Artificial Intelligence

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Evolution

"... no limit to this power of slowly and beautifully adapting each form to the most complex relations of life ... "

——— ***Charles Darwin***

Example

Maximum $f(x) = x^2$, $x \in [1, 31]$

- **Representation**

  $x \in \{0,1\}^5$

- **Initialization**

1st generation	01101,	11000,	01000,	10011
Interpretation	13,	24,	8,	19
Fitness	169,	576,	64,	361

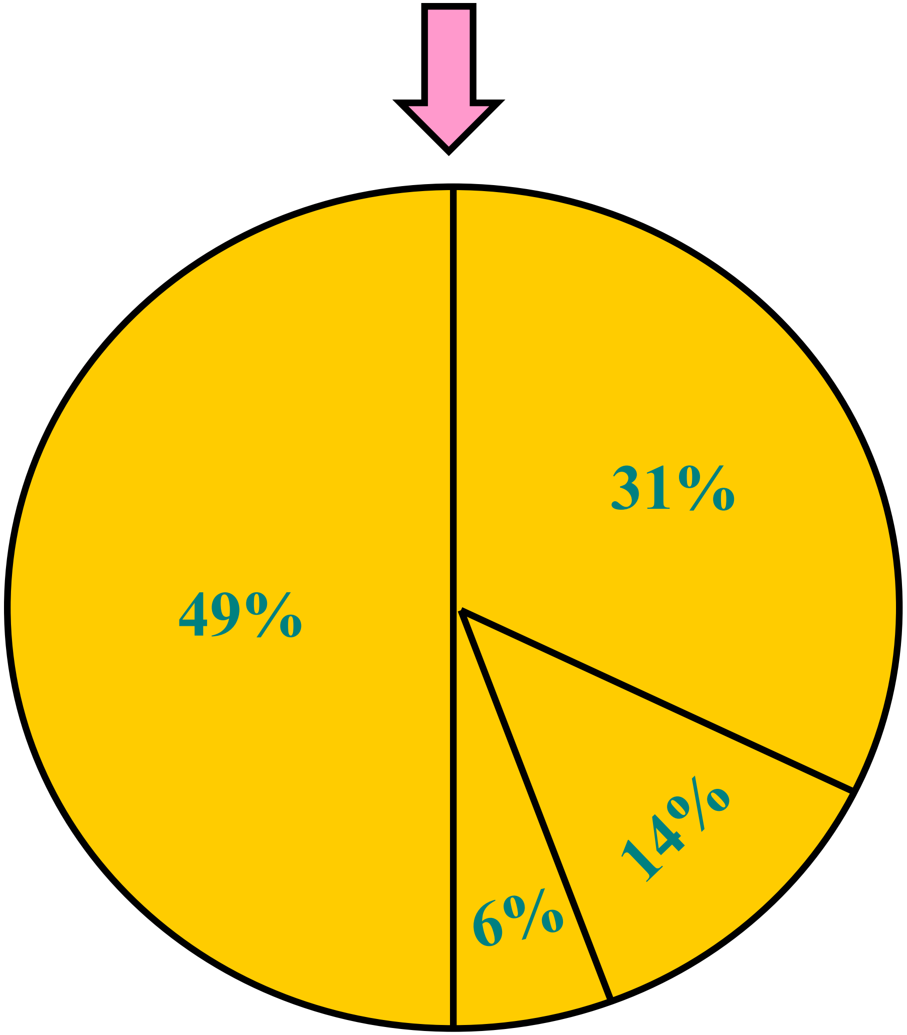
Example

Maximum $f(x) = x^2, x \in [1, 31]$

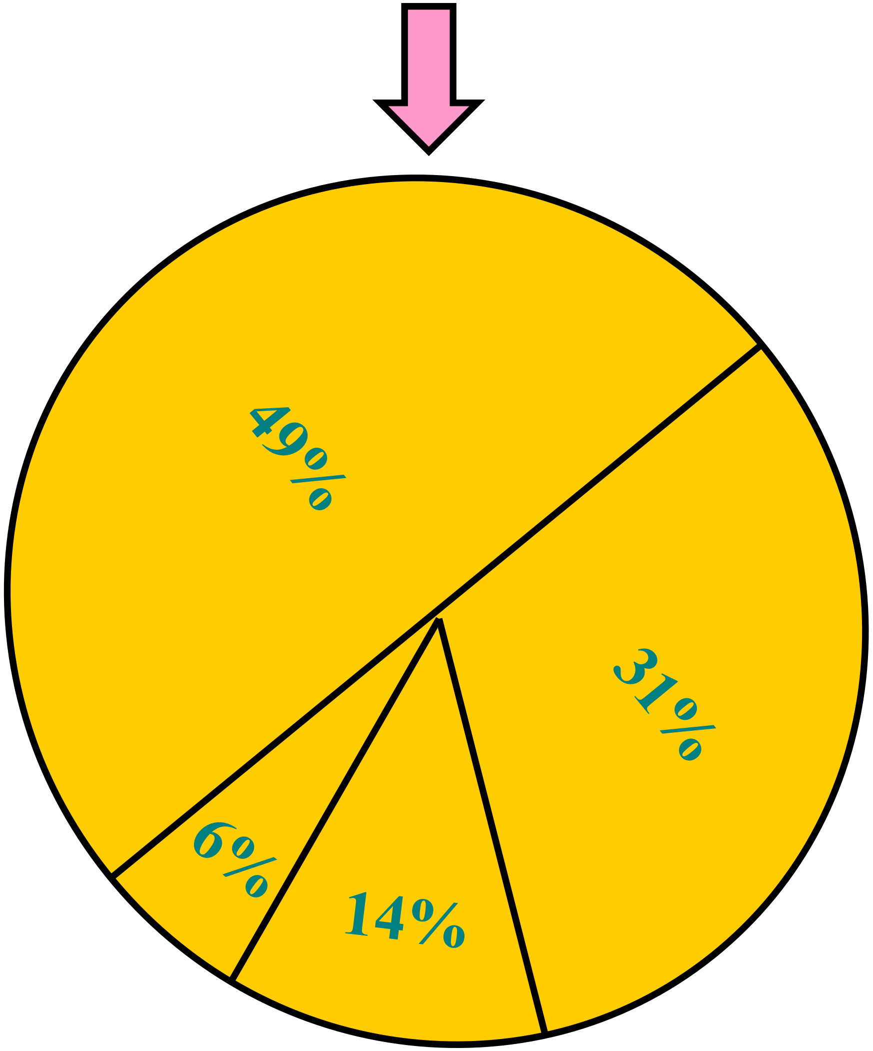
- **Selection**

Individual	01101,	11000,	01000,	10011
Fitness	169,	576,	64,	361 = 1170
Probability	0.14,	0.49,	0.06,	0.31 = 1.0
Result	01101,	11000,	11000,	10011

