

Blood Flow: Bridging the Micro-Macro Gap Using Scientific Machine Learning

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- ① Familiarization with properties and modeling of Blood Flow
- ② Derivation of Navier-Stokes Equation from classical mechanics
- ③ Rheological Non-Newtonian models for blood

- 1 Familiarization with properties and modeling of Blood Flow
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- Blood consist of a suspension of elastic particulate cells:
 - **Red blood cells** ($\approx 40\text{-}45\%$ of blood volume)
 - White blood cells
 - Platelets
- in a liquid known as plasma ($\approx 55\%$ of blood volume)
- Properties of Blood e.g. viscosity depend on physiological flow conditions, blood composition properties e.g. hematocrit, temperature, shear rate, cell aggregation, cell shape, cell deformation, orientation ...

- While plasma has nearly Newtonian behaviour, whole blood exhibits non-Newtonian characteristics, particularly at low shear rates
- The Non-Newtonian characteristics of blood are:
 - ① Shear thinning
 - ② Yield stress (viscoplasticity)
 - ③ Viscoelastic properties
 - ④ Thixotropic behaviour

- Non-Newtonian characteristics occur due to:
 - Aggregation of RBC's at low shear rates (Rouleaux formation)
 - Thixotropic behaviour due to finite time needed for RBC aggregation and disaggregation.
 - Deformability of RBC and their tendency to align with flow field at high shear rates

Low shear rates $\approx < 1 \frac{1}{s}$

apparent viscosity increases
yield stress

thixotropic

RBCs aggregate are solid-like bodies, and has ability to store elastic energy (viscoelastic)

High shear rates $\approx > 400 \frac{1}{s}$

apparent viscosity decreases
RBC's rotate & accomodate flow

RBCs behave fluid-like bodies

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Review

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Soft Matter

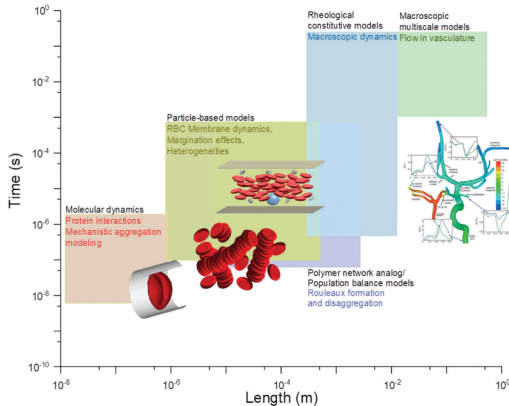


Figure 1: Length and time scales involved in blood flow modeling

- Classical Mechanics \Rightarrow Continuum hypothesis!
- Starting point are mass and momentum conservation for incompressible fluid ($\rho \approx \text{const}$)

①

$$\nabla \cdot \mathbf{u} = 0$$

②

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u} \cdot \nabla) \mathbf{u} = \nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{f}$$

- Newtonian assumption i.e.
 $\boldsymbol{\sigma} = \boldsymbol{\sigma}(\nabla \mathbf{u}, p) = -p\mathbf{I} + 2\eta \mathbf{D}(\nabla \mathbf{u}) = -p\mathbf{I} + \eta(\nabla \mathbf{u} + \nabla \mathbf{u}^T)$
with $\eta = \text{const}$ leads to well-known Navier-Stokes Equation

- Newtonian behaviour is a limitation usually accepted for blood flow in large arteries
- Non-Newtonian characteristics \Rightarrow In small size vessels or in regions of stable recirculation, (e.g. venous system) and parts of arterial vasculature where geometry has been altered and RBC aggregates become more stable, like downstream a stenosis or inside a saccular aneurysm

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- In order to account for Non-Newtonian effects, we can
 - ① use **linear elastic constitutive equation** to close system and make the viscosity dependent of the deformation rate tensor (time-independent models)
 - ② use a (nonlinear) **viscoelastic constitutive equation** \Rightarrow more complex constitutive equations must be solved simultaneously along with the equations of conservation of mass and momentum (time dependent models)

Linear elastic constitutive equation (time independent)

- Express the viscosity as function of the strain rate \Rightarrow Generalised Newtonian Model

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}(\nabla \mathbf{u}, \rho) = -p\mathbf{I} + 2\eta(\dot{\gamma}) \mathbf{D}(\nabla \mathbf{u}) \quad (2)$$

- Examples are e.g. Power law model $\eta = k\dot{\gamma}^{n-1}$, Extension of the power-law model from Walburn and Schneck (considered the dependence of η on the haematocrit and total protein minus albumin content), Cassons equation (account for yield stress) ...
- Computationally inexpensive to implement but cannot predict accurately transient changes which are relevant as blood flows naturally under pulsatile conditions \Rightarrow viscoelastic model

Viscoelastic constitutive equation (time-dependent)

- Split stress tensor approach: Decompose total stress tensor into its non-Newtonian (RBC influence) and Newtonian (plasma) parts
- Different models (Maxwell model, Oldroyd-B model ...) considering e.g. viscoelasticity, viscoelasticity + yield stress, thixotropic elasto-visco-plastic behavior of blood ...
- We get additional equations (e.g. ODE's) to solve for

Questions

- Do we want to examine a specific phenomena (e.g aneurysma) s.t. we can focus from the variety of non-newtonian models (including shear thinning, plasticity, viscoelasticity, thixotropy, all together, etc.) on a specific group
 - What is the relevant diamter which we want to examine?
 - What are our regions of interest?
 - Heat transfer?
 - I will neglect biochemical processes?
 - FSI?