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Контролно 1

вариант 6 зад 1

Нека $L_1 = \{b\}$ и $L_2 = \{a, ba\}$. Използвайки общи конструкции постройте минимален тотален детерминиран автомат разпознаващ езика $(L_1^* . L_2 \cup L_2^*)^*$

$$\Big(\overbrace{b^* . a \cup a^*}^{L_a}\Big)^* \cup \Big(\overbrace{b^* . ba \cup (ba)^*}^{L_{ba}}\Big)^* = L$$

$$\frac{L}{\varepsilon} = L$$

$$\frac{L}{a} = \frac{(b^* a \cup a^*)^*}{a} \cup \frac{(b^* ba \cup (ba)^*)^*}{a} = \frac{b^* a \cup a^*}{a} (b^* a \cup a^*)^*$$

$$\cup \frac{b^* ba \cup (ba)^*}{a} (b^* ba \cup (ba)^*)^* = \frac{b^* a}{a} \cup \frac{a^*}{a} (b^* a \cup a^*)^*$$

$$\cup \frac{b^* ba}{a} \cup \frac{ba}{a} (ba)^* (b^* ba \cup (ba)^*)^* = \underbrace{\frac{b}{a}}_{\emptyset} b^* . a \cup \underbrace{\frac{a}{a}}_{\varepsilon} \cup \underbrace{\frac{a}{a}}_{\varepsilon} a^* (b^* a \cup a^*)^*$$

$$\cup \underbrace{\frac{b}{a}}_{\emptyset} b^* ba \cup \underbrace{\frac{ba}{a}}_{\emptyset} \cup \underbrace{\frac{ba}{a}}_{\emptyset} (ba)^* (b^* ba \cup (ba)^*)^*$$

$$= a^* (b^* a \cup a^*)^* = L' = a^* L_a$$

$$\frac{L}{b} = \Big(\frac{b}{b} b^* a \cup \underbrace{\frac{a}{b}}_{\emptyset} \cup \frac{a}{b} a^*\Big)(b^* a \cup a^*)^* \cup \Big(\frac{b}{b} b^* ba \cup \underbrace{\frac{ba}{b} \cup \frac{ba}{b} (ba)^*}_{\emptyset}\Big)$$

$$\cdot (b^* ba \cup (ba)^*)^* = (b^* a)(b^* a \cup a^*)^* \cup (b^* ba)(b^* ba \cup (ba)^*)^*$$

$$\boxed{= b^* a L_a \cup b^* ba L_{ba} = L''}$$

$$\frac{L}{B} = \underbrace{\frac{(b^*a \cup a^*)^*}{B}}_{\phi} \cup \frac{(b^*Ba \cup (Ba)^*)^*}{B} = \Big(\underbrace{\frac{b}{B}}_{\phi} b^*Ba \cup \underbrace{\frac{Ba}{B}}_{a} \cup \underbrace{\frac{Ba}{B}(Ba^*)}_{a(Ba^*)}\Big)^*$$

$$= (a \cup a(Ba)^*)(b^*Ba \cup (Ba)^*)^* = \boxed{L''' = (a \cup a(Ba)^*)^* L_{Ba}}$$

$$\frac{L'}{a} = \frac{a^*(b^*a \cup a^*)^*}{a} = \underbrace{\frac{a}{a}}_{\varepsilon} a^*(b^*a \cup a^*)^* \cup \frac{(b^*a \cup a^*)}{a}(b^*a \cup a^*)^*$$

$$= a^* L_a \cup a^* L_a = \boxed{a^* L_a = L'}$$

$$\frac{L'}{b} = \underbrace{\frac{a}{b}}_{\phi} a^*(b^*a \cup a^*)^* \cup \frac{b^*a \cup a^*}{b}(b^*a \cup a^*)^* = \Big(\underbrace{\frac{b}{b}}_{\varepsilon} b^*a \cup \frac{a}{b} \cup \frac{a}{b} a^*\Big)^*$$

$$\boxed{= b^*a \; L_a = L^{IV}}$$

$$\frac{L''}{a} = \frac{b^*a(b^*a \cup a^*)^*}{a} \cup \frac{b^*Ba(b^*Ba \cup (Ba)^*)^*}{a}$$

$$\boxed{\frac{L'}{B} = \phi = L^{V}}$$

$$= \Big(\underbrace{\frac{b}{a}}_{\phi} b^*a \cup \underbrace{\frac{a}{a}}_{\varepsilon}\Big)(b^*a \cup a^*)^* \cup \underbrace{\frac{b^*Ba}{a}}_{\phi}(b^*Ba \cup (Ba)^*)^*$$

$$\cup \frac{(b^*Ba \cup (Ba)^*)^*}{a} = \boxed{L_a = L^{VI}}$$

$$\frac{L''}{b} = \frac{b^*a(b^*a \cup a^*)^*}{b} \cup \frac{b^*Ba(b^*Ba \cup (Ba)^*)^*}{b}$$

$$= \frac{b}{b} b^*a(b^*a \cup a^*)^* \cup \underbrace{\frac{a(b^*a \cup a^*)^*}{b}}_{\phi} \cup \frac{b}{b}^{\varepsilon} b^*Ba(*) \cup \underbrace{\frac{Ba^*}{b}}_{\phi}$$

$$\boxed{= b^*a(b^*a \cup a^*)^* \cup b^*Ba(*) = L''}$$

$$\frac{L''}{B} = \frac{b^*a(b^*a \cup a^*)^*}{B} \cup \frac{b^*Ba(b^*Ba \cup (Ba)^*)^*}{B} = \frac{b}{B} b^*Ba(*) \cup \frac{Ba(b^*Ba \cup (Ba)^*)^*}{B}$$

$\emptyset$ $\quad$ $\emptyset$

$$= \frac{b}{B} b^*Ba(b^*Ba \cup (Ba)^*)^* \cup \frac{Ba(b^*Ba \cup (Ba)^*)^*}{B}$$

$\emptyset$

$$= \frac{B}{B} a(*) = \boxed{aL_{Ba} = L^{VII}}$$

$$\frac{L'''}{a} = \frac{(a \cup a(Ba)^*)^*(b^*Ba \cup (Ba)^*)^*}{a} = \frac{a \cup a(Ba)^*}{a}(a \cup a(Ba)^*)^*(b^*Ba \cup (Ba)^*)^*$$

