(102) да се построи минимален КДА A с език $L(A)=\{a,b,c\}^* \circ \{ab, ba\} \circ \{a,b,c\}^*$

$\frac{L}{\varepsilon} = L$

$L = \{a \cup b \cup c\}^* \circ \{ab \cup ba\} \circ \{a \cup b \cup c\}^*$

$\frac{L}{a} = \frac{\{a \cup b \cup c\}^* \circ \{ab \cup ba\} \circ \{a \cup b \cup c\}^*}{a} = \frac{\{a \cup b \cup c\}}{a} \{a \cup b \cup c\}^* \circ$

$\{ab \cup ba\} \circ \{a \cup b \cup c\}^* \cup \frac{ab \cup ba}{a} \{a \cup b \cup c\}^*$

$= (\varepsilon \cup \phi \cup \phi) \Rightarrow \varepsilon \circ \{a \cup b \cup c\}^* \circ \{ab \cup ba\} \circ \{a \cup b \cup c\}^* \cup b\{a \cup b \cup c\}^*$

$= \boxed{L \cup b\{a \cup b \cup c\}^* = L_1}$

$\frac{L}{b} = \boxed{L \cup a\{a \cup b \cup c\}^* = L_2}$

$\frac{L}{c} = \boxed{L}$

$\frac{L_1}{a} = \boxed{\frac{L}{a} \cup \frac{b\{a \cup b \cup c\}^*}{a} = L_1}$ (ϕ)

$\frac{L_1}{b} = \boxed{\frac{L}{b} \cup \{a \cup b \cup c\}^* = L_3}$

$\frac{L}{b} = L_2$, $= L_2 \cup \{a \cup b \cup c\}^*$

$\frac{L_1}{c} = \frac{L}{c} \cup \frac{b\{a \cup b \cup c\}^*}{c} = L$

$\frac{L}{c} = L$, $\frac{b\{a \cup b \cup c\}^*}{c} = \phi$

$\frac{L_2}{a} = \frac{L}{a} \cup \frac{a\{a \cup b \cup c\}^*}{a}$

$= L_1 \cup \{a \cup b \cup c\}^* = L_4$

$\frac{L_2}{b} = \frac{L}{b} \cup \phi = L_2$

$\frac{L_2}{c} = \frac{L}{c} \cup \phi = L$

$\frac{L_3}{a} = \frac{L_2}{a} \cup \frac{a \cup b \cup c}{a} \{a \cup b \cup c\}^*$

$= L_4 \cup \{a \cup b \cup c\}^* = L_4$

$L_1 \cup \{a \cup b \cup c\}^*$

$\frac{L_3}{b} = \frac{L_2}{b} \cup \frac{a \cup b \cup c}{b} \{a \cup b \cup c\}^*$

$= L_2 \cup \{a \cup b \cup c\}^* = L_3$

$$\frac{L_3}{c} = \frac{L_2}{c} \cup \frac{\{a \cup b \cup c\}^*}{c} = \boxed{L \cup \{a \cup b \cup c\}^* = L_5}$$

$$\frac{L_4}{a} = \frac{L_1}{a} \cup \frac{\{a \cup b \cup c\}^*}{a} = L_1 \cup \{a \cup b \cup c\}^* = L_4$$

$$\frac{L_4}{b} = \frac{L_1}{b} \cup \frac{\{a \cup b \cup c\}^*}{b} = L_3 \cup \{a \cup b \cup c\}^* = L_6$$

$$\frac{L_4}{c} = \frac{L_1}{c} \cup \frac{\{a \cup b \cup c\}^*}{c} = L \cup \{a \cup b \cup c\}^* = L_5$$

