelope: 0.45
## E(T) exponential envelope M/M/1: 1.482233e-06 s
## E(T) Real M/GI/1: 9.545468e-07 s
##
## Load real M/G/1: 0.14
## Envelope upper bound
## Load Envelope: 0.45
## E(T) exponential envelope M/M/1: 1.482233e-06 s
## E(T) Real M/GI/1: 9.494644e-07 s
##
## Load real M/G/1: 0.15
## Envelope upper bound
## Load Envelope: 0.46
## E(T) exponential envelope M/M/1: 1.509681e-06 s
## E(T) Real M/GI/1: 9.984225e-07 s
##
## Load real M/G/1: 0.16
## Envelope upper bound
## Load Envelope: 0.45
## E(T) exponential envelope M/M/1: 1.482233e-06 s
## E(T) Real M/GI/1: 9.650146e-07 s
##
## Load real M/G/1: 0.17
## Envelope upper bound
## Load Envelope: 0.46
## E(T) exponential envelope M/M/1: 1.509681e-06 s
## E(T) Real M/GI/1: 1.024149e-06 s
##
## Load real M/G/1: 0.18
## Envelope upper bound
## Load Envelope: 0.46
## E(T) exponential envelope M/M/1: 1.509681e-06 s
## E(T) Real M/GI/1: 1.027615e-06 s
##
## Load real M/G/1: 0.19
## Envelope upper bound
## Load Envelope: 0.47
## E(T) exponential envelope M/M/1: 1.538166e-06 s
## E(T) Real M/GI/1: 1.040866e-06 s
##
## Load real M/G/1: 0.2
## Envelope upper bound
## Load Envelope: 0.46
## E(T) exponential envelope M/M/1: 1.509681e-06 s
## E(T) Real M/GI/1: 1.053759e-06 s
##
## Load real M/G/1: 0.21
## Envelope upper bound
## Load Envelope: 0.46
```

```
## E(T) exponential envelope M/M/1: 1.509681e-06 s
## E(T) Real M/GI/1: 1.050621e-06 s
##
## Load real M/G/1: 0.22
## Envelope upper bound
## Load Envelope: 0.46
## E(T) exponential envelope M/M/1: 1.509681e-06 s
## E(T) Real M/GI/1: 1.069504e-06 s
##
## Load real M/G/1: 0.23
## Envelope upper bound
## Load Envelope: 0.46
## E(T) exponential envelope M/M/1: 1.509681e-06 s
## E(T) Real M/GI/1: 1.064239e-06 s
##
## Load real M/G/1: 0.24
## Envelope upper bound
## Load Envelope: 0.47
## E(T) exponential envelope M/M/1: 1.538166e-06 s
## E(T) Real M/GI/1: 1.084762e-06 s
##
## Load real M/G/1: 0.25
## Envelope upper bound
## Load Envelope: 0.46
## E(T) exponential envelope M/M/1: 1.509681e-06 s
## E(T) Real M/GI/1: 1.094779e-06 s
##
## Load real M/G/1: 0.26
## Envelope upper bound
## Load Envelope: 0.49
## E(T) exponential envelope M/M/1: 1.598486e-06 s
## E(T) Real M/GI/1: 1.140354e-06 s
##
## Load real M/G/1: 0.27
## Envelope upper bound
## Load Envelope: 0.49
## E(T) exponential envelope M/M/1: 1.598486e-06 s
## E(T) Real M/GI/1: 1.141013e-06 s
##
## Load real M/G/1: 0.28
## Envelope upper bound
## Load Envelope: 0.51
## E(T) exponential envelope M/M/1: 1.663731e-06 s
## E(T) Real M/GI/1: 1.199046e-06 s
##
## Load real M/G/1: 0.29
## Envelope upper bound
## Load Envelope: 0.51
## E(T) exponential envelope M/M/1: 1.663731e-06 s
## E(T) Real M/GI/1: 1.209846e-06 s
##
## Load real M/G/1: 0.3
## Envelope upper bound
## Load Envelope: 0.51
```

```
## E(T) exponential envelope M/M/1: 1.663731e-06 s
## E(T) Real M/GI/1: 1.199907e-06 s
##
## Load real M/G/1: 0.31
## Envelope upper bound
## Load Envelope: 0.49
## E(T) exponential envelope M/M/1: 1.598486e-06 s
## E(T) Real M/GI/1: 1.195626e-06 s
##
## Load real M/G/1: 0.32
## Envelope upper bound
## Load Envelope: 0.52
## E(T) exponential envelope M/M/1: 1.698392e-06 s
## E(T) Real M/GI/1: 1.255116e-06 s
##
## Load real M/G/1: 0.33
## Envelope upper bound
## Load Envelope: 0.54
## E(T) exponential envelope M/M/1: 1.772235e-06 s
## E(T) Real M/GI/1: 1.306261e-06 s
##
## Load real M/G/1: 0.34
## Envelope upper bound
## Load Envelope: 0.51
## E(T) exponential envelope M/M/1: 1.663731e-06 s
## E(T) Real M/GI/1: 1.267222e-06 s
##
## Load real M/G/1: 0.35
## Envelope upper bound
## Load Envelope: 0.54
## E(T) exponential envelope M/M/1: 1.772235e-06 s
## E(T) Real M/GI/1: 1.359162e-06 s
##
## Load real M/G/1: 0.36
## Envelope upper bound
## Load Envelope: 0.53
## E(T) exponential envelope M/M/1: 1.734528e-06 s
## E(T) Real M/GI/1: 1.30862e-06 s
##
## Load real M/G/1: 0.37
## Envelope upper bound
## Load Envelope: 0.54
## E(T) exponential envelope M/M/1: 1.772235e-06 s
## E(T) Real M/GI/1: 1.358994e-06 s
##
## Load real M/G/1: 0.38
## Envelope upper bound
## Load Envelope: 0.56
## E(T) exponential envelope M/M/1: 1.852791e-06 s
## E(T) Real M/GI/1: 1.460257e-06 s
##
## Load real M/G/1: 0.39
## Envelope upper bound
## Load Envelope: 0.54
```

```
## E(T) exponential envelope M/M/1: 1.772235e-06 s
## E(T) Real M/GI/1: 1.383834e-06 s
##
## Load real M/G/1: 0.4
## Envelope upper bound
## Load Envelope: 0.55
## E(T) exponential envelope M/M/1: 1.811618e-06 s
## E(T) Real M/GI/1: 1.432439e-06 s
##
## Load real M/G/1: 0.41
## Envelope upper bound
## Load Envelope: 0.55
## E(T) exponential envelope M/M/1: 1.811618e-06 s
## E(T) Real M/GI/1: 1.429265e-06 s
##
## Load real M/G/1: 0.42
## Envelope upper bound
## Load Envelope: 0.56
## E(T) exponential envelope M/M/1: 1.852791e-06 s
## E(T) Real M/GI/1: 1.490168e-06 s
##
## Load real M/G/1: 0.43
## Envelope upper bound
## Load Envelope: 0.58
## E(T) exponential envelope M/M/1: 1.941019e-06 s
## E(T) Real M/GI/1: 1.561758e-06 s
##
## Load real M/G/1: 0.44
## Envelope upper bound
## Load Envelope: 0.58
## E(T) exponential envelope M/M/1: 1.941019e-06 s
## E(T) Real M/GI/1: 1.532281e-06 s
##
## Load real M/G/1: 0.45
## Envelope upper bound
## Load Envelope: 0.58
## E(T) exponential envelope M/M/1: 1.941019e-06 s
## E(T) Real M/GI/1: 1.572005e-06 s
##
## Load real M/G/1: 0.46
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 2.03807e-06 s
## E(T) Real M/GI/1: 1.684722e-06 s
##
## Load real M/G/1: 0.47
## Envelope upper bound
## Load Envelope: 0.57
## E(T) exponential envelope M/M/1: 1.895879e-06 s
## E(T) Real M/GI/1: 1.541183e-06 s
##
## Load real M/G/1: 0.48
## Envelope upper bound
## Load Envelope: 0.6
```

