Artificial Intelligence

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Introduction

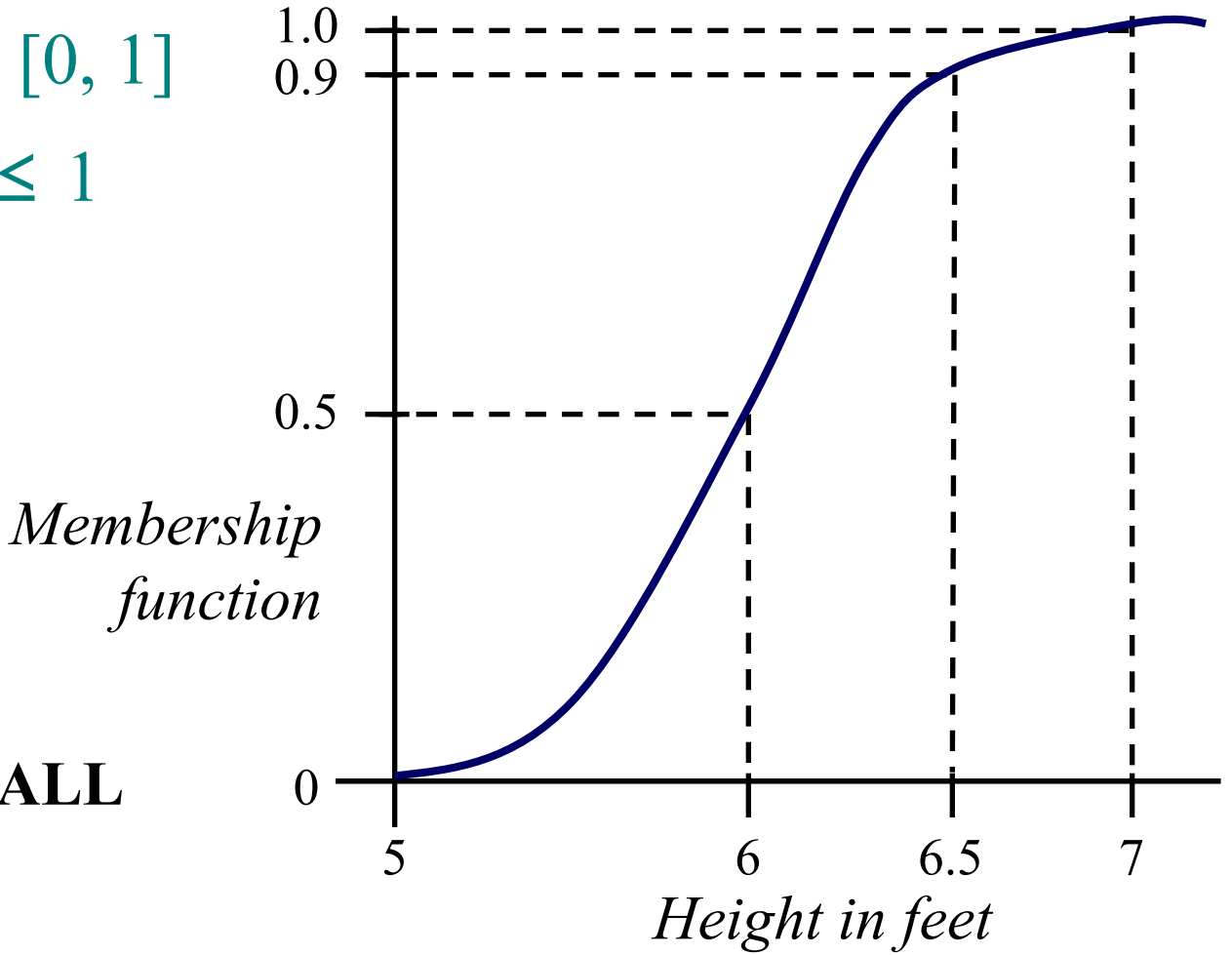
- Classical ***Bivalent*** or ***two-valued*** logic.
- L. A. Zadeh's ***fuzzy*** set.
- Pawlak's ***Rough*** set.

Membership function

Membership or compatibility function

$\mu_A(x)$: $x \rightarrow [0, 1]$

$0 \leq \mu_A(x) \leq 1$



Fuzzy set **TALL**

Information table

U	a	b	c	d	e
(1)	1	2	2	1	2
(2)	3	2	2	2	1
(3)	3	2	3	3	3
(4)	2	1	3	1	2
(5)	2	3	2	1	3
(6)	1	2	2	2	3
(7)	2	1	2	1	2
(8)	3	3	1	1	1

Rough set

- Let $I = (U, A)$ be an ***information system*** (attribute-value system), where U is a non-empty set of finite objects (the universe) and A is a non-empty, finite set of attributes such that $a: U \rightarrow V_a$ for every $a \in A$. V_a is the set of values that attribute a may take.
- The information table assigns a value in V_a to each attribute a of each object in universe U.