Russian roulette



Russian roulette



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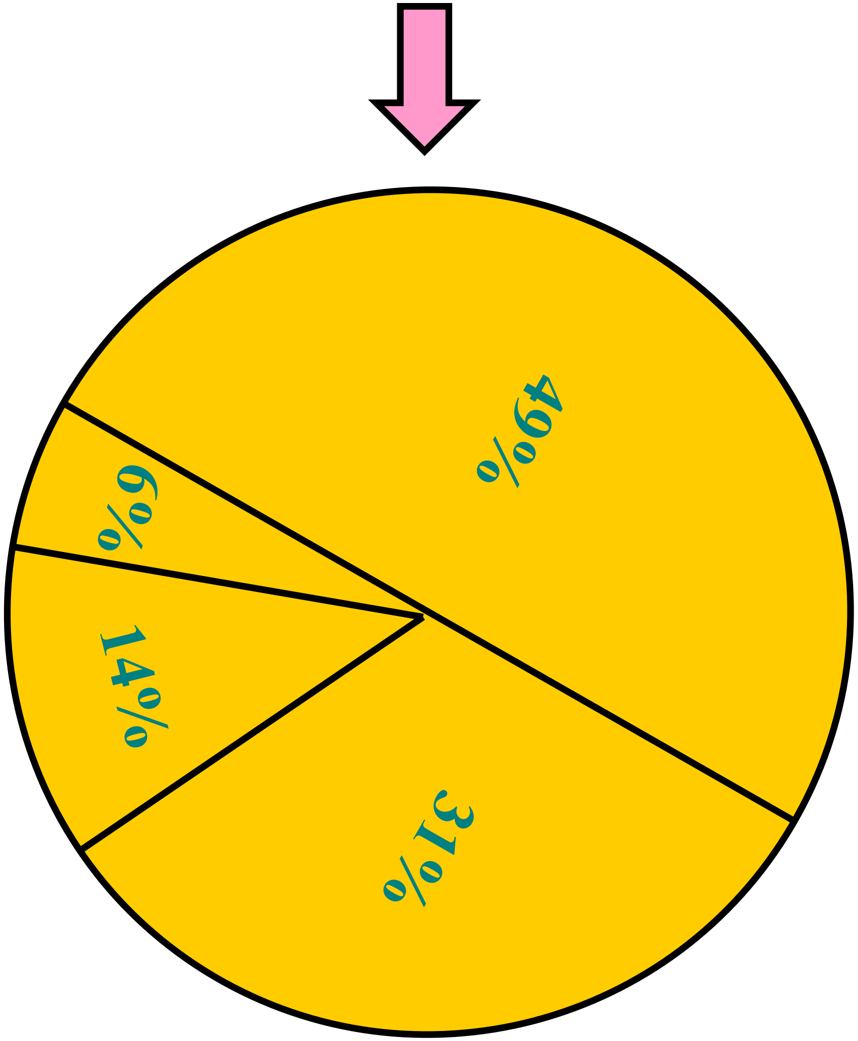

