

Logistic Regression

Finding the Parameters

Finding the Parameters

- Like linear regression, this is done by optimizing a cost function.
- By minimizing a cost function.
- Cost function sometimes called an objective function.

Finding the Parameters

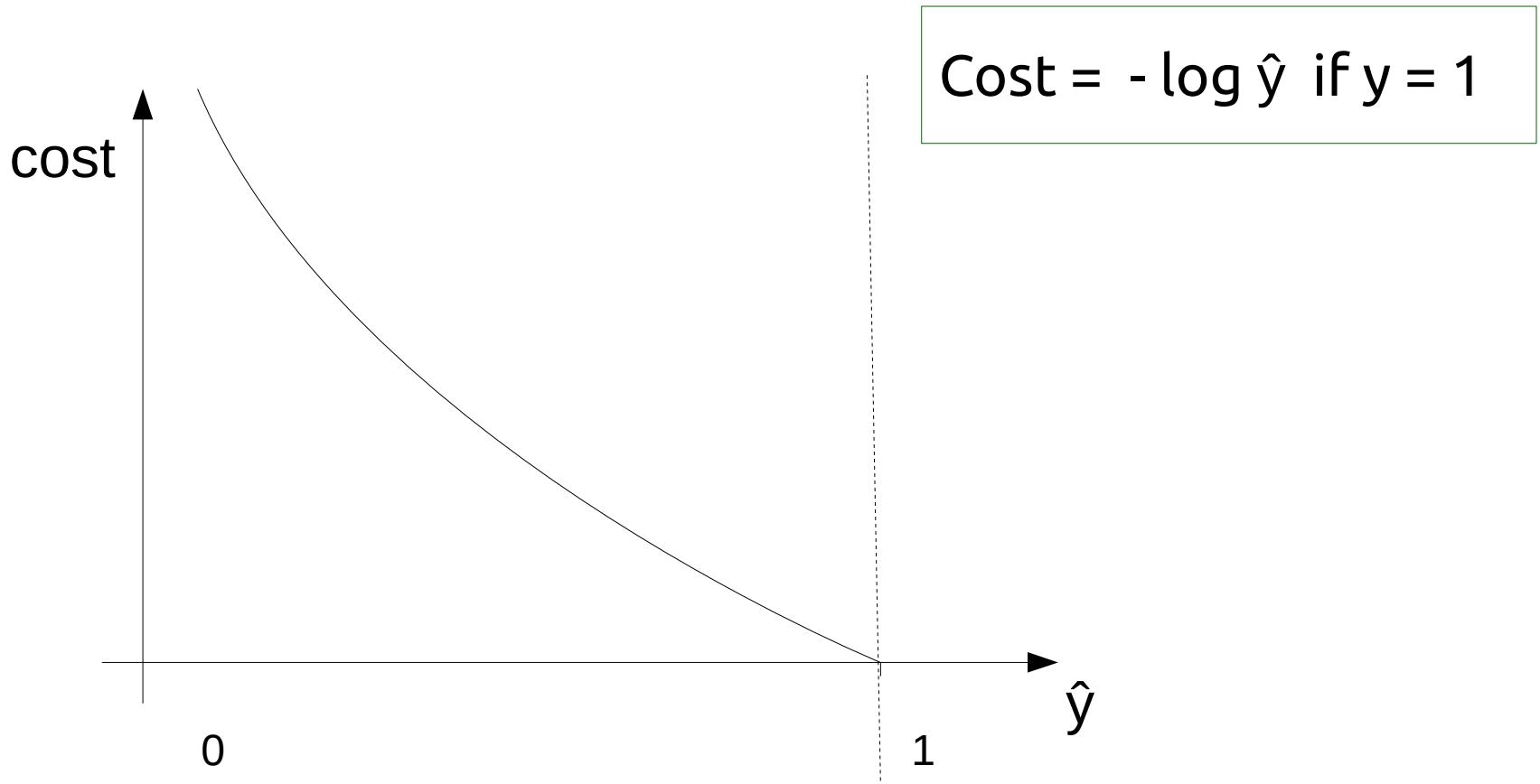
- n - number of factors
- m - number of training instances
- y^i - y values for the training set
- \hat{y}^i - predicted y values for the training set
- parameters $a, b_1, b_2, ..$

