

# Artificial Intelligence

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# Introduction

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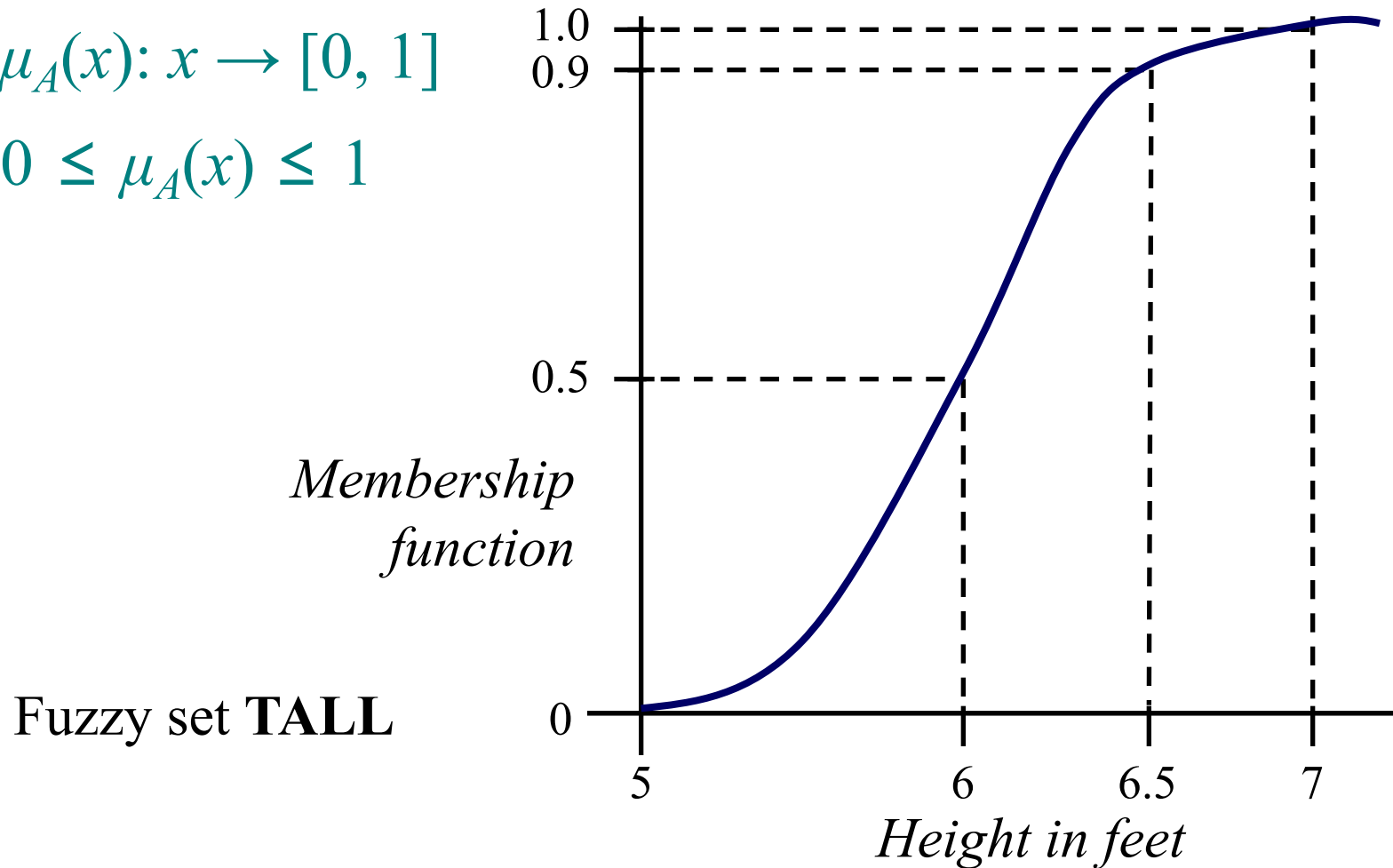
- Classical *Bivalent* or *two-valued* logic.
- L. A. Zadeh's *fuzzy* set.
- Pawlak's *Rough* set.

# Membership function

Membership or compatibility function

$$\mu_A(x): x \rightarrow [0, 1]$$

$$0 \leq \mu_A(x) \leq 1$$



# Information table

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$U$	$a$	$b$	$c$	$d$	$e$
(1)	1	2	2	1	2
(2)	3	2	2	2	1
(3)	3	2	3	3	3
(4)	2	1	3	1	2
(5)	2	3	2	1	3
(6)	1	2	2	2	3
(7)	2	1	2	1	2
(8)	3	3	1	1	1

# Rough set

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- Let  $I = (U, A)$  be an *information system* (attribute-value system), where  $U$  is a non-empty set of finite objects (the universe) and  $A$  is a non-empty, finite set of attributes such that  $a: U \rightarrow V_a$  for every  $a \in A$ .  $V_a$  is the set of values that attribute  $a$  may take.
- The information table assigns a value in  $V_a$  to each attribute  $a$  of each object in universe  $U$ .