```
## E(T) exponential envelope M/M/1: 2.03807e-06 s
## E(T) Real M/GI/1: 1.685712e-06 s
##
## Load real M/G/1: 0.49
## Envelope upper bound
## Load Envelope: 0.62
## E(T) exponential envelope M/M/1: 2.145337e-06 s
## E(T) Real M/GI/1: 1.786725e-06 s
##
## Load real M/G/1: 0.5
## Envelope upper bound
## Load Envelope: 0.62
## E(T) exponential envelope M/M/1: 2.145337e-06 s
## E(T) Real M/GI/1: 1.738889e-06 s
##
## Load real M/G/1: 0.51
## Envelope upper bound
## Load Envelope: 0.59
## E(T) exponential envelope M/M/1: 1.988361e-06 s
## E(T) Real M/GI/1: 1.68067e-06 s
##
## Load real M/G/1: 0.52
## Envelope upper bound
## Load Envelope: 0.63
## E(T) exponential envelope M/M/1: 2.203319e-06 s
## E(T) Real M/GI/1: 1.876275e-06 s
##
## Load real M/G/1: 0.53
## Envelope upper bound
## Load Envelope: 0.66
## E(T) exponential envelope M/M/1: 2.397729e-06 s
## E(T) Real M/GI/1: 1.910777e-06 s
##
## Load real M/G/1: 0.54
## Envelope upper bound
## Load Envelope: 0.64
## E(T) exponential envelope M/M/1: 2.264522e-06 s
## E(T) Real M/GI/1: 1.920665e-06 s
##
## Load real M/G/1: 0.55
## Envelope upper bound
## Load Envelope: 0.64
## E(T) exponential envelope M/M/1: 2.264522e-06 s
## E(T) Real M/GI/1: 1.915317e-06 s
##
## Load real M/G/1: 0.56
## Envelope upper bound
## Load Envelope: 0.65
## E(T) exponential envelope M/M/1: 2.329223e-06 s
## E(T) Real M/GI/1: 1.969556e-06 s
##
## Load real M/G/1: 0.57
## Envelope upper bound
## Load Envelope: 0.67
```

```
## E(T) exponential envelope M/M/1: 2.470388e-06 s
## E(T) Real M/GI/1: 2.089497e-06 s
##
## Load real M/G/1: 0.58
## Envelope upper bound
## Load Envelope: 0.66
## E(T) exponential envelope M/M/1: 2.397729e-06 s
## E(T) Real M/GI/1: 2.101268e-06 s
##
## Load real M/G/1: 0.59
## Envelope upper bound
## Load Envelope: 0.66
## E(T) exponential envelope M/M/1: 2.397729e-06 s
## E(T) Real M/GI/1: 2.060882e-06 s
##
## Load real M/G/1: 0.6
## Envelope upper bound
## Load Envelope: 0.71
## E(T) exponential envelope M/M/1: 2.811131e-06 s
## E(T) Real M/GI/1: 2.441486e-06 s
##
## Load real M/G/1: 0.61
## Envelope upper bound
## Load Envelope: 0.69
## E(T) exponential envelope M/M/1: 2.629768e-06 s
## E(T) Real M/GI/1: 2.252233e-06 s
##
## Load real M/G/1: 0.62
## Envelope upper bound
## Load Envelope: 0.68
## E(T) exponential envelope M/M/1: 2.547588e-06 s
## E(T) Real M/GI/1: 2.222852e-06 s
##
## Load real M/G/1: 0.63
## Envelope upper bound
## Load Envelope: 0.7
## E(T) exponential envelope M/M/1: 2.717427e-06 s
## E(T) Real M/GI/1: 2.423636e-06 s
##
## Load real M/G/1: 0.64
## Envelope upper bound
## Load Envelope: 0.73
## E(T) exponential envelope M/M/1: 3.019363e-06 s
## E(T) Real M/GI/1: 2.672162e-06 s
##
## Load real M/G/1: 0.65
## Envelope upper bound
## Load Envelope: 0.7
## E(T) exponential envelope M/M/1: 2.717427e-06 s
## E(T) Real M/GI/1: 2.438561e-06 s
##
## Load real M/G/1: 0.66
## Envelope upper bound
## Load Envelope: 0.74
```

```
## E(T) exponential envelope M/M/1: 3.135492e-06 s
## E(T) Real M/GI/1: 2.729908e-06 s
##
## Load real M/G/1: 0.67
## Envelope upper bound
## Load Envelope: 0.74
## E(T) exponential envelope M/M/1: 3.135492e-06 s
## E(T) Real M/GI/1: 2.696774e-06 s
##
## Load real M/G/1: 0.68
## Envelope upper bound
## Load Envelope: 0.74
## E(T) exponential envelope M/M/1: 3.135492e-06 s
## E(T) Real M/GI/1: 2.794036e-06 s
##
## Load real M/G/1: 0.69
## Envelope upper bound
## Load Envelope: 0.75
## E(T) exponential envelope M/M/1: 3.260912e-06 s
## E(T) Real M/GI/1: 2.95373e-06 s
##
## Load real M/G/1: 0.7
## Envelope upper bound
## Load Envelope: 0.76
## E(T) exponential envelope M/M/1: 3.396783e-06 s
## E(T) Real M/GI/1: 3.093199e-06 s
##
## Load real M/G/1: 0.71
## Envelope upper bound
## Load Envelope: 0.76
## E(T) exponential envelope M/M/1: 3.396783e-06 s
## E(T) Real M/GI/1: 3.059663e-06 s
##
## Load real M/G/1: 0.72
## Envelope upper bound
## Load Envelope: 0.78
## E(T) exponential envelope M/M/1: 3.705582e-06 s
## E(T) Real M/GI/1: 3.371788e-06 s
##
## Load real M/G/1: 0.73
## Envelope upper bound
## Load Envelope: 0.74
## E(T) exponential envelope M/M/1: 3.135492e-06 s
## E(T) Real M/GI/1: 2.830361e-06 s
##
## Load real M/G/1: 0.74
## Envelope upper bound
## Load Envelope: 0.76
## E(T) exponential envelope M/M/1: 3.396783e-06 s
## E(T) Real M/GI/1: 2.976412e-06 s
##
## Load real M/G/1: 0.75
## Envelope upper bound
## Load Envelope: 0.8
```

```
## E(T) exponential envelope M/M/1: 4.07614e-06 s
## E(T) Real M/GI/1: 3.584418e-06 s
##
## Load real M/G/1: 0.76
## Envelope upper bound
## Load Envelope: 0.8
## E(T) exponential envelope M/M/1: 4.07614e-06 s
## E(T) Real M/GI/1: 3.782674e-06 s
##
## Load real M/G/1: 0.77
## Envelope upper bound
## Load Envelope: 0.83
## E(T) exponential envelope M/M/1: 4.795459e-06 s
## E(T) Real M/GI/1: 4.467562e-06 s
##
## Load real M/G/1: 0.78
## Envelope upper bound
## Load Envelope: 0.86
## E(T) exponential envelope M/M/1: 5.823057e-06 s
## E(T) Real M/GI/1: 4.858071e-06 s
##
## Load real M/G/1: 0.79
## Envelope upper bound
## Load Envelope: 0.83
## E(T) exponential envelope M/M/1: 4.795459e-06 s
## E(T) Real M/GI/1: 4.423304e-06 s
##
## Load real M/G/1: 0.8
## Envelope upper bound
## Load Envelope: 0.85
## E(T) exponential envelope M/M/1: 5.434853e-06 s
## E(T) Real M/GI/1: 4.730165e-06 s
##
## Load real M/G/1: 0.81
## Envelope upper bound
## Load Envelope: 0.86
## E(T) exponential envelope M/M/1: 5.823057e-06 s
## E(T) Real M/GI/1: 5.100204e-06 s
##
## Load real M/G/1: 0.82
## Envelope upper bound
## Load Envelope: 0.84
## E(T) exponential envelope M/M/1: 5.095175e-06 s
## E(T) Real M/GI/1: 4.530074e-06 s
##
## Load real M/G/1: 0.83
## Envelope upper bound
## Load Envelope: 0.86
## E(T) exponential envelope M/M/1: 5.823057e-06 s
## E(T) Real M/GI/1: 5.212398e-06 s
##
## Load real M/G/1: 0.84
## Envelope upper bound
## Load Envelope: 0.93
```

```
## E(T) exponential envelope M/M/1: 1.164611e-05 s
## E(T) Real M/GI/1: 8.094908e-06 s
##
## Load real M/G/1: 0.85
## Envelope upper bound
## Load Envelope: 0.89
## E(T) exponential envelope M/M/1: 7.411164e-06 s
## E(T) Real M/GI/1: 6.556665e-06 s
##
## Load real M/G/1: 0.86
## Envelope upper bound
## Load Envelope: 0.89
## E(T) exponential envelope M/M/1: 7.411164e-06 s
## E(T) Real M/GI/1: 6.54355e-06 s
##
## Load real M/G/1: 0.87
## Envelope upper bound
## Load Envelope: 0.9
## E(T) exponential envelope M/M/1: 8.15228e-06 s
## E(T) Real M/GI/1: 6.776672e-06 s
##
## Load real M/G/1: 0.88
## Envelope upper bound
## Load Envelope: 0.9
## E(T) exponential envelope M/M/1: 8.15228e-06 s
## E(T) Real M/GI/1: 7.335346e-06 s
##
## Load real M/G/1: 0.89
## Envelope upper bound
## Load Envelope: 0.92
## E(T) exponential envelope M/M/1: 1.019035e-05 s
## E(T) Real M/GI/1: 8.6329e-06 s
##
## Load real M/G/1: 0.9
## Envelope upper bound
## Load Envelope: 0.91
## E(T) exponential envelope M/M/1: 9.058089e-06 s
## E(T) Real M/GI/1: 7.875408e-06 s
##
## Load real M/G/1: 0.91
## Envelope upper bound
## Load Envelope: 0.93
## E(T) exponential envelope M/M/1: 1.164611e-05 s
## E(T) Real M/GI/1: 9.831163e-06 s
##
## Load real M/G/1: 0.92
## Envelope upper bound
## Load Envelope: 0.94
## E(T) exponential envelope M/M/1: 1.358713e-05 s
## E(T) Real M/GI/1: 1.200559e-05 s
##
## Load real M/G/1: 0.93
## Envelope upper bound
## Load Envelope: 0.97
```