Universe $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$

Associated equivalence relation R:

$E_1 = \{x_1, x_4\}$

$E_2 = \{x_3, x_8\}$

$E_3 = \{x_2, x_5, x_7\}$

$E_4 = \{x_6\}$

Partition $U/R = \{E_1, E_2, E_3, E_4\}$, and $X = \{x_1, x_4, x_2\}$

Lower approximation $R_*(X) = \{x_1, x_4\}$

Upper approximation $R^*(X) = \{x_1, x_4, x_2, x_5, x_7\}$

Universe $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$

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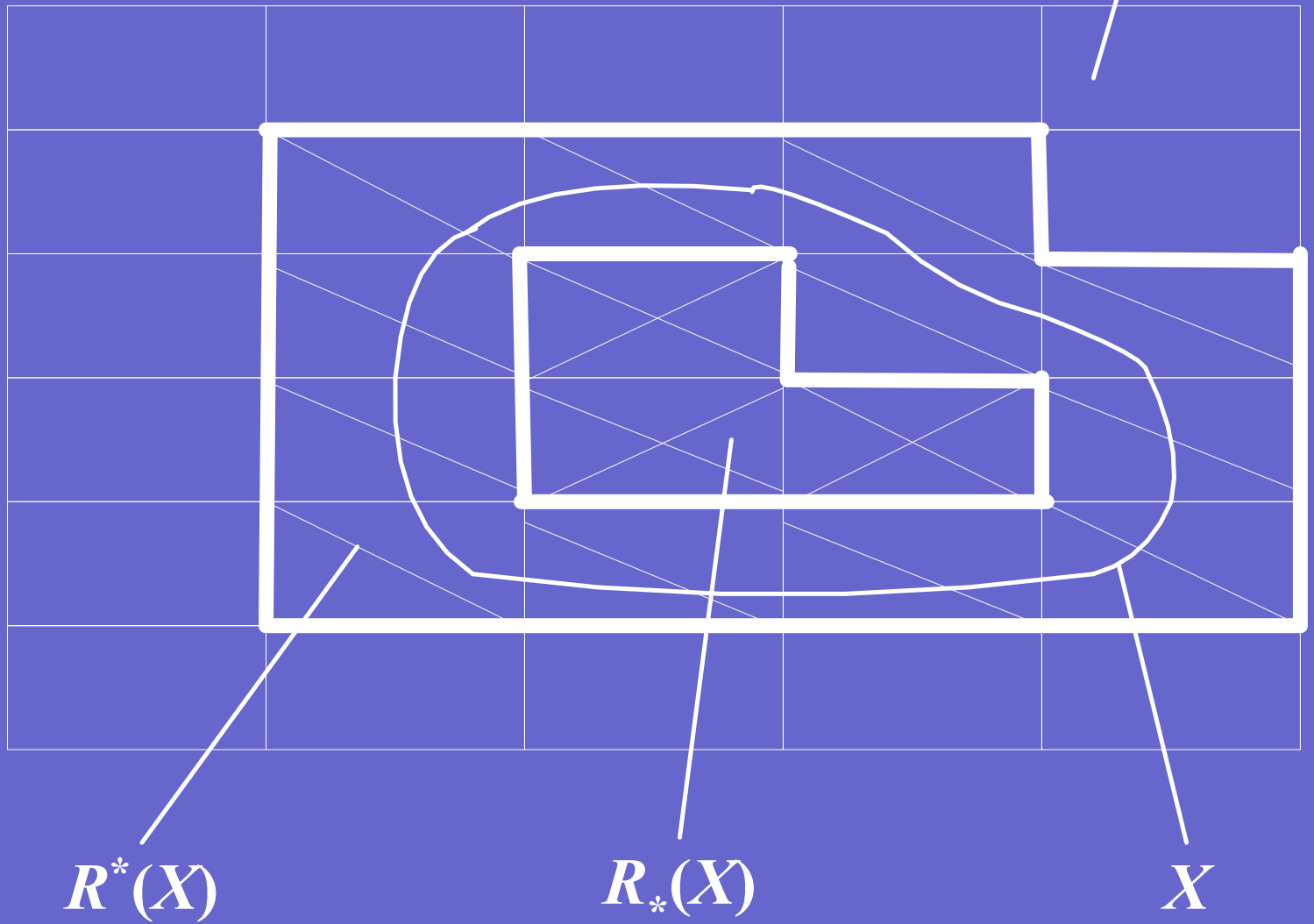