**Universe**  $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$

**Associated equivalence relation**  $R$ :

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$$E_1 = \{x_1, x_4\}$$

$$E_2 = \{x_3, x_8\}$$

$$E_3 = \{x_2, x_5, x_7\}$$

$$E_4 = \{x_6\}$$

**Partition**  $U/R = \{E_1, E_2, E_3, E_4\}$ , and  $X = \{x_1, x_4, x_2\}$

**Lower approximation**  $R_*(X) = \{x_1, x_4\}$

**Upper approximation**  $R^*(X) = \{x_1, x_4, x_2, x_5, x_7\}$

**Universe**  $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$

**Associated equivalence relation**  $R$ :

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$$E_1 = \{x_1, x_4\}$$

$$E_2 = \{x_3, x_8\}$$

$$E_3 = \{x_2, x_5, x_7\}$$

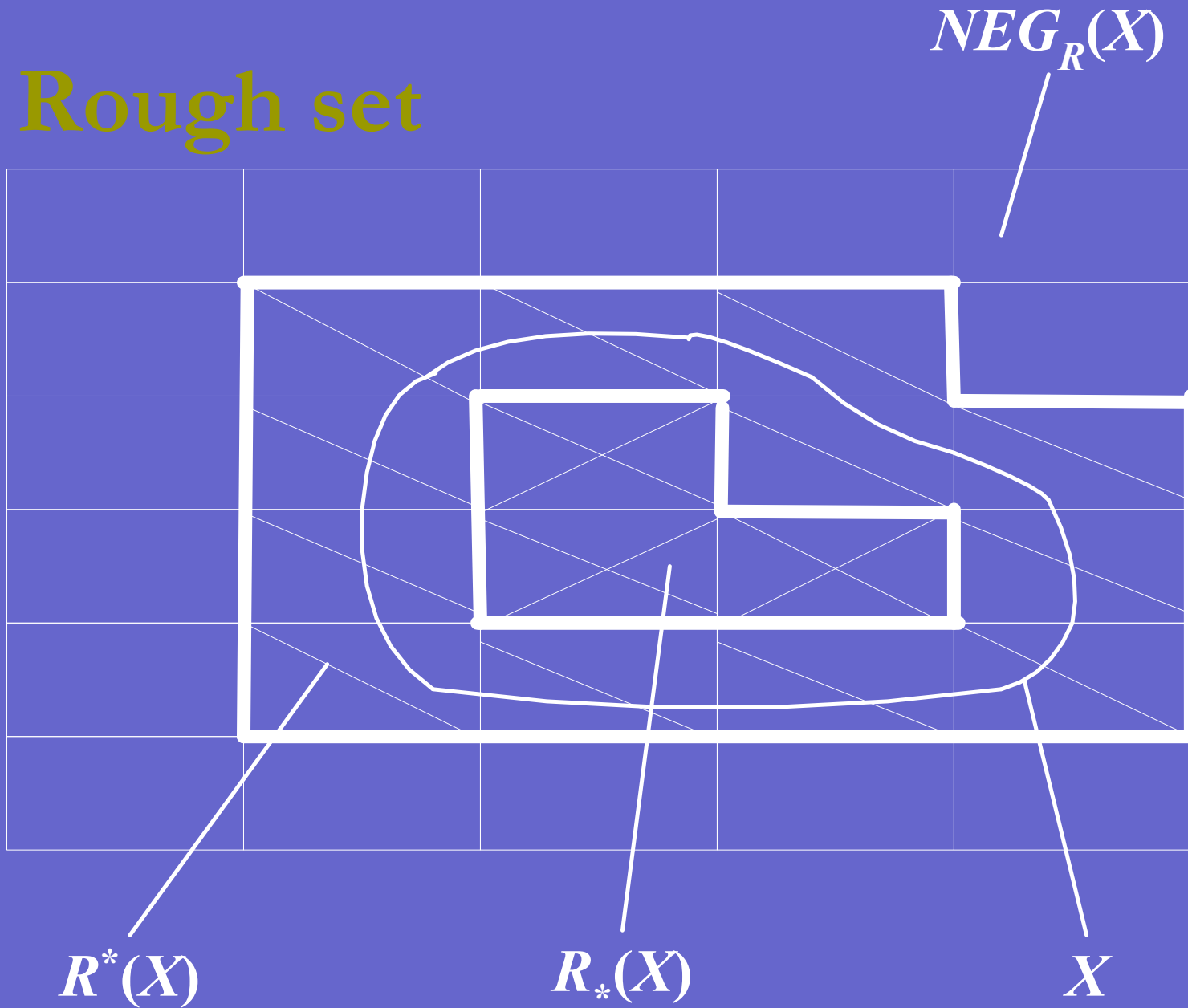
$$E_4 = \{x_6\}$$

**Partition**  $U/R = \{E_1, E_2, E_3, E_4\}$ , and  $X = \{x_1, x_4, x_2\}$

**Lower approximation**  $R_*(X) = \{x_1, x_4\}$

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*Lower approximation*  $R_*(X) = \{x_1, x_4\}$

