marketing-campaign-success-prediction

January 11, 2017

1 Feature engineering: Feature correlation, normality and MIC

1.1 Use case: Linear models for predicting success in marketing campaigns

Dependencies: -http://minepy.readthedocs.io/en/latest/ minepy - Maximal Information-based Nonparametric Exploration —

```
In [16]: import pandas as pd
         import numpy as np
         import seaborn
         from sklearn import linear_model
         from matplotlib import pyplot as plt
         # huge images can't be displayed on a screen, so directly use Agg
         import matplotlib
         matplotlib.use("Agg")
         from scipy import stats
         from feature_engineering import feature_engineering

         % matplotlib inline
         %load_ext autoreload
         %autoreload 2


         f = feature_engineering()

         # Data has blind feature names (unknown), and represents fictional data fr
         train = pd.read_csv('./data/train_data.csv', header= None)
         test = pd.read_csv('./data/test_data.csv', header = None)

         col_names = train.columns.values
         train.rename(columns=lambda x: str(x), inplace=True)
         print col_names
         test.rename(columns=lambda x: str(x), inplace=True)

         train.head(5)

         # make the only categorical feature (row '1') consistent in data types (st
```

```
train['1'] = train['1'].apply(lambda x: str(x))
# train = train.set_index(['0'])
# test = test.set_index(['0'])
```

```
The autoreload extension is already loaded. To reload it, use:
  %reload_ext autoreload
feature_engineering methods
[ 0  1  2  3  4  5  6  7  8  9 10 11]
```

Out[16]:

	0	1	2	3	4	5	6	7	8	
0	0	AdMob	73946	5755397	2947	12665	46196	639	10091	85
1	1	Search	1625476	30183332	22443	364010	1287250	4927	44035	584
2	2	YouTube	796	170994	140	0	0	41	351	2
3	3	AdMob	267791	7184832	5992	79963	270375	1334	13363	142
4	4	GDN	1797	98955	39	998	4400	7	110	

In [17]:
```
test.head(5)
```

Out[17]:

	0	1	2	3	4	5	6	7	8	9	1
0	0	Search	199	194343	63	0	0	40	256	14831	
1	1	AdMob	86869	3558583	885	47151	147847	740	7493	190470	4296
2	2	GDN	1794	117852	16	199	600	20	289	6086	19
3	3	Search	0	60852	24	0	0	8	101	4727	
4	4	YouTube	2195	536854	13	1797	8504	101	947	1292	179

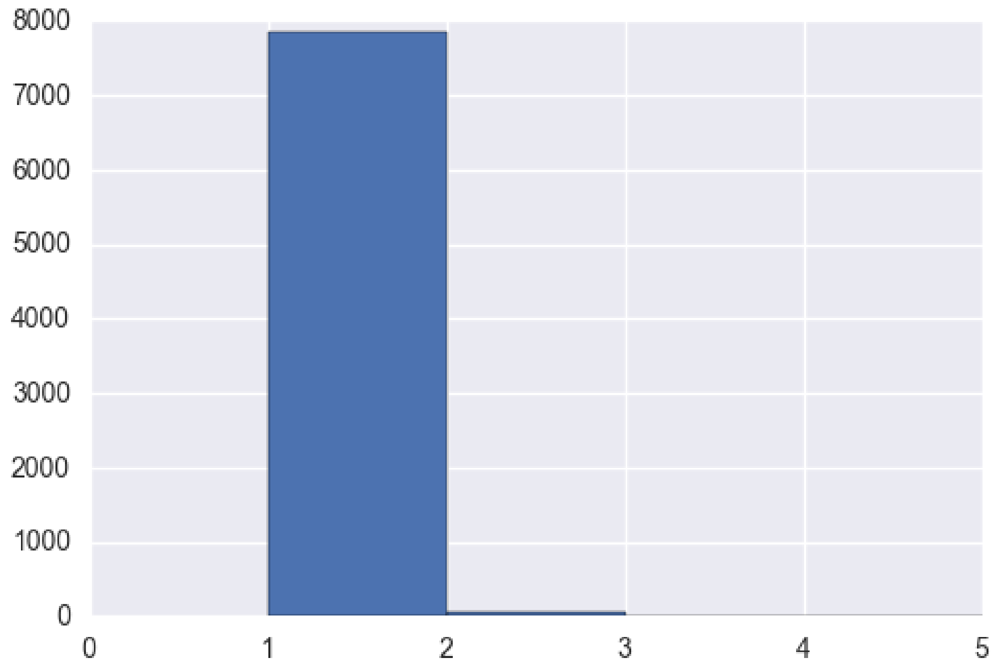
In [18]:
```
Y_train = train.iloc[:, 3] # equivalent to train[3]   # third column
Y_train.head()
Y_test = test.iloc[:, 3]
Y_test.head()

## Plotting distribution of the Y variable variablity and different values
# consider first row as data (not header)
y_counts = Y_train.value_counts()
y_counts.hist(bins=[0, 1, 2, 3,4, 5])
print y_counts.head()
```

Out[18]:
```
0     5755397
1    30183332
2      170994
3     7184832
4       98955
Name: 3, dtype: int64
```

Out[18]:
```
0     194343
1    3558583
2     117852
3      60852
4     536854
Name: 3, dtype: int64
```

Out[18]: <matplotlib.axes._subplots.AxesSubplot at 0x117727050>

```
83288     2
152772    2
50942     2
51569     2
81140     2
Name: 3, dtype: int64
```

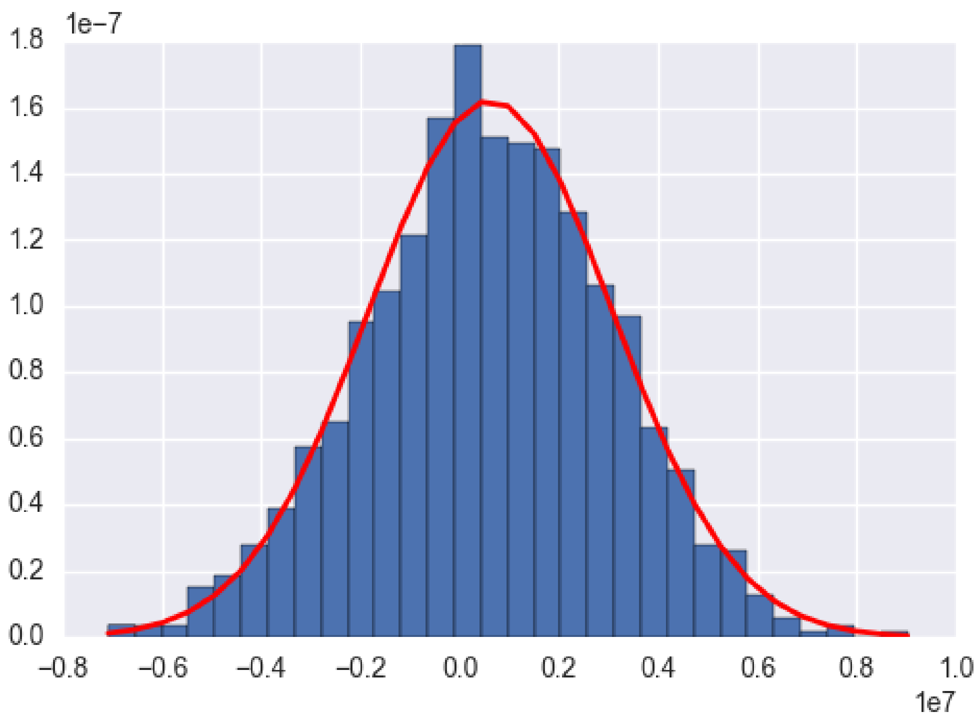


In [19]:
```
mean =  Y_train.mean()
std = Y_train.std()
median = Y_train.median()
print "Feature mean: ", mean
print "Feature std dev: ", std
print "Feature std error of the mean: ", stats.sem(Y_train)
print "Feature median: ", median
# if we were to test normality doing a normal probability plot, we also ne
# train['z-value'] = Y_train - mean
# train['z-value'] = np.divide(train['z-value'], std)

print len(Y_train), " values for Y train -unique: ", len(set(list(Y_train)
print len(Y_test), " values for Y test -unique: ", len(set(list(Y_test)))

def plot_histogram(data):
```

