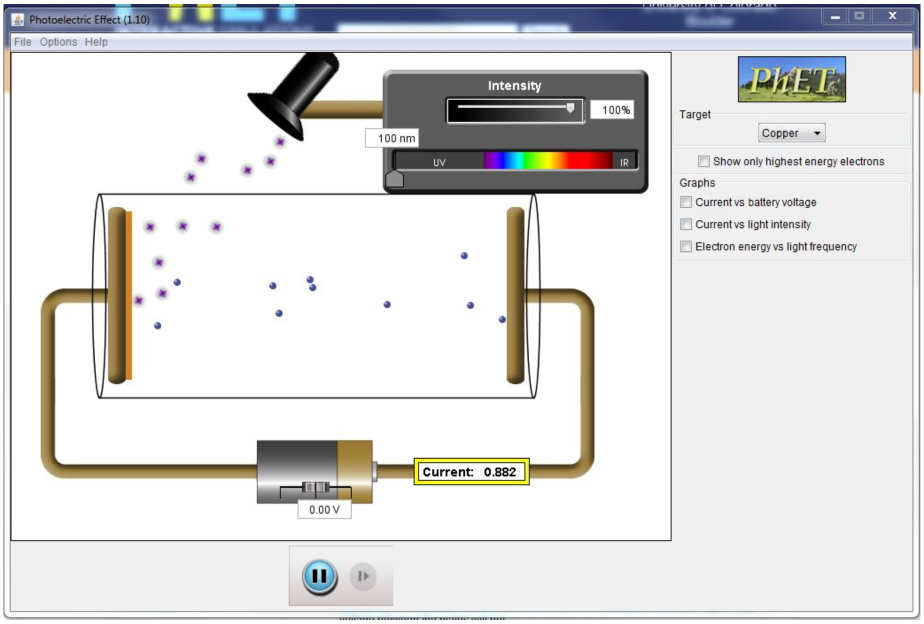
Quantum Systems

Last updated: February 25$^{\text{th}}$ 2019, at 11.45am

Contents

1 Where are the Photons?

1.1 The Photoelectric Effect

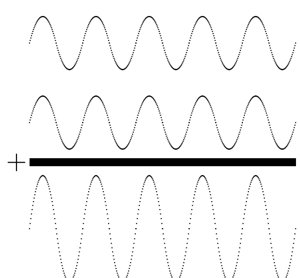


http://phet.colorado.edu/sims/photoelectric/photoelectric_en.jnlp

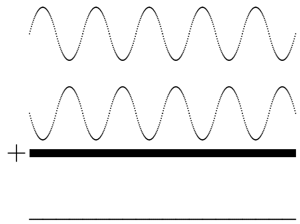
1.2 Double Slit Experiment

1.2.1 Interference

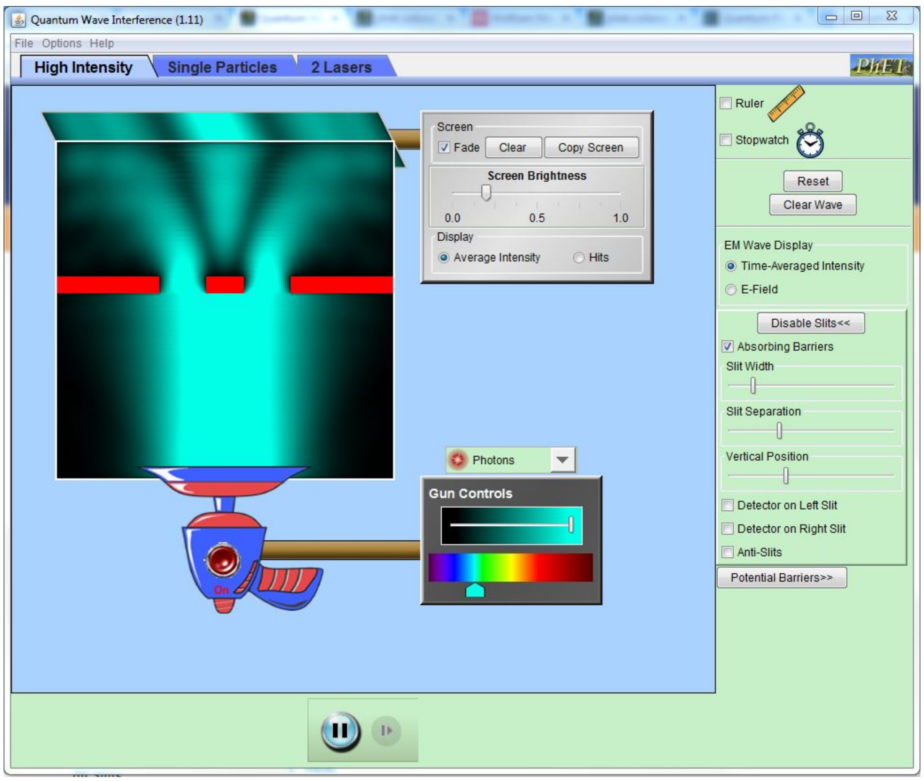
Waves in phase



Waves out of phase



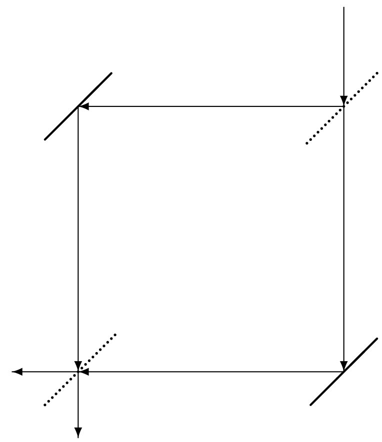
1.2.2 The Experiment



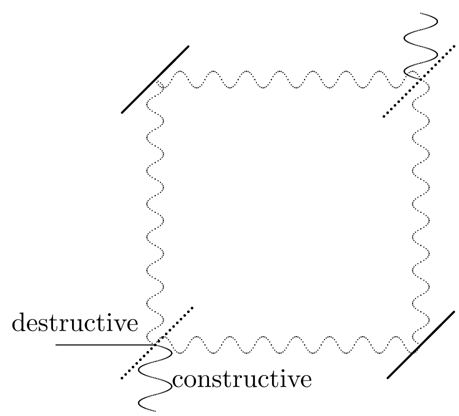
http://phet.colorado.edu/sims/quantum-wave-interference/quantum-wave-interference_en.jnlp