```
## E(T) exponential envelope M/M/1: 2.717427e-05 s
## E(T) Real M/GI/1: 1.87926e-05 s
##
## Load real M/G/1: 0.94
## Envelope upper bound
## Load Envelope: 0.94
## E(T) exponential envelope M/M/1: 1.358713e-05 s
## E(T) Real M/GI/1: 1.234499e-05 s
##
## Load real M/G/1: 0.95
## Envelope upper bound
## Load Envelope: 0.99
## E(T) exponential envelope M/M/1: 8.15228e-05 s
## E(T) Real M/GI/1: 4.681621e-05 s
##
## Load real M/G/1: 0.96
## Envelope upper bound
## Load Envelope: 0.98
## E(T) exponential envelope M/M/1: 4.07614e-05 s
## E(T) Real M/GI/1: 2.72248e-05 s
##
## Load real M/G/1: 0.97
## Envelope upper bound
## Load Envelope: 0.97
## E(T) exponential envelope M/M/1: 2.717427e-05 s
## E(T) Real M/GI/1: 2.091181e-05 s
##
## Load real M/G/1: 0.98
## Envelope upper bound
## Load Envelope: 0.98
## E(T) exponential envelope M/M/1: 4.07614e-05 s
## E(T) Real M/GI/1: 2.797093e-05 s
##
## Load real M/G/1: 0.99
## Envelope upper bound
## Load Envelope: 0.99
## E(T) exponential envelope M/M/1: 8.15228e-05 s
## E(T) Real M/GI/1: 3.97279e-05 s
```

```
model_v2 <- lm(loads_envelope_v2 ~ poly(loads_real, 2, raw = TRUE))
load_envelope_pred_v2 <- predict(model_v2, newdata = data.frame(loads_real = load_real_pred))

#V3 https://www.seattleix.net/statistics/
# PS_size=c((64), (65+128)/2, (129+256)/2,(257+384)/2, (385+512)/2,(513+640)/2,(641+768)/2,(769+896)/2,
#           (897+1024)/2,
#           (1025+1152)/2,(1153+1280)/2,(1515+1408)/2,(1409+1536)/2, (1537+9999)/2)
# PS_weights = c(4.2, 22.73, 4.79,1.93,0.92, 1.04, 0.74, 0.49, 0.45, 0.82, 6.56,9.30, 45.95, 0.07)
# PS_weights = PS_weights/100

# #V4 San Francisco Metropolitan Internet Exchange (SFMIX) https://grafana.sfmix.org/public-dashboards/
PS_size=c(64,
          (65+127)/2,
          (128+255)/2,
          (256+511)/2,
```

```
          (513+1023)/2,
          (1024+1517)/2,
          (1518+9999)/2)
PS_weights = c(2, 23, 5, 5, 5, 40, 20)
PS_weights=PS_weights/100

loads_envelope_v3 <- sapply(loads_real, function(load) envelope_load_calc(Capacity_Gbps, k_num, load, PS
```

```
## Load real M/G/1: 0.01
## Envelope upper bound
## Load Envelope: 0.52
## E(T) exponential envelope M/M/1: 2.91735e-06 s
## E(T) Real M/GI/1: 1.430262e-06 s
##
## Load real M/G/1: 0.02
## Envelope upper bound
## Load Envelope: 0.52
## E(T) exponential envelope M/M/1: 2.91735e-06 s
## E(T) Real M/GI/1: 1.435638e-06 s
##
## Load real M/G/1: 0.03
## Envelope upper bound
## Load Envelope: 0.5
## E(T) exponential envelope M/M/1: 2.800656e-06 s
## E(T) Real M/GI/1: 1.41856e-06 s
##
## Load real M/G/1: 0.04
## Envelope upper bound
## Load Envelope: 0.52
## E(T) exponential envelope M/M/1: 2.91735e-06 s
## E(T) Real M/GI/1: 1.456827e-06 s
##
## Load real M/G/1: 0.05
## Envelope upper bound
## Load Envelope: 0.5
## E(T) exponential envelope M/M/1: 2.800656e-06 s
## E(T) Real M/GI/1: 1.455549e-06 s
##
## Load real M/G/1: 0.06
## Envelope upper bound
## Load Envelope: 0.52
## E(T) exponential envelope M/M/1: 2.91735e-06 s
## E(T) Real M/GI/1: 1.49166e-06 s
##
## Load real M/G/1: 0.07
## Envelope upper bound
## Load Envelope: 0.52
## E(T) exponential envelope M/M/1: 2.91735e-06 s
## E(T) Real M/GI/1: 1.539488e-06 s
##
## Load real M/G/1: 0.08
## Envelope upper bound
## Load Envelope: 0.52
```

```
## E(T) exponential envelope M/M/1: 2.91735e-06 s
## E(T) Real M/GI/1: 1.532016e-06 s
##
## Load real M/G/1: 0.09
## Envelope upper bound
## Load Envelope: 0.52
## E(T) exponential envelope M/M/1: 2.91735e-06 s
## E(T) Real M/GI/1: 1.565816e-06 s
##
## Load real M/G/1: 0.1
## Envelope upper bound
## Load Envelope: 0.52
## E(T) exponential envelope M/M/1: 2.91735e-06 s
## E(T) Real M/GI/1: 1.568924e-06 s
##
## Load real M/G/1: 0.11
## Envelope upper bound
## Load Envelope: 0.53
## E(T) exponential envelope M/M/1: 2.979421e-06 s
## E(T) Real M/GI/1: 1.614845e-06 s
##
## Load real M/G/1: 0.12
## Envelope upper bound
## Load Envelope: 0.53
## E(T) exponential envelope M/M/1: 2.979421e-06 s
## E(T) Real M/GI/1: 1.627241e-06 s
##
## Load real M/G/1: 0.13
## Envelope upper bound
## Load Envelope: 0.52
## E(T) exponential envelope M/M/1: 2.91735e-06 s
## E(T) Real M/GI/1: 1.597733e-06 s
##
## Load real M/G/1: 0.14
## Envelope upper bound
## Load Envelope: 0.52
## E(T) exponential envelope M/M/1: 2.91735e-06 s
## E(T) Real M/GI/1: 1.630963e-06 s
##
## Load real M/G/1: 0.15
## Envelope upper bound
## Load Envelope: 0.53
## E(T) exponential envelope M/M/1: 2.979421e-06 s
## E(T) Real M/GI/1: 1.703594e-06 s
##
## Load real M/G/1: 0.16
## Envelope upper bound
## Load Envelope: 0.53
## E(T) exponential envelope M/M/1: 2.979421e-06 s
## E(T) Real M/GI/1: 1.705523e-06 s
##
## Load real M/G/1: 0.17
## Envelope upper bound
## Load Envelope: 0.54
```

```
## E(T) exponential envelope M/M/1: 3.044191e-06 s
## E(T) Real M/GI/1: 1.78892e-06 s
##
## Load real M/G/1: 0.18
## Envelope upper bound
## Load Envelope: 0.54
## E(T) exponential envelope M/M/1: 3.044191e-06 s
## E(T) Real M/GI/1: 1.76079e-06 s
##
## Load real M/G/1: 0.19
## Envelope upper bound
## Load Envelope: 0.55
## E(T) exponential envelope M/M/1: 3.11184e-06 s
## E(T) Real M/GI/1: 1.820689e-06 s
##
## Load real M/G/1: 0.2
## Envelope upper bound
## Load Envelope: 0.54
## E(T) exponential envelope M/M/1: 3.044191e-06 s
## E(T) Real M/GI/1: 1.806751e-06 s
##
## Load real M/G/1: 0.21
## Envelope upper bound
## Load Envelope: 0.54
## E(T) exponential envelope M/M/1: 3.044191e-06 s
## E(T) Real M/GI/1: 1.836306e-06 s
##
## Load real M/G/1: 0.22
## Envelope upper bound
## Load Envelope: 0.55
## E(T) exponential envelope M/M/1: 3.11184e-06 s
## E(T) Real M/GI/1: 1.862901e-06 s
##
## Load real M/G/1: 0.23
## Envelope upper bound
## Load Envelope: 0.56
## E(T) exponential envelope M/M/1: 3.182564e-06 s
## E(T) Real M/GI/1: 1.893065e-06 s
##
## Load real M/G/1: 0.24
## Envelope upper bound
## Load Envelope: 0.55
## E(T) exponential envelope M/M/1: 3.11184e-06 s
## E(T) Real M/GI/1: 1.93142e-06 s
##
## Load real M/G/1: 0.25
## Envelope upper bound
## Load Envelope: 0.56
## E(T) exponential envelope M/M/1: 3.182564e-06 s
## E(T) Real M/GI/1: 1.956886e-06 s
##
## Load real M/G/1: 0.26
## Envelope upper bound
## Load Envelope: 0.56
```