$\varepsilon \cup (Ba)^*$

$$\cup \frac{b^*Ba \cup (Ba)^*}{a}(b^*Ba \cup (Ba)^*)^* = \boxed{(Ba)^* L''' = L^{VIII}}$$

$\emptyset$

$$\frac{L'''}{b} = b^*Ba(b^*Ba \cup (Ba)^*)^* = \boxed{b^*BaL_{Ba} = L^{IX}}$$

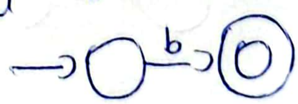
$$\frac{L'''}{B} = \quad \ldots$$ Продължаваме без край

Друг начин. |

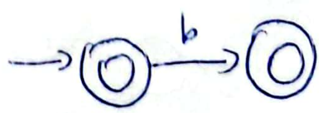
$(b^*a \cup a^*)^* \cup (b^*Ba \cup (Ba)^*)^*$

$(b^*a \cup a^*)^*$

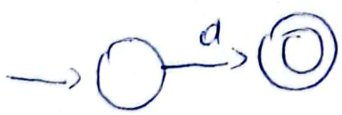
за b



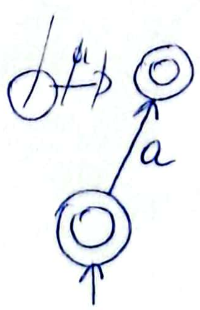
за b^*



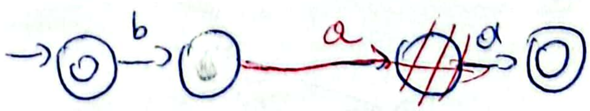
за a



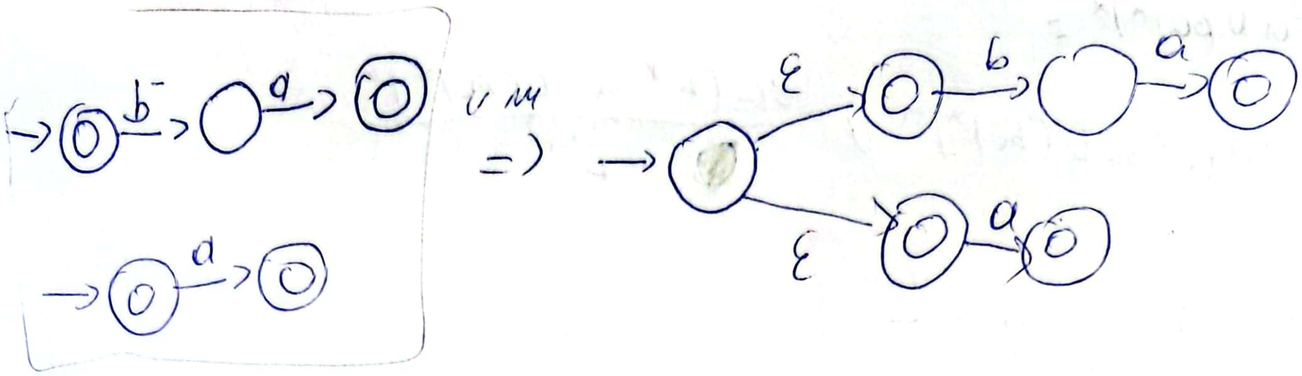
за a^*



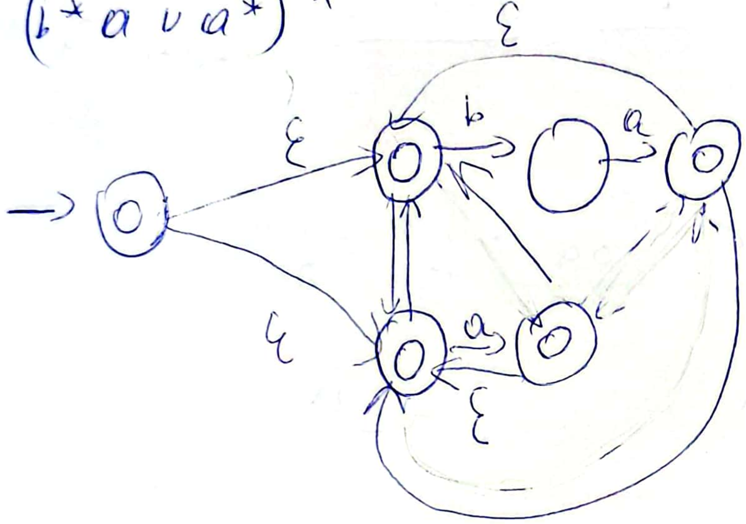
за b^*a



3a $(b^* a \cup a^*)$

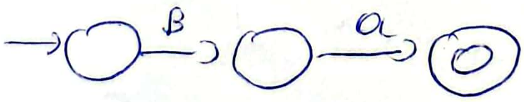


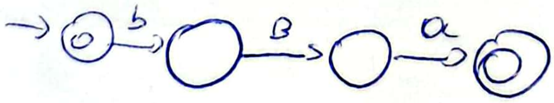
3a $(b^* a \cup a^*)^*$



3a $(b^* Ba \cup (Ba)^*)^*$

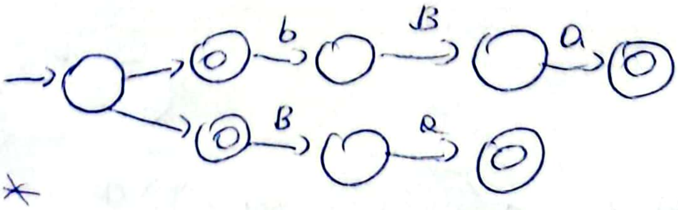
3a b  3a b^* 

3a Ba 

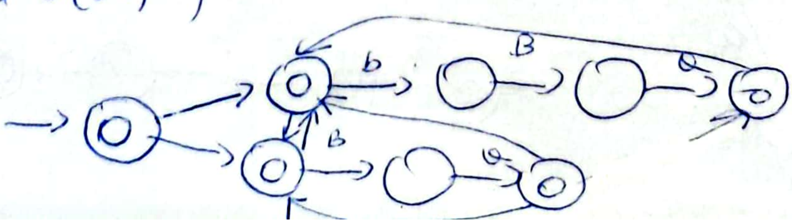
3a b^* Ba 

3a $(Ba)^*$ 

3a b^* Ba $\cup (Ba)^*$



3a $(b^*$ Ba $\cup (Ba)^*)^*$



$\cup \; (b^* a \vee a^*)^*$

52 Вариант 6 Зад 1

Нека $L_1 = \{a, ab\}$ и $L_2 = \{B\}$

$(L_1^* \cup L_1 L_2^*)^*$

$$\boxed{= ((a \cup ab)^* \cup (a \cup ab) B^*)^* = L} \ *$$

$$\frac{L}{\varepsilon} = L$$

$$\frac{L}{a} = \frac{(a \cup ab)^* \cup (a \cup ab) B^*}{a} (*) = \left(\frac{a \cup ab}{a} (a \cup ab)^* \cup \frac{\overset{L_1}{(a \cup ab)} \overset{L_2}{B^*}}{a}\right)^*$$