$$\frac{L_5}{a} = \frac{L}{a} \cup \frac{-''-}{a} = L_4$$

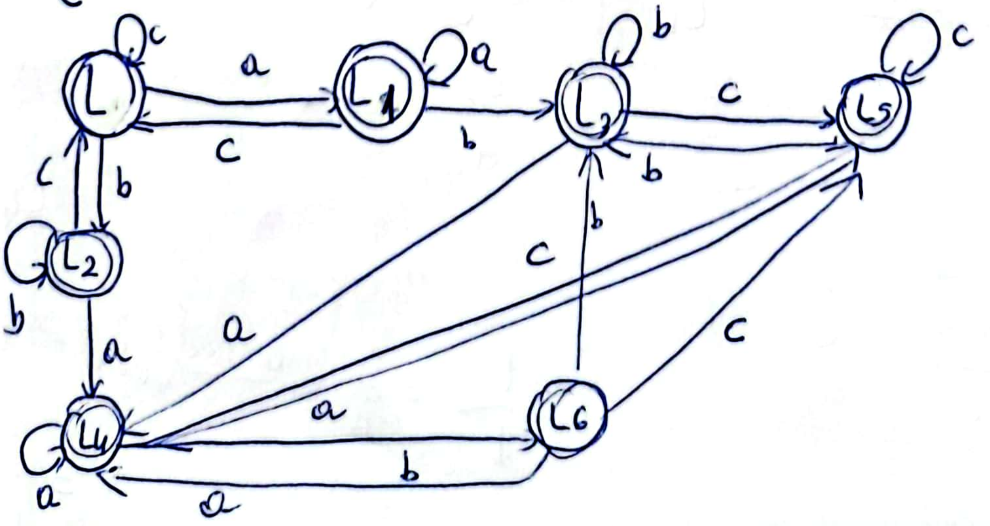
$$\frac{L_5}{b} = \frac{L}{b} \cup \frac{-''-}{b} = L_2 \cup \{a \cup b \cup c\}^* = L_3$$

$$\frac{L_5}{c} = L \cup \{a \cup b \cup c\}^* = L_5$$

$$\frac{L_6}{a} = \frac{L_3}{a} \cup -''- = L_4$$

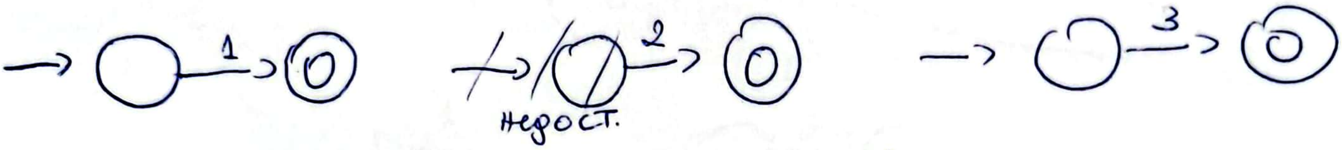
$$\frac{L_6}{b} = \frac{L_3}{b} \cup -''- = L_3$$

$$\frac{L_6}{c} = \frac{L_3}{c} \cup -''- = L_5$$

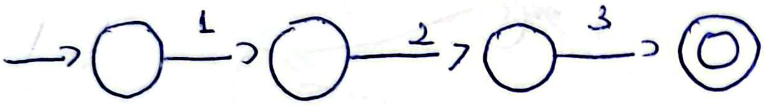


(103) Да се построи КА с азбука $\Sigma = \{1, 2, 3\}$ разпознаващ езика

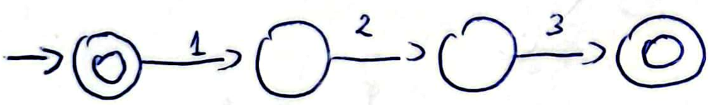
$$L = \Sigma^* | (\{1,2\}\Sigma^* \cap \Sigma^*\{3\}) \cup \{123\}^*$$



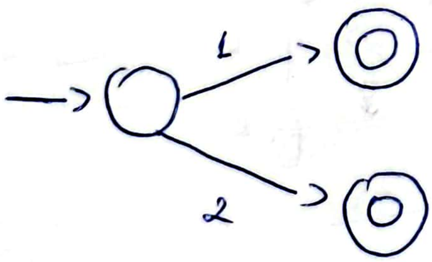
За $1 \cdot 2 \cdot 3$ имаме конкатенация