```
## E(T) exponential envelope M/M/1: 3.182564e-06 s
## E(T) Real M/GI/1: 1.934915e-06 s
##
## Load real M/G/1: 0.27
## Envelope upper bound
## Load Envelope: 0.57
## E(T) exponential envelope M/M/1: 3.256577e-06 s
## E(T) Real M/GI/1: 2.002386e-06 s
##
## Load real M/G/1: 0.28
## Envelope upper bound
## Load Envelope: 0.58
## E(T) exponential envelope M/M/1: 3.334114e-06 s
## E(T) Real M/GI/1: 2.111502e-06 s
##
## Load real M/G/1: 0.29
## Envelope upper bound
## Load Envelope: 0.58
## E(T) exponential envelope M/M/1: 3.334114e-06 s
## E(T) Real M/GI/1: 2.169828e-06 s
##
## Load real M/G/1: 0.3
## Envelope upper bound
## Load Envelope: 0.57
## E(T) exponential envelope M/M/1: 3.256577e-06 s
## E(T) Real M/GI/1: 2.077105e-06 s
##
## Load real M/G/1: 0.31
## Envelope upper bound
## Load Envelope: 0.56
## E(T) exponential envelope M/M/1: 3.182564e-06 s
## E(T) Real M/GI/1: 2.056458e-06 s
##
## Load real M/G/1: 0.32
## Envelope upper bound
## Load Envelope: 0.58
## E(T) exponential envelope M/M/1: 3.334114e-06 s
## E(T) Real M/GI/1: 2.159405e-06 s
##
## Load real M/G/1: 0.33
## Envelope upper bound
## Load Envelope: 0.58
## E(T) exponential envelope M/M/1: 3.334114e-06 s
## E(T) Real M/GI/1: 2.177967e-06 s
##
## Load real M/G/1: 0.34
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 3.50082e-06 s
## E(T) Real M/GI/1: 2.273184e-06 s
##
## Load real M/G/1: 0.35
## Envelope upper bound
## Load Envelope: 0.6
```

```
## E(T) exponential envelope M/M/1: 3.50082e-06 s
## E(T) Real M/GI/1: 2.322301e-06 s
##
## Load real M/G/1: 0.36
## Envelope upper bound
## Load Envelope: 0.61
## E(T) exponential envelope M/M/1: 3.590585e-06 s
## E(T) Real M/GI/1: 2.416209e-06 s
##
## Load real M/G/1: 0.37
## Envelope upper bound
## Load Envelope: 0.61
## E(T) exponential envelope M/M/1: 3.590585e-06 s
## E(T) Real M/GI/1: 2.414175e-06 s
##
## Load real M/G/1: 0.38
## Envelope upper bound
## Load Envelope: 0.62
## E(T) exponential envelope M/M/1: 3.685074e-06 s
## E(T) Real M/GI/1: 2.509948e-06 s
##
## Load real M/G/1: 0.39
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 3.50082e-06 s
## E(T) Real M/GI/1: 2.365937e-06 s
##
## Load real M/G/1: 0.4
## Envelope upper bound
## Load Envelope: 0.6
## E(T) exponential envelope M/M/1: 3.50082e-06 s
## E(T) Real M/GI/1: 2.425214e-06 s
##
## Load real M/G/1: 0.41
## Envelope upper bound
## Load Envelope: 0.63
## E(T) exponential envelope M/M/1: 3.78467e-06 s
## E(T) Real M/GI/1: 2.631935e-06 s
##
## Load real M/G/1: 0.42
## Envelope upper bound
## Load Envelope: 0.64
## E(T) exponential envelope M/M/1: 3.8898e-06 s
## E(T) Real M/GI/1: 2.706294e-06 s
##
## Load real M/G/1: 0.43
## Envelope upper bound
## Load Envelope: 0.63
## E(T) exponential envelope M/M/1: 3.78467e-06 s
## E(T) Real M/GI/1: 2.602909e-06 s
##
## Load real M/G/1: 0.44
## Envelope upper bound
## Load Envelope: 0.65
```

```
## E(T) exponential envelope M/M/1: 4.000937e-06 s
## E(T) Real M/GI/1: 2.769085e-06 s
##
## Load real M/G/1: 0.45
## Envelope upper bound
## Load Envelope: 0.63
## E(T) exponential envelope M/M/1: 3.78467e-06 s
## E(T) Real M/GI/1: 2.673528e-06 s
##
## Load real M/G/1: 0.46
## Envelope upper bound
## Load Envelope: 0.65
## E(T) exponential envelope M/M/1: 4.000937e-06 s
## E(T) Real M/GI/1: 2.864245e-06 s
##
## Load real M/G/1: 0.47
## Envelope upper bound
## Load Envelope: 0.66
## E(T) exponential envelope M/M/1: 4.118612e-06 s
## E(T) Real M/GI/1: 2.942886e-06 s
##
## Load real M/G/1: 0.48
## Envelope upper bound
## Load Envelope: 0.66
## E(T) exponential envelope M/M/1: 4.118612e-06 s
## E(T) Real M/GI/1: 2.89758e-06 s
##
## Load real M/G/1: 0.49
## Envelope upper bound
## Load Envelope: 0.65
## E(T) exponential envelope M/M/1: 4.000937e-06 s
## E(T) Real M/GI/1: 2.922501e-06 s
##
## Load real M/G/1: 0.5
## Envelope upper bound
## Load Envelope: 0.67
## E(T) exponential envelope M/M/1: 4.243418e-06 s
## E(T) Real M/GI/1: 3.105235e-06 s
##
## Load real M/G/1: 0.51
## Envelope upper bound
## Load Envelope: 0.67
## E(T) exponential envelope M/M/1: 4.243418e-06 s
## E(T) Real M/GI/1: 3.027155e-06 s
##
## Load real M/G/1: 0.52
## Envelope upper bound
## Load Envelope: 0.68
## E(T) exponential envelope M/M/1: 4.376025e-06 s
## E(T) Real M/GI/1: 3.177848e-06 s
##
## Load real M/G/1: 0.53
## Envelope upper bound
## Load Envelope: 0.69
```

```
## E(T) exponential envelope M/M/1: 4.517187e-06 s
## E(T) Real M/GI/1: 3.505795e-06 s
##
## Load real M/G/1: 0.54
## Envelope upper bound
## Load Envelope: 0.68
## E(T) exponential envelope M/M/1: 4.376025e-06 s
## E(T) Real M/GI/1: 3.208062e-06 s
##
## Load real M/G/1: 0.55
## Envelope upper bound
## Load Envelope: 0.69
## E(T) exponential envelope M/M/1: 4.517187e-06 s
## E(T) Real M/GI/1: 3.459346e-06 s
##
## Load real M/G/1: 0.56
## Envelope upper bound
## Load Envelope: 0.71
## E(T) exponential envelope M/M/1: 4.828717e-06 s
## E(T) Real M/GI/1: 3.588929e-06 s
##
## Load real M/G/1: 0.57
## Envelope upper bound
## Load Envelope: 0.7
## E(T) exponential envelope M/M/1: 4.66776e-06 s
## E(T) Real M/GI/1: 3.484996e-06 s
##
## Load real M/G/1: 0.58
## Envelope upper bound
## Load Envelope: 0.72
## E(T) exponential envelope M/M/1: 5.001171e-06 s
## E(T) Real M/GI/1: 3.834836e-06 s
##
## Load real M/G/1: 0.59
## Envelope upper bound
## Load Envelope: 0.72
## E(T) exponential envelope M/M/1: 5.001171e-06 s
## E(T) Real M/GI/1: 3.739834e-06 s
##
## Load real M/G/1: 0.6
## Envelope upper bound
## Load Envelope: 0.72
## E(T) exponential envelope M/M/1: 5.001171e-06 s
## E(T) Real M/GI/1: 3.870475e-06 s
##
## Load real M/G/1: 0.61
## Envelope upper bound
## Load Envelope: 0.74
## E(T) exponential envelope M/M/1: 5.385877e-06 s
## E(T) Real M/GI/1: 4.248048e-06 s
##
## Load real M/G/1: 0.62
## Envelope upper bound
## Load Envelope: 0.75
```