Cost Function for Logistic Regression

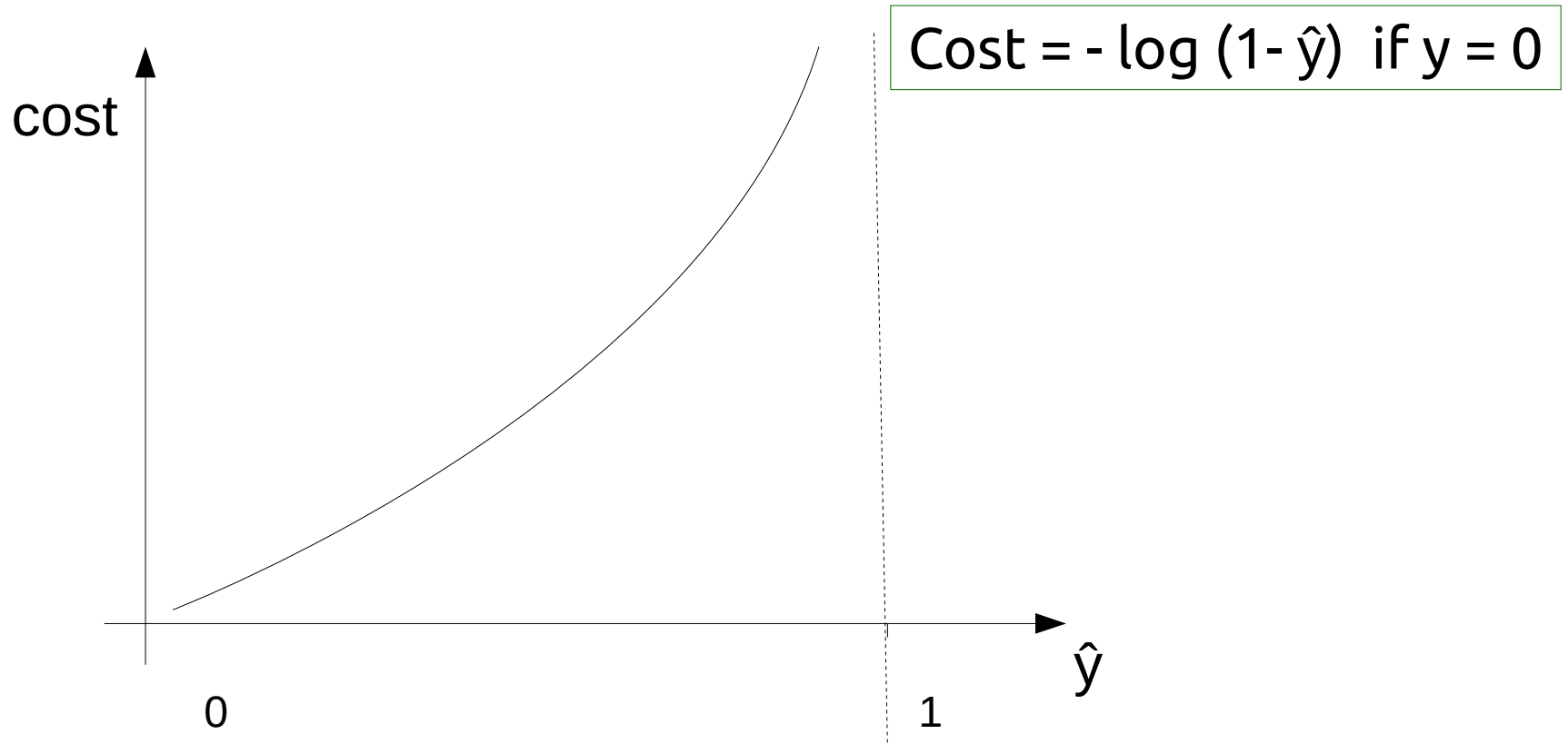
$$\begin{aligned} \text{cost}(\hat{y}, y) &= -\log \hat{y} \text{ if } y=1 \\ \text{cost}(\hat{y}, y) &= -\log(1 - \hat{y}) \text{ if } y=0 \end{aligned}$$