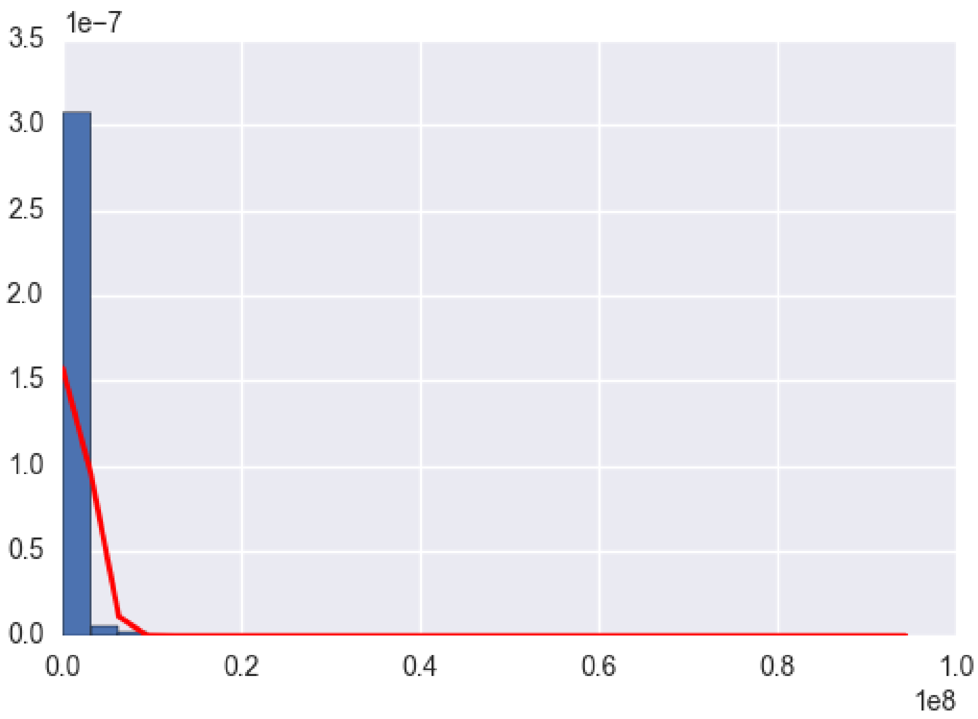
```
        plt.hist(data) #bins=[0, 1, 2, 3,4,5]
        plt.title('Feature histogram')
        plt.xlabel('Y train histogram')
        plt.ylabel('Value count')
        plt.show()

    #plot_histogram(Y_train)
```

```
Feature mean:  619903.597645
Feature std dev:  2460998.64349
Feature std error of the mean:  27544.0824483
Feature median:  160219.0
7983  values for Y train -unique:  7913
754  values for Y test -unique:  753
```

In [20]:
```
# testing normality
def visualize_normality_with_simulated_normal(mu, sigma, Y_train):
    #mu, sigma = 0, 0.1 # mean and standard deviation
    s = np.random.normal(mu, sigma, 1000)
    #Display the histogram of the samples, along with the probability dens
    count, bins, ignored = plt.hist(s, 30, normed=True)
    plt.plot(bins, 1/(sigma * np.sqrt(2 * np.pi)) *
        np.exp( - (bins - mu)**2 / (2 * sigma**2) ),
    linewidth=2, color='r')
    plt.show()
visualize_normality_with_simulated_normal(mean, std, Y_train)
```



```
In [21]: def visualize_normality_for_sample(x):
             #Display the histogram of the samples, along with the probability dens
             mu =  x.mean()
             sigma = x.std()
             count, bins, ignored = plt.hist(x, 30, normed=True)
             plt.plot(bins, 1/(sigma * np.sqrt(2 * np.pi)) *
                 np.exp( - (bins - mu)**2 / (2 * sigma**2) ),
             linewidth=2, color='r')
             plt.show()

         visualize_normality_for_sample(Y_train)
```




```
In [22]: import scipy.stats as stats
         from scipy.stats import mstats

         def test_normality(data):
             """
             Tests whether a sample differs from a normal distribution. Returns a 2
             and the associated p-value. Given the null hypothesis that x came from
             If the p-val is very small (alpha level of 0.05 normally), it means it
             Other possible way: normal probab. plot and https://docs.scipy.org/doc
```

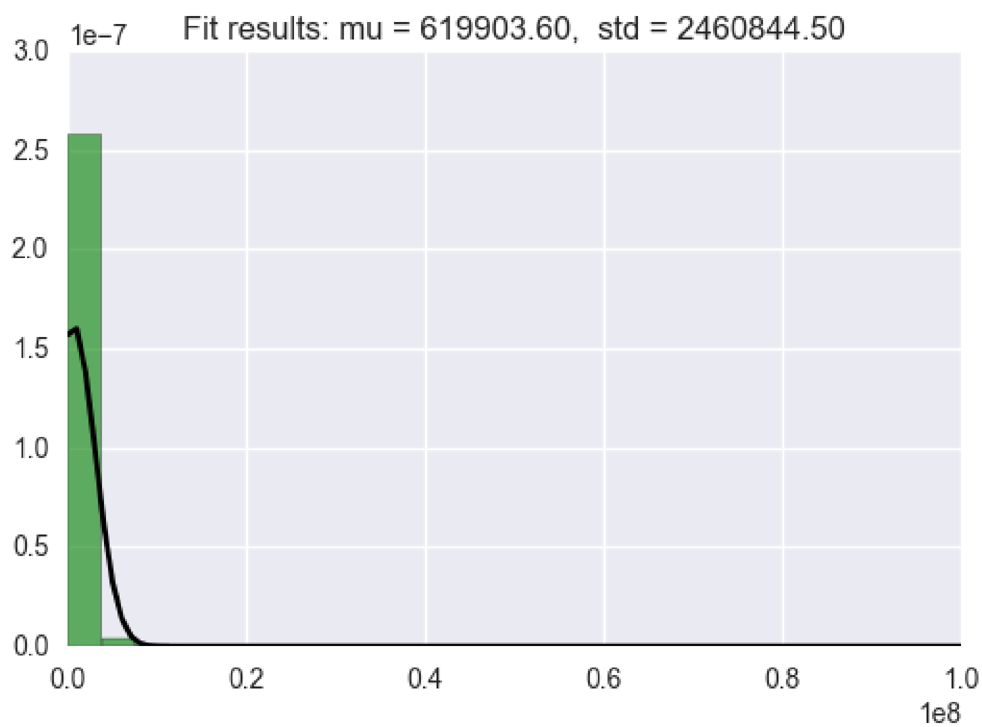
```
        """
        # equivalent: print stats.normaltest(data)
        print "z value and p value: "#, z, pval
        z,pval = mstats.normaltest(data)
        if(pval < 0.05):
            print "Not normal distribution"
        return z, pval

    mu, sigma = 0, 0.1
    s = np.random.normal(mu, sigma, 10000)
    print " Normal distrib. test: ",test_normality(s)  #(1.0491016699730547, (
    print " Y_train test: ", test_normality(Y_train)

    Y_train.shape
```

```
 Normal distrib. test:  z value and p value:
(1.4176249725444574, 0.4922283784874727)
 Y_train test:  z value and p value:
Not normal distribution
(16758.153738777055, 0.0)
```

Out[22]: (7983,)

In [23]:
```
import numpy as np
from scipy.stats import norm
import matplotlib.pyplot as plt
# Generate some data for this demonstration.
#data = norm.rvs(10.0, 2.5, size=500)
data = Y_train

# Fit a normal distribution to the data:
mu, std = norm.fit(data)

# Plot the histogram.
plt.hist(data, bins=25, normed=True, alpha=0.6, color='g')

# Plot the PDF.
xmin, xmax = plt.xlim()
x = np.linspace(xmin, xmax, 100)
p = norm.pdf(x, mu, std)
plt.plot(x, p, 'k', linewidth=2)
title = "Fit results: mu = %.2f,  std = %.2f" % (mu, std)
plt.title(title)

plt.show()

#plt.plot(bin_edges, scipy.stats.norm.pdf(bin_edges, loc=Y_train.mean(), s
```

```
Out[23]: (array([  2.58036245e-07,   4.11689326e-09,   1.42763234e-09,
                    4.64810529e-10,   1.32803008e-10,   3.65208273e-10,
                    9.96022563e-11,   9.96022563e-11,   6.64015042e-11,
                    3.32007521e-11,   3.32007521e-11,   3.32007521e-11,
                    3.32007521e-11,   0.00000000e+00,   0.00000000e+00,
                    0.00000000e+00,   3.32007521e-11,   0.00000000e+00,
                    0.00000000e+00,   3.32007521e-11,   0.00000000e+00,
                    0.00000000e+00,   0.00000000e+00,   0.00000000e+00,
                    3.32007521e-11]),
          array([  7.15300000e+03,   3.78014556e+06,   7.55313812e+06,
                    1.13261307e+07,   1.50991232e+07,   1.88721158e+07,
                    2.26451084e+07,   2.64181009e+07,   3.01910935e+07,
                    3.39640860e+07,   3.77370786e+07,   4.15100712e+07,
                    4.52830637e+07,   4.90560563e+07,   5.28290488e+07,
                    5.66020414e+07,   6.03750340e+07,   6.41480265e+07,
                    6.79210191e+07,   7.16940116e+07,   7.54670042e+07,
                    7.92399968e+07,   8.30129893e+07,   8.67859819e+07,
                    9.05589744e+07,   9.43319670e+07]),
          <a list of 25 Patch objects>)

Out[23]: [<matplotlib.lines.Line2D at 0x117554f50>]

Out[23]: <matplotlib.text.Text at 0x11147b710>
```