1.3 Beamsplitters

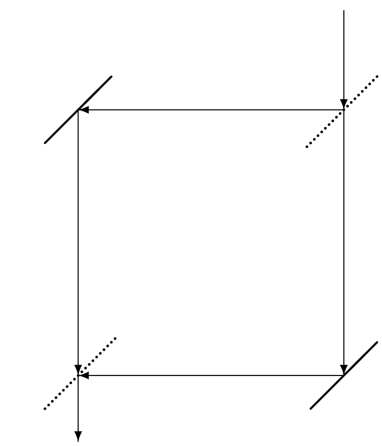
1.3.1 The Set-Up



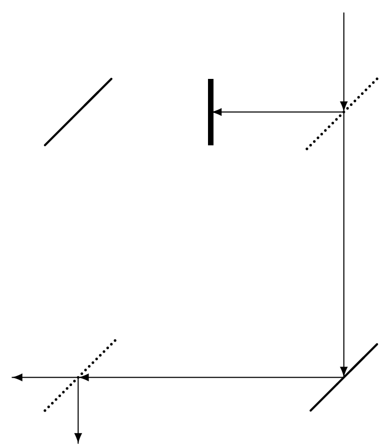
1.3.2 The Waves



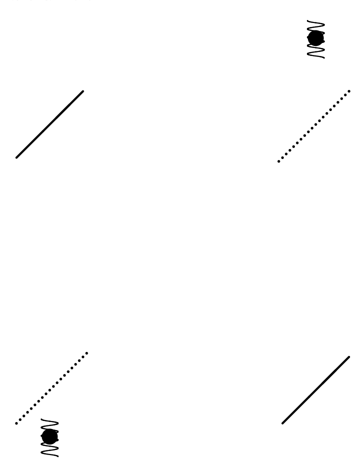
1.3.3 The result



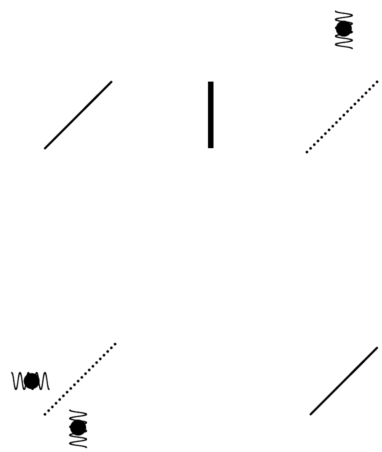
1.3.4 Adding a barrier



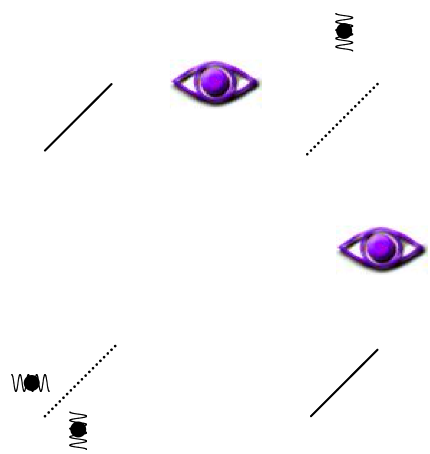
1.3.5 Single photon source



1.3.6 Single photon source with a barrier



1.3.7 Single photon source with observation



2 Why Does This Happen?

2.1 Superposition

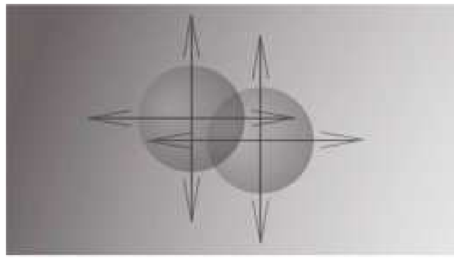
Photon is “on both paths at the same time”

2.2 Decoherence

Superposition “collapses” to single state (with a certain probability)

2.3 Entanglement

Two (or more) superpositions are connected



Two entangled photons. . .

. . . move apart. . .

. . . then decohere

WANTED

Dead ~~or~~ and Alive

Schrödinger's Cat

3 How Can We Model It?

3.1 Maths

3.1.1 Superposition

α	β	γ	δ
$\begin{bmatrix}1\\0\end{bmatrix}$	$\begin{bmatrix}0\\1\end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$	$\begin{bmatrix}-3- & i\\ - & 2i\end{bmatrix}$
100% zero	100% one	50% zero, 50%one	71.4% zero, 28.6% one

The probability (or weighting) of a particular value can be calculated by taking the square of the magnitude of the corresponding entry in the matrix, and dividing it by the sum of the squares of the magnitudes of all entries. E.g., for matrix δ:

n	$\delta_{n,0}$	$\lvert\delta_{n_0}\rvert$	$\lvert\delta_{n,0}\rvert^2$
0	$-3-i$	$\sqrt{(-3)^2+(-1)^2}$	$(-3)^2+(-1)^2{=}10$
1	$-2i$	$\sqrt{(-2)^2}$	$(-2)^2{=}\ 4$
			total: 14

so the weighting of entry 1 is $\frac{4}{14} \approx 0.286$.

Note: we will normally design our matrices so that the divisor is 1. E.g. rather than δ we would have