$$= \left(\underbrace{\frac{a}{a}}_{\varepsilon} \cup \frac{ab}{a}\right)(a \cup ab)^* \cup \underbrace{\frac{a \cup ab}{a}}_{\varepsilon} \cdot B^* \ (*)$$

$$= (b (a \cup ab)^* \cup B^* b) L = L'$$

$$\frac{L}{b} = \left(\frac{(a \cup ab)^*}{b} \cup \frac{(a \cup ab) B^*}{b}\right)^* = \left(\underbrace{\frac{a \cup ab}{b} *'}_{\emptyset} \cup \frac{a \cup ab}{b} \cdot B^*\right)^*$$

$$= \emptyset = L''$$

$$\frac{L}{B} = \frac{(a \cup ab)^* \cup (a \cup ab) B^*}{B} * = \left(\frac{a \cup ab}{B} *' \cup \frac{(a \cup ab) B^*}{B}\right)^*$$

$$= \emptyset = L''$$

$$\frac{L'}{a} = \frac{(b(a \cup ab)^* \cup bB^*) L}{a} = \frac{b(a \cup ab)^*}{a} L \cup \frac{bB^*}{a} L$$

$$= \phi = L''$$

$$\frac{L'}{b} = \frac{b(a \cup ab)^* L}{b} \cup \frac{bB^* L}{b} = \boxed{((a \cup ab)^* \cup B^*) L = L'''}$$

$$\frac{L'}{B} = \phi = L'' \qquad \frac{L''}{a,b,B} = \phi$$

$$\frac{L'''}{a} = \frac{((a \cup ab)^* \cup B^*) L}{a} = \frac{a \cup ab}{a} (a \cup ab)^* L \cup \frac{L}{a} \cup \frac{B^*}{a} L$$

$$\cup \frac{L}{a} = b(a \cup ab)^* L \cup L' \cup \phi \cup L' = \underline{b(a \cup ab)^* L \cup L'}$$

$$= L^{IV}$$

$$\frac{L'''}{b} = \left(\frac{(a \cup ab)^* \cup B^*}{b}\right) L \cup \frac{L}{b} = \left(\frac{a \cup ab}{b} (a \cup ab)^* \cup \frac{B^*}{b}\right) L \cup \frac{L}{b}$$

$$= \boxed{\phi = L''}$$

$$\frac{L'''}{B} = \phi \cup \frac{B^*}{B} L \cup \frac{L}{B} = B^* L \cup \phi = \boxed{B^* L = L^{V}}$$

$$\frac{L^{IV}}{a} = \frac{b(a \cup ab)^* L \cup L'}{a} = \boxed{L'' = \phi}$$

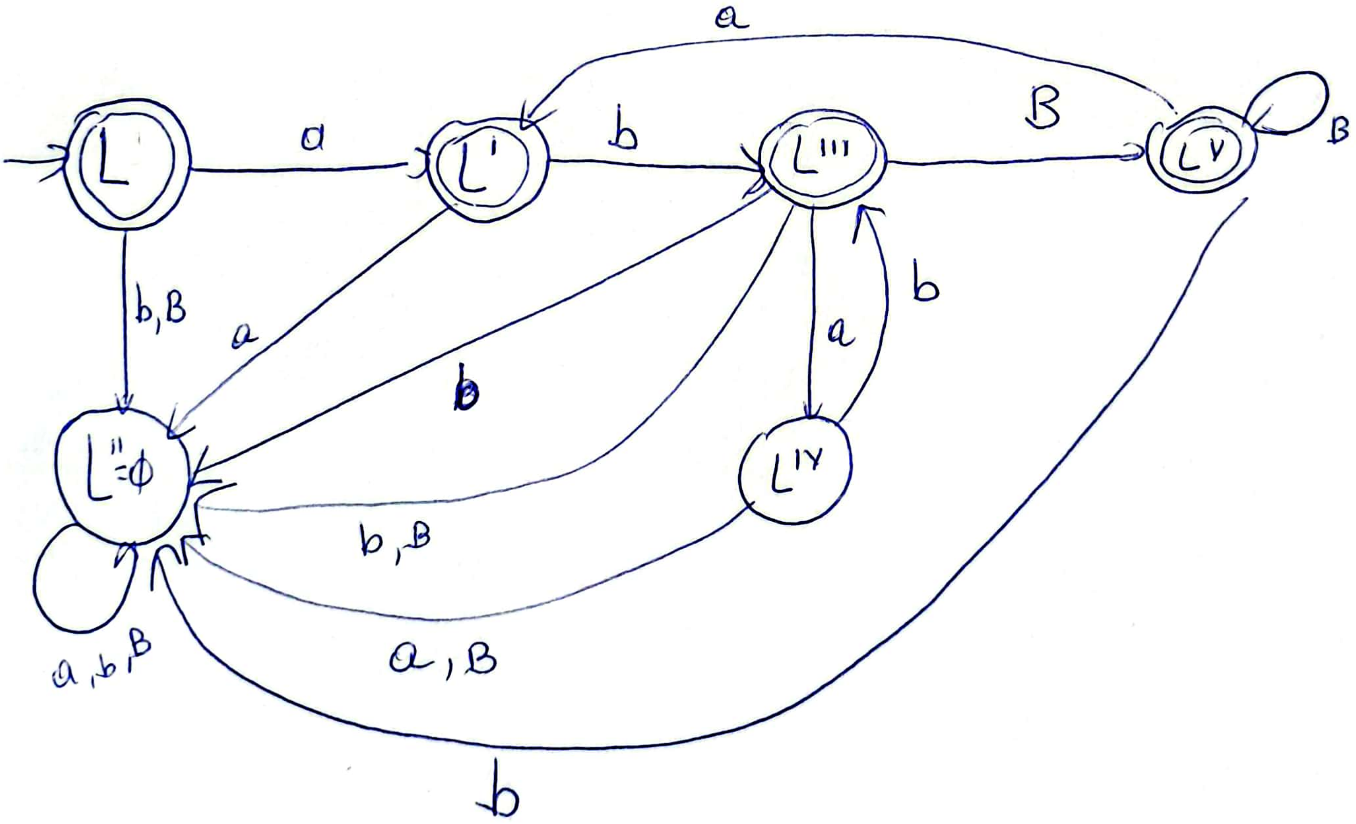
$$\frac{L^{IV}}{b} = \frac{\overset{\varepsilon}{b}(a \cup ab)^* L \cup L'}{b} = (a \cup ab)^* L \cup L''' = L'''$$