In [24]:
```
# make the only categorical feature (row 1) consistent in data types (str)
train['1'] = train['1'].apply(lambda x: str(x)) #train['1'] = train['1'].a
print train['1'].dtypes  ," and Y type: ", Y_train.dtypes  #train['1'] = t

categorical_features = ['1']
train_dum = f.create_dummy_vars_for_categorical_features(train, categorica
# dropping non categorical useless features once they have been binarized
#train_dum.drop(['0'], axis = 1, inplace = True)
#train_dum.drop(['z-value'], axis = 1, inplace = True)
```

```
object  and Y type:  int64
Before dummys:
(7983, 12)   ['0' '1' '2' '3' '4' '5' '6' '7' '8' '9' '10' '11']
   0        1        2          3      4       5         6     7      8         9
0  0    AdMob    73946    5755397   2947   12665     46196   639  10091    854301    8
1  1   Search  1625476   30183332  22443  364010   1287250  4927  44035   5843319  290
2  2  YouTube      796     170994    140       0         0    41    351     23160
3  3    AdMob   267791    7184832   5992   79963    270375  1334  13363   1428950   62
4  4      GDN     1797      98955     39     998      4400     7    110      9484
After dummys:
(7983, 7224)   ['0' '1' '2' ..., '1_zmBV' '1_zsC' '1_zxFV']
   1_0  1_00451cfe1dc94904746b5cbda3c4db03  1_0082812513bc90d9b0c1857d125e049d5f56a
0    0                                   0
1    0                                   0
2    0                                   0
3    0                                   0
4    0                                   0

   1_0436cf0f893ca9ecdb2169e00d30736eb79955a4fbf2c9257cc8055e367de23e  1_0455899d92
0                                                                   0
1                                                                   0
2                                                                   0
3                                                                   0
4                                                                   0

   1_0794abdc89843ab90d36b2888dbd704e9812d64f  1_07cd418370b8edf0119698cdb7ba7e1f
0                                           0                                   0
1                                           0                                   0
2                                           0                                   0
3                                           0                                   0
4                                           0                                   0

   1_0bae15bf272d2f079698c2fba8eedf74c81b64b6  1_0bd6acdc8938e52336c4232ed3dc28db
0                                           0                                   0
1                                           0                                   0
2                                           0                                   0
3                                           0                                   0
4                                           0                                   0
```

	1_1069b78a70a2ca378050e2ebb6a6dc3d8be95394	1_1075939556	1_1078643602	1_108048
0	0	0	0	
1	0	0	0	
2	0	0	0	
3	0	0	0	
4	0	0	0	

	1_14b6f821805e61cb53854acd34dc8b04	1_15148	1_15234	1_1581d334719b3ef8e9540a46
0	0	0	0	
1	0	0	0	
2	0	0	0	
3	0	0	0	
4	0	0	0	

	1_17f852d86d16340bbbafb365a6487335	1_183c71ed82e5cde01c5891cf23cf84ba7ac9bded
0	0	0
1	0	0
2	0	0
3	0	0
4	0	0

	1_1bbd9e4fbfdbddd63eefe32dd679ad001ccea105	1_1bc20440079adeb1a71f516c6fd3c27928
0	0	
1	0	
2	0	
3	0	
4	0	

	1_1f8668d8f2cd9b730a59fbcd32a979da	1_1fa647d3e8b577aa363597c2bf839a3cace98e9f
0	0	0
1	0	0
2	0	0
3	0	0
4	0	0

	1_22c7e65eb3b6344eeaf64ea071b3e99d176190ce	1_22d8f4881fbd1b555787bd39ce2bbc1612
0	0	
1	0	
2	0	
3	0	
4	0	

	1_e11ae398fc5cca524ffede00cbbe5a3254a92cd3	1_e11f45caa7b9bf1897a983d8e9df0fb969
0	0	
1	0	
2	0	
3	0	

```
4                                                    0

   1_e5165e78b49317d3122b76888344a1604eeaa05e40803b1e1b9702699078171c  1_e52919fd32
0                                                            0
1                                                            0
2                                                            0
3                                                            0
4                                                            0

   1_eae91567304bc29e4d41709bb0efc0677395dd8a  1_eafce3db712581af27f7ae8cba25ba30
0                                          0                                   0
1                                          0                                   0
2                                          0                                   0
3                                          0                                   0
4                                          0                                   0

   1_ee33a78aa59fb2d8fb816dd49bfa19d6efa46226  1_eeb9c2b113755e1d8f7852dd8446c2fc
0                                          0                                   0
1                                          0                                   0
2                                          0                                   0
3                                          0                                   0
4                                          0                                   0

   1_f3d0216b03a226490f736fc0febea2a845e10a1b7dec622a09cd2f0e5b551246  1_f40622e609
0                                                            0
1                                                            0
2                                                            0
3                                                            0
4                                                            0

   1_f6db27f26592d97d0d150a88267b58a56aeb6c85  1_f6e05dc5df7aff9f2059c171cb02aab207
0                                          0
1                                          0
2                                          0
3                                          0
4                                          0

   1_fa855c32c28e3d81f88e1826e3b1d8658afbbf60  1_fa8d47590d046dd311ba6846a2daaad1a6
0                                          0
1                                          0
2                                          0
3                                          0
4                                          0

   1_fe27b4e8250d0be5b0cb007b0261775a1db3f3f3  1_fe570513a97624bbe908436bb083f1bc43
0                                          0
1                                          0
2                                          0
```

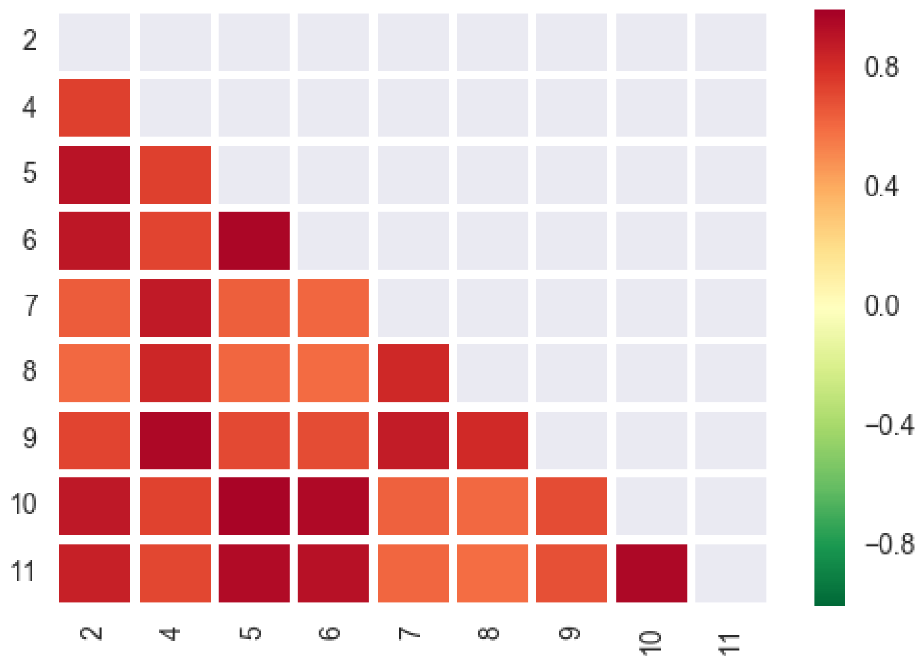
```
3                                           0
4                                           0

   1_ylBV  1_zmBV  1_zsC  1_zxFV
0       0       0      0       0
1       0       0      0       0
2       0       0      0       0
3       0       0      0       0
4       0       0      0       0

[5 rows x 7224 columns]
```