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Membership function

$$\mu_X^R(x) = card(\boldsymbol{X} \cap \boldsymbol{R}(x)) \,/\, card(\boldsymbol{R}(x))$$

$$\mu_X^R(x_1) = 2 / 2 = 1.00$$
$$\mu_X^R(x_2) = 1 / 3 = 0.33$$
$$\mu_X^R(x_3) = 0 / 2 = 0.00$$

Universe $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$

A set of ***Associated equivalence relation*** $R = \{ P, Q, W \}$

$$U/P = \{\{x_1, x_3, x_4, x_5, x_6, x_7\}, \{x_2, x_8\}\}$$

$$U/Q = \{\{x_1, x_3, x_4, x_5\}, \{x_2, x_6, x_7, x_8\}\}$$

$$U/W = \{\{x_1, x_5, x_6\}, \{x_2, x_7, x_8\}, \{x_3, x_4\}\}$$

Then $U/IND(R) = \{\{x_1, x_5\}, \{x_3, x_4\}, \{x_2, x_8\}, \{x_6\}, \{x_7\}\}$

Associated equivalence relation S (***Decision Attribute***)

$$U/S = \{\{x_1, x_5, x_6\}, \{x_3, x_4\}, \{x_2, x_7\}, \{x_8\}\}$$

Positive region:

$$POS_R(S) = \{x_1, x_3, x_4, x_5, x_6, x_7\}$$

Universe $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$

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$$POS_R(S) = \{x_1, x_3, x_4, x_5, x_6, x_7\}$$

Information table

U	a	b	c	d	e
(1)	1	0	2	1	1
(2)	2	1	0	1	0
(3)	2	1	2	0	2
(4)	1	2	2	1	1
(5)	1	2	0	0	2

Rules:

$a_1 b_0 c_2 \rightarrow d_1 e_1$

$a_2 b_1 c_0 \rightarrow d_1 e_0$

$a_2 b_1 c_2 \rightarrow d_0 e_2$

$a_1 b_2 c_2 \rightarrow d_1 e_1$

$a_1 b_2 c_0 \rightarrow d_0 e_2$

Problem

- An interesting question is whether there are attributes in the information system which are more ***important*** to the knowledge represented in the equivalence class structure than other attributes.
- We wonder whether there is a subset of attributes which can, by itself, fully characterize the knowledge in the database; such an attribute set is called a ***reduct***.

Remove attribute a

U	b	c	d	e
(1)	0	2	1	1
(2)	1	0	1	0
(3)	1	2	0	2
(4)	2	2	1	1
(5)	2	0	0	2

Legitimate reduct

$a_2b_1c_2 \quad d_0e_2$

$a_1b_2c_2 \rightarrow d_1e_1$

$a_1b_2c_0 \rightarrow d_0e_2$

Remove attribute b

U	a	c	d	e
(1)	1	2	1	1
(2)	2	0	1	0
(3)	2	2	0	2
(4)	1	2	1	1
(5)	1	0	0	2

Legitimate reduct

$a_2c_2 \quad d_0e_2$

$a_1b_2c_2 \rightarrow d_1e_1$

$a_1b_2c_0 \rightarrow d_0e_2$

Remove attribute c

U	a	b	d	e
(1)	1	0	1	1
(2)	2	1	1	0
(3)	2	1	0	2
(4)	1	2	1	1
(5)	1	2	0	2

Rules:

$a_1 b_0 c_2 \rightarrow d_1 e_1$

$a_2 b_1 c_0 \rightarrow d_1 e_0$

$a_2 b_1 c_2 \rightarrow d_0 e_2$

$a_1 b_2 c_2 \rightarrow d_1 e_1$

$a_1 b_2 c_0 \rightarrow d_0 e_2$

Remove attribute c

U	a	b	d	e
(1)	1	0	1	1
(2)	**2**	**1**	**1**	**0**
(3)	**2**	**1**	**0**	**2**
(4)	1	2	1	1
(5)	1	2	0	2

Causing collapse

$a_2b_1c_2 \quad d_0e_2$

$a_1b_2c_2 \rightarrow d_1e_1$

$a_1b_2c_0 \rightarrow d_0e_2$

Remove attribute c

U	a	b	d	e
(1)	1	0	1	1
(2)	2	1	1	0
(3)	2	1	0	2
(4)	**1**	**2**	**1**	**1**
(5)	**1**	**2**	**0**	**2**