*Upper approximation*  $R^*(X) = \{x_1, x_4, x_2, x_5, x_7\}$

# Membership function

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$$\mu_X^R(x) = \text{card}(X \cap R(x)) / \text{card}(R(x))$$

$$\mu_X^R(x_1) = 2 / 2 = 1.00$$

$$\mu_X^R(x_2) = 1 / 3 = 0.33$$

$$\mu_X^R(x_3) = 0 / 2 = 0.00$$

**Universe**  $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$

A set of **Associated equivalence relation**  $R = \{P, Q, W\}$

$$U/P = \{\{x_1, x_3, x_4, x_5, x_6, x_7\}, \{x_2, x_8\}\}$$

$$U/Q = \{\{x_1, x_3, x_4, x_5\}, \{x_2, x_6, x_7, x_8\}\}$$

$$U/W = \{\{x_1, x_5, x_6\}, \{x_2, x_7, x_8\}, \{x_3, x_4\}\}$$

Then  $U/IND(R) = \{\{x_1, x_5\}, \{x_3, x_4\}, \{x_2, x_8\}, \{x_6\}, \{x_7\}\}$

**Associated equivalence relation**  $S$  (**Decision Attribute**)

$$U/S = \{\{x_1, x_5, x_6\}, \{x_3, x_4\}, \{x_2, x_7\}, \{x_8\}\}$$

**Positive region:**

$$POS_R(S) = \{x_1, x_3, x_4, x_5, x_6, x_7\}$$

*Universe*  $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$

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# Information table

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$U$	$a$	$b$	$c$	$d$	$e$
(1)	1	0	2	1	1
(2)	2	1	0	1	0
(3)	2	1	2	0	2
(4)	1	2	2	1	1
(5)	1	2	0	0	2

## Rules:

$$a_1 b_0 c_2 \rightarrow d_1 e_1$$

$$a_2 b_1 c_0 \rightarrow d_1 e_0$$

$$a_2 b_1 c_2 \rightarrow d_0 e_2$$

$$a_1 b_2 c_2 \rightarrow d_1 e_1$$

$$a_1 b_2 c_0 \rightarrow d_0 e_2$$

# Problem

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- An interesting question is whether there are attributes in the information system which are more *important* to the knowledge represented in the equivalence class structure than other attributes.
- We wonder whether there is a subset of attributes which can, by itself, fully characterize the knowledge in the database; such an attribute set is called a *reduct*.