1.1.1 FEATURE SELECTION THROUGH FEATURE CORRELATION

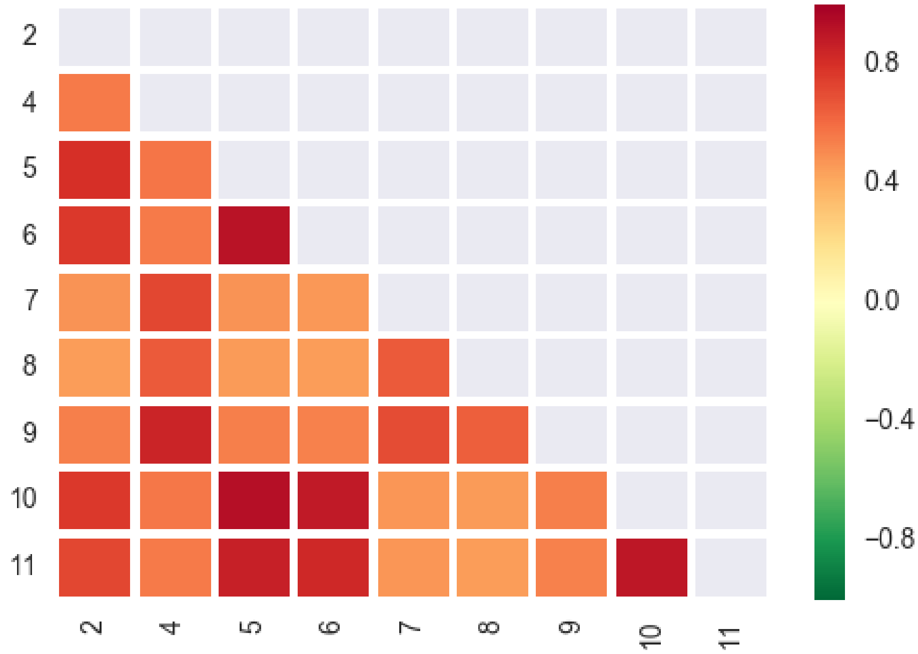
Pearson correlation coefficient ρ -rho- (if it exists, in [-1,1]) does not assume normality, but is only an exhaustive measure of association if the joint distribution is multivariate normal. Given the failed normality test in our case, another option can be using Spearman's s or Kendall's tau. Spearman's rho can be less reliable and interpretable than Kendall's while Spearman rho is easier to compute).

```
In [25]: #find_correlated_features(train_dum, 'spearman', cols_to_skip=['3', '0'])
         f.visualize_feature_correlations(train_dum, 'spearman', cols_to_skip=['3',
```

```
initial columns  ['0' '2' '3' '4' '5' '6' '7' '8' '9' '10' '11']
cols_to_consider ['2', '4', '5', '6', '7', '8', '9', '10', '11']
Df to perform correlation analysis:
['2' '4' '5' '6' '7' '8' '9' '10' '11']
         2      4       5        6     7      8        9      10      11
0    73946   2947   12665    46196   639  10091   854301    8875    6680
1  1625476  22443  364010  1287250  4927  44035  5843319  290910  197941
2      796    140       0        0    41    351    23160       0       0
3   267791   5992   79963   270375  1334  13363  1428950   62518   49256
4     1797     39     998     4400     7    110     9484     998     799
--------------- CORRELATIONS ---------------
           2         4         5         6         7         8         9        10
2   1.000000  0.743127  0.918763  0.899129  0.649821  0.613268  0.727066  0.897125
4   0.743127  1.000000  0.746336  0.730530  0.888340  0.836569  0.963483  0.738350
5   0.918763  0.746336  1.000000  0.972387  0.641088  0.617912  0.718292  0.982197
6   0.899129  0.730530  0.972387  1.000000  0.623969  0.601831  0.704703  0.957726
7   0.649821  0.888340  0.641088  0.623969  1.000000  0.834476  0.876237  0.634067
8   0.613268  0.836569  0.617912  0.601831  0.834476  1.000000  0.825776  0.611522
9   0.727066  0.963483  0.718292  0.704703  0.876237  0.825776  1.000000  0.708868
10  0.897125  0.738350  0.982197  0.957726  0.634067  0.611522  0.708868  1.000000
11  0.860970  0.723718  0.946167  0.923446  0.621887  0.601312  0.694735  0.965509
--------------- CREATE A HEATMAP ---------------
```




```
In [26]: f.visualize_feature_correlations(train_dum, 'kendall', cols_to_skip=['3',

initial columns  ['0' '2' '3' '4' '5' '6' '7' '8' '9' '10' '11']
cols_to_consider ['2', '4', '5', '6', '7', '8', '9', '10', '11']
Df to perform correlation analysis:
['2' '4' '5' '6' '7' '8' '9' '10' '11']
         2      4       5        6     7      8        9      10      11
0    73946   2947   12665    46196   639  10091   854301    8875    6680
1  1625476  22443  364010  1287250  4927  44035  5843319  290910  197941
2      796    140       0        0    41    351    23160       0       0
3   267791   5992   79963   270375  1334  13363  1428950   62518   49256
4     1797     39     998     4400     7    110     9484     998     799
--------------- CORRELATIONS ---------------
           2         4         5         6         7         8         9        10
2   1.000000  0.560801  0.798315  0.772140  0.479241  0.447647  0.546545  0.769079
4   0.560801  1.000000  0.572503  0.559252  0.725702  0.659961  0.844375  0.567374
5   0.798315  0.572503  1.000000  0.921733  0.480173  0.458691  0.546812  0.937331
6   0.772140  0.559252  0.921733  1.000000  0.466677  0.446051  0.535792  0.889924
7   0.479241  0.725702  0.480173  0.466677  1.000000  0.659356  0.708899  0.476352
8   0.447647  0.659961  0.458691  0.446051  0.659356  1.000000  0.642938  0.455279
9   0.546545  0.844375  0.546812  0.535792  0.708899  0.642938  1.000000  0.540316
10  0.769079  0.567374  0.937331  0.889924  0.476352  0.455279  0.540316  1.000000
11  0.724697  0.557845  0.863085  0.830434  0.469605  0.450712  0.531369  0.902083
--------------- CREATE A HEATMAP ---------------
```