→ Recall \hat{y} is between 0 and 1

Cost function



Cost function



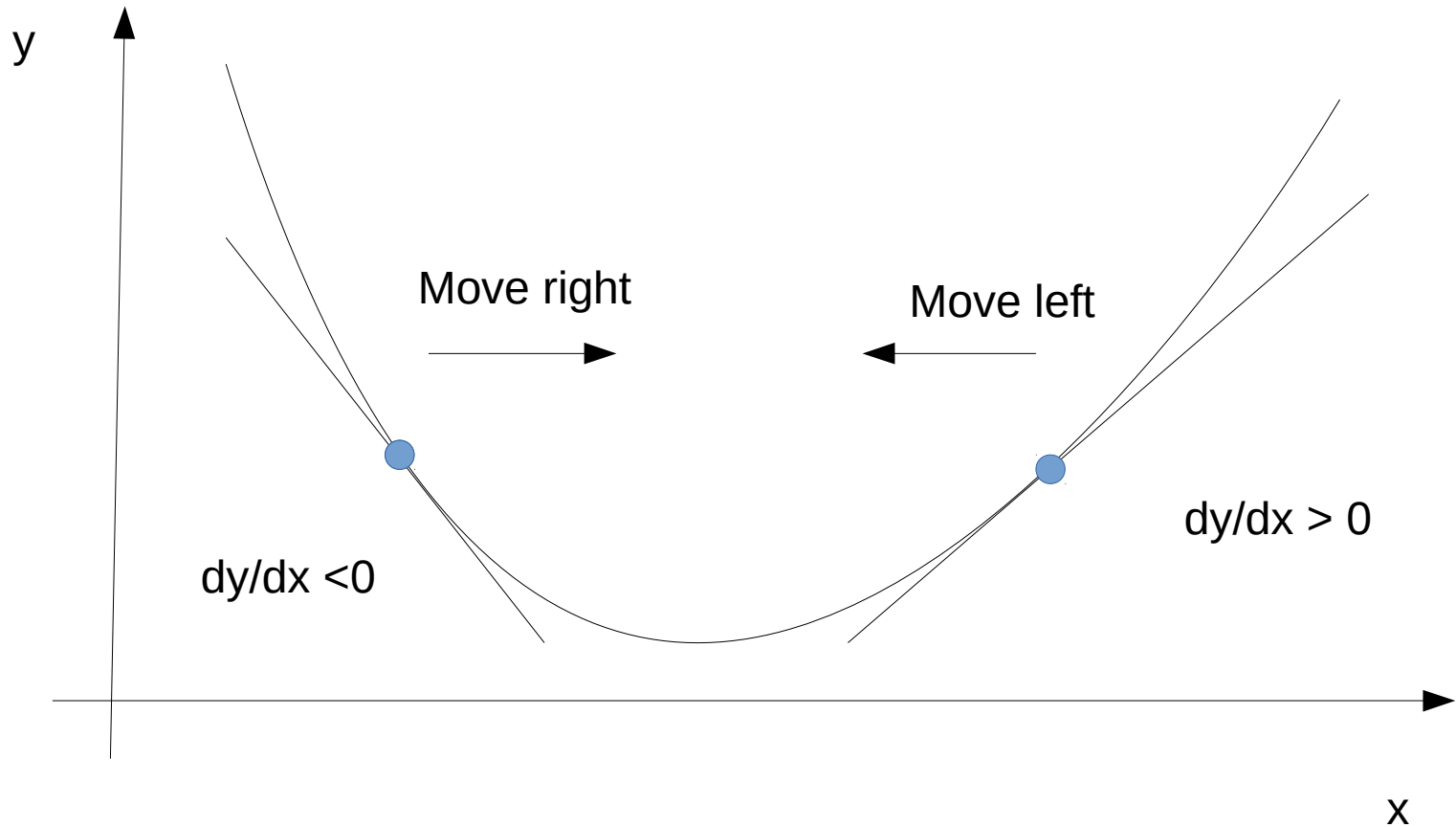
Cost function

- If $y = 1$ and we predict a value close to 0, then there is a heavy cost (penalty)
- If $y = 0$ and we predict a value close to 1, then there is also a heavy cost (penalty)