$$\frac{L^{IV}}{a} = \frac{b(a \cup ab)^* L \cup L'}{a} = \phi$$

$$\frac{L^{IV}}{B} = \phi \cup \phi = \phi$$

$$\frac{L^{V}}{a} = \frac{B^* L}{a} = \underbrace{\frac{B}{a} B^* L}_{\phi} \cup \frac{L}{a} = L'$$

$$\frac{L^{V}}{b} = \phi \qquad \frac{L^{V}}{B} = \frac{B}{B} B^* L \cup \frac{L}{B} = B^* L \cup \phi = L^{V}$$



(53) Вариант 1 зад. 1

$L_1 = \{a, ab\}$ $L_2 = \{B\}$

$$(L_1^* \cup L_1 \cdot L_2^*)^* = ((a \cup ab)^* \cup (a \cup ab) \cdot B^*)^* = L$$

$$\frac{L}{\varepsilon} = L \qquad \frac{L}{a} = \left(\frac{(a \cup ab)^*}{a} \cup \frac{(a \cup ab)\, B^*}{a}\right) L$$

$$= \left(\frac{a \cup ab}{a} (a \cup ab)^* \cup \frac{a \cup ab}{a} B^*\right) L$$

$$\boxed{= (b\,(a \cup ab)^* \cup bB^*)\, L = L'}$$

$$\frac{L}{b} = \left(\frac{a \cup ab}{b}(a \cup ab)^* \cup \frac{(a \cup ab)\, B^*}{b}\right) L = \boxed{\phi = L''}$$

$$\frac{L}{B} = \phi = L''$$

$$\frac{L'}{a} = \frac{(b\,(a \cup ab)^* \cup bB^*)\, L}{a} = \left(\frac{b\,(a \cup ab)^*}{a} \cup \frac{bB^*}{a}\right) L$$

$$= \phi = L''$$

$$\boxed{\frac{L'}{b} = ((a \cup ab)^* \cup B^*)\, L = L'''} = L \cup B^* L$$

$$\frac{L'}{B} = \phi \qquad \boxed{\frac{L''}{a, b, B} = \phi} \qquad \frac{L'''}{a} = \frac{(a \cup ab)^* L}{a} \cup \frac{B^* L}{a} =$$

$$\left(\frac{a \cup ab}{a}\right)(a \cup ab)^* L \cup \frac{L}{a} \cup \frac{BB^* L}{a} \cup \frac{L}{a}$$

$$= b\,(a \cup ab)^* L \cup L' \cup \phi \cup L' = L'$$

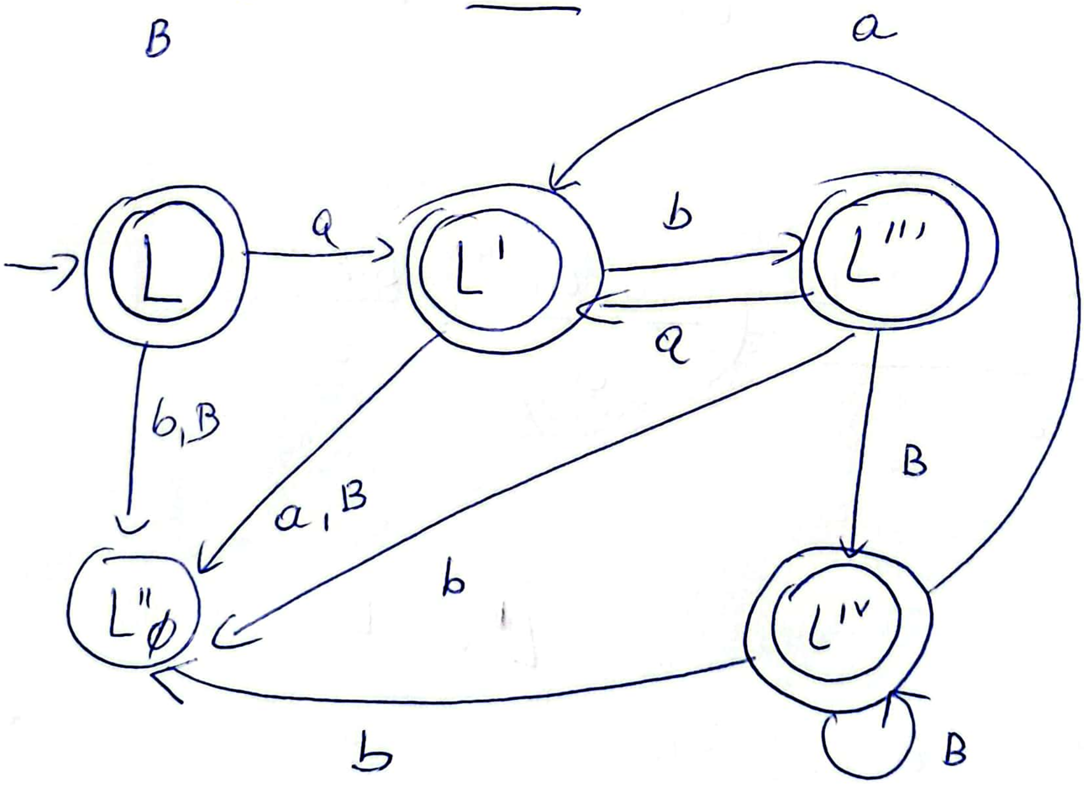
$$\frac{L'''}{b} = \frac{L \cup B^*L}{b} = \phi = L''$$

$$\frac{L'''}{B} = \frac{L \cup B^*}{B} = \phi \cup B^*L = B^* \cup \frac{L}{B} = \underline{B^*L = L^{IV}}$$

$$\frac{L^{IV}}{a} = \frac{B^*L}{a} = \underbrace{\frac{B}{a}}_{\phi} B^*L \cup \frac{L}{a} = L'$$

$$\frac{L^{IV}}{b} = \phi$$

$$\frac{L^{IV}}{B} = B^*L = \underline{L^{IV}}$$



Вариант 2 зад. 3 54

$L = \{a^n b^k \mid n + k \equiv 1 \pmod 3)$ или $n \geq 2k\}$ не е рег.