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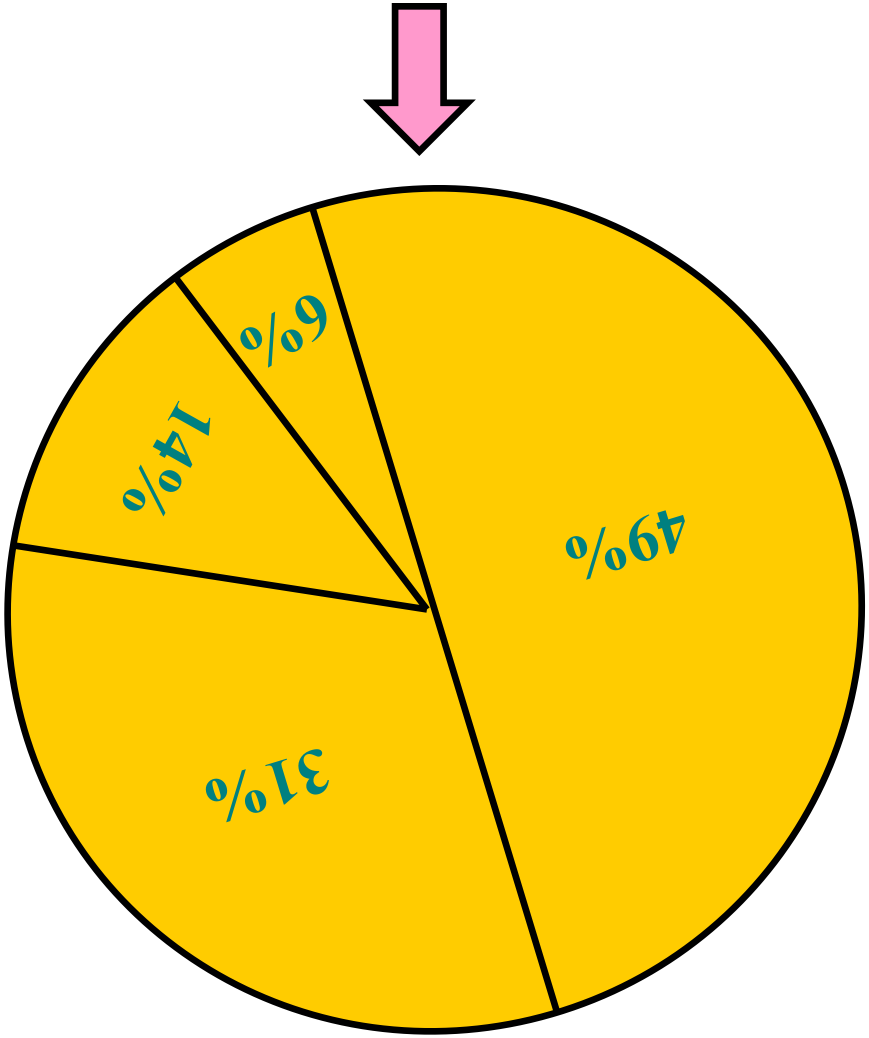
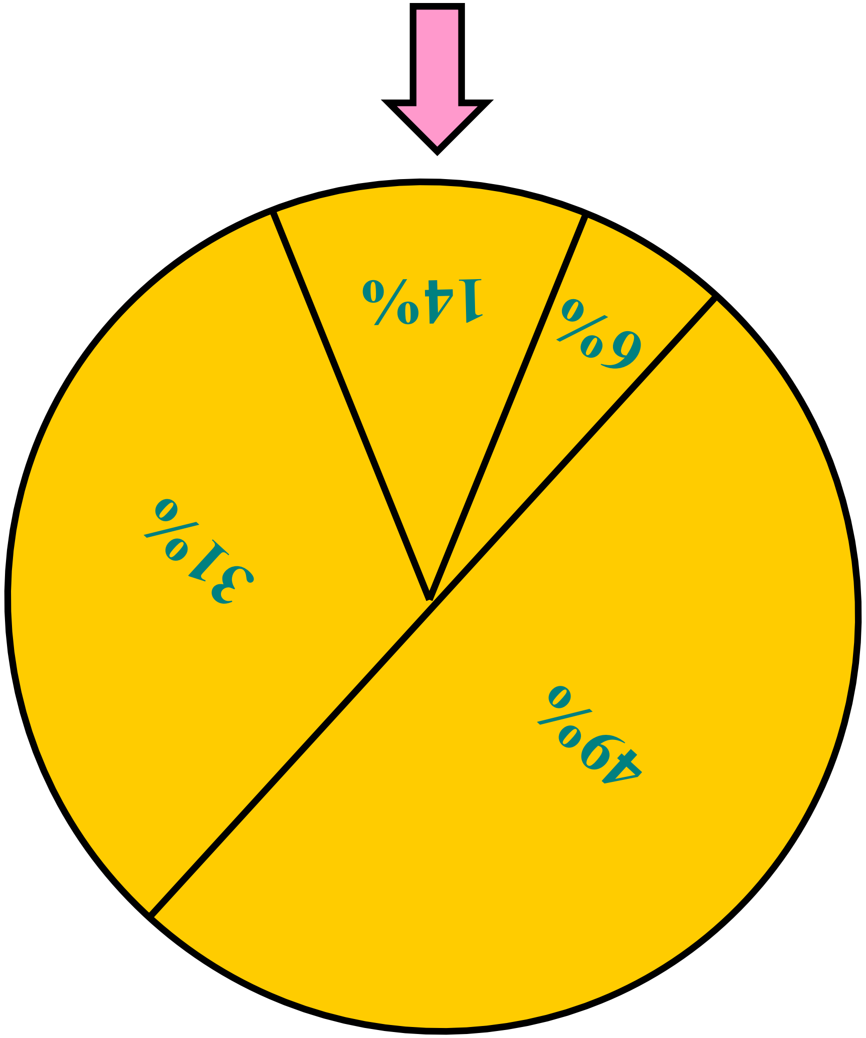


Russian roulette



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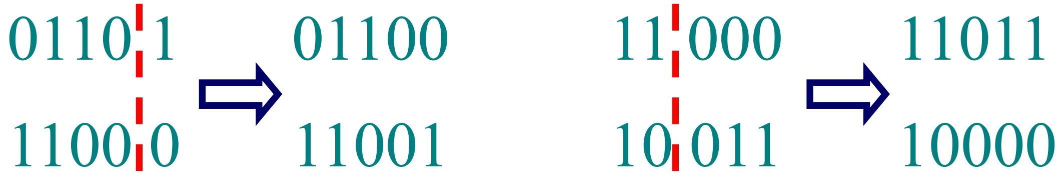
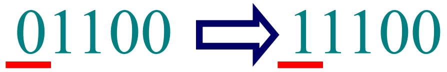
- **Crossover**



- **Mutation**



Genetic algorithm

```
begin
 set time $t = 0$
 initialize the population $P(t)$
 while the termination condition is not met do
  begin
   evaluate fitness of each member of the population $P(t)$;
   select members from population $P(t)$ based on fitness;
   produce the offspring of these pairs using genetic operators;
   replace  candidates of $P(t)$, with these offspring;
   set time $t = t + 1$
  end
end
```