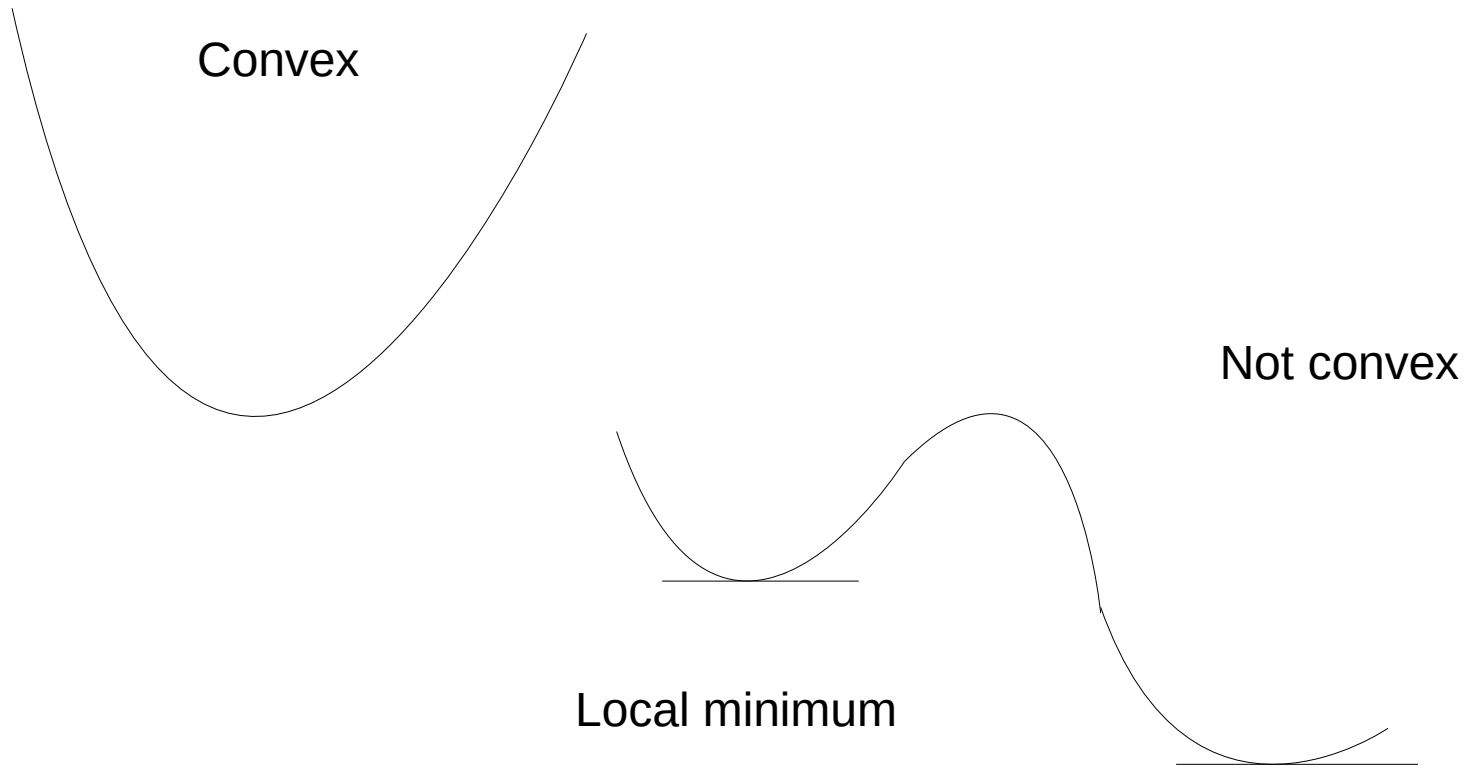
Gradient Descent

- Gradient Descent
- If dy/dx is negative move right
- If positive move left.
- Finds a local minimum.
- It turns out that the cost function defined above is convex, so gradient descent finds a absolute minimum.

Gradient Descent



Convex Cost Function



Gradient Descent

- ➔ Gradient Descent is used to find the values of the parameters a and b 's that minimize the value of the cost function for the training data.
- ➔ This is implemented in R.

Logistic Regression - Summary

- Find the equation of the line that best separates the classes.
 - $z = a_1 * x_1 + a_2 * x_2 + \dots + a_n * x_n + b$
 - This is done by minimising the cost function defined above.
- To predict the class of a new instance:-
 - Calculate the values of z given above
 - Find $g(z)$ where g is the logistic function.
 - This gives the probability (of a positive).