Q5,

Definition:

1. Every set of k clauses, where k>=1, in a slithy boolean expression includes a variable that appears only once among those clauses
2. Slithy must be CNF, hence the clauses are made up of disjunctions

Base step:

- Let the number of variables n, be equal to 1, to create a Boolean expression with one variable. The expression involves a single clause, with only a lone literal. By assigning the lone literal either True or False, the expression will eventually have a true value, thus making it satisfiable. The base case Boolean expression at n = 1 variables satisfies being slithy and satisfiable.

Inductive Hypothesis

Statement: Let n = k , and assume all slithy boolean expressions (SBE) with at most k number of variables has at most k clauses and is satisfiable, where k >=1.

Inductive Step:

- Let E be a SBE with k + 1 variables.
- As defined before, there exists a variable "m" that exists in one clause only in the SBE.
- To decompose E to yield a new expression that is sized at most k variables, we can remove all instances of m from the expression:

Case 1: there are lone literals as independent clauses

The variable m which acts as an independent clause can be removed from E. What remains is a subset of E sized at k variables.

Case 2: A clause contains at least one lone literal

Assume the variable m is located within a multivariable clause. What remains is a subset of E that does not have m nor the other variables included. This yields an expression sized at most k number of variables.

- Now that we have a smaller expression than E that contains at most k number of variables, as per the inductive hypothesis this expression has the upper bound of k number of clauses, as we have removed at least one clause to decompose E. By our inductive hypothesis, E has the upper bound k+1 number of clauses.
- Additionally, what remains is another slithy expression after the decomposition. This is as defined previously, as all greater than 1 sized subsets must be slithy.
- Furthermore, as per our inductive hypothesis, there exists a scenario where all the singular clauses in the smaller expression are true at the same time and so be satisfiable when bound by and operators. Since m only appeared once and independent from the rest of the clauses in E, their truth values will not affect the other clauses' truth values and if true, will disregard any false values that accompany m in a disjunction clause to make the clause true. This keeps E remaining satisfiable.
- This concludes the inductive step.

Conclusion

Using mathematical induction, we are able to prove the statement "All slithy boolean expressions (SBE) with at most k number of variables has at most k clauses and is satisfiable" as true. Hence, a slithy Boolean expression in CNF with at most n variables has at most n clauses and is satisfiable.