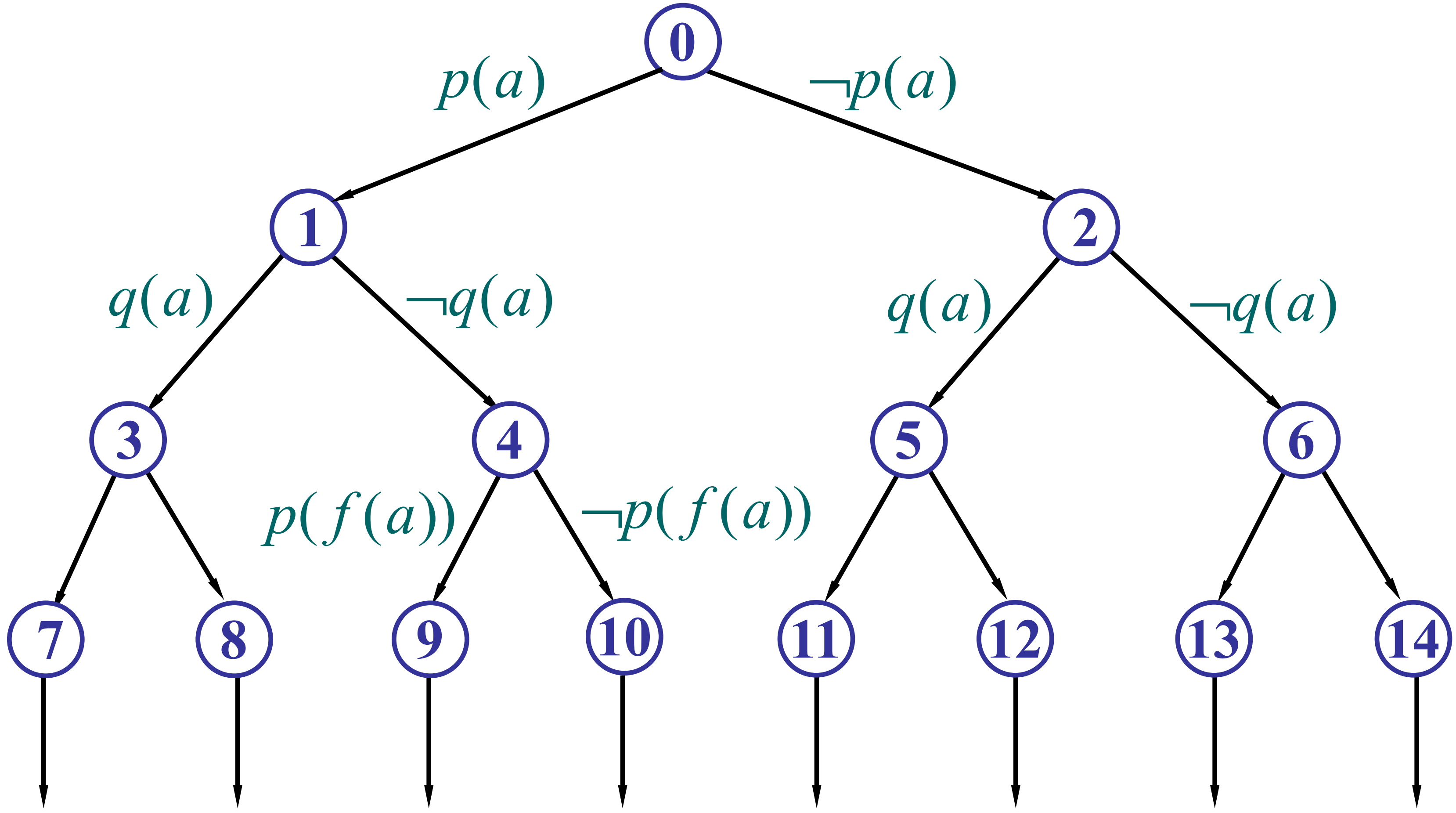
Knapsack problem

A thief robbing a store finds n items; the ith item is worth v_i dollars and weights w_i pounds, where v_i and w_i are integers. He wants to take as valuable a load as possible, but he can carry at most W pounds in his knapsack for some integer W.

Which items should he take?

Semantic tree

$\{p(x), \neg p(x) \vee q(x), \neg q(f(a))\}$



CNF-satisfaction problem

The ***conjunctive normal form*** (CNF) satisfiability problem is straightforward: an expression of propositions is in conjunctive normal form when it is a sequence of clauses joined by an **AND** relation. Each of these clauses is in the form of a disjunction, the **OR** of literals.

$$(\neg a \vee c) \wedge (\neg a \vee c \vee \neg e) \wedge (\neg b \vee c \vee d \vee \neg e) \wedge$$
$$(a \vee \neg b \vee c) \wedge (\neg e \vee v)$$

Analysis of traveling salesman problem

Given a finite number of "cities" along with the cost of travel between each pair of them, find the ***cheapest way*** of visiting each city exactly once and finishing at the city he starts from.

- 19 ***Cities***
- ***Possible routes*** = 18! = 6.40237×10^{15}
- ***life time*** = 80 × 365 × 24 × 60 × 60
  = 2.52288×10^{9}
- ***Computer speed*** = 10000 ***routes/second***

253.77 ***Generation!***

Traveling salesman problem

Crossover

$p_1 = (\ 1\ \ 9\ \ 2 \mid 4\ \ 6\ \ 5\ \ 7 \mid 8\ \ 3\ )$

$p_2 = (\ 4\ \ 5\ \ 9 \mid 1\ \ 8\ \ 7\ \ 6 \mid 2\ \ 3\ )$

Start from the second cut point of one parent, the cities from the other parent are copied in the same order, omitting cities already present. When the end of the string is reached, continue on from the beginning. The sequence of cities from p_2 is :