Causing collapse

$a_2b_1c_2 \quad d_0e_2$

$a_1b_2c_2 \rightarrow d_1e_1$

$a_1b_2c_0 \rightarrow d_0e_2$

Information table after removing a

U	b	c	d	e
(1)	0	2	1	1
(2)	1	0	1	0
(3)	1	2	0	2
(4)	2	2	1	1
(5)	2	0	0	2

Rules:

$b_0c_2 \rightarrow d_1e_1$

$b_1c_0 \rightarrow d_1e_0$

$b_1c_2 \rightarrow d_0e_2$

$b_2c_2 \rightarrow d_1e_1$

$b_2c_0 \rightarrow d_0e_2$

Information table after removing b

U	a	c	d	e
(1)	1	2	1	1
(2)	2	0	1	0
(3)	2	2	0	2
(4)	1	2	1	1
(5)	1	0	0	2

Rules:

$a_1c_2 \rightarrow d_1e_1$

$a_2c_0 \rightarrow d_1e_0$

$a_2c_2 \rightarrow d_0e_2$

$a_1c_2 \rightarrow d_1e_1$

$a_1c_0 \rightarrow d_0e_2$

Information table after removing b

U	a	c	d	e
(1)	**1**	**2**	**1**	**1**
(2)	2	0	1	0
(3)	2	2	0	2
(4)	**1**	**2**	**1**	**1**
(5)	1	0	0	2

Rules:

$a_1c_2 \rightarrow d_1e_1$

$a_2c_0 \rightarrow d_1e_0$

$a_2c_2 \rightarrow d_0e_2$

$a_1c_2 \rightarrow d_1e_1$

$a_1c_0 \rightarrow d_0e_2$

Information table after removing b

U	a	c	d	e
(1)	1	2	1	1
(2)	2	0	1	0
(3)	2	2	0	2
(5)	1	0	0	2

Rules:

$a_1c_2 \rightarrow d_1e_1$

$a_2c_0 \rightarrow d_1e_0$

$a_2c_2 \rightarrow d_0e_2$

$a_1c_0 \rightarrow d_0e_2$

Records

序号	住宅区名	地段	房型	面积 (m^2)	结构	价格 (元/ m^2)
1	站前路小区	站前路	2/2, 3/2	90~140	一般	10000
2	高新区	高新区	2/2, 3/2, 4/2	83~170	框架	7000
3	远东花园	洪城路	3/2, 4/2	120以上	复式	7800
4	曙光小区	洪都南大道	3/2	103以上	一般	9000
5	怡鑫花园	洪都中大道	2/1, 2/2, 3/2, 4/2	87~230	一般	12000
6	文化大楼	沿江路	2/1, 3/1, 3/2, 2/2	99~180	框架	18000
7	玉达住宅	二七北路	2/2	97~107	一般	9800
8	洪都新村	洛阳东路	2/1, 3/1, 3/2, 1/1	53~123	一般	7000

Discretization

- 地段分类：站前路和沿江路被数字化为1；洪都大道和二七路被数字化为2；洪城路、高新区和洛阳东路被数字化为3。
- 房型分类：房型单一被数字化为1；房型中等被数字化为2；房型多样化被数字化为3。
- 面积分类：80m^2以下面积被数字化为1；80~120m^2为中等面积，被数字化为2；120m^2以上为大面积，被数字化为3。
- 结构分类：一般结构被数字化为1；框架结构被数字化为2；复式结构被数字化为3。
- 价格分类：9000元/m^2以下被数字化为1；9000~12000元/m^2被数字化为2；12000元/m^2被数字化为3。

Information table

U	a	b	c	d	e
(1)	1	2	2	1	2
(2)	3	2	2	2	1
(3)	3	2	3	3	3
(4)	2	1	3	1	2
(5)	2	3	2	1	3
(6)	1	2	2	2	3
(7)	2	1	2	1	2
(8)	3	3	1	1	1

Rules:

$a_1b_2c_2d_1 \rightarrow e_2$

$a_3b_2c_2d_2 \rightarrow e_1$

$a_3b_2c_3\ d_3 \rightarrow e_3$

$a_2b_1c_3\ d_1 \rightarrow e_2$

$a_2b_3c_2\ d_1 \rightarrow e_3$

$a_1b_2c_2\ d_2 \rightarrow e_3$

$a_2b_1c_2\ d_1 \rightarrow e_2$

$a_3b_3c_1\ d_1 \rightarrow e_1$

Remove attribute *a*

U	*b*	*c*	*d*	*e*
(1)	2	2	1	2
(2)	2	2	2	1
(3)	2	3	3	3
(4)	1	3	1	2
(5)	3	2	1	3
(6)	2	2	2	3
(7)	1	2	1	2
(8)	3	1	1	1