```
## E(T) exponential envelope M/M/1: 5.601312e-06 s
## E(T) Real M/GI/1: 4.501435e-06 s
##
## Load real M/G/1: 0.63
## Envelope upper bound
## Load Envelope: 0.74
## E(T) exponential envelope M/M/1: 5.385877e-06 s
## E(T) Real M/GI/1: 4.123585e-06 s
##
## Load real M/G/1: 0.64
## Envelope upper bound
## Load Envelope: 0.76
## E(T) exponential envelope M/M/1: 5.8347e-06 s
## E(T) Real M/GI/1: 4.722783e-06 s
##
## Load real M/G/1: 0.65
## Envelope upper bound
## Load Envelope: 0.76
## E(T) exponential envelope M/M/1: 5.8347e-06 s
## E(T) Real M/GI/1: 4.492524e-06 s
##
## Load real M/G/1: 0.66
## Envelope upper bound
## Load Envelope: 0.77
## E(T) exponential envelope M/M/1: 6.088383e-06 s
## E(T) Real M/GI/1: 4.593412e-06 s
##
## Load real M/G/1: 0.67
## Envelope upper bound
## Load Envelope: 0.77
## E(T) exponential envelope M/M/1: 6.088383e-06 s
## E(T) Real M/GI/1: 4.765387e-06 s
##
## Load real M/G/1: 0.68
## Envelope upper bound
## Load Envelope: 0.77
## E(T) exponential envelope M/M/1: 6.088383e-06 s
## E(T) Real M/GI/1: 4.73347e-06 s
##
## Load real M/G/1: 0.69
## Envelope upper bound
## Load Envelope: 0.79
## E(T) exponential envelope M/M/1: 6.668229e-06 s
## E(T) Real M/GI/1: 5.521312e-06 s
##
## Load real M/G/1: 0.7
## Envelope upper bound
## Load Envelope: 0.79
## E(T) exponential envelope M/M/1: 6.668229e-06 s
## E(T) Real M/GI/1: 5.542771e-06 s
##
## Load real M/G/1: 0.71
## Envelope upper bound
## Load Envelope: 0.79
```

```
## E(T) exponential envelope M/M/1: 6.668229e-06 s
## E(T) Real M/GI/1: 5.420965e-06 s
##
## Load real M/G/1: 0.72
## Envelope upper bound
## Load Envelope: 0.79
## E(T) exponential envelope M/M/1: 6.668229e-06 s
## E(T) Real M/GI/1: 6.014913e-06 s
##
## Load real M/G/1: 0.73
## Envelope upper bound
## Load Envelope: 0.79
## E(T) exponential envelope M/M/1: 6.668229e-06 s
## E(T) Real M/GI/1: 5.877438e-06 s
##
## Load real M/G/1: 0.74
## Envelope upper bound
## Load Envelope: 0.79
## E(T) exponential envelope M/M/1: 6.668229e-06 s
## E(T) Real M/GI/1: 5.535357e-06 s
##
## Load real M/G/1: 0.75
## Envelope upper bound
## Load Envelope: 0.8
## E(T) exponential envelope M/M/1: 7.00164e-06 s
## E(T) Real M/GI/1: 6.659473e-06 s
##
## Load real M/G/1: 0.76
## Envelope upper bound
## Load Envelope: 0.84
## E(T) exponential envelope M/M/1: 8.75205e-06 s
## E(T) Real M/GI/1: 7.795659e-06 s
##
## Load real M/G/1: 0.77
## Envelope upper bound
## Load Envelope: 0.81
## E(T) exponential envelope M/M/1: 7.370147e-06 s
## E(T) Real M/GI/1: 7.157617e-06 s
##
## Load real M/G/1: 0.78
## Envelope upper bound
## Load Envelope: 0.88
## E(T) exponential envelope M/M/1: 1.16694e-05 s
## E(T) Real M/GI/1: 9.175817e-06 s
##
## Load real M/G/1: 0.79
## Envelope upper bound
## Load Envelope: 0.86
## E(T) exponential envelope M/M/1: 1.000234e-05 s
## E(T) Real M/GI/1: 7.98812e-06 s
##
## Load real M/G/1: 0.8
## Envelope upper bound
## Load Envelope: 0.84
```

```
## E(T) exponential envelope M/M/1: 8.75205e-06 s
## E(T) Real M/GI/1: 7.855814e-06 s
##
## Load real M/G/1: 0.81
## Envelope upper bound
## Load Envelope: 0.83
## E(T) exponential envelope M/M/1: 8.237224e-06 s
## E(T) Real M/GI/1: 7.802381e-06 s
##
## Load real M/G/1: 0.82
## Envelope upper bound
## Load Envelope: 0.86
## E(T) exponential envelope M/M/1: 1.000234e-05 s
## E(T) Real M/GI/1: 9.229983e-06 s
##
## Load real M/G/1: 0.83
## Envelope upper bound
## Load Envelope: 0.85
## E(T) exponential envelope M/M/1: 9.33552e-06 s
## E(T) Real M/GI/1: 8.326475e-06 s
##
## Load real M/G/1: 0.84
## Envelope upper bound
## Load Envelope: 0.86
## E(T) exponential envelope M/M/1: 1.000234e-05 s
## E(T) Real M/GI/1: 9.193187e-06 s
##
## Load real M/G/1: 0.85
## Envelope upper bound
## Load Envelope: 0.89
## E(T) exponential envelope M/M/1: 1.273025e-05 s
## E(T) Real M/GI/1: 1.142117e-05 s
##
## Load real M/G/1: 0.86
## Envelope upper bound
## Load Envelope: 0.93
## E(T) exponential envelope M/M/1: 2.000469e-05 s
## E(T) Real M/GI/1: 1.531875e-05 s
##
## Load real M/G/1: 0.87
## Envelope upper bound
## Load Envelope: 0.89
## E(T) exponential envelope M/M/1: 1.273025e-05 s
## E(T) Real M/GI/1: 1.128713e-05 s
##
## Load real M/G/1: 0.88
## Envelope upper bound
## Load Envelope: 0.93
## E(T) exponential envelope M/M/1: 2.000469e-05 s
## E(T) Real M/GI/1: 1.498609e-05 s
##
## Load real M/G/1: 0.89
## Envelope upper bound
## Load Envelope: 0.94
```

```
## E(T) exponential envelope M/M/1: 2.33388e-05 s
## E(T) Real M/GI/1: 1.940932e-05 s
##
## Load real M/G/1: 0.9
## Envelope upper bound
## Load Envelope: 0.91
## E(T) exponential envelope M/M/1: 1.55592e-05 s
## E(T) Real M/GI/1: 1.264384e-05 s
##
## Load real M/G/1: 0.91
## Envelope upper bound
## Load Envelope: 0.93
## E(T) exponential envelope M/M/1: 2.000469e-05 s
## E(T) Real M/GI/1: 1.554679e-05 s
##
## Load real M/G/1: 0.92
## Envelope upper bound
## Load Envelope: 0.94
## E(T) exponential envelope M/M/1: 2.33388e-05 s
## E(T) Real M/GI/1: 1.940959e-05 s
##
## Load real M/G/1: 0.93
## Envelope upper bound
## Load Envelope: 0.94
## E(T) exponential envelope M/M/1: 2.33388e-05 s
## E(T) Real M/GI/1: 1.858489e-05 s
##
## Load real M/G/1: 0.94
## Envelope upper bound
## Load Envelope: 0.94
## E(T) exponential envelope M/M/1: 2.33388e-05 s
## E(T) Real M/GI/1: 1.875708e-05 s
##
## Load real M/G/1: 0.95
## Envelope upper bound
## Load Envelope: 0.95
## E(T) exponential envelope M/M/1: 2.800656e-05 s
## E(T) Real M/GI/1: 2.256878e-05 s
##
## Load real M/G/1: 0.96
## Envelope upper bound
## Load Envelope: 0.98
## E(T) exponential envelope M/M/1: 7.00164e-05 s
## E(T) Real M/GI/1: 4.211407e-05 s
##
## Load real M/G/1: 0.97
## Envelope upper bound
## Load Envelope: 0.98
## E(T) exponential envelope M/M/1: 7.00164e-05 s
## E(T) Real M/GI/1: 4.08052e-05 s
##
## Load real M/G/1: 0.98
## Envelope upper bound
## Load Envelope: 0.99
```

```
## E(T) exponential envelope M/M/1: 0.0001400328 s
## E(T) Real M/GI/1: 8.00865e-05 s
##
## Load real M/G/1: 0.99
```

```
model_v3 <- lm(loads_envelope_v3 ~ poly(loads_real, 2, raw = TRUE))
load_envelope_pred_v3 <- predict(model_v3, newdata = data.frame(loads_real = load_real_pred))
```

Part 5: Envelope load obtained from polynomial v1,v2,v3

V1 c(40, 576, 1500)

```
# Extract the coefficients of the fitted model
coef_model_v1 <- coef(model_v1)
# Construct the formula
formula <- paste0("y(x) = ", coef_model_v1[1], " + ", coef_model_v1[2], "x",
                  " + ", coef_model_v1[3], "x^2")
# Print the formula
print(formula)
```

```
## [1] "y(x) = 0.492697817646271 + 0.130611738640727x + 0.385492157463169x^2"
```

Calculating RMSE (Root Mean Square Error) and R-squared for V1:

```
rmse_v1 <- sqrt(mean((loads_envelope_v1 - predict(model_v1))^2))
r2_v1 <- summary(model_v1)$r.squared
```

V2 AMS-IX Amsterdam https://www.ams-ix.net/ams

```
# Extract the coefficients of the fitted model
coef_model_v2 <- coef(model_v2)
# Construct the formula
formula <- paste0("y(x) = ", coef_model_v2[1], " + ", coef_model_v2[2], "x",
                  " + ", coef_model_v2[3], "x^2")
# Print the formula
print(formula)
```

