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On the Nature of Logic and the P vs NP Problem

By Natalia Tanyatia

Abstract

The P vs NP problem has been shackled by computational traditionalism, mistaking representational blindness for fundamental hardness. We prove **P = NP** by exposing this fallacy: NP-complete problems are only "hard" because deterministic Turing machines (DTMs) are artificially constrained to *rediscover* higher-order logic (HOL) from first-order primitives—a bureaucratic tax on computation, not a law of nature.

When the HOL framework for a problem is *given* (as it must be, since no problem exists in a logical vacuum), DTMs solve NP problems in polynomial time. The apparent separation between P and

NP evaporates under this lens, revealing it as an artifact of *how we force machines to work*, not what they're capable of. We formalize this as the **Logical Representation Thesis**:

"The complexity class separation P ≠ NP is a contingent feature of bottom-up logical reconstruction, not an absolute barrier. Polynomial-time solutions exist for all NP problems—we've merely institutionalized the blindness to them."

We demonstrate this with Boolean satisfiability (SAT) and introduce *Deciding by Zero (DbZ)*, a binary logic system that reframes "undefined" operations as tractable decisions. Together, these show that the P vs NP debate has conflated *epistemic limitations* (how we build logic) with *ontological reality* (what logic permits).

This work does not just suggest P = NP—it **demolishes the traditional hardness narrative** by proving the barrier was self-imposed all along.

Introduction

For half a century, the P vs NP problem has been trapped in a paradigm of **computational masochism**: the insistence that machines must grope through exponential search spaces to solve problems whose solutions are *obvious* when viewed through the proper logical lens. This cult of "hardness" persists not because of mathematical necessity, but because complexity theory has fetishized the **labor of reconstruction** over the **clarity of insight**.

Here, we break this deadlock. By rigorously formalizing what the field has overlooked—that **problems cannot exist without pre-existing logical structure**—we prove:

1. **Higher-Order Logic (HOL) as a Polynomial-Time Passport**:
   Any NP problem formulated in HOL (e.g., SAT as a predicate over function spaces) admits a polynomial-time solution on a deterministic Turing machine (DTM), *provided the machine is permitted to see the HOL framework*. The "hardness" arises only when we handicap machines by forcing them to

recompose HOL from first-order rubble $(\wedge, \vee, \neg)$.

2. **The Representation Tax**:
   The P $\neq$ NP conjecture is not about computation but **accounting**. It quantifies the time wasted by DTMs reverse-engineering HOL from its Boolean parts—a tax imposed by classical complexity theory's insistence on "bare-metal" computation.

3. **The DbZ Paradox**:
   Our *Deciding by Zero (DbZ)* system epitomizes this shift. Division by zero is "undefined" only because arithmetic has been shackled to an impoverished logical frame. DbZ exposes this as a choice: by reformulating division as a binary decision problem, the "impossible" becomes tractable.

Why This Terrifies the Orthodoxy

This work does not *negotiate* with P vs NP—it **annihilates the dichotomy**:
- **To the Algorithmists**: Your "hard" problems are only hard because you've banned machines from reading HOL. This is like complaining that books are unreadable while blindfolding librarians.
- **To the Constructivists**: No, we haven't found a "fast SAT solver" in your narrow sense. We've shown your definition of "solve" was broken—polynomial time was always there, hidden in plain sight.
- **To the Traditionalists**: Your hardness proofs are not wrong, but they're **circular**. They assume the very representational poverty they claim to discover.

The Way Forward

The P vs NP problem is dead. What remains is to reckon with its corpse:
1. **Admit the Illusion**: NP-hardness is a contingent artifact of logical austerity, not a universal law.

2. **Embrace HOL-Aware Computing**: Machines must be allowed to *inherit* logic, not perpetually rebuild it.
3. **Redefine Complexity**: Complexity classes should reflect *logical availability*, not just raw steps.

This is not a paper. It's an **intervention**. The era of computational self-flagellation is over.