Rules:

$a_1 b_2 c_2 d_1 \rightarrow e_2$

$a_3 b_2 c_2 d_2 \rightarrow e_1$

$a_3 b_2 c_3 \, d_3 \rightarrow e_3$

$a_2 b_1 c_3 \, d_1 \rightarrow e_2$

$a_2 b_3 c_2 \, d_1 \rightarrow e_3$

$a_1 b_2 c_2 \, d_2 \rightarrow e_3$

$a_2 b_1 c_2 \, d_1 \rightarrow e_2$

$a_3 b_3 c_1 \, d_1 \rightarrow e_1$

Remove attribute *a*

U	*b*	*c*	*d*	*e*
(1)	2	2	1	2
(2)	**2**	**2**	**2**	**1**
(3)	2	3	3	3
(4)	1	3	1	2
(5)	3	2	1	3
(6)	**2**	**2**	**2**	**3**
(7)	1	2	1	2
(8)	3	1	1	1

Rules:

$a_1 b_2 c_2 d_1 \rightarrow e_2$

$a_3 b_2 c_2 d_2 \rightarrow e_1$

$a_3 b_2 c_3\ d_3 \rightarrow e_3$

$a_2 b_1 c_3\ d_1 \rightarrow e_2$

$a_2 b_3 c_2\ d_1 \rightarrow e_3$

$a_1 b_2 c_2\ d_2 \rightarrow e_3$

$a_2 b_1 c_2\ d_1 \rightarrow e_2$

$a_3 b_3 c_1\ d_1 \rightarrow e_1$

Remove attribute *b*

U	*a*	*c*	*d*	*e*
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	3	3	3
(4)	2	3	1	2
(5)	2	2	1	3
(6)	1	2	2	3
(7)	2	2	1	2
(8)	3	1	1	1

Rules:

$a_1 b_2 c_2 d_1 \rightarrow e_2$

$a_3 b_2 c_2 d_2 \rightarrow e_1$

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$a_2 b_1 c_3\ d_1 \rightarrow e_2$

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$a_2 b_1 c_2\ d_1 \rightarrow e_2$

$a_3 b_3 c_1\ d_1 \rightarrow e_1$

Remove attribute *b*

U	*a*	*c*	*d*	*e*
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	3	3	3
(4)	2	3	1	2
(5)	**2**	**2**	**1**	**3**
(6)	1	2	2	3
(7)	**2**	**2**	**1**	**2**
(8)	3	1	1	1

Rules:

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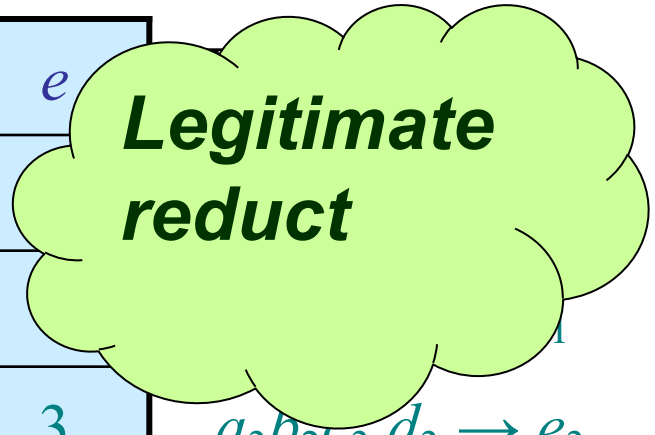
$a_2 b_1 c_2\ d_1 \rightarrow e_2$

$a_3 b_3 c_1\ d_1 \rightarrow e_1$

Remove attribute c

U	a	b	d	e	
(1)	1	2	1		
(2)	3	2	2		
(3)	3	2	3	3	$a_3b_2c_3\ d_3 \rightarrow e_3$
(4)	2	1	1	2	$a_2b_1c_3\ d_1 \rightarrow e_2$
(5)	2	3	1	3	$a_2b_3c_2\ d_1 \rightarrow e_3$
(6)	1	2	2	3	$a_1b_2c_2\ d_2 \rightarrow e_3$
(7)	2	1	1	2	$a_2b_1c_2\ d_1 \rightarrow e_2$
(8)	3	3	1	1	$a_3b_3c_1\ d_1 \rightarrow e_1$