2 3 4 5 9 1 8 7 6

$c_1 = (\ 2\ \ 3\ \ 9 \mid 4\ \ 6\ \ 5\ \ 7 \mid 1\ \ 8\ )$

$c_2 = (\ 3\ \ 9\ \ 2 \mid 1\ \ 8\ \ 7\ \ 6 \mid 4\ \ 5\ )$

Traveling salesman problem

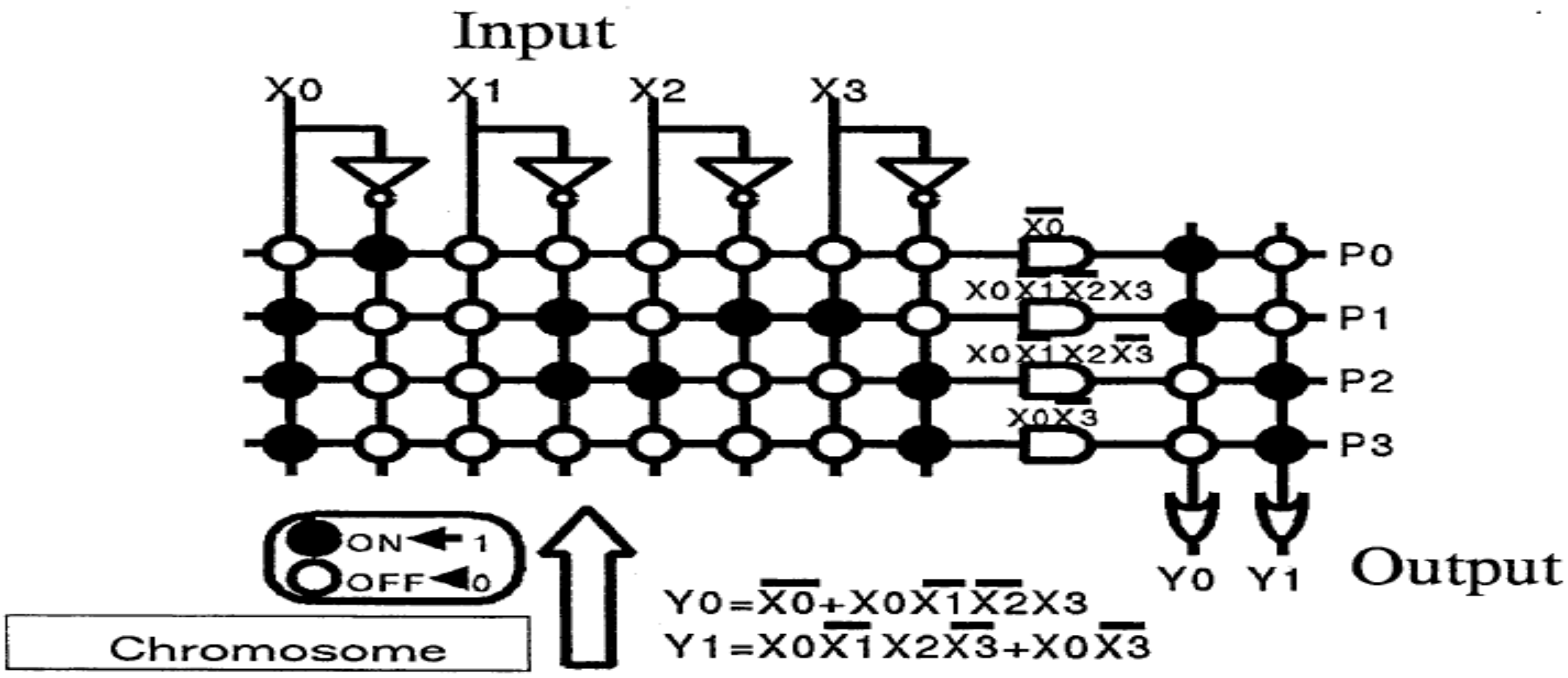
Mutation

$$c_1 = (\,2\;\;3\;\;9\;|\;4\;\;6\;\;5\;\;7\;|\;1\;\;8\,)$$

$$c_1 = (\,2\;\;3\;\;9\;|\;\underbrace{7\;\;5\;\;6\;\;4}_{\textit{Inversion}}\;|\;1\;\;8\,)$$

Evolutionary hardware

PLA (Programmable Logic Array)



$Y0=\overline{X0}+X0\overline{X1}\overline{X2}X3$

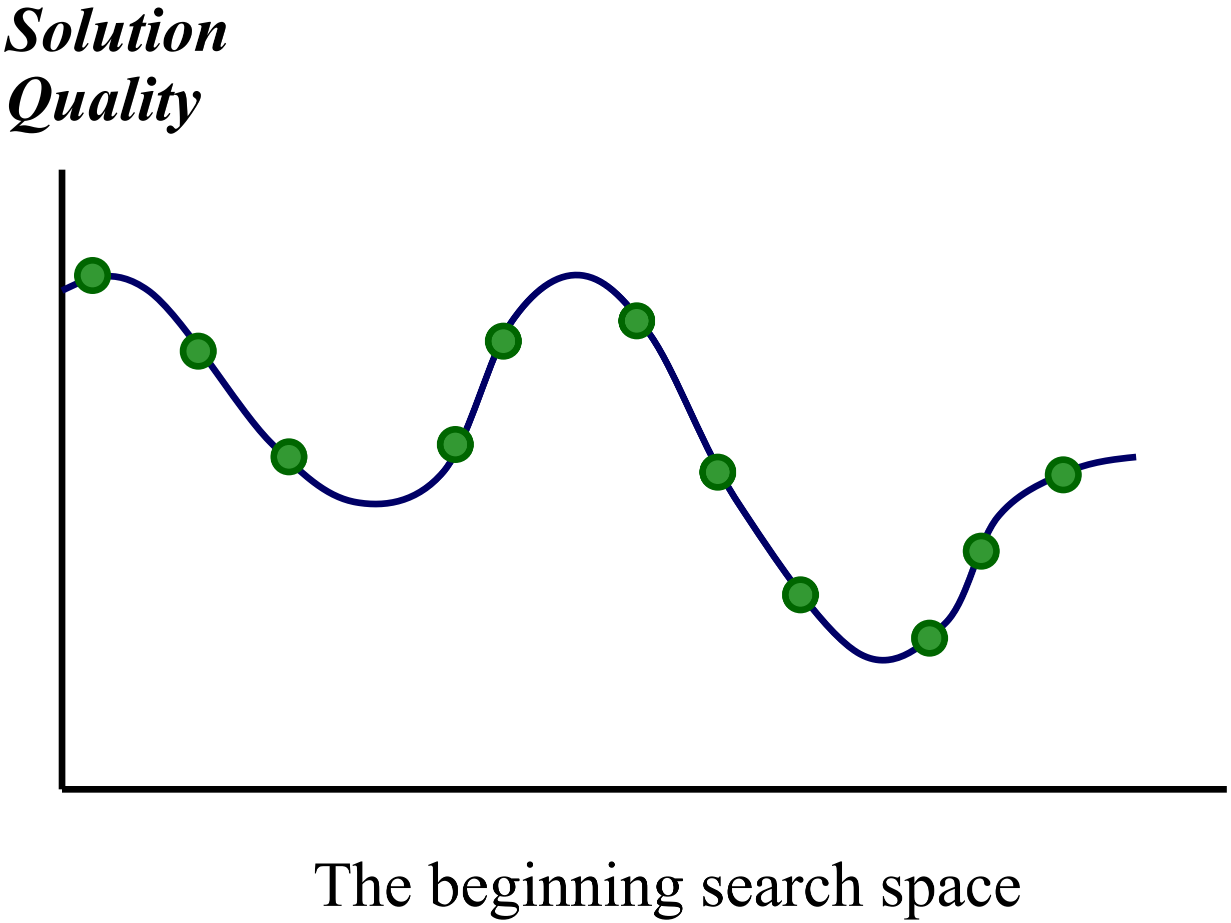
$Y1=X0\overline{X1}X2\overline{X3}+X0\overline{X3}$