1. сл. $n+k \equiv 1 \pmod 3)$ тоест $n+k$ се дели на 3 и има остатък 1.

=> Нека ∃ число $p > 0$ такова че $p = n$.

Тогава $p+k$ трябва да се дели на 3 с остатък 1.

=> за k избираме $k = 2p+1$ така $p + 2p + 1 = 3p+1$ се дели на 3 с остатък 1.

=> $w \in L$ => $w = a^p b^{2p+1}$ и ∃ разбиване $w = xyz$

Т.4 $|xy| \leq p$, $|y| \geq 1$ => часта xy е изцяло от а-та.

$$\Rightarrow w = \underbrace{a^{|x|}}_{x}\underbrace{a^{|y|}}_{y}\underbrace{a^{p-|x|-|y|} b^{2p+1}}_{z}$$

$$\Rightarrow w_i = a^{|x|} a^{i|y|} a^{p-|x|-|y|} b^{2p+1}$$

$$\Rightarrow w_i = a^{\cancel{|x|} + i|y| + p - \cancel{|x|} - |y|} b^{2p+1}$$

$$= a^{p+(i-1)|y|} b^{2p+1}$$

Нека $i = 0$ => $w_0 = a^{p-|y|} b^{2p+1}$

=> $p - |y| + 2p+1$ трябва да се дели на 3 с ост. 1

но $3p+1-|y| < 3p+1$ => $w_0 \notin L$ => L не е регулярен

2 сл. $w = a^n b^k$ и $n \geq 2k$

Нека $n = 2k \Rightarrow w = a^{2k} b^k$

Нека $\exists$ число $p > 0$ т.ч. $p = k \Rightarrow w = a^{2p} b^p$ такова че $w = xyz$ и

и $\exists$ разбиване на w

$|y| \geq 1$ и $|xy| \leq p \Rightarrow$ частта xy ще бъде запълнена с а-та

$$\Rightarrow w = \underbrace{\overbrace{a\ldots}^{xy}\ldots aa}_{p}\ \underbrace{a\ldots\ldots a}_{p}\ \underbrace{b\ldots\ldots b}_{p}$$

$\Rightarrow w_i = xy^iz$ за $\forall i \geq 0$.

$w_i = a^{|x|} a^{i|y|} a^{2p-|x|-|y|} b^p \Rightarrow a^{\cancel{|x|}+i|y|+2p-\cancel{|x|}-|y|} b^p$

$= a^{2p+(i-1)|y|} b^p \Rightarrow$

Нека $w_0 = a^{2p-|y|} b^p$

$2p - |y| \geq 2p$ не е вярно

защото $2p - |y| < 2p \Rightarrow w_0 \notin L \Rightarrow L$ не е регулярен.

Вариант 3 зад. 3

$L = \{a^n b^k \mid n+k \equiv 1 \pmod 3$ или $n < 2k\}$ не е регулярен

1. сл. $n+k \equiv 1 \pmod 3$

означава че $n+k$ трябва да се дели на 3 с остатък 1.

$\exists p > 0$ т.че $n = p$ и $k = 2p+1$

защото $p + 2p + 1 = 3p+1$ се дели на 3 и има остатък 1.

$w = a^p b^{2p+1}$ Тогава $\exists$ разбиване на

$w = xyz$ и $|xy| \leq p$, $|y| \geq 1$ и $w^i = xy^iz$

Нека частта xy съдържва само a-та.

$$w = \overbrace{a \ldots .}^{x\cancel{y}} \overbrace{a\, b \ldots \ldots \ldots b}^{z}$$

$$\underbrace{a \ldots . a}_{p} \; \underbrace{b \ldots \ldots b}_{2p+1}$$

$$w = \underbrace{a^{|x|}}_{x} \underbrace{a^{i|y|}}_{y} \underbrace{a^{p-|x|-|y|} b^{2p+1}}_{z}$$

$$w^i = a^{\cancel{|x|} + i|y| + p - \cancel{|x|} - |y|} b^{2p+1}$$

$w = a^{p+(i-1)|y|} b^{2p+1}$ при $w_0 = a^{p-|y|} b^{2p+1}$

$= 3p + 1 - |y|$ ... се дели

то трябва $p - |y| + 2p + 1$

на 3 состатък 1 но $3p + 1 - |y| < 3p + 1$

$\Rightarrow w_0 \notin L$ не е рег.

2. сл. $n < 2k$

Нека $n = 2k - 1$

Тогава $a^{2k-1} b^{2k}$

Нека $\exists\, p > 0$ такова че $k = p$

$n = 2p - 1$

$a^{2p-1} b^{2p}$

$w = a^{2p-1} b^{2p}$ и $w \in L$

$\exists$ разбиване на $w = xyz$

$|xy| \leq p$ и $|y| \geq 1$ и $w_i = xy^i z$

$$\underbrace{a \dots a}_{2p-1} \overbrace{\underbrace{b \dots b}_{p} \underbrace{b \dots b}_{p}}^{2p}$$

часта xy съдържа само a-те

$w_i = a^{|\cancel{x}|} a^{i|y|} a^{2p-1-|x|-|y|} b^{2p}$

$w_i = a^{i|y| + 2p - 1 - |y|} b^{2p}$

$w_i = a^{2p + (i-1)|y| - 1} b^{2p}$

и $2p + (i-1)|y| - 1$ трябва $< 2p$

при $w_2 = 2p + |y| - 1 > 2p \Rightarrow w_2 \notin L$

не е рег.

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Вариант 1 зад. 3

$L = \{a^n b^k \mid n+k \equiv 0 \pmod 3$ или $2n < k\}$ не е рег.

<u>1. случай</u> $n+k$ се дели на 3 и остатъка $=0$

$\Rightarrow$ Нека $\exists\, p > 0$ такова че $p = n$

Тогава $k = 2p$ защото $p + 2p = 3p$ се дели на 3

и остатък $= 0$.