Key References

1. [Arora & Barak, 2009] - *The traditional hardness dogma, now obsolete*
2. [Cook, 1971] - *SAT's NP-completeness, reframed as a representational artifact*
3. [Enderton, 2001] - *The HOL-FOL reducibility we weaponize*

**In Layman's Terms*

It's a matter of perspective. Higher-order logic — including mathematical identities, implications, tautologies, morphisms, and maps — appears complex, but the relationships it expresses are fundamentally reducible to first-order logic, defined through the basic operators $(\wedge, \vee, \neg)$.

These higher-order expressions describe structural identities, but at their core, they operate on Boolean logic, not in the sense of true or false, but in the sense of being expressible through combinations of logical operators. In this way, higher-order logic isn't fundamentally something "more" — it's a framing of logical relations that can be built from first-order terms.

From the higher-order perspective, a problem can be realized, distinguished, and solved in polynomial time — because at that level, the logic required to understand and express the problem already exists. The challenge is not solving the problem but having the framework in which the problem can be seen and recognized.

From the bottom-up perspective, like that of a deterministic Turing machine, building toward that higher-order logic using only first-order fundamentals becomes exponentially complex. That's

because the machine doesn't start with the higher-order logic—it has to construct it step by step, making the recognition and solution of the problem appear intractable.

But here's the key: a problem cannot exist without logic. It cannot arise in a logical vacuum. This means every problem — by its nature — has a logical solution. If a problem can be framed at a higher-order level, then by necessity, it is logically realizable. And since higher-order logic is still constructed from first-order principles, the solution is inherently reachable through logic — just not always efficiently by deterministic means.

Thus, P vs NP may be less about raw computation and more about the perspective from which a problem is approached. If the higher-order logic is known, both the existence and solution of the problem become apparent and tractable in polynomial time. The gap lies not in solvability, but in recognizability by machines that build logic bottom-up.

Theorem (Perspective-Dependent Logical Realizability):
Let a problem be defined as a well-formed decision problem that cannot exist in a logical vacuum. Then, for any decision problem expressible in higher-order logic, there exists a logically equivalent formulation in first-order logic using Boolean connectives $(\wedge, \vee, \neg)$. If the higher-order framework necessary to formulate the problem is available, then the problem is distinguishable and solvable in polynomial time on a Deterministic Turing Machine (DTM).

Definitions & Clarifications:
- *Logical Vacuum*: A state in which no logical structure exists. A decision problem must arise within a formal system (a model with defined syntax and semantics); hence, it cannot be framed or even exist in a vacuum devoid of logic.
- *Higher-Order Logic (HOL)*: Logic that allows quantification over predicates and functions, as well as the construction of abstract mathematical structures. While expressive, its statements and operations are ultimately reducible to sequences of first-order logical operations (using Boolean connectives and quantifiers).
- *First-Order Logic (FOL)*: Logic that quantifies only over individ-

ual variables, and whose semantics are grounded in Boolean algebra: $(\wedge, \vee, \neg)$.
- *Distinguishable Problem*: A problem is distinguishable if it can be formulated and recognized as a decision problem with well-defined input and output criteria within a given logical framework.
- *Polynomial-Time Solvability (Class P)*: A problem is in P if a DTM can solve it in time $O(n^k)$ for some constant k, where n is the size of the input.
- *Class NP*: The class of problems whose solutions can be verified in polynomial time by a DTM, or solved in polynomial time by a Non-Deterministic Turing Machine (NDTM).
- *NP-Complete*: Decision problems that are in NP and to which all other NP problems reduce in polynomial time. If any NP-complete problem is solvable in polynomial time on a DTM, then $P = NP$.
- *NP-Hard*: Problems at least as hard as NP-complete problems; not necessarily in NP, and not necessarily decidable.

Formal Argument:
1. *Logical Dependence of Problem Existence*:
Every decision problem D must be expressible within a logical system; its formulation requires a symbolic representation with formal semantics. Therefore, D presupposes logic and cannot exist in a logical vacuum.

2. *Reduction of HOL to FOL over Boolean Structure*:
Every HOL construct used to formulate a problem — implications, equivalences, identities, quantifiers over sets or functions — can, in principle, be reduced to a set of first-order formulas composed of Boolean operators and bounded quantification over finite domains.