0100000010 1001011010 1001100101 1000000101

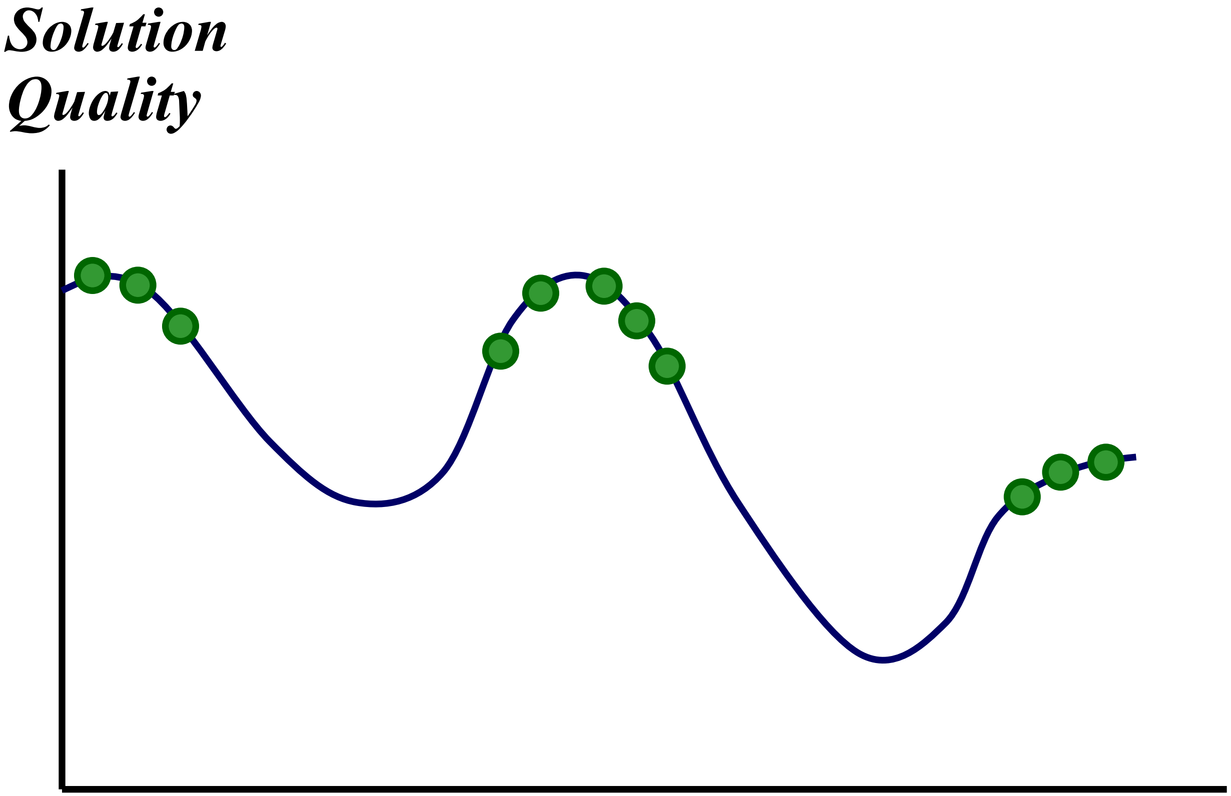
Gray coding

Binary	Gray	Binary	Gray
0000	0000	1000	1100
0001	0001	1001	1101
0010	0011	1010	1111
0011	0010	1011	1110
0100	0110	1100	1010
0101	0111	1101	1011
0110	0101	1110	1001
0111	0100	1111	1000