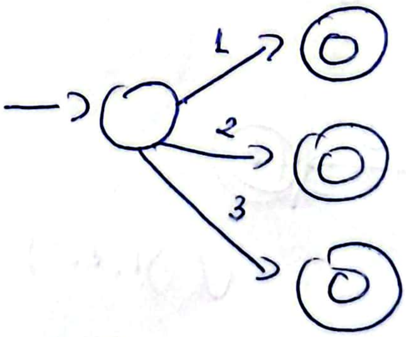
За $\{123\}^*$



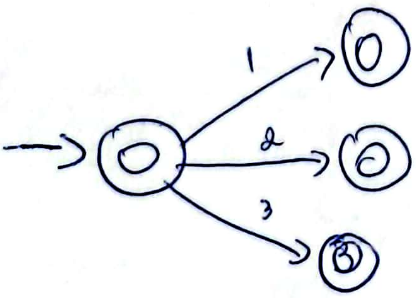
За 1 и 2



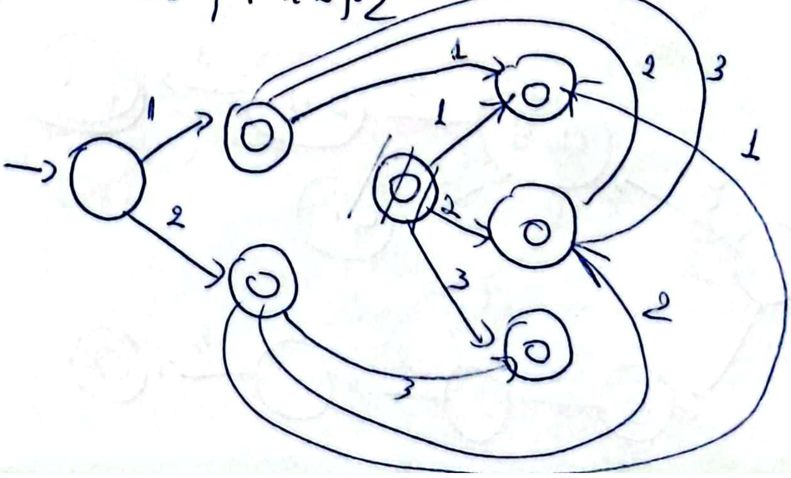
За 1 и 2 и 3 = Σ



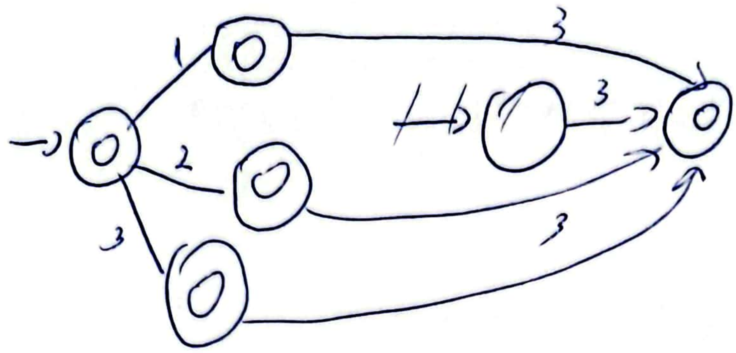
За $\{1$ и 2 и $3\}^*$



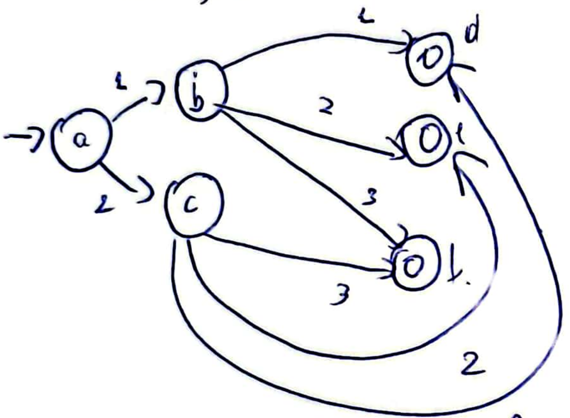
За $\{1$ и $2\} \cdot \Sigma^*$

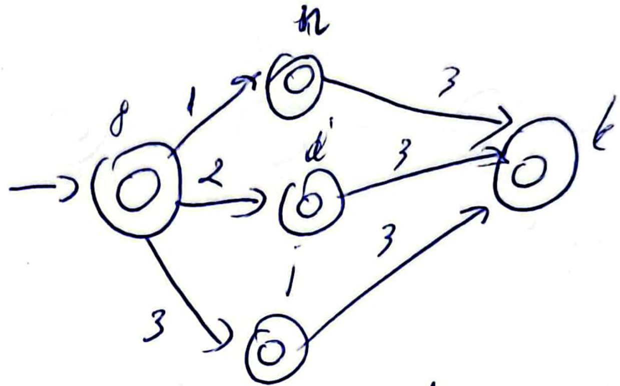


$\Sigma^* \cdot \{3\}$



3a $\{1;2\}\Sigma^* \cap \Sigma^* \cdot \{3\}$





Нач. със. $a, \xrightarrow{1} b$ $\quad a \xrightarrow{2} c \quad a \xrightarrow{3} \emptyset$