Remove attribute *d*

U	*a*	*b*	*c*	*e*
(1)	1	2	2	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	3	2
(5)	2	3	2	3
(6)	1	2	2	3
(7)	2	1	2	2
(8)	3	3	1	1

Rules:

$a_1 b_2 c_2 d_1 \rightarrow e_2$

$a_3 b_2 c_2 d_2 \rightarrow e_1$

$a_3 b_2 c_3\ d_3 \rightarrow e_3$

$a_2 b_1 c_3\ d_1 \rightarrow e_2$

$a_2 b_3 c_2\ d_1 \rightarrow e_3$

$a_1 b_2 c_2\ d_2 \rightarrow e_3$

$a_2 b_1 c_2\ d_1 \rightarrow e_2$

$a_3 b_3 c_1\ d_1 \rightarrow e_1$

Remove attribute *d*

U	*a*	*b*	*c*	*e*
(1)	**1**	**2**	**2**	**2**
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	3	2
(5)	2	3	2	3
(6)	**1**	**2**	**2**	**3**
(7)	2	1	2	2
(8)	3	3	1	1

Rules:

$a_1 b_2 c_2 d_1 \rightarrow e_2$

$a_3 b_2 c_2 d_2 \rightarrow e_1$

$a_3 b_2 c_3\ d_3 \rightarrow e_3$

$a_2 b_1 c_3\ d_1 \rightarrow e_2$

$a_2 b_3 c_2\ d_1 \rightarrow e_3$

$a_1 b_2 c_2\ d_2 \rightarrow e_3$

$a_2 b_1 c_2\ d_1 \rightarrow e_2$

$a_3 b_3 c_1\ d_1 \rightarrow e_1$

Information table after removing C

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(7)	2	1	1	2
(8)	3	3	1	1

Information table after removing C

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	**2**	**1**	**1**	**2**
(5)	2	3	1	3
(6)	1	2	2	3
(7)	**2**	**1**	**1**	**2**
(8)	3	3	1	1

Reduction of attribute value

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

Rules:

$a_1b_2d_1 \rightarrow e_2$

$a_3b_2d_2 \rightarrow e_1$

$a_3b_2\,d_3 \rightarrow e_3$

$a_2b_1d_1 \rightarrow e_2$

$a_2b_3d_1 \rightarrow e_3$

$a_1b_2d_2 \rightarrow e_3$

$a_3b_3d_1 \rightarrow e_1$

Analyze rule (1):

$[1]_a = \{1, 6\},$

$[1]_b = \{1, 2, 3, 6\},$

$[1]_d = \{1, 4, 5, 8\},$

$[1]_e = \{1, 4\}$

Reduction of attribute value

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

Rules:

$a_1b_2d_1 \rightarrow e_2$

$a_3b_2d_2 \rightarrow e_1$

$a_3b_2\,d_3 \rightarrow e_3$

$a_2b_1d_1 \rightarrow e_2$

$a_2b_3d_1 \rightarrow e_3$

$a_1b_2d_2 \rightarrow e_3$

$a_3b_3d_1 \rightarrow e_1$

Analyze rule (1):

$[1]_a = \{1, 6\}$,

$[1]_b = \{1, 2, 3, 6\}$,

$[1]_d = \{1, 4, 5, 8\}$,

$[1]_e = \{1, 4\}$

$[1]_{\{a, b\}} = [1]_a \cap [1]_b = \{1, 6\} \not\subset [1]_e$

d_1 can not be removed.

Reduction of attribute value

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

Rules:

$a_1b_2d_1 \rightarrow e_2$

$a_3b_2d_2 \rightarrow e_1$

$a_3b_2\,d_3 \rightarrow e_3$

$a_2b_1d_1 \rightarrow e_2$

$a_2b_3d_1 \rightarrow e_3$

$a_1b_2d_2 \rightarrow e_3$

$a_3b_3d_1 \rightarrow e_1$

Analyze rule (1):

$[1]_a = \{1, 6\}$,

$[1]_b = \{1, 2, 3, 6\}$,

$[1]_d = \{1, 4, 5, 8\}$,

$[1]_e = \{1, 4\}$

$[1]_{\{a,\,d\}} = [1]_a \cap [1]_d = \{1\} \subseteq [1]_e$

b_2 can be removed.