Strength of the genetic algorithms



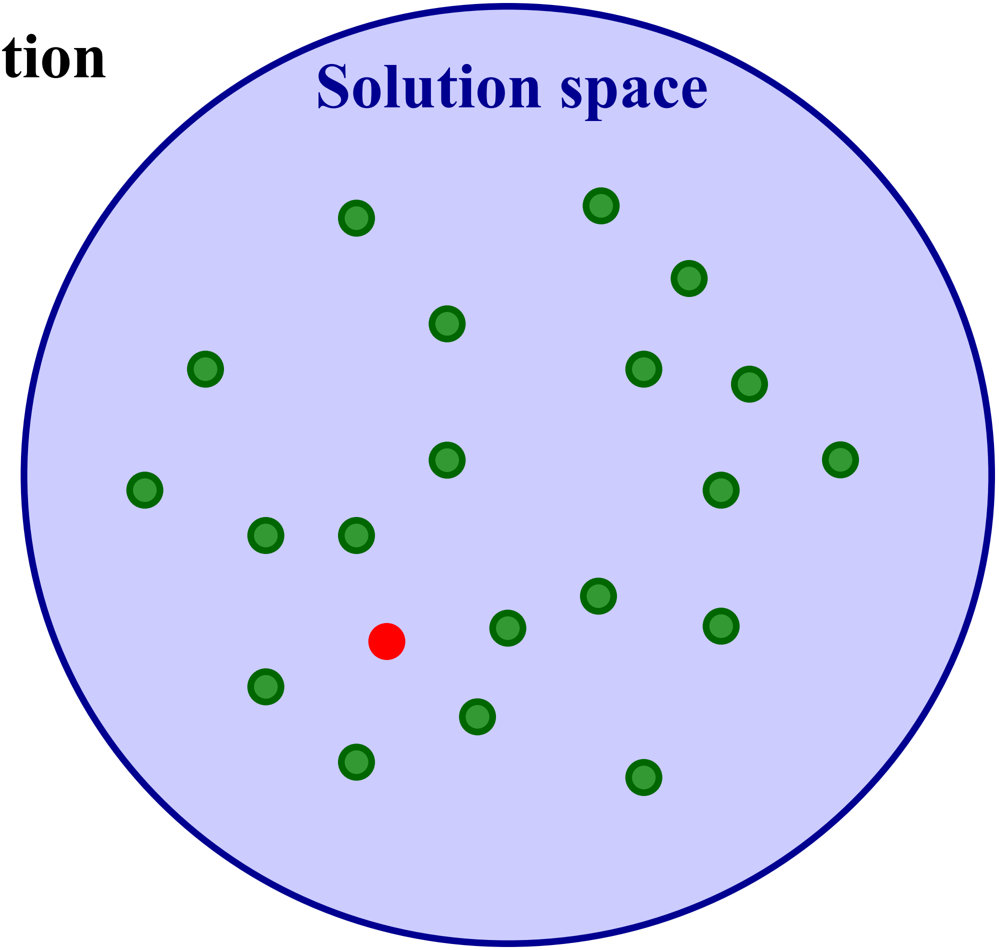
Strength of the genetic algorithms



The search space after *n* generations

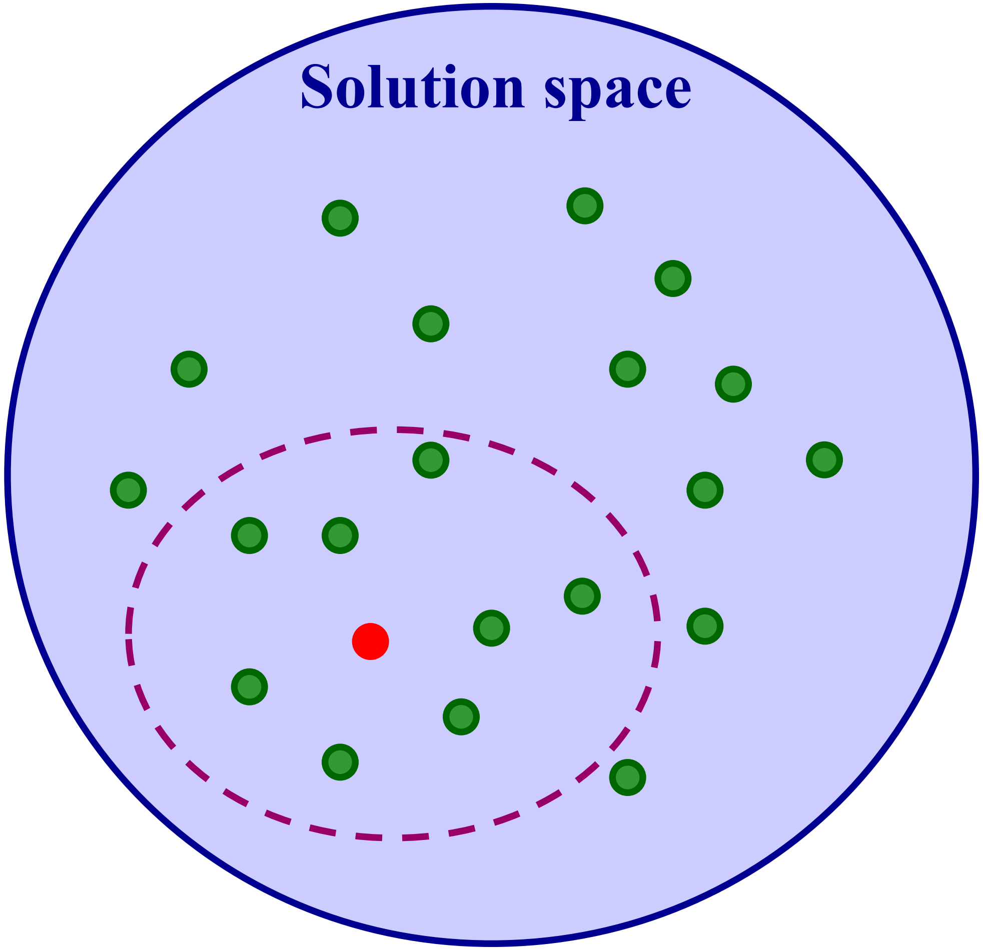
Random research

Initialization



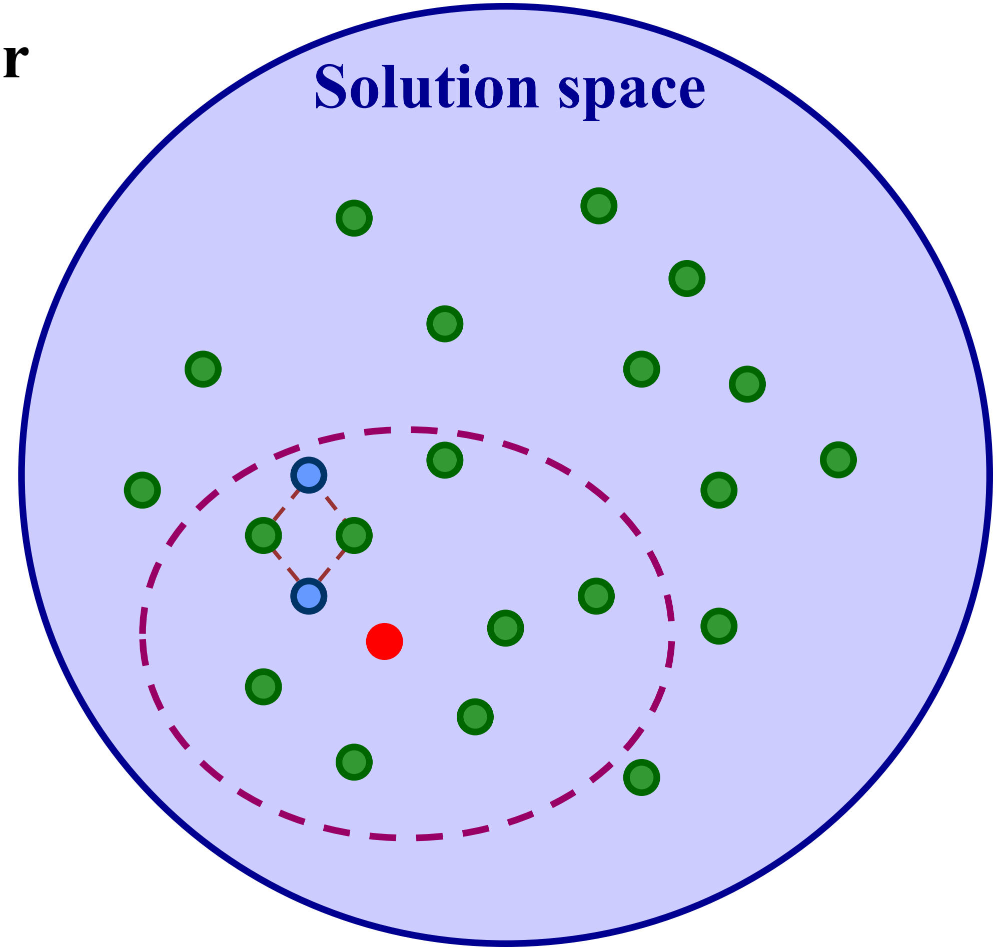
Random research

Selection



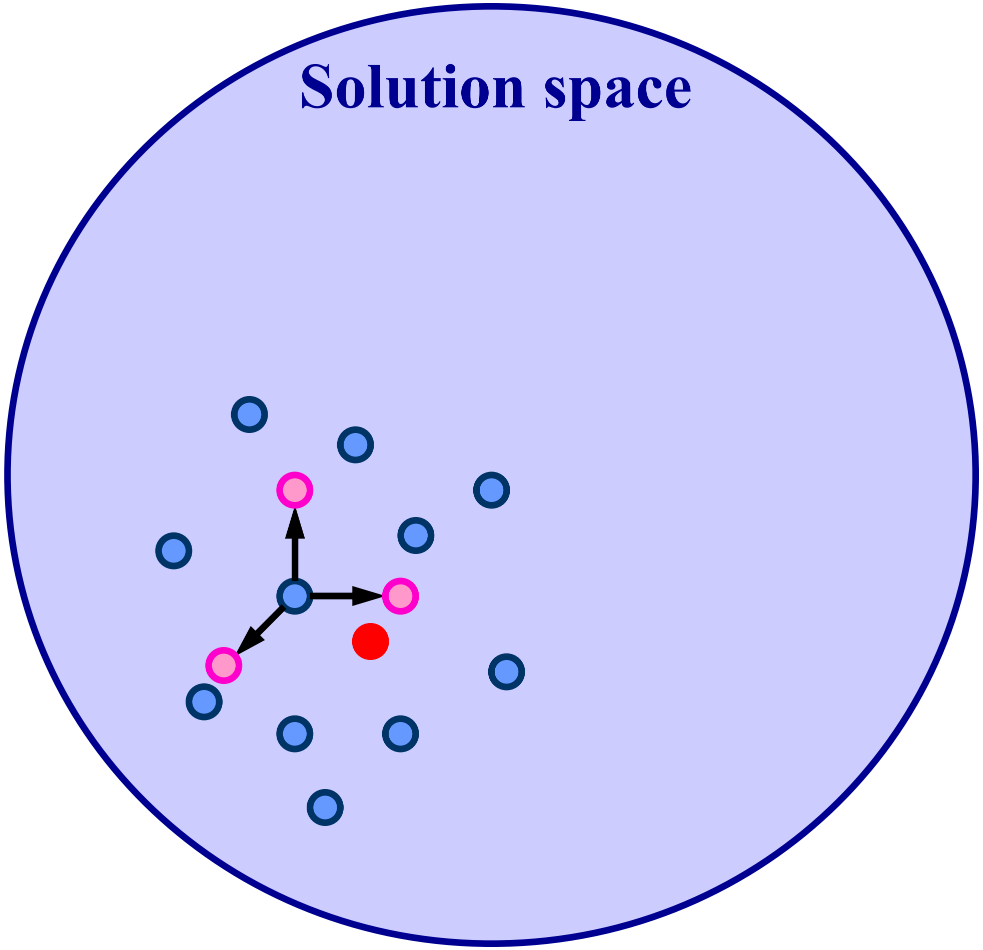
Random research

Crossover



Random research

Mutation



Schema

- The schema (1 * * 0 * 1) describes the set of all strings of length 6 with 1's at positions 1 and 6 and a 0 at position 4. The * is a ***wildcard*** symbol, which means that positions 2, 3 and 5 can have a value of either 1 or 0.
- The ***order*** of a schema, $O(s)$, is defined as the number of ***fixed positions*** in the template, while the defining ***length*** $\delta(s)$ is the distance between the first and last specific positions. The order of (1 * * 0 * 1) is 3 and its defining length is 5.
- The ***fitness*** of a schema is the ***average*** fitness of all strings matching the schema. The fitness of a string is a measure of the value of the encoded problem solution, as computed by a problem-specific evaluation function.

Schema theorem

The probability of the string *i* that is selected:

$$p_i = \frac{f(i)}{\sum_{j=1}^{n} f(j)}$$

m(*s*,*t*): the number of strings belonging to schema *s* at generation *t*.

$$E\left[m(s,t+1)\right] = m(s,t) \cdot n \cdot \frac{\overline{f}(s)}{\sum_{j=1}^{n} f(j)}$$

Schema theorem

$\overline{f}$ is the average fitness at generation t.

$$\overline{f} = \frac{\sum_{j=1}^{n} f(j)}{n}$$

Then,

$$E\left[m(s,t+1)\right] = m(s,t) \cdot \frac{\overline{f}(s)}{\overline{f}}$$