$g \xrightarrow{1} h \quad g \xrightarrow{2} i \quad g \xrightarrow{3} j$

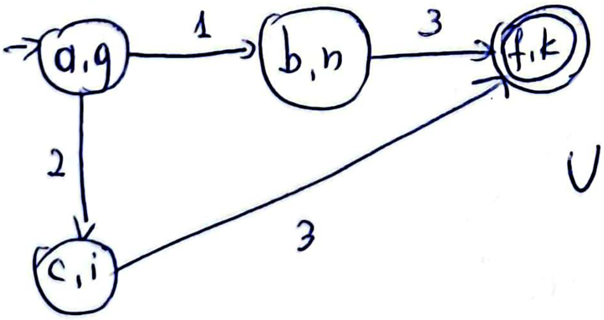
$b,n \xrightarrow{1} d\ \emptyset$

$\rightarrow \emptyset$

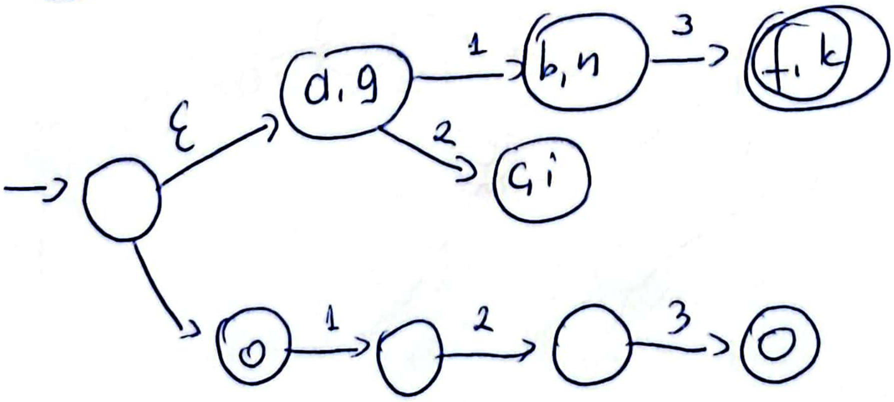
$b,n \xrightarrow{2} \emptyset$ няма

$b,n \xrightarrow{3} f,k$

$c,i \xrightarrow{3} f,k$

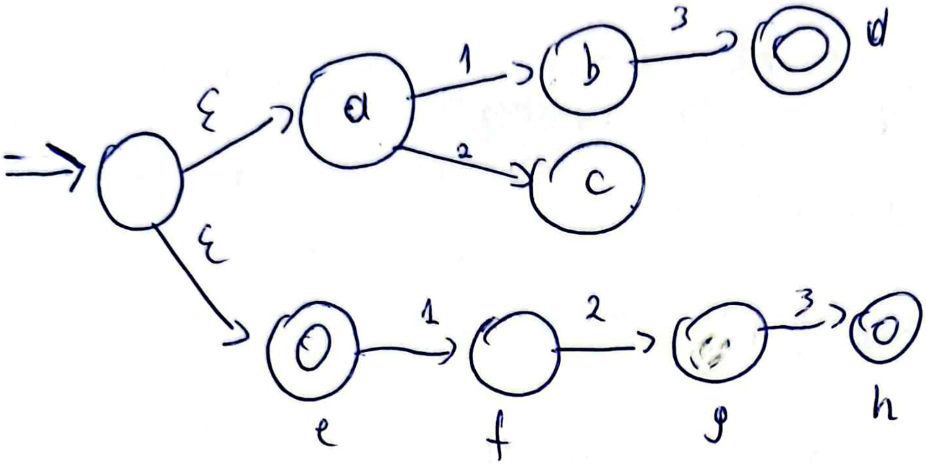


$\cup \{1,2,3\}^*$



продължение на 103

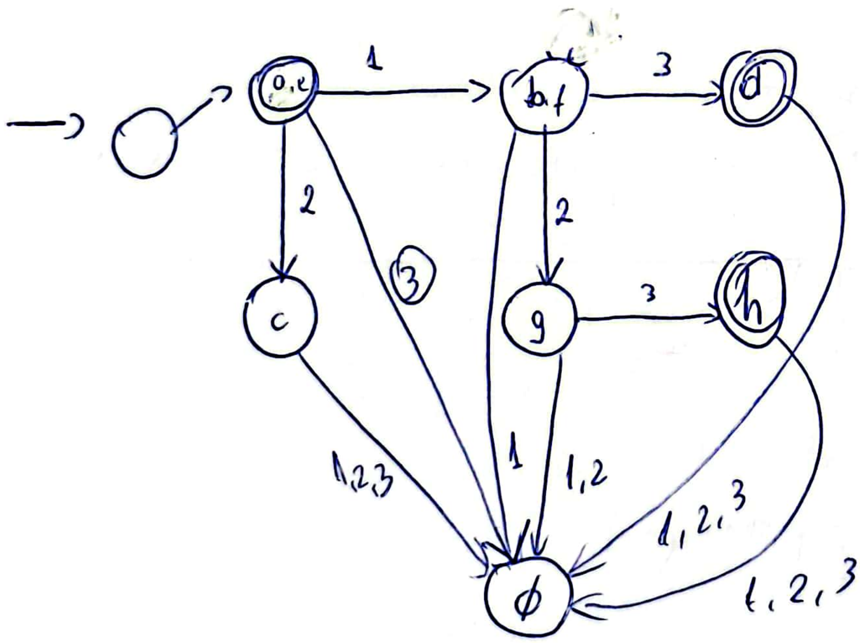
$\Sigma^* \setminus$ -||-

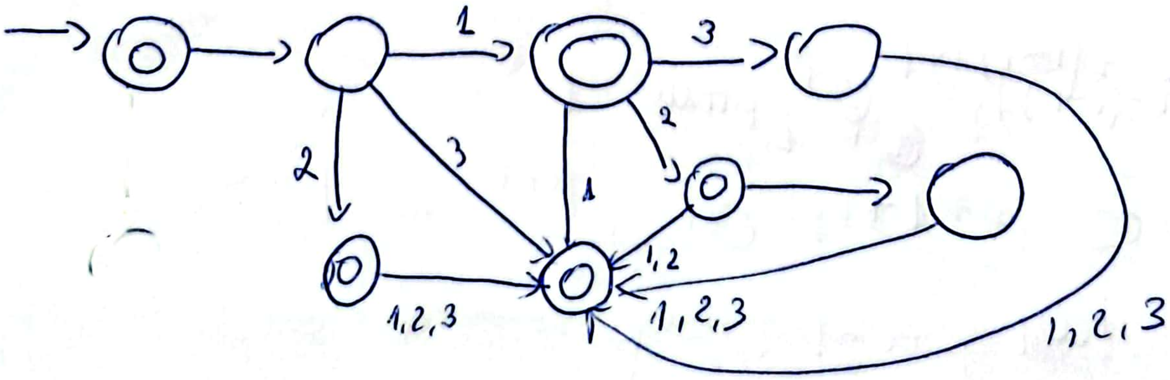


$a \xrightarrow{1} b$

$d \xrightarrow{2} c$

$e \xrightarrow{1} f$





□

(104) Да се докаже че $L = \{ w 0^n w^R \mid w \in \{0,1\}^*, |w| > n \}$ не е регулярен

Нека допуснем че има ДКА A който разпознава L.

Нека $|w| = i$ $n = j$

Тогава $L = 1^i 0^j 1^i$ за примерно $w = 1$

$0 \leq j < i$

Нека $\exists!\ p > 0$ такова че :