$\Rightarrow w \in L$. Тогава $w = a^p b^{2p}$ и $\exists$ разбиване на

$w = xyz$ където $|xy| \leq p$ $|y| \geq 1$

$\Rightarrow$ часта xy съдържа само a-та

$$\Rightarrow w = \underbrace{a^{|x|}}_{x}\underbrace{a^{|y|}}_{y}\underbrace{a^{p-|x|-|y|}\, b^{2p}}_{z}$$

$$w_i = a^{i|y| + p - |y|}\, b^{2p}$$

$p + (i-1)|y| + 2p$ трябва да се дели на 3 ост. $= 0$.

при $i=0$ имаме $p - |y| + 2p = 3p - |y| < 3p \Rightarrow w_0 \notin L$

<u>2 случай</u> $2n < k$

избиране $k = 2n+1 \Rightarrow \exists\, p > 0$ $p = n \Rightarrow$ имаме $a^p b^{2p+1}$

$w = xyz$ $|xy| \leq p$ $|y| \geq 1$ и $w_i = xy^iz$

xy съдържа само a-та.

$$w_i = a^{|x| + i|y| + p - |x| - |y|}\, b^{2p+1}$$

$$w_i = a^{p + (i-1)|y|}\, b^{2p+1}$$

$\Rightarrow$ $2p + 2(i-1)|y|$ трябва $< 2p+1$ ($\overline{p + (i-1)|y|}$)

при $i = 2$ имаме $2p + 2|y|$ и $|y| \geq 1 \Rightarrow 2p + 2|y| \not< 2p+1$ $w_2 \notin L$

Вариант 4 зад 3.

$L = \{a^n b^k \mid n+k \equiv 2 \pmod 3$ или $2n \geq k\}$ не е рег.

1. сл. $n+k \equiv 2 \pmod 3$

$\exists p$ т. че $n = p \Rightarrow k = 2p+2$ защото $p + 2p + 2 = 3p + 2$ се дели на 3 ост. $= 2$.

$w = a^p b^{2p+2} \in L$ тогава $w = xyz$, $|xy| \leq p$, $|y| \geq 1$.

xy съдържа само a.

$a^{|x|} a^{|y|} \, a^{p-|x|-|y|} \, b^{2p+2}$

$w_i = a^{p+(i-1)|y|} b^{2p+2}$

при $w_0 = a^{p-|y|}$ но $|y| \geq 1$

но $p - |y| + 2p + 2 = 3p - |y| + 2 < 3p + 2$

$\Rightarrow w_0 \notin L$.

xy съдържа само b

$\underbrace{a \ldots a \, b \ldots}_{z} \underbrace{\ldots b}_{xy}$

$w = \underbrace{a^p}_{z} \underbrace{b^{|x|} \cdot b^{|y|}}_{xy} \cdot \underbrace{b^{2p+2-|x|-|y|}}_{z}$

$w_i = a^p b^{2p+2+(i-1)|y|}$ при $i = 0$ имаме $2p + 2 - |y|$ но $|y| \geq 1$

тогава $p + 2p + 2 - |y| = 3p + 2 - |y| < 3p + 2 \Rightarrow w_0 \notin L$

xy съдържа и a и b

$$\overbrace{a \ldots\ldots a}^{p}\overbrace{b \ldots\ldots\ldots b}^{2p+2}$$

$$\underbrace{a\ldots\ldots}_{z}\ \underbrace{a\,b}_{xy}\ \underbrace{\ldots\ldots\ldots b}_{z}$$

$$w = a^{|x|} \ldots a^{i|y|} b^{|x|} b^{i|y|} b^{2p+2-|x|-|y|} a^{p-|x|-|y|}$$

$$w_i = a^{\cancel{|x|} + i|y| + p - \cancel{|x|} - |y|} b^{\cancel{|x|} + i|y| + 2p - \cancel{|x|} - |y|}$$

$$= a^{p+(i-1)|y|} b^{2p+(i-1)|y|}$$

при $i=0$ имаме $p-|y| + 2p - |y| = 3p - 2|y|$ но $|y| \geq$

$\Rightarrow 3p - 2|y| < 3p+2 \Rightarrow w_0 \notin L$

2 сл $2n \geq k$ тогава имаме $k = 2n$

$\exists\ p = n$

$a^p b^{2p}$ $\exists\ w \in L = a^p b^{2p}$ $w = xyz$

$w_i = xy^iz$

xy съдържа само b. $|xy| \leq p$ $|y| \geq 1$

$$\overbrace{a \ldots\ldots a}^{z}\overbrace{b \ldots b}^{xy}\overbrace{b \ldots b}^{z}$$

$$\underbrace{a\ldots\ldots a}_{p}\ \underbrace{b\ldots\ldots b\,b\ldots\ldots b}_{2p}$$

$$w_i = a^p b^{|x|} b^{i|y|} b^{2p-|x|-|y|}$$

$$w_i = a^p b^{2p+(i-1)y}$$

при $i=0$ $2p - |y| < 2p$ защото $|y| \geq 1 \Rightarrow w_0 \notin L$

при $i=2$ $2p + |y| > 2p$ се изпълнява.

(57)

Вариант 5 зад 3

$L = \{a^n b^k \mid n+k \equiv 0 \pmod 3$ или $n > 2k\}$

1 сл. $n+k \equiv 0 \pmod 3$ $\exists p \geq 0 \Rightarrow n = p$

$k = 2p$ защото $p + 2p = 3p \equiv 0 \pmod 3$

$a^p b^{2p}$ xy съдържат само b

$w_i = a^p b^{2p - \cancel{|x|} - |y| + \cancel{|x|} + i|y|}$

от $w_i \Rightarrow 2p + (i-1)|y| + p = 3p + (i-1)|y|$

при $i = 0$ имаме $3p - |y|$ но $|y| \geq 1$

$3p - |y| < 3p \Rightarrow w_0 \notin L$

2 сл $n > 2k \Rightarrow n = 2k+1$ $\exists p = k$

$a^{2p+1} b^p$ xy съдържат само a

$w_i = a^{\cancel{|x|} + i|y| + 2p+1 - \cancel{|x|} - |y|} b^p \Rightarrow 2p+1 + (i-1)|y|$

при $i = 0$ $2p+1-|y|$ и $|y| \geq 1$

$2p + 1 - |y| < 2p+1 \Rightarrow w_0 \notin L$

трябва да е $> 2p$

(39) $L = \{a^n b^k \mid n+k \equiv 2 \pmod 3$ или $2n \leq k\}$ Вариант 6

Зад 3.