# Remove attribute $a$

$U$	$b$	$c$	$d$	$e$
(1)	0	2	1	1
(2)	1	0	1	0
(3)	1	2	0	2
(4)	2	2	1	1
(5)	2	0	0	2

**Legitimate  
reduct**

$$a_2b_1c_2 \rightarrow d_0e_2$$

$$a_1b_2c_2 \rightarrow d_1e_1$$

$$a_1b_2c_0 \rightarrow d_0e_2$$

# Remove attribute $b$

$U$	$a$	$c$	$d$	$e$
(1)	1	2	1	1
(2)	2	0	1	0
(3)	2	2	0	2
(4)	1	2	1	1
(5)	1	0	0	2

**Legitimate  
reduct**

$$a_2b_1c_2 \rightarrow d_0e_2$$

$$a_1b_2c_2 \rightarrow d_1e_1$$

$$a_1b_2c_0 \rightarrow d_0e_2$$

# Remove attribute $c$

$U$	$a$	$b$	$d$	$e$
(1)	1	0	1	1
(2)	2	1	1	0
(3)	2	1	0	2
(4)	1	2	1	1
(5)	1	2	0	2

## Rules:

$$a_1b_0c_2 \rightarrow d_1e_1$$

$$a_2b_1c_0 \rightarrow d_1e_0$$

$$a_2b_1c_2 \rightarrow d_0e_2$$

$$a_1b_2c_2 \rightarrow d_1e_1$$

$$a_1b_2c_0 \rightarrow d_0e_2$$

# Remove attribute $c$

$U$	$a$	$b$	$d$	$e$
(1)	1	0	1	1
(2)	2	1	1	0
(3)	2	1	0	2
(4)	1	2	1	1
(5)	1	2	0	2

**Causing  
collapse**

$a_2b_1c_2 \rightarrow d_0e_2$

$a_1b_2c_2 \rightarrow d_1e_1$

$a_1b_2c_0 \rightarrow d_0e_2$

# Remove attribute $c$

$U$	$a$	$b$	$d$	$e$
(1)	1	0	1	1
(2)	2	1	1	0
(3)	2	1	0	2
(4)	1	2	1	1
(5)	1	2	0	2

**Causing  
collapse**

$a_2b_1c_2 \rightarrow d_0e_2$

$a_1b_2c_2 \rightarrow d_1e_1$

$a_1b_2c_0 \rightarrow d_0e_2$



# Information table after removing $a$

$U$	$b$	$c$	$d$	$e$
(1)	0	2	1	1
(2)	1	0	1	0
(3)	1	2	0	2
(4)	2	2	1	1
(5)	2	0	0	2

## Rules:

$$b_0c_2 \rightarrow d_1e_1$$

$$b_1c_0 \rightarrow d_1e_0$$

$$b_1c_2 \rightarrow d_0e_2$$

$$b_2c_2 \rightarrow d_1e_1$$

$$b_2c_0 \rightarrow d_0e_2$$

# Information table after removing $b$

$U$	$a$	$c$	$d$	$e$
(1)	1	2	1	1
(2)	2	0	1	0
(3)	2	2	0	2
(4)	1	2	1	1
(5)	1	0	0	2

## Rules:

$$a_1c_2 \rightarrow d_1e_1$$

$$a_2c_0 \rightarrow d_1e_0$$

$$a_2c_2 \rightarrow d_0e_2$$

$$a_1c_2 \rightarrow d_1e_1$$

$$a_1c_0 \rightarrow d_0e_2$$

# Information table after removing $b$

$U$	$a$	$c$	$d$	$e$
(1)	1	2	1	1
(2)	2	0	1	0
(3)	2	2	0	2
(4)	1	2	1	1
(5)	1	0	0	2

## Rules:

$$a_1c_2 \rightarrow d_1e_1$$

$$a_2c_0 \rightarrow d_1e_0$$

$$a_2c_2 \rightarrow d_0e_2$$

$$a_1c_2 \rightarrow d_1e_1$$

$$a_1c_0 \rightarrow d_0e_2$$

# Information table after removing $b$

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$U$	$a$	$c$	$d$	$e$
(1)	1	2	1	1
(2)	2	0	1	0
(3)	2	2	0	2
(5)	1	0	0	2

## Rules:

$$a_1c_2 \rightarrow d_1e_1$$

$$a_2c_0 \rightarrow d_1e_0$$

$$a_2c_2 \rightarrow d_0e_2$$

$$a_1c_0 \rightarrow d_0e_2$$

# Records

序号	住宅区名	地段	房型	面积 (m <sup>2</sup> )	结构	价格 (元/ m <sup>2</sup> )
1	站前路小区	站前路	2/2, 3/2	90~140	一般	10000
2	高新区	高新区	2/2, 3/2, 4/2	83~170	框架	7000
3	远东花园	洪城路	3/2, 4/2	120以上	复式	7800
4	曙光小区	洪都南大道	3/2	103以上	一般	9000
5	怡鑫花园	洪都中大道	2/1, 2/2, 3/2, 4/2	87~230	一般	12000
6	文化大楼	沿江路	2/1, 3/1, 3/2, 2/2	99~180	框架	18000
7	玉达住宅	二七北路	2/2	97~107	一般	9800
8	洪都新村	洛阳东路	2/1, 3/1, 3/2, 1/1	53~123	一般	7000

# Discretization

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- 地段分类：站前路和沿江路被数字化为1；洪都大道和二七路被数字化为2；洪城路、高新区和洛阳东路被数字化为3。
- 房型分类：房型单一被数字化为1；房型中等被数字化为2；房型多样化被数字化为3。
- 面积分类：80m<sup>2</sup>以下面积被数字化为1；80~120m<sup>2</sup>为中等面积，被数字化为2；120m<sup>2</sup>以上为大面积，被数字化为3。
- 结构分类：一般结构被数字化为1；框架结构被数字化为2；复式结构被数字化为3。
- 价格分类：9000元/m<sup>2</sup>以下被数字化为1；9000~12000元/m<sup>2</sup>被数字化为2；12000元/m<sup>2</sup>被数字化为3。

# Information table

$U$	$a$	$b$	$c$	$d$	$e$
(1)	1	2	2	1	2
(2)	3	2	2	2	1
(3)	3	2	3	3	3
(4)	2	1	3	1	2
(5)	2	3	2	1	3
(6)	1	2	2	2	3
(7)	2	1	2	1	2
(8)	3	3	1	1	1