$$\gamma = 1^{p+1} 0^p 1^{p+1}$$

$\exists$ разбиване на $\gamma = xyz$ $|xy| \leq p$ $p > 0$ и

$\gamma' = xy^i z$.

Ще разгледаме следните случаи.

xy съдържа само 1 . примерно от 1 част на думата γ.

$$\gamma = \underbrace{\overbrace{1 \ldots}^{z} \overbrace{\ldots 1}^{xy}}_{p+1} \overbrace{\underbrace{0 \ldots 0}_{p} 1 \ldots 1}^{z}$$

$$\gamma_i = 1^{|x|} 1^{|y|} 1^{|p+1| - |x+y|} 1^{|p+1|} 0^{|p|}$$

$$= 1^{|x| + i|y| + p+1 - |x| - |y| + p+1} 0^p 1^{p+1} \text{ отделно}$$

$$\gamma_i = 1^{p+1-(i-1)|y|} 0^p 1^{p+1}$$

при $i = 2 \to \underbrace{1^{p+1-|y|}}_{w} 0^p \underbrace{1^{p+1}}_{w}$ но $|y| > 0$

и точка $p+1-|y| < p \Rightarrow \gamma_i \notin L$

но лявата страна $|w| \neq |w^R| \to \gamma_i \notin L$

L не е регулярен.

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Продължение на 104