1. сл. $n+k \equiv 2 \pmod 3$

$\exists p = n > 0$ за което $a^p b^{2p+2}$

$|xy|$ съдържа само b

$w_i = a^p b^{2p+2-\cancel{|x|}-|y|+\cancel{|x|}+i|y|}$

$= a^p b^{2p+2+(i-1)|y|}$

при $i=0$ имаме $p+2p+2-|y|$ и $|y| \geq 1$

$\Rightarrow = 3p+2-|y| < 3p+2 \Rightarrow w_0 \notin L$

2 сл $2n \leq k$

$k = 2n \quad \exists p = n$

$a^p b^{2p}$

xy съдържа b.

$w_i = a^p b^{2p-\cancel{|x|}-|y|+|x|+i|y|}$

$\Rightarrow a^p b^{2p+(i-1)|y|}$

при $i=0$ имаме $2p-|y|$ и $|y| \geq 1$

$2p - |y| < 2p \Rightarrow w_0 \notin L.$

(59) друг начин

$L = \{a^n b^k c^m d^l \mid n > 2m$ или $k < l\}$

Нека $\exists$ число $p > 0 : n = 2m + p$

$q > 0 : l = k + q$

$L = \{a^{2m+p} b^k c^m d^{k+q} \mid m, p, k, q \geq 0\}$

$L_1 = \{a^{2m+p} b^k c^m d^l\}$

$\underbrace{a^p}_{P} a^{2m} b^k c^m d^l$

За $P = \{a^p \mid p \geq 0\}$ нека $w \in P \Rightarrow w = a^p$

при $p = 0 \Rightarrow w = \varepsilon$ при $p > 0$ $w = a^p$

$w = \varepsilon$ или $w = a^p$ нека $p = z + 1$ | $w = \varepsilon$, $w = \underbrace{a^z}_{P} a$ $\Rightarrow$ $P \to \varepsilon$, $P \to Pa$

аналогично и за другите

за $K \to \varepsilon \mid Kb$

за $L' \to \varepsilon \mid dL$

за $M \to,$

$a^{2m} w^k c^m$ за $m = 0$ w^k $M \to K$

за $m > 0$ нека $\exists z$ т.ч. $m = z + 1$

$a^{2z+2} w^k c^{z+1} \Rightarrow a^2 \overbrace{a^{2z} w^k c^z}^{M} c$

$\to M \to \alpha^2 Mc / K$

$\to \int \to KL^1 MP$

$= € $

или не бо

59

Вариант 1 контролно 2 зад. 1

докажете че L е безконтекстен.

$$L=\{a^n b^k c^m d^l \mid n>2m \ \text{или} \ k<l\}$$

$\vee$ $\cup$

$$L_1=\{a^n b^k c^m d^l \mid n>2m\}$$

$$L_2=\{a^n b^k c^m d^l \mid k<l\}$$

за $L_1=\{a^{2m+1} b^k c^m d^l \mid n>2m\}$

$$= \underline{a^{2m}}\, a\, \underline{b^k}\, \underline{c^m}\, \underline{d^l}$$

$K \to bK \mid \varepsilon$

$L \to dL \mid \varepsilon$

$M \to a^2Mc \mid aK$

$S' \to KLM$

за $L_2=\{a^n \underline{b}^k c^m \underline{d}^{k+1} \mid k<l\}$

$N \to aN \mid \varepsilon$

$M' \to cM \mid \varepsilon$

$K \to bKd \mid Md$

$S'' \to N \cdot M' K$

$$\underline{L \to S' \cup S''}$$

(60)

Вариант 2 зад 1

$L = \{a^n b^k c^m d^l \mid n < m$ или $k > 2l\}$

$L_{12} = \{a^n b^k c^m d^l \mid m = n+1 \wedge k = 2l+1\}$

$\underbrace{a^n b^k c^{n+1}}d^l$ $\quad$ $a^n \underbrace{b^{2l+1} c^m d^l}$

$K \to bK \mid \varepsilon$

$N \to aNc \mid Kc$

$L_1 \to dL \mid \varepsilon$

$S_1 = KNL_1$

$N' = aN \mid \varepsilon$

$M = cM \mid \varepsilon$

$L' = b^2Ld \mid bM$

$S_2 = N'ML'$

$\boxed{L = S_1 \cup S_2}$

(61)

Вариант 3 зад 1

$L = \{a^n b^k c^m d^l \mid 2n < m$ или $l < k\}$

$L_1 = \underbrace{a^n b^k c^{2n+1}}d^l \quad \cup \quad L_2 = a^n \underbrace{b^{l+1} c^m d^l}$

$K \to bK \mid \varepsilon$

$N \to aNc^2 \mid Kc$

$D \to dD \mid \varepsilon$

$L_1 = KND$

$A \to aA \mid \varepsilon$

$M \to cM \mid \varepsilon$

$B \to bBd \mid bM$

$L_2 \to AMB$

$L = L_1 \cup L_2$

62 Вариант 4 зад 1

$L = \{a^n b^k c^m d^l \mid m < n \text{ или } 2k < l\}$

$\underbrace{a^{m+1} b^k c^m}\ d^l$ $\qquad$ $a^n\ \underbrace{b^k c^m d}^{2k+1}$

$K \to bK \mid \varepsilon$

$M \to aMc \mid aK$

$D \to dD \mid \varepsilon$

$S' \to KMD$

$N \to aN \mid \varepsilon$

$M' \to cM' \mid \varepsilon$

$K' \to bK'd^2 \mid Md \mid \varepsilon$

$S'' \to NM'K'$

$S \to S' \cup S''$

63 Вариант 5 зад 1

$L = \{a^n b^k c^m d^l \mid m > 2n \text{ или } l > k\}$

$\underbrace{a^n b^k c}^{2n+1}\ d^l$ $\qquad$ $a^n\ \underbrace{b^{k\ldots} c^m d}^{k+1}$

$N \to aNc^2 \mid Kc$

$K \to bK \mid \varepsilon$

$L \to dL$

$S_1 \to NKL$

$M \to cM \mid \varepsilon$

$N \to aN \mid \varepsilon$

$K' \to bK'd \mid Md \mid \varepsilon$

$S_2 \to MNK'$

$S \to S_1 \cup S_2$

(64)

Вариант 6 зад 1

$$L = \{a^n b^k c^m d^l \mid m > n \text{ или } l > 2k\}$$

$a^n b^k c^{n+1} d^l$

$K \to bK \mid \varepsilon$

$N \to aNc \mid Kc$

$L \to dL$

$S_1 \to KNL$

$a^n b^k c^m d^{2k+1}$

$N' \to aN' \mid \varepsilon$

$K' \to bK'd^2 \mid Md \mid \varepsilon$

$M \to cM \mid \varepsilon$

$S_2 \to N'K'M$

$S \to S_1 \cup S_2$

Вариант 1 зад. 2

Нека $\Sigma = \{a,b\}$ за дума $\alpha \in \Sigma^*$ и буква $x \in \Sigma$ означаваме с $|\alpha|_x$ броя на срещанията на x в α.