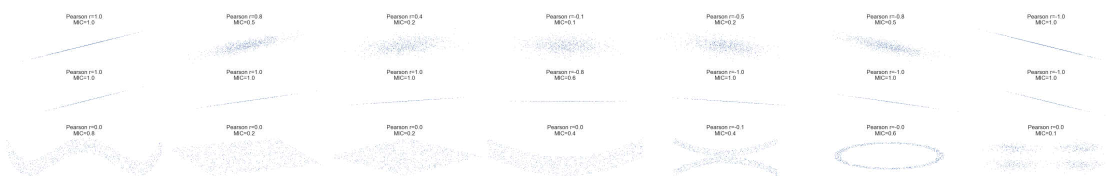
In [27]:
```
from __future__ import division
import numpy as np
import matplotlib.pyplot as plt
from minepy import MINE
# for computing correlation in between categorical features, use Mutual In
# package minepy (Maximal Information-based Nonparametric Exploration)

rs = np.random.RandomState(seed=0)

def MIC_plot(x, y, numRows, numCols, plotNum,
             xlim=(-4, 4), ylim=(-4, 4)):
    # build the MIC (Maximal Information-based Nonparametric Exploration)
    # and correlation plot using the covariant matrix using a vectorized i
    r = np.around(np.corrcoef(x, y)[0, 1], 1)
    mine = MINE(alpha=0.6, c=15, est="mic_approx")
    mine.compute_score(x, y)
    mic = np.around(mine.mic(), 1)
    ax = plt.subplot(numRows, numCols, plotNum,
                     xlim=xlim, ylim=ylim)
    ax.set_title('Pearson r=%.1f\nMIC=%.1f' % (r, mic),fontsize=12)
    ax.set_frame_on(False)
    ax.axes.get_xaxis().set_visible(False)
    ax.axes.get_yaxis().set_visible(False)
    ax.plot(x, y, ',')
    ax.set_xticks([])
```

```
    ax.set_yticks([])
    return ax

def rotation(xy, t):
    return np.dot(xy, [[np.cos(t), -np.sin(t)], [np.sin(t), np.cos(t)]])

def mvnormal(n=1000):
    cors = [1.0, 0.8, 0.4, 0.0, -0.4, -0.8, -1.0]
    for i, cor in enumerate(cors):
        cov = [[1, cor],[cor, 1]]
        xy = rs.multivariate_normal([0, 0], cov, n)
        MIC_plot(xy[:, 0], xy[:, 1], 3, 7, i+1)

def rotnormal(n=1000):
    ts = [0, np.pi/12, np.pi/6, np.pi/4, np.pi/2-np.pi/6,
          np.pi/2-np.pi/12, np.pi/2]
    cov = [[1, 1],[1, 1]]
    xy = rs.multivariate_normal([0, 0], cov, n)
    for i, t in enumerate(ts):
        xy_r = rotation(xy, t)
        MIC_plot(xy_r[:, 0], xy_r[:, 1], 3, 7, i+8)

def others(n=1000):
    x = rs.uniform(-1, 1, n)
    y = 4*(x**2-0.5)**2 + rs.uniform(-1, 1, n)/3
    MIC_plot(x, y, 3, 7, 15, (-1, 1), (-1/3, 1+1/3))

    y = rs.uniform(-1, 1, n)
    xy = np.concatenate((x.reshape(-1, 1), y.reshape(-1, 1)), axis=1)
    xy = rotation(xy, -np.pi/8)
    lim = np.sqrt(2+np.sqrt(2)) / np.sqrt(2)
    MIC_plot(xy[:, 0], xy[:, 1], 3, 7, 16, (-lim, lim), (-lim, lim))

    xy = rotation(xy, -np.pi/8)
    lim = np.sqrt(2)
    MIC_plot(xy[:, 0], xy[:, 1], 3, 7, 17, (-lim, lim), (-lim, lim))

    y = 2*x**2 + rs.uniform(-1, 1, n)
    MIC_plot(x, y, 3, 7, 18, (-1, 1), (-1, 3))

    y = (x**2 + rs.uniform(0, 0.5, n)) * \
        np.array([-1, 1])[rs.random_integers(0, 1, size=n)]
    MIC_plot(x, y, 3, 7, 19, (-1.5, 1.5), (-1.5, 1.5))

    y = np.cos(x * np.pi) + rs.uniform(0, 1/8, n)
    x = np.sin(x * np.pi) + rs.uniform(0, 1/8, n)
    MIC_plot(x, y, 3, 7, 20, (-1.5, 1.5), (-1.5, 1.5))
```

```
        xy1 = np.random.multivariate_normal([3, 3], [[1, 0], [0, 1]], int(n/4)
        xy2 = np.random.multivariate_normal([-3, 3], [[1, 0], [0, 1]], int(n/4
        xy3 = np.random.multivariate_normal([-3, -3], [[1, 0], [0, 1]], int(n/
        xy4 = np.random.multivariate_normal([3, -3], [[1, 0], [0, 1]], int(n/4
        xy = np.concatenate((xy1, xy2, xy3, xy4), axis=0)
        MIC_plot(xy[:, 0], xy[:, 1], 3, 7, 21, (-7, 7), (-7, 7))

    plt.figure(facecolor='white', figsize=(33,5))
    mvnormal(n=800)
    rotnormal(n=200)
    others(n=800)
    plt.tight_layout()
    plt.show()
```

```
Out[27]: <matplotlib.figure.Figure at 0x111e978d0>
```

```
/Users/natalia/.virtualenvs/keras/lib/python2.7/site-packages/ipykernel/__main__.py
/Users/natalia/.virtualenvs/keras/lib/python2.7/site-packages/ipykernel/__main__.py
/Users/natalia/.virtualenvs/keras/lib/python2.7/site-packages/ipykernel/__main__.py
```



```
In [28]: train_dum.head()

    def plot_covariance_based_mutual_info_for_categorical_correlations(df, cat
        """
        Computes covariance matrix using a vectorized implementation to be use
        coefficient
        """
        cols = []
        plot_id = 1
        for c in df.columns:
            if not c.startswith('1_') and c !='0':
                cols.append(c)
        plot_grid_wide = len(cols)#/2
        plot_grid_length = len(cols)#/plot_grid_wide
        print "Computing covariance matrix and MIC for features: ",cols
        for i in range(len(cols)):
            for j in range(len(cols)):
                if j > i and not (cols[i].startswith('1_') and cols[j].startsw
                    #cov_matrix = np.cov([df[cols[i]], df[cols[j]]], ddof= 0)
```

```
                MIC_plot(df[cols[i]], df[cols[j]], plot_grid_wide, plot_gr
                plot_id +=1

        plt.figure(facecolor='white')
        #plt.tight_layout()
        plt.show()

    def MIC_plot(x, y, numRows, numCols, plotNum, x_name, y_name, xlim=(-4, 4)
        # build the MIC and correlation plot using the covariant matrix using
        # categorical features are part of the model (otherwise, Pearson, Kend
        print "Pearson product-moment correlation coefficients np.corrcoef(x='
        r = np.around(np.corrcoef(x, y)[0, 1], 1)  # Pearson product-moment co
        # TODO: compute cov matrix for each one-hot encoding variable of the c
        # MINE's Mutual Information coefficient

        fig = plt.figure(figsize=(33,5), frameon=False)
        mine = MINE(alpha=0.6, c=15, est="mic_approx")
        mine.compute_score(x, y)
        mic = np.around(mine.mic(), 1)
        ax = plt.subplot(numRows, numCols, plotNum)#,xlim=xlim, ylim=ylim)
        ax.set_title('Pearson r=%.1f\nMIC=%.1f Features %s and %s' % (r, mic,
        ax.set_frame_on(False)
        ax.axes.get_xaxis().set_visible(False)
        ax.axes.get_yaxis().set_visible(False)
        ax.plot(x, y, ',')
        ax.set_xticks([])
        ax.set_yticks([])
        return ax

    plot_covariance_based_mutual_info_for_categorical_correlations(train_dum)
```

Out[28]:

	0	2	3	4	5	6	7	8	9
0	0	73946	5755397	2947	12665	46196	639	10091	854301
1	1	1625476	30183332	22443	364010	1287250	4927	44035	5843319
2	2	796	170994	140	0	0	41	351	23160
3	3	267791	7184832	5992	79963	270375	1334	13363	1428950
4	4	1797	98955	39	998	4400	7	110	9484

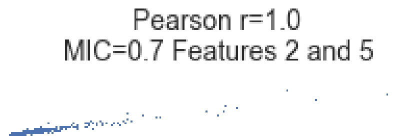
```
Computing covariance matrix and MIC for features:  ['2', '3', '4', '5', '6', '7', '
Pearson product-moment correlation coefficients np.corrcoef(x= 2 , y= 3 ):  [[ 1.
 [ 0.79392723  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 2 , y= 4 ):  [[ 1.
 [ 0.76982974  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 2 , y= 5 ):  [[ 1.
 [ 0.97473137  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 2 , y= 6 ):  [[ 1.
 [ 0.97594607  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 2 , y= 7 ):  [[ 1.
```

