Blood Flow: Bridging the Micro-Macro Gap Using Scientific Machine Learning

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1. Familiarization with properties and modeling of Blood Flow
2. Derivation of Navier-Stokes Equation from classical mechanics
3. Rheological Non-Newtonian models for blood

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- Blood consist of a suspension of elastic particulate cells:
  - **Red blood cells** ($\approx$ 40-45 % of blood volume)
  - White blood cells
  - Platelets

  in a liquid known as plasma ($\approx$ 55% of blood volume)
- Properties of Blood e.g. viscosity depend on physiological flow conditions, blood composition properties e.g. hematocrit, temperature, shear rate, cell aggregation, cell shape, cell deformation, orientation ...

- While plasma has nearly Newtonian behaviour, whole blood exhibits non-Newtonian characteristics, particulary at low shear rates
- The Non-Newtonian characteristics of blood are:
  1. Shear thinning
  2. Yield stress (viscoplasticity)
  3. Viscoelastic properties
  4. Thixotropic behaviour

- Non-Newtonian characteristics occur due to:
    - Aggregation of RBC's at low shear rates (Rouleaux formation)
    - Thixotropic behaviour due to finite time required for RBC aggregation and disaggregation.
    - Deformability of RBC and their tendency to align with flow field at high shear rates

Low shear rates $\approx < 1 - 100\frac{1}{s}$	High shear rates $\approx > 400\frac{1}{s}$
apparent viscosity increases	apparent viscosity decreases
yield stress	RBC's rotate & accomadate flow
thixotropic	
RBCs aggregate are solid-like bodies, and has ability to store elastic energy (viscoelastic)	RBCs behave fluid-like bodies

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Review

Soft Matter

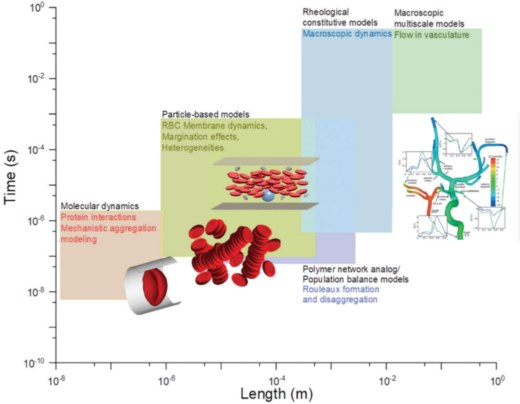

Figure 1: Length and time scales involved in blood flow modeling

- Classical Mechanics $\Rightarrow$ Continuum hypothesis!
- Starting point are mass and momentum conservation for incompressible fluids ($\rho \approx$ const) and a constitutive equation, which describes material behaviour

1. $$\nabla \cdot \boldsymbol{u} = 0$$
2. $$\rho \frac{\partial \boldsymbol{u}}{\partial t} + \rho(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = \nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{f}$$
3. $$\boldsymbol{\sigma} = \underbrace{-p\boldsymbol{I}}_{\text{volumetric stress tensor}} + \underbrace{\boldsymbol{\tau}}_{\text{deviatoric stress tensor}}$$

- Newtonian assumption i.e. $\boldsymbol{\sigma} = \boldsymbol{\sigma}(\boldsymbol{\nabla u}, p) = -p\boldsymbol{I} + 2\eta \boldsymbol{D}(\boldsymbol{\nabla u}) = -p\boldsymbol{I} + \eta(\boldsymbol{\nabla u} + (\boldsymbol{\nabla u})^T)$ with $\eta =$ const leads to well-known Navier-Stokes Equation

- Newtonian behaviour is a limitation usually accepted for blood flow in large arteries
- Non-Newtonian characteristics $\Rightarrow$ In small size vessels or in regions of stable recirculation, (e.g. venous system) and parts of arterial vasculature where geometry has been altered and RBC aggregates become more stable, like downstream a stenosis or inside a saccular aneurysm

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- In order to account for Non-Newtonian effects, we can
  1. use **time-independent constitutive equation** to close system and make the viscosity dependent of the deformation rate tensor
  2. use **time-dependent models** i.e. a (nonlinear) viscoelastic constitutive equation or thixotropic models $\Rightarrow$ more complex constitutive equations must be solved simultaneously along with the equations of conservation of mass and momentum

Time-independent constitutive equation

- Express the viscosity as function of the strain rate $\Rightarrow$ Generalised Newtonian Model

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}(\nabla \boldsymbol{u}, p) = -p\boldsymbol{I} + 2\eta(\dot{\gamma})\ \boldsymbol{D}(\nabla \boldsymbol{u}) \tag{2}$$

with $\dot{\gamma} = \sqrt{2\boldsymbol{D} : \boldsymbol{D}}$

- Examples are e.g. Power law model $\eta = k\dot{\gamma}^{n-1}$, Extension of power-law model from Walburn and Schneck (considered dependence of η on haematocrit and TPMA content), Cassons equation (account for yield stress) ...
- Computationally inexpensive to implement but cannot predict accurately transient changes which are relevant as blood flows naturally under pulsatile conditions $\Rightarrow$ time-dependent models

While exhibiting shear-dependent viscosity these fluids are essentially Newtonian in the following aspects: the structure of the stress tensor of generalized Newtonian fluids in a particular flow is the same as in their Newtonian counterparts, and their velocity fields adjust instantaneously to changes in stresses. Many complex fluids behave quite differently. One of the key features of viscoelastic fluids is the presence of memory; stresses in such fluids depend on the flow history. Another is stress anisotropy. Generally, a viscoelastic fluid generates stresses that are absent in a Newtonian fluid subjected to the same deformation history (Morozov, Alexander and Spagnolie, Saverio E., 2015)

Time-dependent constitutive equation

- Regarding Viscoelasticity: Different models to account for elastic effects e.g.
    - Maxwell model (based on spring-dashpot analogy)
    - Oldroyd-B model (split stress tensor approach: Decompose total stress tensor into its non-Newtonian (RBC influence) and Newtonian (plasma) parts)
- Different complexities of models e.g. considering viscoelasticity, thixotropy, thixotropic elasto-visco-plastic behavior of blood ...
- We get additional equations (PDE/ODE) to solve for

Questions

- Do we want to examine a specific phenomena (e.g aneurysma) s.t. we can focus from the variety of non-newtonian models (including shear thinning, plasticity, viscoelasticity, thixotropy, all together, etc.) on a specific group
  - What is the relevant diamter which we want to examine?
  - What are our regions of interest?
  - Heat transfer?
  - I will neglect biochemical processes?
  - FSI?