## Rules:

$$a_1 b_2 c_2 d_1 \rightarrow e_2$$

$$a_3 b_2 c_2 d_2 \rightarrow e_1$$

$$a_3 b_2 c_3 d_3 \rightarrow e_3$$

$$a_2 b_1 c_3 d_1 \rightarrow e_2$$

$$a_2 b_3 c_2 d_1 \rightarrow e_3$$

$$a_1 b_2 c_2 d_2 \rightarrow e_3$$

$$a_2 b_1 c_2 d_1 \rightarrow e_2$$

$$a_3 b_3 c_1 d_1 \rightarrow e_1$$

# Remove attribute $a$

$U$	$b$	$c$	$d$	$e$
(1)	2	2	1	2
(2)	2	2	2	1
(3)	2	3	3	3
(4)	1	3	1	2
(5)	3	2	1	3
(6)	2	2	2	3
(7)	1	2	1	2
(8)	3	1	1	1

## Rules:

$$a_1 b_2 c_2 d_1 \rightarrow e_2$$

$$a_3 b_2 c_2 d_2 \rightarrow e_1$$

$$a_3 b_2 c_3 d_3 \rightarrow e_3$$

$$a_2 b_1 c_3 d_1 \rightarrow e_2$$

$$a_2 b_3 c_2 d_1 \rightarrow e_3$$

$$a_1 b_2 c_2 d_2 \rightarrow e_3$$

$$a_2 b_1 c_2 d_1 \rightarrow e_2$$

$$a_3 b_3 c_1 d_1 \rightarrow e_1$$



# Remove attribute $a$

$U$	$b$	$c$	$d$	$e$
(1)	2	2	1	2
(2)	2	2	2	1
(3)	2	3	3	3
(4)	1	3	1	2
(5)	3	2	1	3
(6)	2	2	2	3
(7)	1	2	1	2
(8)	3	1	1	1

## Rules:

$$a_1 b_2 c_2 d_1 \rightarrow e_2$$

$$a_3 b_2 c_2 d_2 \rightarrow e_1$$

$$a_3 b_2 c_3 d_3 \rightarrow e_3$$

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$$a_2 b_3 c_2 d_1 \rightarrow e_3$$

$$a_1 b_2 c_2 d_2 \rightarrow e_3$$

$$a_2 b_1 c_2 d_1 \rightarrow e_2$$

$$a_3 b_3 c_1 d_1 \rightarrow e_1$$

# Remove attribute $b$

$U$	$a$	$c$	$d$	$e$
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	3	3	3
(4)	2	3	1	2
(5)	2	2	1	3
(6)	1	2	2	3
(7)	2	2	1	2
(8)	3	1	1	1

## Rules:

$$a_1 b_2 c_2 d_1 \rightarrow e_2$$

$$a_3 b_2 c_2 d_2 \rightarrow e_1$$

$$a_3 b_2 c_3 d_3 \rightarrow e_3$$

$$a_2 b_1 c_3 d_1 \rightarrow e_2$$

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$$a_2 b_1 c_2 d_1 \rightarrow e_2$$

$$a_3 b_3 c_1 d_1 \rightarrow e_1$$

# Remove attribute $b$

$U$	$a$	$c$	$d$	$e$
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	3	3	3
(4)	2	3	1	2
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## Rules:

$$a_1 b_2 c_2 d_1 \rightarrow e_2$$

$$a_3 b_2 c_2 d_2 \rightarrow e_1$$

$$a_3 b_2 c_3 d_3 \rightarrow e_3$$

$$a_2 b_1 c_3 d_1 \rightarrow e_2$$

$$a_2 b_3 c_2 d_1 \rightarrow e_3$$

$$a_1 b_2 c_2 d_2 \rightarrow e_3$$

$$a_2 b_1 c_2 d_1 \rightarrow e_2$$

$$a_3 b_3 c_1 d_1 \rightarrow e_1$$

# Remove attribute $c$

$U$	$a$	$b$	$d$	$e$
(1)	1	2	1	
(2)	3	2	2	
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(7)	2	1	1	2
(8)	3	3	1	1

**Legitimate  
reduct**

$$a_3 b_2 c_3 d_3 \rightarrow e_3$$

$$a_2 b_1 c_3 d_1 \rightarrow e_2$$

$$a_2 b_3 c_2 d_1 \rightarrow e_3$$

$$a_1 b_2 c_2 d_2 \rightarrow e_3$$

$$a_2 b_1 c_2 d_1 \rightarrow e_2$$

$$a_3 b_3 c_1 d_1 \rightarrow e_1$$

# Remove attribute $d$

$U$	$a$	$b$	$c$	$e$
(1)	1	2	2	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	3	2
(5)	2	3	2	3
(6)	1	2	2	3
(7)	2	1	2	2
(8)	3	3	1	1