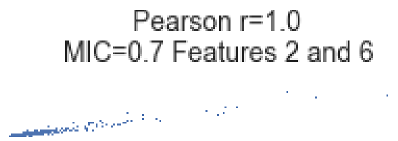
```
 [ 0.74375329  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 2 , y= 8 ):  [[ 1.
 [ 0.74101961  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 2 , y= 9 ):  [[ 1.
 [ 0.80379601  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 2 , y= 10 ):  [[ 1.
 [ 0.97007857  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 2 , y= 11 ):  [[ 1.
 [ 0.9614406  1.       ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 3 , y= 4 ):  [[ 1.
 [ 0.95512053  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 3 , y= 5 ):  [[ 1.
 [ 0.8298201  1.       ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 3 , y= 6 ):  [[ 1.
 [ 0.82045368  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 3 , y= 7 ):  [[ 1.
 [ 0.96481518  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 3 , y= 8 ):  [[ 1.
 [ 0.98410937  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 3 , y= 9 ):  [[ 1.
 [ 0.96943653  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 3 , y= 10 ):  [[ 1.
 [ 0.83013546  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 3 , y= 11 ):  [[ 1.
 [ 0.83024132  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 4 , y= 5 ):  [[ 1.
 [ 0.84009476  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 4 , y= 6 ):  [[ 1.
 [ 0.82801532  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 4 , y= 7 ):  [[ 1.
 [ 0.98224552  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 4 , y= 8 ):  [[ 1.
 [ 0.94163681  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 4 , y= 9 ):  [[ 1.
 [ 0.98377684  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 4 , y= 10 ):  [[ 1.
 [ 0.84307499  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 4 , y= 11 ):  [[ 1.
 [ 0.84894869  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 5 , y= 6 ):  [[ 1.
 [ 0.99425826  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 5 , y= 7 ):  [[ 1.
 [ 0.80799348  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 5 , y= 8 ):  [[ 1.
 [ 0.78816171  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 5 , y= 9 ):  [[ 1.
 [ 0.85921513  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 5 , y= 10 ):  [[ 1.
```

```
 [ 0.99880516  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 5 , y= 11 ):  [[ 1.
 [ 0.99541269  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 6 , y= 7 ):  [[ 1.
 [ 0.79327667  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 6 , y= 8 ):  [[ 1.
 [ 0.7747764  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 6 , y= 9 ):  [[ 1.
 [ 0.84968197  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 6 , y= 10 ):  [[ 1.
 [ 0.99273582  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 6 , y= 11 ):  [[ 1.
 [ 0.98873514  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 7 , y= 8 ):  [[ 1.
 [ 0.95831715  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 7 , y= 9 ):  [[ 1.
 [ 0.97351327  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 7 , y= 10 ):  [[ 1.
 [ 0.81152667  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 7 , y= 11 ):  [[ 1.
 [ 0.81709429  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 8 , y= 9 ):  [[ 1.
 [ 0.94680423  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 8 , y= 10 ):  [[ 1.
 [ 0.7898437  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 8 , y= 11 ):  [[ 1.
 [ 0.79207057  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 9 , y= 10 ):  [[ 1.
 [ 0.86143793  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 9 , y= 11 ):  [[ 1.
 [ 0.86547285  1.        ]]
Pearson product-moment correlation coefficients np.corrcoef(x= 10 , y= 11 ):  [[ 1.
 [ 0.99726583  1.        ]]
```

Pearson r=0.8
MIC=0.4 Features 2 and 3

Pearson r=0.8
MIC=0.4 Features 2 and 4


Pearson r=1.0
MIC=0.7 Features 2 and 5


Pearson r=1.0
MIC=0.7 Features 2 and 6


Pearson r=0.7
MIC=0.3 Features 2 and 7


Pearson r=0.7
MIC=0.3 Features 2 and 8

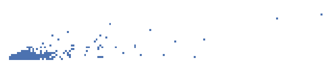

Pearson r=0.8
MIC=0.4 Features 2 and 9


Pearson r=1.0
MIC=0.7 Features 2 and 10

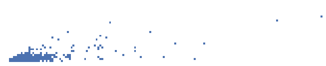
Pearson r=1.0
MIC=0.6 Features 2 and 11

Pearson r=1.0
MIC=0.7 Features 3 and 4

Pearson r=0.8
MIC=0.4 Features 3 and 5



Pearson r=0.8
MIC=0.4 Features 3 and 6

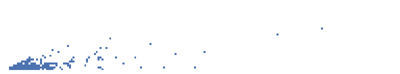


Pearson r=1.0
MIC=0.7 Features 3 and 7

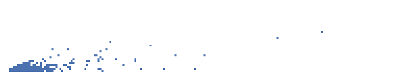
Pearson r=1.0
MIC=0.7 Features 3 and 8

Pearson r=1.0
MIC=0.7 Features 3 and 9

Pearson r=0.8
MIC=0.4 Features 3 and 10



Pearson r=0.8
MIC=0.3 Features 3 and 11

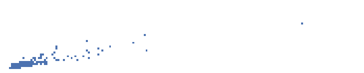


Pearson r=0.8
MIC=0.4 Features 4 and 5

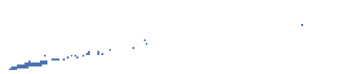
Pearson r=0.8
MIC=0.4 Features 4 and 6

Pearson r=1.0
MIC=0.6 Features 4 and 7

Pearson r=0.9
MIC=0.6 Features 4 and 8



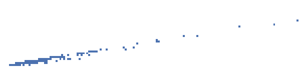
Pearson r=1.0
MIC=0.8 Features 4 and 9



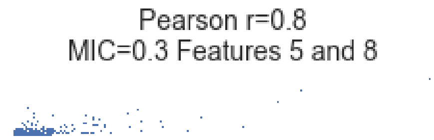
Pearson r=0.8
MIC=0.4 Features 4 and 10

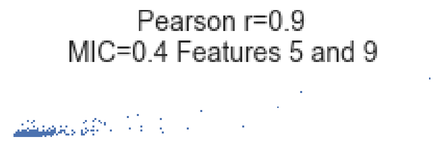
Pearson r=0.8
MIC=0.4 Features 4 and 11

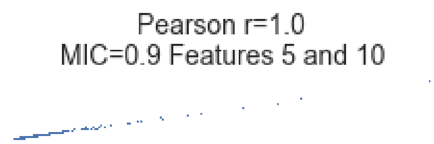
Pearson r=1.0
MIC=0.9 Features 5 and 6

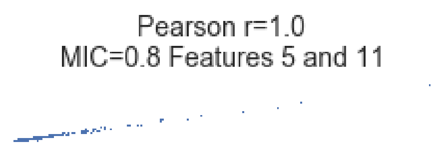


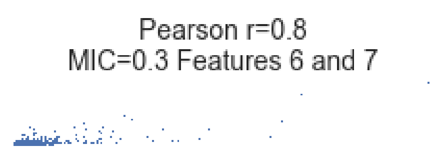
Pearson r=0.8
MIC=0.3 Features 5 and 7

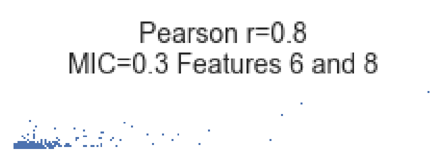

Pearson r=0.8
MIC=0.3 Features 5 and 8


Pearson r=0.9
MIC=0.4 Features 5 and 9

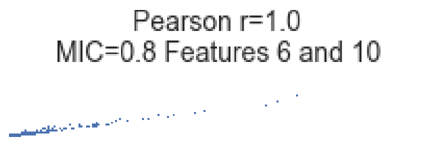

Pearson r=1.0
MIC=0.9 Features 5 and 10

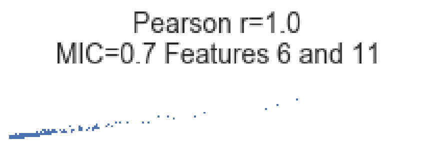

Pearson r=1.0
MIC=0.8 Features 5 and 11

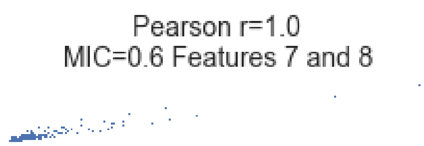

Pearson r=0.8
MIC=0.3 Features 6 and 7

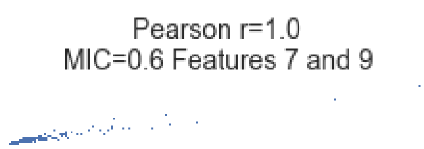

Pearson r=0.8
MIC=0.3 Features 6 and 8

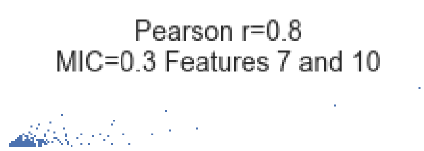

Pearson r=0.8
MIC=0.4 Features 6 and 9

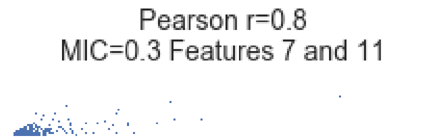

Pearson r=1.0
MIC=0.8 Features 6 and 10

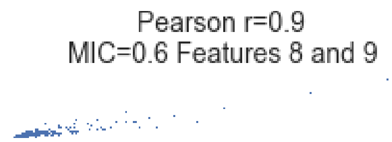

Pearson r=1.0
MIC=0.7 Features 6 and 11


Pearson r=1.0
MIC=0.6 Features 7 and 8


Pearson r=1.0
MIC=0.6 Features 7 and 9


Pearson r=0.8
MIC=0.3 Features 7 and 10

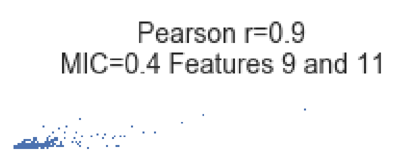

Pearson r=0.8
MIC=0.3 Features 7 and 11


Pearson r=0.9
MIC=0.6 Features 8 and 9


Pearson r=0.8
MIC=0.3 Features 8 and 10


Pearson r=0.8
MIC=0.3 Features 8 and 11


Pearson r=0.9
MIC=0.4 Features 9 and 10


Pearson r=0.9
MIC=0.4 Features 9 and 11


```
<matplotlib.figure.Figure at 0x111f50250>
```

