Blood Flow: Bridging the Micro-Macro Gap Using Scientific Machine Learning

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1. Familiarization with properties and modeling of Blood Flow
2. Derivation of Navier-Stokes Equation from classical mechanics
3. Rheological Non-Newtonian models for blood

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- Blood consist of a suspension of elastic particulate cells:
    - **Red blood cells** ($\approx$ 40-45 % of blood volume)
    - White blood cells
    - Platelets

  in a liquid known as plasma ($\approx$ 55% of blood volume)
- Properties of Blood e.g. viscosity depend on physiological flow conditions, blood composition properties e.g. hematocrit, temperature, shear rate, cell aggregation, cell shape, cell deformation, orientation ...

- While plasma has nearly Newtonian behaviour, whole blood exhibits non-Newtonian characteristics, particulary at low shear rates
- The Non-Newtonian characteristics of blood are:
  1. Shear thinning
  2. Yield stress (viscoplasticity)
  3. Viscoelastic properties
  4. Thixotropic behaviour

- Non-Newtonian characteristics occur due to:
  - Aggregation of RBC's at low shear rates (Rouleaux formation)
  - Thixotropic behaviour due to finite time needed for RBC aggregation and disaggregation.
  - Deformability of RBC and their tendency to align with flow field at high shear rates

Low shear rates $\approx < 1\frac{1}{s}$	High shear rates $\approx > 400\frac{1}{s}$
apparent viscosity increases	apparent viscosity decreases
yield stress	RBC's rotate & accomadate flow
thixotropic	
RBCs aggregate are solid-like bodies, and has ability to store elastic energy (viscoelastic)	RBCs behave fluid-like bodies

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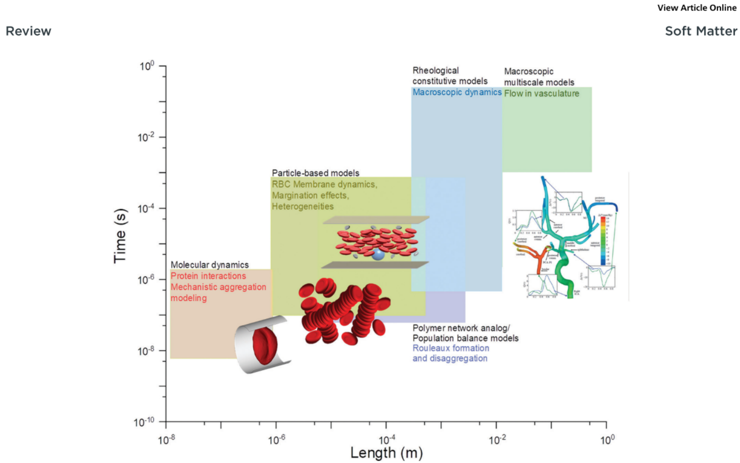

Figure 1: Length and time scales involved in blood flow modeling

- Classical Mechanics $\Rightarrow$ Continuum hypothesis!
- Starting point are mass and momentum conservation for incompressible fluids ($\rho \approx$ const) and a constitutive equation, which describes material behaviour

1. $$\nabla \cdot \boldsymbol{u} = 0$$
2. $$\rho \frac{\partial \boldsymbol{u}}{\partial t} + \rho(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = \nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{f}$$
3. $$\boldsymbol{\sigma} = \underbrace{-p\boldsymbol{I}}_{\text{volumetric stress tensor}} + \underbrace{\boldsymbol{\tau}}_{\text{deviatoric stress tensor}}$$

- Newtonian assumption i.e.
$\boldsymbol{\sigma} = \boldsymbol{\sigma}(\nabla \boldsymbol{u}, p) = -p\boldsymbol{I} + 2\eta \boldsymbol{D}(\nabla \boldsymbol{u}) = -p\boldsymbol{I} + \eta(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)$
with $\eta =$ const leads to well-known Navier-Stokes Equation

- Newtonian behaviour is a limitation usually accepted for blood flow in large arteries
- Non-Newtonian characteristics $\Rightarrow$ In small size vessels or in regions of stable recirculation, (e.g. venous system) and parts of arterial vasculature where geometry has been altered and RBC aggregates become more stable, like downstream a stenosis or inside a saccular aneurysm

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- In order to account for Non-Newtonian effects, we can
  1. use **time-independent constitutive equation** to close system and make the viscosity dependent of the deformation rate tensor
  2. use a (nonlinear) **viscoelastic constitutive equation** $\Rightarrow$ more complex constitutive equations must be solved simultaneously along with the equations of conservation of mass and momentum (time dependent models)

time-independent constitutive equation

- Express the viscosity as function of the strain rate $\Rightarrow$ Generalised Newtonian Model

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}(\nabla \boldsymbol{u}, p) = -p\boldsymbol{I} + 2\eta(\dot{\gamma})\ \boldsymbol{D}(\nabla \boldsymbol{u}) \tag{2}$$

- Examples are e.g. Power law model $\eta = k\dot{\gamma}^{n-1}$, Extension of the power-law model from Walburn and Schneck (considered the dependence of η on the haematocrit and total protein minus albumin content), Cassons equation (account for yield stress) ...
- Computationally inexpensive to implement but cannot predict accurately transient changes which are relevant as blood flows naturally under pulsatile conditions $\Rightarrow$ viscoelastic model

Viscoelastic constitutive equation (time-dependent)

- Split stress tensor approach: Decompose total stress tensor into its non-Newtonian (RBC influence) and Newtonian (plasma) parts
- Different models (Maxwell model, Oldroyd-B model ...) considering e.g. viscoelasticity, viscoelasticity + yield stress, thixotropic elasto-visco-plastic behavior of blood ...
- We get additional equations (e.g. ODE's) to solve for

Questions

- Do we want to examine a specific phenomena (e.g aneurysma) s.t. we can focus from the variety of non-newtonian models (including shear thinning, plasticity, viscoelasticity, thixotropy, all together, etc.) on a specific group
    - What is the relevant diamter which we want to examine?
    - What are our regions of interest?
    - Heat transfer?
    - I will neglect biochemical processes?
    - FSI?