## Rules:

$$a_1 b_2 c_2 d_1 \rightarrow e_2$$

$$a_3 b_2 c_2 d_2 \rightarrow e_1$$

$$a_3 b_2 c_3 d_3 \rightarrow e_3$$

$$a_2 b_1 c_3 d_1 \rightarrow e_2$$

$$a_2 b_3 c_2 d_1 \rightarrow e_3$$

$$a_1 b_2 c_2 d_2 \rightarrow e_3$$

$$a_2 b_1 c_2 d_1 \rightarrow e_2$$

$$a_3 b_3 c_1 d_1 \rightarrow e_1$$

# Remove attribute $d$

$U$	$a$	$b$	$c$	$e$
(1)	1	2	2	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	3	2
(5)	2	3	2	3
(6)	1	2	2	3
(7)	2	1	2	2
(8)	3	3	1	1

## Rules:

$$a_1 b_2 c_2 d_1 \rightarrow e_2$$

$$a_3 b_2 c_2 d_2 \rightarrow e_1$$

$$a_3 b_2 c_3 d_3 \rightarrow e_3$$

$$a_2 b_1 c_3 d_1 \rightarrow e_2$$

$$a_2 b_3 c_2 d_1 \rightarrow e_3$$

$$a_1 b_2 c_2 d_2 \rightarrow e_3$$

$$a_2 b_1 c_2 d_1 \rightarrow e_2$$

$$a_3 b_3 c_1 d_1 \rightarrow e_1$$

# Information table after removing $C$

---

$U$	$a$	$b$	$d$	$e$
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(7)	2	1	1	2
(8)	3	3	1	1

# Information table after removing $C$

$U$	$a$	$b$	$d$	$e$
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(7)	2	1	1	2
(8)	3	3	1	1



# Reduction of attribute value

$U$	$a$	$b$	$d$	$e$
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

**Rules:**

$$a_1 b_2 d_1 \rightarrow e_2$$

$$a_3 b_2 d_2 \rightarrow e_1$$

$$a_3 b_2 d_3 \rightarrow e_3$$

$$a_2 b_1 d_1 \rightarrow e_2$$

$$a_2 b_3 d_1 \rightarrow e_3$$

$$a_1 b_2 d_2 \rightarrow e_3$$

$$a_3 b_3 d_1 \rightarrow e_1$$

**Analyze rule (1):**

$$[1]_a = \{1, 6\},$$

$$[1]_b = \{1, 2, 3, 6\},$$

$$[1]_d = \{1, 4, 5, 8\},$$

$$[1]_e = \{1, 4\}$$

# Reduction of attribute value

$U$	$a$	$b$	$d$	$e$
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

**Rules:**

$$a_1 b_2 d_1 \rightarrow e_2$$

$$a_3 b_2 d_2 \rightarrow e_1$$

$$a_3 b_2 d_3 \rightarrow e_3$$

$$a_2 b_1 d_1 \rightarrow e_2$$

$$a_2 b_3 d_1 \rightarrow e_3$$

$$a_1 b_2 d_2 \rightarrow e_3$$

$$a_3 b_3 d_1 \rightarrow e_1$$

**Analyze rule (1):**

$$[1]_a = \{1, 6\},$$

$$[1]_b = \{1, 2, 3, 6\},$$

$$[1]_d = \{1, 4, 5, 8\},$$

$$[1]_e = \{1, 4\}$$

$$[1]_{\{a, b\}} = [1]_a \cap [1]_b = \{1, 6\} \not\subseteq [1]_e$$

$d_1$  can not be removed.

# Reduction of attribute value

$U$	$a$	$b$	$d$	$e$
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

**Rules:**

$$a_1 b_2 d_1 \rightarrow e_2$$

$$a_3 b_2 d_2 \rightarrow e_1$$

$$a_3 b_2 d_3 \rightarrow e_3$$

$$a_2 b_1 d_1 \rightarrow e_2$$

$$a_2 b_3 d_1 \rightarrow e_3$$

$$a_1 b_2 d_2 \rightarrow e_3$$

$$a_3 b_3 d_1 \rightarrow e_1$$

**Analyze rule (1):**

$$[1]_a = \{1, 6\},$$

$$[1]_b = \{1, 2, 3, 6\},$$

$$[1]_d = \{1, 4, 5, 8\},$$

$$[1]_e = \{1, 4\}$$

$$[1]_{\{a, d\}} = [1]_a \cap [1]_d = \{1\} \subseteq [1]_e$$

$b_2$  can be removed.

# Reduction of attribute value

$U$	$a$	$b$	$d$	$e$
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

**Rules:**

$$a_1 b_2 d_1 \rightarrow e_2$$

$$a_3 b_2 d_2 \rightarrow e_1$$

$$a_3 b_2 d_3 \rightarrow e_3$$

$$a_2 b_1 d_1 \rightarrow e_2$$

$$a_2 b_3 d_1 \rightarrow e_3$$

$$a_1 b_2 d_2 \rightarrow e_3$$

$$a_3 b_3 d_1 \rightarrow e_1$$

**Analyze rule (1):**

$$[1]_a = \{1, 6\},$$

$$[1]_b = \{1, 2, 3, 6\},$$

$$[1]_d = \{1, 4, 5, 8\},$$

$$[1]_e = \{1, 4\}$$

$$[1]_{\{b, d\}} = [1]_b \cap [1]_d = \{1\} \subseteq [1]_e$$

$a_1$  can be removed.