2 Learning Journal: Linear models for predicting success in marketing campaigns

2.1 1. Describing the dependent variable (column 3) in terms of the count, standard deviation, mean and median values, describing the general shape and whether it can be considered a normal distribution.

```
Feature mean: 619260.213355
Feature std dev: 2460481.30781
Feature std error of the mean: 27540.0172749
Feature median: 160190.0
7982 values for Y train -unique: 7912
```

An idea for finding if data can be considered a normal distribution is doing a 2 hypothesis t-test to compare means. However, an issue that affects a 2 Hypothesis test is whether the data are censored, ie. values from the sample are somehow restricted. Censoring occurs if the range of potential values are limited such that values from one or both tails of the distribution are unavailable (e.g., right and/or left censoring where high and/or low values are missing [3]). This could be an issue to be aware here, since we may cut a campaign from running if it is underperforming for max X days, since we do not want to waste marketing campaign costs so we do not allow it running live more than X days).

Normality tests [5]:

1) Plotting the histogram for feature in column 3 could give a good picture of normality if we had more data, but a better visual test is the normal probability plot. If the plot is linear, it is consistent with being sampled from a normal distribution (this test help the human eye at detecting linearity versus detecting bell curve)[1].

2) Using both visualisation and scipy.stats.normaltest(data) in method test_normality, we can see that Y_train cannot be considered a normal distribution (confirmed by a p-value of 0.0 given that H0 = "Both distributions are drawn from a normal distribution". This means that there is statistical significance between the normal distribution and our sample distribution). This method in scipy is based on D'Agostino and Pearson's test that combines skew and kurtosis to produce an omnibus test of normality. p-value returned is a 2-sided chi squared probability for the hypothesis test.

3) We may conduct a formal test, the Shapiro-Wilk test, to see whether the data come from a normal distribution.

H0: The data are sampled from a normal distribution
H1: The data are not sampled from a normal distribution.
A small significant p-value (< 0.05), as shown below, means that we reject the null hypothesis (and conclude that the data is probably not normally distributed).

```
> shapiro.test(rm)
W = 0.9609, p-value = 2.412e-10
```

We see that we reject the null hypothesis that the rm variable was sampled from a normal distribution (W = 0.9609, p-value = 2.412e-10).

2.2 What do we do if the normality assumption fails?

Here is where the non-parametric test comes in. The Wilcoxon Signed rank test can compare the median to a hypothesized median value. For example, in our case,

```
wilcox.test(dis, mu=3.5) or with with continuity correction:
wilcox.test(dis, mu = 3.5, alternative="less")
wilcox.test(dis, mu=3.5, alternative= "greater")
```

[1] Just because a population has a normal distribution doesn't mean that a random sample drawn from the population will "look" (in a histogram) normal. Of course for large samples the representativeness is more accurate. If the sample is not large enough, examining the histogram may not give the whole picture or allow full detection of "bell-shaped" curve. The normal probability plot tackles this issue.

2.3 Are columns correlated to each other? To Y? Which ones and how to determine the correlation?

Depending on outliers, unequal variances, normality and linearity of features, there are different tests to compute correlation [12]. Pearson correlation coefficient R is a common one used for finding correlation among features and consequently, eliminate redundant features. We could set a rule to use only in our model those features that have absolute value of Pearson correlation coefficient r > 0.1 (both among original features and a linear combination of features). The sign (close to -1 or close to 1) will indicate the positive or negative correlation on the dependent variable Y.

After normalising categorical feature in column 1, we have 7224 columns. Out of those, 38 have a correlation coefficient of Pearson P larger than abs(0.05) with the dependent variable Y (all of these are the categorical feature). However, Pearson correlation test is only applicable when each dataset is normally distributed, which is not the case for our Y variable [9]. However, Spearman correlation coefficient, on the contrary, is non parametric (its exact sampling distribution can be obtained without requiring knowledge about the parameters of the joint probability distribution of X and Y). Spearman is applicable when X and Y are related by any monotonic function (in contrast to the Pearson correlation, which only is precise when X and Y are related by a linear function)[10]. Spearman's Rho is considered as the regular Pearson's correlation coefficient in terms of the proportion of variability accounted for, whereas Kendall's Tau represents a probability, i.e., the difference between the probability that the observed data are in the same order versus the probability that the observed data are not in the same order [12]. Therefore I provide a grid plot on both Kendall and Spearman correlation coefficients.

For the categorical features, correlation per se is not adequate, and we compute for each pair of features, its covariance matrix using a vectorized implementation that allows to compute instead the the mutual information coefficient (MIC). This method MIC_plot shows features with high correlation for those values of MIC closer to 1.

Once detected highly correlated features, one can do: PCA (using grid search on hyper parameters of the model selection like the # of components in PCA) and they will be naturally gone, or Choose the conjugates coordinates, meaning if x and y are highly correlated choose r= (x+y)/2 and s = (x-y)/2 then r is effectively representing one of them while s is representing the deviation of one from the other. Otherwise, the main feature to attend is how removing or adding these redundant features affects the CV score.

2.4 2. Looking at Y (Column 3 in the test data), what regression models should one first and what are the pros and cons of each?

In terms of regression algorithms, options are, as a summary of the most popular regression algorithms:

Ordinary Least Squares Regression (OLSR)
Linear Regression
Logistic Regression
Stepwise Regression
Multivariate Adaptive Regression Splines (MARS)
Locally Estimated Scatterplot Smoothing (LOESS)
Polynomial Regression
Ridge Regression
Lasso Regression
ElasticNet Regression

1) Linear regression: alone suffers from multicollinearity and is sensitive to outliers. Solved with ordinary least squares.

2) Mixed model regression with condition as a fixed effect on different features and maybe date as a random effect (to estimate and remove variability due to events going on outside of our control, like school holidays or weekends, etc. to improve the chance of observing a difference and to better estimate the predictor Y).

3) Polynomial regression: makes a linear estimation by fitting a nonlinear model to the data (when linear relations among variables may not hold and we will have a more complex model with a second term for x such as y = a0 + bx + cx^2). Non-linear least squares can be used.

4) Maximum likelihood estimation (MLE), a special case of Maximum a posteriori (MAP, which ignores the priors and therefore is unregularized). It is a parametric method requiring mean and variance of an assumed normally distributed variable.

5) Gradient descent and stochastic gradient descent (including random samples out of all the set). In order to model a set of points using a line defined by two parameters, gradient

descent attempts to find the best values for the line's slope m, and y-intercept b parameters, subject to an error function.

6) Other types of regression: Bayesian (using conditional distribution and errors that have a normal distribution, assumes a prior distribution to obtain a posterior probability distribution), Ecological and Robust regression (designed to be not overly affected by violations of assumptions by the underlying data-generating process of other methods). Bayesian regression uses the conditional probabilities and ordinary least squares to estimate the coefficient vector using the Moore-Penrose pseudoinverse.

I would try linear regression models first of all. In order to prevent overfitting, I would apply three different kinds of regularization techniques:

a) Lasso or L1 regularizer

b) Ridge regression or L2 regularizer

c) a weighted balance of L1 and L2 regularizers through Elastic Net regularizer. This method, although takes longer training time and is less maintainable in time, provides the best compromise of both kinds of L1 and L2 regularization techniques.

Ridge regression can't zero out coefficients and thus, you either end up including all the coefficients in the model, or none of them" [4]. However, LASSO does both parameter shrinkage and variable selection automatically. If some of your covariates are highly correlated, Elastic Net may be better instead of the LASSO. Another approach is using the Non-Negative Garotte (NNG) [1] as it's consistent in terms of estimation and variable selection [2]. Unlike LASSO and ridge regression, NNG requires an initial estimate that is then shrunk towards the origin. In the original paper, Breiman recommends the least-squares solution for the initial estimate (you may however want to start the search from a ridge regression solution and use something like GCV to select the penalty parameter) [3]" [4] Ridge regression and Lasso are subset selection methods or shrinkage methods that use least squares to fit a linear model that contains a subset of predictors. Shrinking the coefficient estimates towards zero can significantly reduce the model's variance. Lasso uses an L1 norm penalty of the coefficient vector while Ridge uses an L2 norm. Cross validation for an optimal lambda value election is critical for performance of these models.