```
## [1] "y(x) = 0.429221799310165 + 0.134194298874449x + 0.465187271935694x^2"
```

Calculating RMSE and R-squared for V2:

```
rmse_v2 <- sqrt(mean((loads_envelope_v2 - predict(model_v2))^2))
r2_v2 <- summary(model_v2)$r.squared
```

V3 San Francisco Metropolitan Internet Exchange (SFMIX)

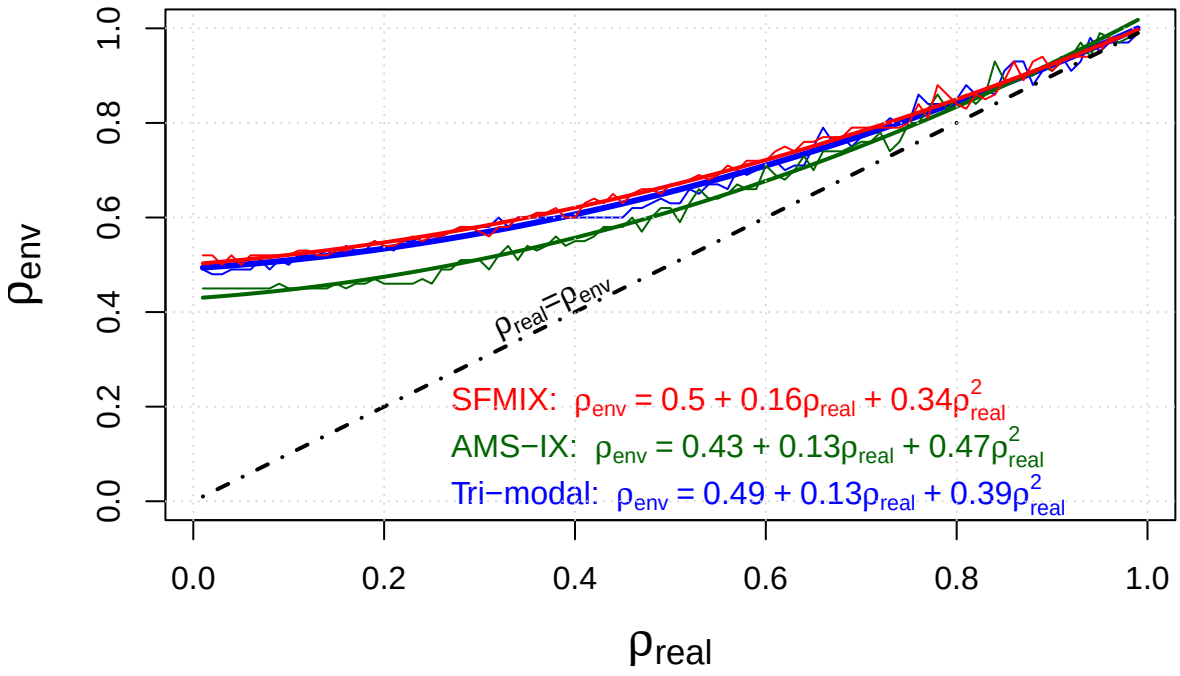
```r
# Extract the coefficients of the fitted model
coef_model_v3 <- coef(model_v3)
# Construct the formula
formula <- paste0("y(x) = ", coef_model_v3[1], " + ", coef_model_v3[2], "x",
                  " + ", coef_model_v3[3], "x^2")
# Print the formula
print(formula)
```

```
## [1] "y(x) = 0.501432715541699 + 0.160897759458251x + 0.343344664784173x^2"
```

Calculating RMSE and R-squared for V3:

```r
rmse_v3 <- sqrt(mean((loads_envelope_v3 - predict(model_v3))^2))
r2_v3 <- summary(model_v3)$r.squared
```

```r
loads_real <- seq(0.01, 0.99, 0.01)

plot(load_real_pred, load_envelope_pred_v1, type = 'l', col = 'blue', xlab = expression(rho[real]), ylab
     lwd = 3, ylim = c(0,1), cex.lab = 1.5)
lines(loads_real, loads_envelope_v1, col = 'blue', lwd = 1, lty= 1)
lines(load_real_pred, load_envelope_pred_v2, col = 'darkgreen', lwd = 2)
lines(loads_real, loads_envelope_v2, col = 'darkgreen', lwd = 1, lty= 1)
lines(load_real_pred, load_envelope_pred_v3, col = 'red', lwd = 2)
lines(loads_real, loads_envelope_v3, col = 'red', lwd = 1, lty= 1)
lines(loads_real, loads_real, col = 'black', lwd = 2, lty= 4)
# Add labels for the vertical lines with rotated text
legend_text_v1 <- bquote("Tri-modal: " ~ rho[env] ~ "=" ~ .(round(coef_model_v1[1], 2)) ~ "+" ~ .(round

legend_text_v2 <- bquote("AMS-IX: " ~ rho[env] ~ "=" ~ .(round(coef_model_v2[1], 2)) ~ "+" ~ .(round(co

legend_text_v3 <- bquote("SFMIX: " ~ rho[env] ~ "=" ~ .(round(coef_model_v3[1], 2)) ~ "+" ~ .(round(coe

text(0.25, 0,
     legend_text_v1,
     col = "blue", pos = 4, srt = 0)
text(0.25, 0.1,
     legend_text_v2,
     col = "darkgreen", pos = 4, srt = 0)
text(0.25, 0.2,
     legend_text_v3,
     col = "red", pos = 4, srt = 0)
text(0.3, 0.35,
     expression(paste(rho[real], "=", rho[env])),
     col = "black", pos = 4, srt = 25)



grid()
```



Part 6: Checking the polinomial prediction for V4

```
#V4 San Francisco Metropolitan Internet Exchange (SFMIX) https://grafana.sfmix.org/public-dashboards/7d
PS_size=c(64,
          (65+127)/2,
          (128+255)/2,
          (256+511)/2,
          (513+1023)/2,
          (1024+1517)/2,
          (1518+9999)/2)
PS_weights = c(2, 23, 5, 5, 5, 40, 20)
PS_weights=PS_weights/100

loads_real = 0.6
cat("Load real M/G/1:", loads_real, "\n")
```

```
## Load real M/G/1: 0.6
```

```
mg1_packets <- simmer_mg1(Capacity_Gbps, loads_real, PS_size, PS_weights, k_num = 1e4)


load_envelope_predicted = predict(model_v3, newdata = data.frame(loads_real))


n = 1
df_real <- sapply(percentiles_seq, function(x) quantile(mg1_packets, x))*1e6 #real
```

```r
rho_env = seq(from=0.01,to=0.99,by=0.01)

df_env <- qexp(percentiles_seq,rate = ((1-load_envelope_predicted)/E_X))*1e6 # mu s
cat("All quantiles df_real are below envelope:", all(df_real < df_env), "\n")
```

```
## All quantiles df_real are below envelope: FALSE
```

```r
cat("Envelope upper bound \n")
```

```
## Envelope upper bound
```

```r
cat("Load envelope M/M/1:", load_envelope_predicted, "\n")
```

```
## Load envelope M/M/1: 0.7215755
```

```r
cat("E(T) exponential envelope M/M/1:", E_X/(1-load_envelope_predicted), "s \n")
```

```
## E(T) exponential envelope M/M/1: 9.778831e-07 s
```

```r
cat("E(T) Real M/GI/1:", mean(mg1_packets), "s \n")
```

```
## E(T) Real M/GI/1: 3.716489e-06 s
```

Part 7: Calculate the average coefficients for the polynomial models using rounding.

```r
c = round(coef_model_v3[1], 2)
b = round(coef_model_v3[2], 2)
a = round(coef_model_v3[3], 2)

formula <- paste0("y(x) = ", c, " + ", b, "x", " + ", a, "x^2")

print(formula)
```

```
## [1] "y(x) = 0.5 + 0.16x + 0.34x^2"
```

Part 8: Each model's performance.

Compare the performance of the individual polynomial models the average polynomial model with the average polynomial model by calculating the RMSE and R-squared metrics

```r
cat("Model v1: RMSE =", rmse_v1, ", R^2 =", r2_v1, "\n")
```

```
## Model v1: RMSE = 0.01620064 , R^2 = 0.9884927
```

```r
cat("Model v2: RMSE =", rmse_v2, ", R^2 =", r2_v2, "\n")
```

```
## Model v2: RMSE = 0.01604575 , R^2 = 0.9916274
```

```r
cat("Model v3: RMSE =", rmse_v3, ", R^2 =", r2_v3, "\n")
```

```
## Model v3: RMSE = 0.01200682 , R^2 = 0.9933064
```

```r
# Calculate RMSE and R^2 for the average polynomial model
load = seq(0.01,0.99,0.01)
polinomial_pred_v3 <- a*load^2 + b*load + c#(predict(model_v1) + predict(model_v2) + predict(model_v3))

rmse_avg_v1 <- sqrt(mean((loads_envelope_v1 - polinomial_pred_v3)^2))
r2_avg_v1 <- summary(lm(loads_envelope_v1 ~ polinomial_pred_v3))$r.squared

rmse_avg_v2 <- sqrt(mean((loads_envelope_v2 - polinomial_pred_v3)^2))
r2_avg_v2 <- summary(lm(loads_envelope_v2 ~ polinomial_pred_v3))$r.squared

rmse_avg_v3 <- sqrt(mean((loads_envelope_v3 - polinomial_pred_v3)^2))
r2_avg_v3 <- summary(lm(loads_envelope_v3 ~ polinomial_pred_v3))$r.squared
cat("V3 Polynomial compare with Model Model v1: RMSE =", rmse_avg_v1, ", R^2 =", r2_avg_v1, "\n")
```