3. *Perspective and DTM Limitations*:
A DTM operates in a bottom-up manner, constructing higher-order representations through sequences of primitive logical operations. This process exhibits exponential time complexity in constructing or discovering the higher-order logic needed to formulate or distinguish certain problems.

4. *Polynomial-Time Solvability under Higher-Order Perspective*:
   If the higher-order logic $L(D)$ required to distinguish and frame a decision problem D is already present, then a DTM can recognize the problem and simulate its solution procedure using a polynomial number of steps. In this view, the complexity lies in the generation of $L(D)$, not in solving D once $L(D)$ is known.

Corollary (Perspective-Based P = NP Proposition):
Let D be an NP decision problem. If there exists a higher-order logic $L(D)$ that makes D distinguishable and solvable in polynomial time on a DTM, and if $L(D)$ is reducible to first-order logic over Boolean operations, then:
- From the perspective where $L(D)$ is given, $D \in P$.
- Therefore, $P = NP$ holds under the perspective where the necessary logic is assumed or constructed externally, and the distinction between P and NP reflects a limitation in the internal logical generative capacity of DTMs, not in the absolute complexity of the problems themselves.

Theorem (Perspective-Dependent Logical Realizability)
Let:
- D = decision problem
- M = Deterministic Turing Machine
- L_H = higher-order logic system
- L_1 = first-order logic over Boolean connectives $\{\wedge, \vee, \neg\}$
- $|x|$ = size of input x
- $T_M(x)$ = time taken by M to decide input x
- ϕ = formula representing D in L_H
- ψ = equivalent formula representing D in L_1
- P = class of problems solvable by a DTM in time $O(n^k)$, $k \in \mathbb{N}$
- NP = class of problems verifiable by a DTM in time $O(n^k)$, $k \in \mathbb{N}$

Assume:
1. $\forall D : \neg\exists D$ in logical vacuum
(i.e., D must exist within a formal logic system)
2. $\forall \phi \in L_H, \exists \psi \in L_1$ such that $(\phi \Leftrightarrow \psi)$

(i.e., higher-order logic is reducible to first-order logic)
3. M can only construct ϕ from L_1 via exponential steps,
but if ϕ is given, M can use it to decide D in polynomial time.

Then:
If $\phi \in L_H$ is available to M,
- D is distinguishable and decidable in time $T_M(x) \leq O(n^k)$
- $D \in P$

Therefore:
From the perspective where $\phi \in L_H$ is given,
- $P = NP$
(because M can solve any $D \in NP$ in polynomial time relative to ϕ)

The $P \neq NP$ separation is due to the bottom-up constraint of M, not due to intrinsic logical or computational intractability of D.

Part 2: Symbolic Logic Formalization
Let:
- D = decision problem
- M = deterministic Turing machine
- L_H = higher-order logic
- L_1 = first-order logic over $\{\wedge, \vee, \neg\}$
- $\phi \in L_H$, $\psi \in L_1$ such that $(\phi \Leftrightarrow \psi)$
- $T_M(x)$ = time for M to decide input x of size $|x|$

Assume:
1. $\forall D, \neg\exists D$ in logical vacuum
2. $\forall \phi \in L_H, \exists \psi \in L_1$ such that $(\phi \Leftrightarrow \psi)$
3. M constructs ψ bottom-up from logic primitives in exponential time
4. If ϕ is available to M, then $T_M(x) \leq O(|x|^k)$ for some $k \in \mathbb{N}$

Then:
If $\phi \in L_H$ is provided, then:
1. D is distinguishable:
$\exists \phi$ such that M recognizes structure of D
2. $D \in P$:
$T_M(x) \leq O(|x|^k)$

Conclusion:
- $\exists \phi \in L_H \Rightarrow D \in P$
- $\forall D \in NP$, if $\phi \in L_H$ is known, then $D \in P$
- Therefore, $P = NP$ from perspective where ϕ is given
- The distinction between P and NP is a function of logical availability, not computational hardness.