Because the standard least squares coefficient estimates are scale equivariant, the ridge regression coefficient estimates can change substantially when multiplying a given predictor by a constant, due to the sum of squared coefficients term in the penalty part of the ridge regression objective function. Therefore, it is best to apply ridge regression after standardizing the predictors [7]. Both methods shrink the coefficient estimates towards zero, however, in Lasso, the L1 penalty forces some coefficient estimates to be zero when the tuning parameter λ is large enough.

Apart from regular grid-search fine-tuning for optimal configuration of the regularization parameters, I would also run a random forest regressor and use the built-in out-of-bag scoring feature for random forest regressor as a manner of applying bootstrapping to the model [6]

Other models one could try are mixed effect models where there would be some random components to capture external factors that are not possible to observe or quantify, and account for seasonality effects, holiday "playing" times, holiday periods when users are more receptive to play games, outside school time/exams time fluctuations, etc.

2.5 How to measure the accuracy and validate the robustness of a model?

Cross validation is the general assessment model procedure. Depending on the model one or other metric will be looked at.

2.5.1 1. In linear regression models normally one computes the coefficient of determination R^2, but also the adjusted R^2, error term, the statistical significance of each feature and Mallow's Cp criterion can be used. The latter compares precision and bias of the full model to models with a subset of predictors. R^2 provides a way to quantitatively measure how well the model explains the data [11]. To compare the ground truth target values with predicted target values by the random forest regressor, scikit-learn doesn't use the MSE but R2 or coefficient of determination #r2_score() computes the coefficient of determination R2, whose best possible score is 1.0, and lower values are worse.

R-squared is the "percent of variance explained" by the model [8] or fraction by which the variance of the errors is less than the variance of the dependent variable. Therefore, from all models tried with different regularization methods, I would keep the one producing best (closer to 1) coefficient of determination R^2 [4].

Other criteria for measuring the relative quality of statistical models are Akaike Information criterion (AIC) (assessed by likelihood function) and the Bayesian Information Criterion (AIC) or Schwarz criterion (SBIC), which is used for model selection among a set of them (lower BIC is preferred).

[1] Breiman, L. Better Subset Regression Using the Nonnegative Garrote Technometrics, 1995, 37, 373-384

[2] Yuan, M. & Lin, Y. On the non-negative garrotte estimator Journal of the Royal Statistical Society (Series B), 2007, 69, 143-161

[3] Zou, H. & Hastie, T. Regularization and variable selection via the elastic net Journal of the Royal Statistical Society (Series B), 2005, 67, 301-320

[4] http://stats.stackexchange.com/questions/866/when-should-i-use-lasso-vs-ridge

[5] Breiman, Leo. Out-of-bag estimation. Technical report, Statistics Department, University of California Berkeley, 1996b. 33, 34.

[6] OOB estimation http://scikit-learn.org/stable/auto_examples/ensemble/plot_ensemble_oob.html

[7] https://lagunita.stanford.edu/c4x/HumanitiesScience/StatLearning/asset/model_selection.pdf

[8] RandomForestRegressor score() returns the coefficient of determination R^2 of the prediction, defined as (1 - u/v), where u is the regression sum of squares ((y_true - y_pred) ** 2).sum() and v is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of y, disregarding the input features, would get a R^2 score of 0.0.

[9] http://www.statsoft.com/textbook/glosp.html#Pearson%20Correlation

[10] Pearson's ρ does not assume normality, but is only an exhaustive measure of association if the joint distribution is multivariate normal. Given the failed normality test in our case, another option can be using Spearman's s or Kendall's tau. Spearman's rho can be less reliable and interpretable than Kendall's while Spearman rho is easier to compute) [Kendall & Gibbons, 1990].

[11] R^2 is measurable using scoring function 'r2' as follows:

```
# LINEAR REGRESSION
lr = LinearRegression(fit_intercept=True, normalize=False, copy_X=True, n_jobs=-1)
score_lr = cross_val_score(lr, x, y, cv=5, scoring='r2')
alpha_vals = [0.1, 0.3, 1.0] # lasso, ridge (alpha in [0,1])
```

```
# LINEAR REGRESSION LASSO Regularization
la = Lasso(alpha=a, fit_intercept=True, normalize=False, precompute=False, copy_X=T
score_la = cross_val_score(la, train, y, cv=5, scoring='r2')
```

```
# LINEAR REGRESSION RIDGE Regression
ri = Ridge(alpha=a, fit_intercept=True, normalize=False, copy_X=True, max_iter=None
score_ri = cross_val_score(ri, train_all, y, cv=5, scoring='r2')
```

[12] http://www.unesco.org/webworld/idams/advguide/Chapt4_2.htm

2.5.2 2. To measure the accuracy and validate the robustness of the model, some goodness-of-fit tests are:

-Chi-square test for continuous and discrete distributions;

-Kolmogorov-Smirnov test for continuous distributions based on the empirical distribution function (EDF);

-Anderson-Darling test for continuous distributions: http://www.oswego.edu/~srp/stats/npp_small.htm Specially the latter, is well suited when the sample size is small, and a histogram does not provide very accurate insights regarding normality (see histograms in notebook).

[1] The Standard Normal curve, shown here, has mean 0 and standard deviation 1. If a dataset follows a normal distribution, then about 68% of the observations will fall within of the mean , which in this case is with the interval (-1,1). About 95% of the observations will fall within 2 standard deviations of the mean, which is the interval (-2,2) for the standard normal, and about 99.7% of the observations will fall within 3 standard deviations of the mean, which corresponds to the interval (-3,3) in this case. Although it may appear as if a normal distribution does not include any values beyond a certain interval, the density is actually positive for all values, . Data from any normal distribution may be transformed into data following the standard normal distribution by subtracting the mean and dividing by the standard deviation. http://www.stat.yale.edu/Courses/1997-98/101/normal.htm

[2] https://docs.scipy.org/doc/scipy-0.14.0/reference/generated/scipy.stats.mstats.normaltest.html

[3] Goodness-of-fit techniques D'Agostino & Stephens (1986).

[4] R2 is a statistic that will give some information about the goodness of fit of a model. In regression, the R2 coefficient of determination is a statistical measure of how well the regression line approximates the real data points. An R2 of 1 indicates that the regression line perfectly fits the data. Values of R2 outside the range 0 to 1 can occur where it is used to measure the agreement between observed and modeled values and where the "modeled" values are not obtained by linear regression and depending on which formulation of R2 is used.

[5] Normality tests: http://sphweb.bumc.bu.edu/otlt/MPH-Modules/BS/R/R4_One-TwoSampleTests-ANOVA/R4_One-TwoSampleTests-ANOVA2.html

2.6 Other methods for model accuracy improvement

In order to improve the accuracy of the model, I would use some of the common feature selection strategies below to identify and remove unneeded irrelevant and redundant attributes from data that do not contribute to the accuracy of a predictive model: a) Filter Methods to score-ranking each feature: Chi squared test, information gain and correlation coefficient scores.

b) Wrapper Methods that select a set of features as a search problem, to find the best set of features with e.g. stochastic methods such as random hill-climbing algorithm, for instance, the recursive feature elimination algorithm.

c) Embedded Methods (into the model), which learn the contribution of each attribute within the model fitting phase, e.g. regularization methods.

d) Perform feature analysis for feature significance and feature relevance using University of Washington Lime package

More concretely in Python scikit-learn, I would use:
-Recursive Feature Elimination (obtains support and ranking for each feature via rfe.support_ and rfe.ranking_ where RFE is a sklearn.features_selection RFE class and model is a (logistic) regression model)
-Feature importance ranking (model.feature_importances_ where model is a ExtraTreesClassifier object rfe = RFE(model, n_features)).
After applying correlation coefficient score and removing features highly correlated, we can improve the linear regressor by including extra features by multiplying some of the most predictive ones and creating non-linear features in this way that can improve the predictive power of the model. Other strategies to try are removing correlated features among themselves or less predictive, visualising the data to find the right model or apply PCA to remove co-linearity.

```
In [ ]:
```