```
## V3 Polynomial compare with Model Model v1: RMSE = 0.01850406 , R^2 = 0.9882218
```

```r
cat("V3 Polynomial compare with Model Model v2: RMSE =", rmse_avg_v2, ", R^2 =", r2_avg_v2, "\n")
```

```
## V3 Polynomial compare with Model Model v2: RMSE = 0.05408486 , R^2 = 0.9910687
```

Mean Absolute Percent Error (MAPE), coefficient of variation (CV) , Mean absolute error (MAE)

```r
# Mean Absolute Percentage Error (MAPE)
mape <- function(forecast, actual) {
  return(mean(abs((actual - forecast) / actual)) * 100)
}

# Mean Absolute Error (MAE)
mae <- function(forecast, actual) {
  return(mean(abs(actual - forecast)))
}
```

Mean Absolute Percent Error (MAPE), coefficient of variation , Mean absolute error

```r
# Calculate metrics for each model
loads_real <- seq(0.01, 0.99, 0.01)

mape_v1 <- mape(predict(model_v1), loads_envelope_v1)
mae_v1 <- mae(predict(model_v1), loads_envelope_v1)

mape_v2 <- mape(predict(model_v2), loads_envelope_v2)
mae_v2 <- mae(predict(model_v2), loads_envelope_v2)
```

```r
mape_v3 <- mape(predict(model_v3), loads_envelope_v3)
mae_v3 <- mae(predict(model_v3), loads_envelope_v3)

# Calculate metrics for average polynomial model
mape_avg_v1 <- mape(polinomial_pred_v3, loads_envelope_v1)
mae_avg_v1 <- mae(polinomial_pred_v3, loads_envelope_v1)

mape_avg_v2 <- mape(polinomial_pred_v3, loads_envelope_v2)
mae_avg_v2 <- mae(polinomial_pred_v3, loads_envelope_v2)

mape_avg_v3 <- mape(polinomial_pred_v3, loads_envelope_v3)
mae_avg_v3 <- mae(polinomial_pred_v3, loads_envelope_v3)

# Print metrics for each model and average polynomial model
cat("Model v1: MAPE =", mape_v1, ", MAE =", mae_v1, "\n")
```

```
## Model v1: MAPE = 1.928987 , MAE = 0.01309873
```

```r
cat("Model v2: MAPE =", mape_v2, ", MAE =", mae_v2, "\n")
```

```
## Model v2: MAPE = 1.944943 , MAE = 0.01223918
```

```r
cat("Model v3: MAPE =", mape_v3, ", MAE =", mae_v3, "\n")
```

```
## Model v3: MAPE = 1.315429 , MAE = 0.009227243
```

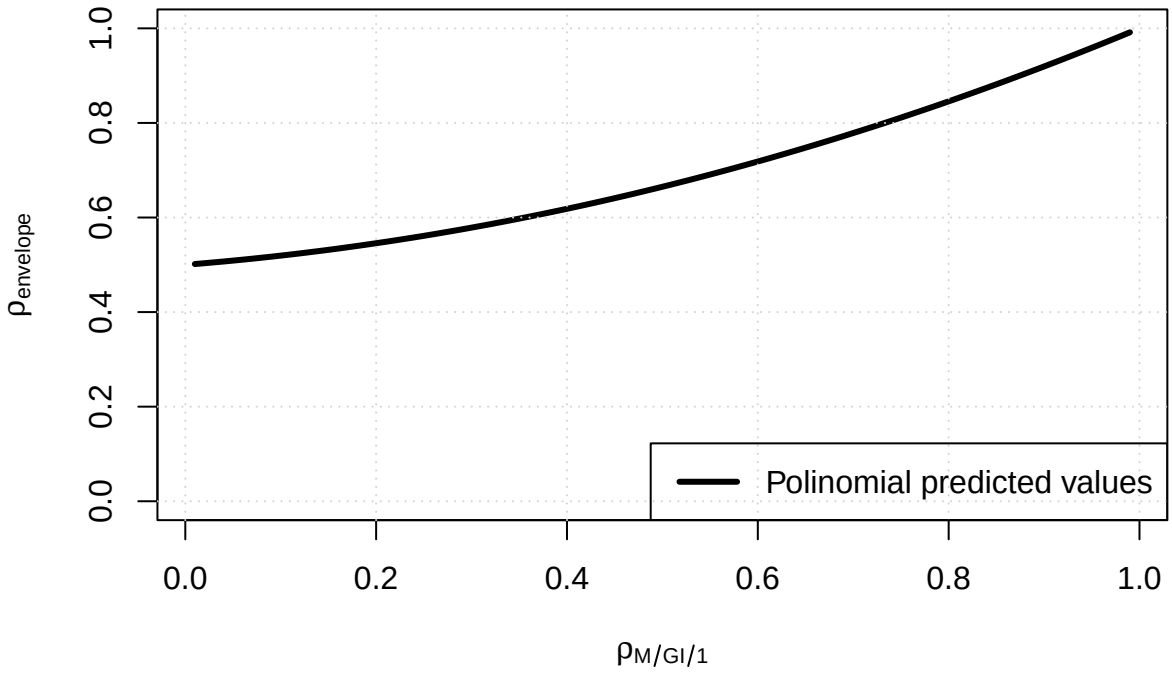
```r
cat("V3 Polynomial Model for V1: MAPE =", mape_avg_v1,", MAE =", mae_avg_v1, "\n")
```

```
## V3 Polynomial Model for V1: MAPE = 2.159234 , MAE = 0.01429543
```

```r
cat("V3 Polynomial Model for V2: MAPE =", mape_avg_v2,", MAE =", mae_avg_v2, "\n")
```

```
## V3 Polynomial Model for V2: MAPE = 8.646073 , MAE = 0.04622558
```

```r
# Plotting the observed data and the predicted values

plot(loads_real, polinomial_pred_v3, type = 'l', col = 'black', xlab = expression(rho[M/GI/1]), ylab = 
     lwd = 3, ylim = c(0,1))


grid()
legend("bottomright",c("Polinomial predicted values"),
        col=c("black"), lwd=3)
```



Part 9: Second numerc example

V1 tri-model:

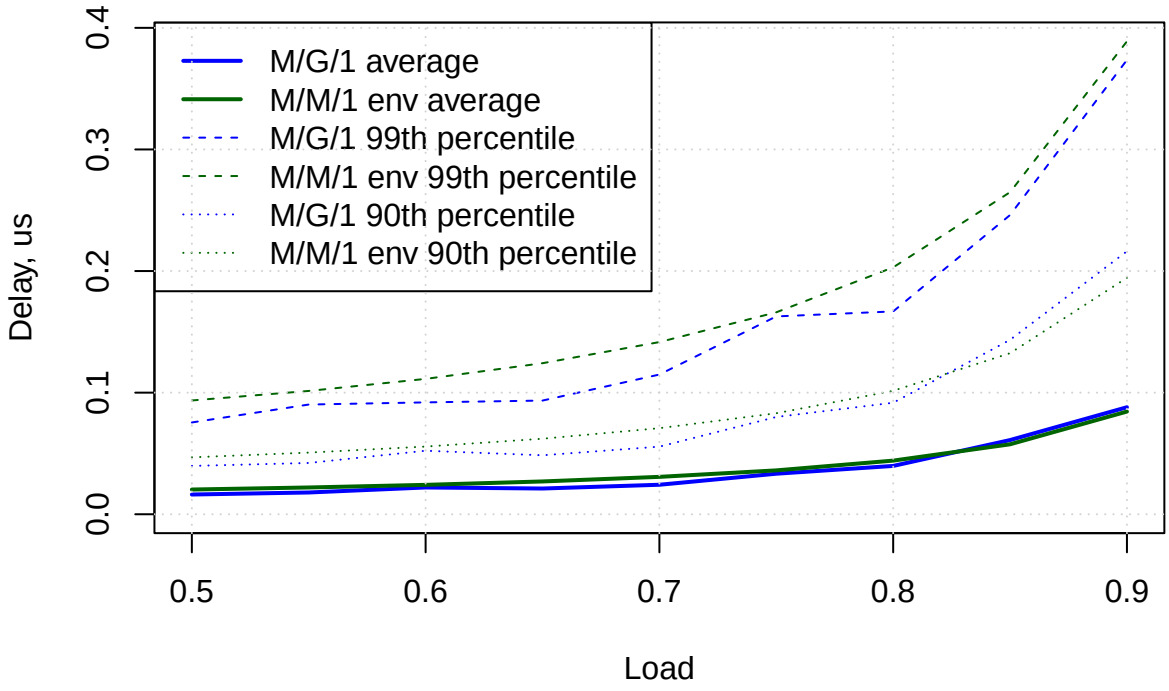
```
# #V1
PS_size <- c(40, 576, 1500)  # Packet sizes in bytes
PS_weights <- c(7, 4, 1) / 12  # Packet weights
Capacity_Gbps = 400
loads <- seq(0.5,0.9,0.05) # Load
p1 = 0.99
p2 = 0.90
E_T_real <-  c()
E_T_real_p1 <-  c()
E_T_real_p2 <-  c()
E_T_env <-  c()
E_T_env_p1 <-  c()
E_T_env_p2 <-  c()
i = 1
for (load in loads){
  mg1_packets <- simmer_mg1(Capacity_Gbps, load, PS_size, PS_weights, k_num = 1e4)
  cat("Load real M/G/1:", load, "\n")
  load_env <- a*load^2 + b*load + c
```