2 случай xy съдържа само 0.

$$\alpha = 1^{p+1} 0^{p} 1^{p+1}$$

$$\underbrace{1\dots1\ 0}_{z}\underbrace{\dots}_{xy}\underbrace{0\ 1\dots1}_{z}$$

$$\gamma_i = 1^{p+1} \cancel{0^{|x|}} 0^{i|y|} 0^{p-\cancel{|x|}-|y|} 1^{p+1}$$

$$\gamma_i = 1^{p+1} 0^{p+(i-1)|y|} 1^{p+1}$$

при $i = 2$ имаме $1^{p+1} 0^{p+|y|} 1^{p+1}$

но $|y| \geq 1$ тогава $p+|y| \geq p+1$

но според условието имаме $p+1 > p$.

$\Rightarrow \gamma_i \notin L$.

3 случай xy съдържа и 0 и 1

$$\underbrace{1\dots1}_{z}\underbrace{1\ 0\dots}_{xy}\underbrace{0\ 1\dots1}_{z}$$

$$\gamma_i = 1^{\cancel{|x|}} 1^{i|y|} 0^{|x|} 0^{i|y|} 1^{p+1-\cancel{|x|}-|y|} 0^{p-|x|-|y|} z^{p+1}$$

$$= 1^{p+1+(i-1)|y|} 0^{p+(i-1)|y|} z^{p+1}$$

при $i = 0$ имаме $1^{p+1-|y|} 0^{p-|y|} z^{p+1}$

$p+1-|y| > p-|y|$ но $|w| \neq |w^R| \Rightarrow \gamma_i \notin L$.