Reduction of attribute value

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

Rules:

$a_1b_2d_1 \rightarrow e_2$

$a_3b_2d_2 \rightarrow e_1$

$a_3b_2\,d_3 \rightarrow e_3$

$a_2b_1d_1 \rightarrow e_2$

$a_2b_3d_1 \rightarrow e_3$

$a_1b_2d_2 \rightarrow e_3$

$a_3b_3d_1 \rightarrow e_1$

Analyze rule (1):

$[1]_a = \{1, 6\}$,

$[1]_b = \{1, 2, 3, 6\}$,

$[1]_d = \{1, 4, 5, 8\}$,

$[1]_e = \{1, 4\}$

$[1]_{\{b,\,d\}} = [1]_b \cap [1]_d = \{1\} \subseteq [1]_e$

a_1 can be removed.

Reduction of attribute value

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

Rules:

$a_1b_2d_1 \rightarrow e_2$

$a_3b_2d_2 \rightarrow e_1$

$a_3b_2\,d_3 \rightarrow e_3$

$a_2b_1d_1 \rightarrow e_2$

$a_2b_3d_1 \rightarrow e_3$

$a_1b_2d_2 \rightarrow e_3$

$a_3b_3d_1 \rightarrow e_1$

Analyze rule (1):

$[1]_a = \{1, 6\}$,

$[1]_b = \{1, 2, 3, 6\}$,

$[1]_d = \{1, 4, 5, 8\}$,

$[1]_e = \{1, 4\}$

d_1 is core value, then we get

$a_1d_1 \rightarrow e_2$

$b_2d_1 \rightarrow e_2$

Reduction of attribute value

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

Rules:

$a_1b_2d_1 \rightarrow e_2$

$a_3b_2d_2 \rightarrow e_1$

$a_3b_2\ d_3 \rightarrow e_3$

$a_2b_1d_1 \rightarrow e_2$

$a_2b_3d_1 \rightarrow e_3$

$a_1b_2d_2 \rightarrow e_3$

$a_3b_3d_1 \rightarrow e_1$

Analyze rule (2):

$[2]_a = \{2, 3, 8\}$,

$[2]_b = \{1, 2, 3, 6\}$,

$[2]_d = \{2, 6\}$,

$[2]_e = \{2, 8\}$

Reduction of attribute value

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

Rules:

$a_1b_2d_1 \rightarrow e_2$

$a_3b_2d_2 \rightarrow e_1$

$a_3b_2\,d_3 \rightarrow e_3$

$a_2b_1d_1 \rightarrow e_2$

$a_2b_3d_1 \rightarrow e_3$

$a_1b_2d_2 \rightarrow e_3$

$a_3b_3d_1 \rightarrow e_1$

Analyze rule (2):

$[2]_a = \{2, 3, 8\}$,

$[2]_b = \{1, 2, 3, 6\}$,

$[2]_d = \{2, 6\}$,

$[2]_e = \{2, 8\}$

$[2]_{\{a, b\}} = [2]_a \cap [2]_b = \{2, 3\} \not\subset [2]_e$

d_2 can not be removed.

Reduction of attribute value

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

Rules:

$a_1b_2d_1 \rightarrow e_2$

$a_3b_2d_2 \rightarrow e_1$

$a_3b_2\,d_3 \rightarrow e_3$

$a_2b_1d_1 \rightarrow e_2$

$a_2b_3d_1 \rightarrow e_3$

$a_1b_2d_2 \rightarrow e_3$

$a_3b_3d_1 \rightarrow e_1$

Analyze rule (2):

$[2]_a = \{2, 3, 8\}$,

$[2]_b = \{1, 2, 3, 6\}$,

$[2]_d = \{2, 6\}$,

$[2]_e = \{2, 8\}$

$[2]_{\{a,\,d\}} = [2]_a \cap [2]_d = \{2\} \subseteq [1]_e$

b_2 can be removed.

Reduction of attribute value

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

Rules:

$a_1b_2d_1 \rightarrow e_2$

$a_3b_2d_2 \rightarrow e_1$

$a_3b_2\,d_3 \rightarrow e_3$

$a_2b_1d_1 \rightarrow e_2$

$a_2b_3d_1 \rightarrow e_3$

$a_1b_2d_2 \rightarrow e_3$

$a_3b_3d_1 \rightarrow e_1$

Analyze rule (2):

$[2]_a = \{2, 3, 8\}$,

$[2]_b = \{1, 2, 3, 6\}$,

$[2]_d = \{2, 6\}$,

$[2]_e = \{2, 8\}$

$[2]_{\{b,\,d\}} = [2]_a \cap [2]_b = \{2, 6\} \not\subset [1]_e$

a_3 can not be removed.