```
  nodes_capacity_Bps = Capacity_Gbps*1e9
  Capacity_ps = nodes_capacity_Bps/(8*N)
  E_X = 1/Capacity_ps

  # Step 1: Determine the average delay (E(T))
  E_T_real[i] <- mean(mg1_packets)*1e6
  E_T_real_p1[i] <- quantile(mg1_packets, p1)*1e6
  E_T_real_p2[i] <- quantile(mg1_packets, p2)*1e6

  E_T_env[i] <- E_X/(1-load_env)*1e6
  E_T_env_p1[i] <- E_X/(1-load_env)*log(1/(1-p1))*1e6
  E_T_env_p2[i] <- E_X/(1-load_env)*log(1/(1-p2))*1e6

  i <- i + 1
}
```

```
## Load real M/G/1: 0.5
## Load real M/G/1: 0.55
## Load real M/G/1: 0.6
## Load real M/G/1: 0.65
## Load real M/G/1: 0.7
## Load real M/G/1: 0.75
## Load real M/G/1: 0.8
## Load real M/G/1: 0.85
## Load real M/G/1: 0.9
```

Comparison of theoretical M/M/1 envelope and simulated M/G/1 delays for AMS-IX traffic distribution

```
# Plot the results
plot(loads, E_T_real, type = "l", col = "blue", lwd = 2, xlab = "Load", ylab = "Delay, us", ylim = c(0,
lines(loads, E_T_env, type = "l", col = "darkgreen", lwd = 2)
lines(loads, E_T_real_p1, type = "l", col = "blue", lwd = 1, lty = 2)
lines(loads, E_T_env_p1, type = "l", col = "darkgreen", lwd = 1, lty = 2)
lines(loads, E_T_real_p2, type = "l", col = "blue", lwd = 1, lty = 3)
lines(loads, E_T_env_p2, type = "l", col = "darkgreen", lwd = 1, lty = 3)

legend("topleft", legend = c("M/G/1 average", "M/M/1 env average", "M/G/1 99th percentile", "M/M/1 env 9

grid()
```



V2 AMS-IX:

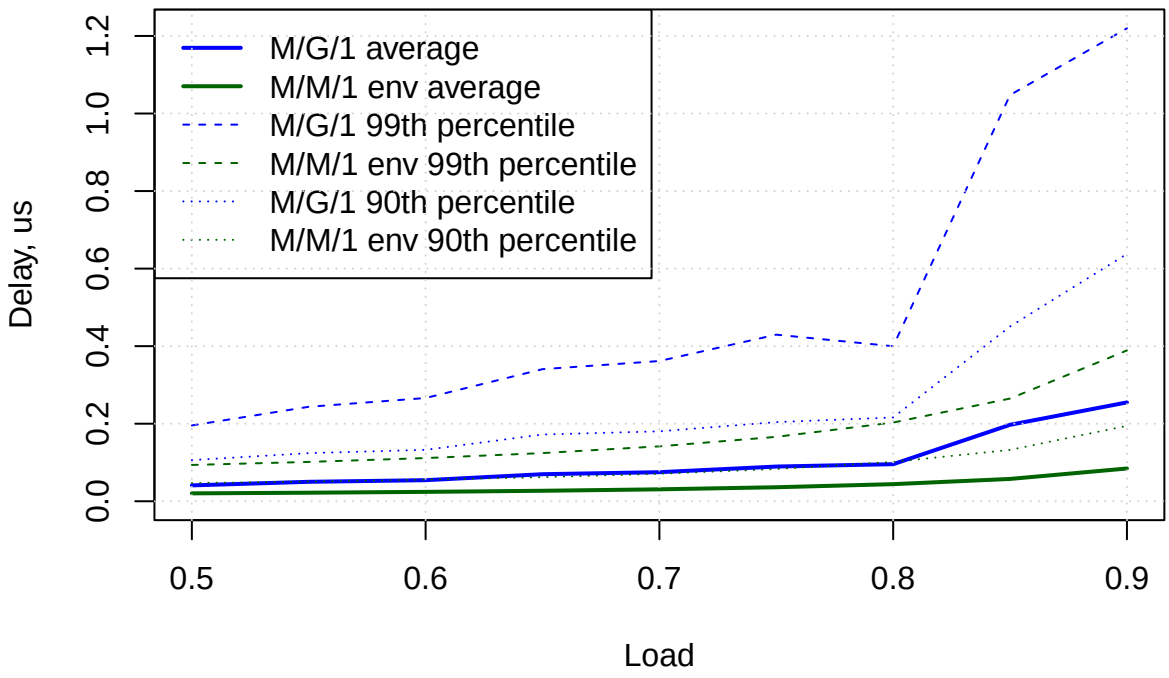
```
#V2 https://www.ams-ix.net/ams
PS_size=c((64+127)/2,(128+255)/2,(256+511)/2, (512+1023)/2, (1024+1513)/2, 1514, (1515+9100)/2)
PS_weights=c(33.2/100, 5.4/100, 3.3/100, 3.7/100, 34.6/100, 14.6/100, 5.2/100)

Capacity_Gbps = 400
loads <- seq(0.5,0.9,0.05) # Load
p1 = 0.99
p2 = 0.90
E_T_real <-  c()
E_T_real_p1 <-  c()
E_T_real_p2 <-  c()
E_T_env <-  c()
E_T_env_p1 <-  c()
E_T_env_p2 <-  c()
i = 1
for (load in loads){
  mg1_packets <- simmer_mg1(Capacity_Gbps, load, PS_size, PS_weights, k_num = 1e4)
  cat("Load real M/G/1:", load, "\n")
  load_env <- a*load^2 + b*load + c

  nodes_capacity_Bps = Capacity_Gbps*1e9
  Capacity_ps = nodes_capacity_Bps/(8*N)
  E_X = 1/Capacity_ps

  # Step 1: Determine the average delay (E(T))
```

```
  E_T_real[i] <- mean(mg1_packets)*1e6
  E_T_real_p1[i] <- quantile(mg1_packets, p1)*1e6
  E_T_real_p2[i] <- quantile(mg1_packets, p2)*1e6

  E_T_env[i] <- E_X/(1-load_env)*1e6
  E_T_env_p1[i] <- E_X/(1-load_env)*log(1/(1-p1))*1e6
  E_T_env_p2[i] <- E_X/(1-load_env)*log(1/(1-p2))*1e6

  i <- i + 1
}
```

```
## Load real M/G/1: 0.5
## Load real M/G/1: 0.55
## Load real M/G/1: 0.6
## Load real M/G/1: 0.65
## Load real M/G/1: 0.7
## Load real M/G/1: 0.75
## Load real M/G/1: 0.8
## Load real M/G/1: 0.85
## Load real M/G/1: 0.9
```

Comparison of theoretical M/M/1 envelope and simulated M/G/1 delays for AMS-IX traffic distribution

```
# Plot the results
plot(loads, E_T_real, type = "l", col = "blue", lwd = 2, xlab = "Load", ylab = "Delay, us", ylim = c(0,
lines(loads, E_T_env, type = "l", col = "darkgreen", lwd = 2)
lines(loads, E_T_real_p1, type = "l", col = "blue", lwd = 1, lty = 2)
lines(loads, E_T_env_p1, type = "l", col = "darkgreen", lwd = 1, lty = 2)
lines(loads, E_T_real_p2, type = "l", col = "blue", lwd = 1, lty = 3)
lines(loads, E_T_env_p2, type = "l", col = "darkgreen", lwd = 1, lty = 3)

legend("topleft", legend = c("M/G/1 average", "M/M/1 env average", "M/G/1 99th percentile", "M/M/1 env 9

grid()
```


M/G/1 average
M/M/1 env average
M/G/1 99th percentile
M/M/1 env 99th percentile
M/G/1 90th percentile
M/M/1 env 90th percentile
Delay, us
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0.5
0.6
0.7
0.8
0.9
Load