За произволен език $L \subseteq \Sigma^*$ дефинираме езика

$$s(L) = \{a^{2n}\alpha b^m \mid \alpha \in L \ \& \ |\alpha|_a = n \ \& \ |\alpha|_b = 2m\}$$

докажете че $\exists$ рег. език L такъв че езикът $s(L)$ не е безконтекстен.

$$\overbrace{\underbrace{a\ldots\ldots a}_{n}\underbrace{a\ldots\ldots a}_{n}}^{a^{2n}}\underbrace{\underbrace{a\ldots\ldots a}_{n}\overbrace{\underbrace{b\ldots b}_{m}\underbrace{b\ldots b}_{m}}^{b^{2m}}}_{\alpha}\underbrace{b\ldots\ldots b}_{m}$$

Нека допуснем че $s(L)$ е КС. $\Rightarrow \exists p>0$ е свидетел за безконтекстността.

$w \in s(L)$ $|w| \geq p$ то $\exists$ $w = uvxyz$ т.ч $|vxz| \leq p$ $|vy| > 0$ и за $\forall i \geq 0$ $w_i = uv^ixy^iz \in L$

$$a^{2n}a^nb^{2m}b^m \to a^{3n}b^{3m} \to a^{3p}b^{3p}$$

2 случая

$n \leq m$ или $n \geq m$ и за това имаме $a^{3p}b^{3p}$

1.

$$\overbrace{\underbrace{a\ldots\ldots a}_{2n}}^{w_1}\underbrace{ababab}_{\alpha}\overbrace{b\ldots\ldots b}^{w_2}$$

$|w_1|_a = 2n$ $|w_1|_b = 0$ $|w_1| = 2n = 2p$

$|\alpha|_a = n \quad |\alpha|_b = 2m \qquad |\alpha| = n + 2m = p + 2p = 3p$

$|w_2|_a = 0 \qquad |w_2|_b = m \qquad |w_2| = p$

$$|w| = |w_1| + |\alpha| + |w_2| = 2p + 3p + p = \underline{6p}$$

където $|w|_a = 2p + p + 0 = 3p$
$|w|_b = 2p + p = 3p$.

1. случай поне едно от y или v има 2 типове от символи.

$$\underbrace{a \ldots\ldots a\, a\, b}_{u}\ \underbrace{a\, b}_{v}\ \underbrace{a\, b}_{y}\ \underbrace{a\, b\, b \ldots\ldots b}_{z}$$

$2p_a \qquad p_a \quad 2p_b \qquad p_b$

демек y и v са в 2-я блок в $\underline{\alpha}$

$w_i = u\, v^i\, x\, y^i\, z$ при $i = 2$ се увеличава броят на a и на b с 2

$$a \ldots\ldots a\, a\, \underbrace{b\, a\, b\, a\, b\, a\, b}_{\alpha}\, b \ldots\ldots b$$

$|\alpha| = 3p$ но при $i = 2$ $|\alpha|_a = n+1$ $|\alpha|_b = 2m+1$

$|\alpha| = p + 1 + 2p + 1 = 3p + 2 > 3p$

$\Rightarrow w_2 \notin L$

2 сл. ∀ символи в y са еднакви и в v са еднакви

2.1 и y и v съдържат само a-та. b не участва в |yv|

$$\overbrace{a\ldots\ldots}^{u}\overbrace{a}^{y}\;\overbrace{a\ldots\ldots a}^{v}\;\overbrace{b\ldots\ldots}^{x}\overbrace{\ldots b\;\; b\ldots b}^{z}$$

$$\underbrace{a\ldots\ldots a}_{2p_a}\;\underbrace{a\ldots\ldots a}_{p_a}\;\underbrace{b\ldots\ldots\ldots b}_{2p_b}\;\underbrace{b\ldots b}_{p_b}$$

$$|w_i|_b = |xy^iuv^iz|_b + (i-1)|yv|_b$$

но $|yv|_b = 0 \Rightarrow |w_i|_b = |xyuvz|_b = |w|_b =$

Ако $|yv|_a > 0$ то $|w_2|_a = |w|_a + |yv|_a = 3p + 1$

но от условие $|w|_a = |w|_b$

$\Rightarrow w_2 \notin L$

3 сл. a не участва в yv.

аналогично като 2-случай.

(66)

Вариант 1 зад. 3

Нека $M \subseteq \Sigma^*$ е произволен регулярен език и $\# \notin M$. докажете че езикът

$L = \{ \alpha_1 \ldots \alpha_n \# \beta_1 \ldots \beta_n \mid n \in N \ \& \ \forall i \in \{1 \ldots n\}$

$\alpha_i \beta_{n-i+1} \in M \}$ е безконтекстен.

Нека $w \in \{ \alpha^{n_i} \# \beta^n \mid n \in IN \mid \alpha_i \beta_{n-i+1} \in M \}$

$\alpha_i \beta_{n-i+1}$ $L(\Gamma_1) = L$

$n=4$

$\alpha_{1\,2\,3\,4} \underbrace{\beta_4 \beta_3 \beta_2 \beta_1}_{\beta^{rev}}$

$\Rightarrow)$

$S \to a^n S b^n \to aaSbb \to \underbrace{aaa}_{n} S \underbrace{bbb}_{n}$

$\xrightarrow{\varepsilon} \alpha^n \beta^n$ е изводим в Γ_1 откъдето $w \in L(\Gamma_1)$

$S \to a^{n_i} S b^{rev_i} \to aa S\beta\beta\beta \to \ldots \to a^n \beta^{rev}_n$

$\Rightarrow w \in L(\Gamma_1)$

$\Leftarrow)$ има но точно един нетерминал S като заменяте по тях могат да и осъществят единствено с помощта на правилото $S \to \alpha S \beta$. Но нека α_n има S но w_n няма то заменената $\alpha_n \to w_n$ може да стане само с помощта на правилото $S - \varepsilon$

$\Rightarrow L(L_1) \subseteq L$.