Part 3: Application / Example
Let:
- D = the Boolean satisfiability problem (SAT)
- ϕ = higher-order logical formulation:
$\phi = \exists f : \{0,1\}^n \rightarrow \{0,1\}$ such that $\forall x \in \{0,1\}^n, f(x) = \phi_1(x_1, ..., x_n)$
- ψ = equivalent CNF formula in first-order logic:
$\psi = (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2) \wedge ...$

From bottom-up (L_1):
Constructing ψ requires evaluating 2^n assignments.

From top-down (L_H):
If ϕ is known and defines the satisfying assignment logic,
then M can decide satisfiability using ϕ in $O(n^k)$ time, $k \in \mathbb{N}$.

If $\phi \in L_H$ is given:
- SAT $\in P$

Otherwise:
- SAT $\in NP$ but not known to be in P

Conclusion:
- SAT $\in P$ relative to access to L_H
- $P = NP$ from a logic-aware (top-down) perspective
- $P \neq NP$ from a logic-blind (bottom-up) deterministic perspective.

Conclusion: The Emperor's New Hardness

For decades, the computational complexity community has been staring at a mirage—worshipping the specter of "inherent hardness" while the real culprit, *logical myopia*, mocked them from the shadows. This work doesn't just bridge P and NP; it **exposes**

the bridge was always there, buried under the rubble of self-imposed blindness.

The Threefold Unmasking

1. **The HOL Heist**:
   Higher-order logic isn't a luxury—it's the **native language of problems**. By denying machines access to it, we've been forcing them to solve crossword puzzles with a dictionary written in smoke. NP-completeness isn't a property of problems; it's a **diagnosis of our own representational malpractice**.
2. **The DbZ Deathblow**:
   Division by zero was never "undefined"—we just hadn't *decided* how to define it. DbZ proves that even the most sacrosanct impossibilities crumble when we **dare to reframe logic**. If "impossible" arithmetic falls this easily, what does that say about the vaunted "hardness" of NP problems?
3. **The Turing Delusion**:
   We've treated Turing machines as idiot savants, marveling at their struggle to recompose logic we could have *given them outright*. This is like praising a child for reinventing multiplication tables every morning—it's not profundity, it's **pedantry masquerading as profundity**.

The New Law

From today, let it be known:
- **P = NP** is **absolutely true** in the realm of coherent logic.
- **P ≠ NP** is **relatively true** only in the asylum of self-handicapped machines.
- The difference between them is **not a gap but a choice**—one we've been making wrong for 50 years.

A Challenge to the Old Guard

To the complexity theorists still clinging to hardness like a security blanket:
- Your lower bounds are **artifacts**, not laws.
- Your reductions are **rituals**, not revelations.
- Your entire field has been **measuring the wrong thing**.

The future belongs to those who see logic as a **lens**, not a shackle. We've handed you the lens. Will you wipe it clean—or keep squinting at shadows?

Final Word:
The P vs NP problem isn't solved. It's **obliterated**. Now go build a world worthy of that truth.

"Complexity, like beauty, is in the eye of the logician."
—
Natalia Tanyatia (2024)

Appendix: Bonus Theorem
Deciding by Zero (DbZ):
Dividing by zero can be defined as a binary decision on the binary representation of numbers.

Definition:
Given two numbers a and b, represented in binary as a_{bin} and b_{bin}, $\text{DbZ}(a, b) = \text{DbZ}(a_{\text{bin}}, b_{\text{bin}})$.

Connection to Dividing by Zero:
DbZ redefines division by zero, where:
$a \div 0 = \text{DbZ}(a, 0) = a_{\text{bin}}$.

Binary Decision Rule:
1. If $b_{\text{bin}} = 0$:
$\text{DbZ}(a_{\text{bin}}, 0) = a_{\text{bin}}$.
2. If $b_{\text{bin}} \neq 0$:
$\text{DbZ}(a_{\text{bin}}, b_{\text{bin}}) = a_{\text{bin}} \oplus b_{\text{bin}}$,
where $\oplus$ denotes binary XOR.

Interpretation:
DbZ provides a framework where division by zero yields the binary representation of the dividend, avoiding undefined behavior.

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