(105) докажете че езикът

$L = \{a^n b^k \mid n+k \equiv 2 \pmod 3$ или $2n \leq k\}$

1 случай

$n+k \equiv 2 \pmod 3$

Нека $\exists p$ $p > 0$ такова че $p = n$

Тогава пак $p+k \equiv 2 \pmod 3$

за k избираме $2p+2$ така че:

$3p+2 \equiv 2 \pmod 3$

$w \in L \Rightarrow a^p b^{2p+2}$ $\exists$ разбиване на $w = xyz$

т.че $|xy| \leq p$ $|y| \geq 1$ $\Rightarrow$ часта от xy има само a

$\underbrace{a \dots a}_{xy} b \dots\dots\dots b$

$a^{|x|} a^{i|y|} a^{p-|x|-|y|} b^{2p+2}$

$a^{p+(i-1)|y|} b^{2p+2}$ Тоест $p+(i-1)|y| + 2p+2$

трябва да се дели на 3 и да има остатък 2 но при $i=0$ имаме

$p-|y|$ но $|y| \geq 1$ и тогава

$p-|y|+2p+2 = 3p+2-|y| < 3p+2$

$\Rightarrow w_0 \notin L$.

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Продължение (105)

2 случай $2n \le k$ Нека $k = 2n$

$\exists!\ p$ за което $n = p$

тогава имаме $a^p b^{2p}$

Нека xy има само b

$\cancel{a^{|x|} a^{i|y|} a^{p-|x|-|y|} b^{2p}}$

$a^p b^{2p - \cancel{|x|} - |y| + \cancel{|x|} + i|y|}$

$a^p b^{2p + (i-1)y}$

$2p + (i-1)y \geq 2p$

при $i = 0$

$2p - |y|$ но $|y| \geq 1$

$\Rightarrow 2p - |y| < 2p$

$\Rightarrow w_0 \notin L$

(106) докажете че L е КС

$$L = \{a^n b^k c^m d^l \mid \underbrace{m > 2n}_{L_1} \text{ или } \underbrace{l > k}_{L_2}\} \quad L_1 \cup L_2$$

$L = \{a^n b^k c^m d^l \mid m > 2n\}$

$m = 2n + p$

$\Rightarrow \underbrace{a^n b^k c^{2n+p}} d^l$

$K \to bK \mid \varepsilon$

$N \to aNc^2 \mid Kc$

$D \to dD$

$\cup$

$L = \{a^n b^k c^m d^l \mid l > k\}$

$l = k + p$

$\Rightarrow a^n b^k c^m d^{k+p}$

$N' \to a$

$K' \to bK'd \mid d^*$

$M \to c$

(107) докажете че L е КС. $L = \{a^n b^k \mid 3n < [illegible] 1\}$

[illegible] $3n + \cancel{2}^{1} = 2k + \cancel{p} + \cancel{1}$

$3n + 1 = 2k$

$2k \equiv 1 \pmod 3$

$k \equiv 2 \pmod 3$

$k = 3k' + 2$

$3n + 1 = 6k' + 4$

$3n + \cancel{1} = 2k + p + \cancel{1}$

$3n = 2k + p$

$3n = 2k + 3p \Rightarrow 2k \equiv 0 \pmod 3 \Rightarrow k \equiv 0 \pmod 3$

$3n = 2k + 3p + 1 \Rightarrow 2k \equiv 2 \pmod 3 \Rightarrow k \equiv 1 \pmod 3$

$3n = 2k + 3p + 2 \Rightarrow 2k \equiv 1 \pmod 3 \Rightarrow k \equiv 2 \pmod 3$