# Reduction of attribute value

$U$	$a$	$b$	$d$	$e$
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

**Rules:**

$$a_1 b_2 d_1 \rightarrow e_2$$

$$a_3 b_2 d_2 \rightarrow e_1$$

$$a_3 b_2 d_3 \rightarrow e_3$$

$$a_2 b_1 d_1 \rightarrow e_2$$

$$a_2 b_3 d_1 \rightarrow e_3$$

$$a_1 b_2 d_2 \rightarrow e_3$$

$$a_3 b_3 d_1 \rightarrow e_1$$

**Analyze rule (1):**

$$[1]_a = \{1, 6\},$$

$$[1]_b = \{1, 2, 3, 6\},$$

$$[1]_d = \{1, 4, 5, 8\},$$

$$[1]_e = \{1, 4\}$$

$d_1$  is core value, then we get

$$a_1 d_1 \rightarrow e_2$$

$$b_2 d_1 \rightarrow e_2$$

# Reduction of attribute value

$U$	$a$	$b$	$d$	$e$
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

**Rules:**

$$a_1 b_2 d_1 \rightarrow e_2$$

$$a_3 b_2 d_2 \rightarrow e_1$$

$$a_3 b_2 d_3 \rightarrow e_3$$

$$a_2 b_1 d_1 \rightarrow e_2$$

$$a_2 b_3 d_1 \rightarrow e_3$$

$$a_1 b_2 d_2 \rightarrow e_3$$

$$a_3 b_3 d_1 \rightarrow e_1$$

**Analyze rule (2):**

$$[2]_a = \{2, 3, 8\},$$

$$[2]_b = \{1, 2, 3, 6\},$$

$$[2]_d = \{2, 6\},$$

$$[2]_e = \{2, 8\}$$

# Reduction of attribute value

$U$	$a$	$b$	$d$	$e$
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

**Rules:**

$$a_1 b_2 d_1 \rightarrow e_2$$

$$a_3 b_2 d_2 \rightarrow e_1$$

$$a_3 b_2 d_3 \rightarrow e_3$$

$$a_2 b_1 d_1 \rightarrow e_2$$

$$a_2 b_3 d_1 \rightarrow e_3$$

$$a_1 b_2 d_2 \rightarrow e_3$$

$$a_3 b_3 d_1 \rightarrow e_1$$

**Analyze rule (2):**

$$[2]_a = \{2, 3, 8\},$$

$$[2]_b = \{1, 2, 3, 6\},$$

$$[2]_d = \{2, 6\},$$

$$[2]_e = \{2, 8\}$$

$$[2]_{\{a, b\}} = [2]_a \cap [2]_b = \{2, 3\} \not\subseteq [2]_e$$

$d_2$  can not be removed.

# Reduction of attribute value

$U$	$a$	$b$	$d$	$e$
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

**Rules:**

$$a_1 b_2 d_1 \rightarrow e_2$$

$$a_3 b_2 d_2 \rightarrow e_1$$

$$a_3 b_2 d_3 \rightarrow e_3$$

$$a_2 b_1 d_1 \rightarrow e_2$$

$$a_2 b_3 d_1 \rightarrow e_3$$

$$a_1 b_2 d_2 \rightarrow e_3$$

$$a_3 b_3 d_1 \rightarrow e_1$$

**Analyze rule (2):**

$$[2]_a = \{2, 3, 8\},$$

$$[2]_b = \{1, 2, 3, 6\},$$

$$[2]_d = \{2, 6\},$$

$$[2]_e = \{2, 8\}$$

$$[2]_{\{a, d\}} = [2]_a \cap [2]_d = \{2\} \subseteq [1]_e$$

$b_2$  can be removed.



# Reduction of attribute value

$U$	$a$	$b$	$d$	$e$
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

**Rules:**

$$a_1 b_2 d_1 \rightarrow e_2$$

$$a_3 b_2 d_2 \rightarrow e_1$$

$$a_3 b_2 d_3 \rightarrow e_3$$

$$a_2 b_1 d_1 \rightarrow e_2$$

$$a_2 b_3 d_1 \rightarrow e_3$$

$$a_1 b_2 d_2 \rightarrow e_3$$

$$a_3 b_3 d_1 \rightarrow e_1$$

**Analyze rule (2):**

$$[2]_a = \{2, 3, 8\},$$

$$[2]_b = \{1, 2, 3, 6\},$$

$$[2]_d = \{2, 6\},$$

$$[2]_e = \{2, 8\}$$

$$[2]_{\{b, d\}} = [2]_a \cap [2]_b = \{2, 6\} \not\subseteq [1]_e$$

$a_3$  can not be removed.