Let $\overline{f}(s) = (1+c)\overline{f}$

$$E\left[m(s,t+1)\right] = m(s,t) \cdot \frac{(1+c)\overline{f}}{\overline{f}}$$

$$= m(s,t) \cdot (1+c) = m(s,0) \cdot (1+c)^{t+1}$$

Schema theorem

The probability of disruption p_s is the probability that crossover will not destroy the schema s.

$$p_s \geq 1 - \frac{\delta(s)}{l-1}$$

p_c is the probability of crossover.

$$p_s \geq 1 - p_c \cdot \frac{\delta(s)}{l-1}$$

Then,

$$E\left[m(s,t+1)\right] = m(s,t) \cdot \frac{\overline{f(s)}}{\overline{f}}\left[1 - p_c \cdot \frac{\delta(s)}{l-1}\right]$$

Schema theorem

The probability of disruption p_s is the probability that mutation will not destroy the schema s, and p_m is the probability of mutation.

$$p_s = \left(1 - p_m\right)^{O(s)}$$

Assuming $p_m << 1$

$$p_s \approx 1 - p_m \cdot O(s)$$

Then,

$$\begin{aligned} E\left[m(s,t+1)\right] &= m(s,t) \cdot \frac{\overline{f}(s)}{\overline{f}} \cdot \left[1 - p_c \cdot \frac{\delta(s)}{l-1}\right] \cdot \left[1 - O(s) p_m\right] \\ &\approx m(s,t) \cdot \frac{\overline{f}(s)}{\overline{f}} \cdot \left[1 - p_c \cdot \frac{\delta(s)}{l-1} - O(s) \cdot p_m\right] \end{aligned}$$

Schema theorem

- Holland's schema theorem is widely taken to be the foundation for explanations of the power of ***genetic algorithms***.
- A ***schema*** is a template that identifies a ***subset*** of strings with similarities at certain string positions.
- The schema theorem states that ***short***, ***low-order***, ***schemata*** with ***above-average fitness*** increase exponentially in successive generations.

Any question?

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