Reduction of attribute value

U	a	b	d	e
(1)	1	2	1	2
(2)	3	2	2	1
(3)	3	2	3	3
(4)	2	1	1	2
(5)	2	3	1	3
(6)	1	2	2	3
(8)	3	3	1	1

Rules:

$a_1b_2d_1 \rightarrow e_2$

$a_3b_2d_2 \rightarrow e_1$

$a_3b_2\,d_3 \rightarrow e_3$

$a_2b_1d_1 \rightarrow e_2$

$a_2b_3d_1 \rightarrow e_3$

$a_1b_2d_2 \rightarrow e_3$

$a_3b_3d_1 \rightarrow e_1$

Analyze rule (2):

$[2]_a = \{2, 3, 8\}$,

$[2]_b = \{1, 2, 3, 6\}$,

$[2]_d = \{2, 6\}$,

$[2]_e = \{2, 8\}$

a_3 and d_2 is core, then we get

$a_3d_2 \rightarrow e_1$

Results of reduction

U	a	b	d	e
(1)	1	–	1	2
(1')	–	2	1	2
(2)	3	–	2	1
(3)	–	–	3	3
(4)	–	1	–	2
(5)	2	3	–	3
(6)	1	–	2	3
(8)	3	–	1	1
(8')	3	3	–	1

Rules:

$a_1 d_1 \rightarrow e_2$ **or** $b_2 d_1 \rightarrow e_2$

$a_3 d_2 \rightarrow e_1$

$d_3 \rightarrow e_3$

$b_1 \rightarrow e_2$

$a_2 b_3 \rightarrow e_3$

$a_1 d_2 \rightarrow e_3$

$a_3 d_1 \rightarrow e_1$ **or** $a_3 b_3 \rightarrow e_1$

Results of reduction

Rules:

$a_1 d_1 \rightarrow e_2$ **or** $b_2 d_1 \rightarrow e_2$

$a_3 d_2 \rightarrow e_1$

$d_3 \rightarrow e_3$

$b_1 \rightarrow e_2$

$a_2 b_3 \rightarrow e_3$

$a_1 d_2 \rightarrow e_3$

$a_3 d_1 \rightarrow e_1$ **or** $a_3 b_3 \rightarrow e_1$

Results of reduction

Rules:

$a_1 d_1 \rightarrow e_2$ **or** $b_2 d_1 \rightarrow e_2$

$a_3 d_2 \rightarrow e_1$

$d_3 \rightarrow e_3$

$b_1 \rightarrow e_2$

$a_2 b_3 \rightarrow e_3$

$a_1 d_2 \rightarrow e_3$

$a_3 d_1 \rightarrow e_1$ **or** $a_3 b_3 \rightarrow e_1$

Final Rules:

$a_3 d_2 \vee a_3 d_1 \vee a_3 b_3 \rightarrow e_1$

$a_1 d_1 \vee b_2 d_1 \vee b_1 \rightarrow e_2$

$d_3 \vee a_2 b_3 \vee a_1 d_2 \rightarrow e_3$

Results of reduction

Rules:

$a_1 d_1 \rightarrow e_2$ **or** $b_2 d_1 \rightarrow e_2$

$a_3 d_2 \rightarrow e_1$

$d_3 \rightarrow e_3$

$b_1 \rightarrow e_2$

$a_2 b_3 \rightarrow e_3$

$a_1 d_2 \rightarrow e_3$

$a_3 d_1 \rightarrow e_1$ **or** $a_3 b_3 \rightarrow e_1$

Final Rules:

$a_3 d_2 \vee a_3 d_1 \vee a_3 b_3 \rightarrow e_1$

$a_1 d_1 \vee b_2 d_1 \vee b_1 \rightarrow e_2$

$d_3 \vee a_2 b_3 \vee a_1 d_2 \rightarrow e_3$

自然语言解释：

(1)3类地段框架结构或3类地段一般结构或3类地段房型多样的地产价格便宜；

(2)1类地段一般结构或房型中等一般结构或房型单一的地产价格适中；

(3)复式结构或2类地段房型多样或1类地段框架结构的地产价格昂贵

Comparison

Attributes	Rough sets	Decision tree	BP
Accuracy	***Medium***	***Medium***	***High***
Learning / Usage time	***Slow / Slow***	***Slow / Medium***	***Slowest / Quick***
Robustness	***Medium***	***Bad***	***Good***
Scalable	***Medium***	***Bad***	***Good***
Understanding	***Good***	***Medium***	***Bad***

Any question?

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