# Reduction of attribute value

$U$	$a$	$b$	$d$	$e$
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

**Rules:**

$$a_1 b_2 d_1 \rightarrow e_2$$

$$a_3 b_2 d_2 \rightarrow e_1$$

$$a_3 b_2 d_3 \rightarrow e_3$$

$$a_2 b_1 d_1 \rightarrow e_2$$

$$a_2 b_3 d_1 \rightarrow e_3$$

$$a_1 b_2 d_2 \rightarrow e_3$$

$$a_3 b_3 d_1 \rightarrow e_1$$

**Analyze rule (2):**

$$[2]_a = \{2, 3, 8\},$$

$$[2]_b = \{1, 2, 3, 6\},$$

$$[2]_d = \{2, 6\},$$

$$[2]_e = \{2, 8\}$$

$a_3$  and  $d_2$  is core, then we get

$$a_3 d_2 \rightarrow e_1$$

# Results of reduction

$U$	$a$	$b$	$d$	$e$
(1)	1	—	1	2
(1')	—	2	1	2
(2)	3	—	2	1
(3)	—	—	3	3
(4)	—	1	—	2
(5)	2	3	—	3
(6)	1	—	2	3
(8)	3	—	1	1
(8')	3	3	—	1

## Rules:

$$a_1d_1 \rightarrow e_2 \textbf{ or } b_2d_1 \rightarrow e_2$$

$$a_3d_2 \rightarrow e_1$$

$$d_3 \rightarrow e_3$$

$$b_1 \rightarrow e_2$$

$$a_2b_3 \rightarrow e_3$$

$$a_1d_2 \rightarrow e_3$$

$$a_3d_1 \rightarrow e_1 \textbf{ or } a_3b_3 \rightarrow e_1$$

# Results of reduction

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## Rules:

$$a_1d_1 \rightarrow e_2 \text{ or } b_2d_1 \rightarrow e_2$$

$$a_3d_2 \rightarrow e_1$$

$$d_3 \rightarrow e_3$$

$$b_1 \rightarrow e_2$$

$$a_2b_3 \rightarrow e_3$$

$$a_1d_2 \rightarrow e_3$$

$$a_3d_1 \rightarrow e_1 \text{ or } a_3b_3 \rightarrow e_1$$

# Results of reduction

---

## Rules:

$$a_1d_1 \rightarrow e_2 \textbf{ or } b_2d_1 \rightarrow e_2$$

$$a_3d_2 \rightarrow e_1$$

$$d_3 \rightarrow e_3$$

$$b_1 \rightarrow e_2$$

$$a_2b_3 \rightarrow e_3$$

$$a_1d_2 \rightarrow e_3$$

$$a_3d_1 \rightarrow e_1 \textbf{ or } a_3b_3 \rightarrow e_1$$

## Final Rules:

$$a_3d_2 \vee a_3d_1 \vee a_3b_3 \rightarrow e_1$$

$$a_1d_1 \vee b_2d_1 \vee b_1 \rightarrow e_2$$

$$d_3 \vee a_2b_3 \vee a_1d_2 \rightarrow e_3$$

# Results of reduction

## Rules:

$$a_1d_1 \rightarrow e_2 \text{ or } b_2d_1 \rightarrow e_2$$

$$a_3d_2 \rightarrow e_1$$

$$d_3 \rightarrow e_3$$

$$b_1 \rightarrow e_2$$

$$a_2b_3 \rightarrow e_3$$

$$a_1d_2 \rightarrow e_3$$

$$a_3d_1 \rightarrow e_1 \text{ or } a_3b_3 \rightarrow e_1$$

## Final Rules:

$$a_3d_2 \vee a_3d_1 \vee a_3b_3 \rightarrow e_1$$

$$a_1d_1 \vee b_2d_1 \vee b_1 \rightarrow e_2$$

$$d_3 \vee a_2b_3 \vee a_1d_2 \rightarrow e_3$$

## 自然语言解释：

(1) 3类地段框架结构或3类地段一般结构或3类地段房型多样的地产价格便宜；

(2) 1类地段一般结构或房型中等一般结构或房型单一的地产价格适中；

(3) 复式结构或2类地段房型多样或1类地段框架结构的地产价格昂贵

# Comparison

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Attributes	Rough sets	Decision tree	BP
Accuracy	<i>Medium</i>	<i>Medium</i>	<i>High</i>
Learning / Usage time	<i>Slow / Slow</i>	<i>Slow / Medium</i>	<i>Slowest / Quick</i>
Robustness	<i>Medium</i>	<i>Bad</i>	<i>Good</i>
Scalable	<i>Medium</i>	<i>Bad</i>	<i>Good</i>
Understanding	<i>Good</i>	<i>Medium</i>	<i>Bad</i>

*Any question?*



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