$$\delta' = \frac{1}{\sqrt{14}}\begin{bmatrix}-3- & i\\ - & 2i\end{bmatrix}$$

3.1.2 Entanglement

Entanglement is a property of two or more objects. For example, consider the state

$$\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

This looks like a superposition, but consider what happens if it decoheres. We will either end up with $|01\rangle$, or with $|10\rangle$. Now consider what happens if we are only looking at the first bit. If it is a $|0\rangle$ then we know that the other must be a $|1\rangle$, and if it is a $|1\rangle$ we know the other must be a $|0\rangle$. The two bits are *entangled*.

What is the mathematical difference between superposition and entanglement? A superposition can be written as a combination of states of its component parts, an entanglement cannot. E.g.

$$\frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$$

is an entanglement, but

$$\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

is just a superposition.

i) **$\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$ is a superposition**
We show that $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$ can be written as a combination (tensor product) of independent states of the two bits:

$$\frac{1}{2}(|00\rangle+|01\rangle+|10\rangle+|11\rangle) = \frac{1}{2}\begin{bmatrix}1\\1\\1\\1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} \otimes \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} = \left|\frac{|0\rangle + |1\rangle}{\sqrt{2}}, \frac{|0\rangle + |1\rangle}{\sqrt{2}}\right\rangle$$

ii) **$\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ is an entanglement**
We show, by contradiction, that $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ cannot be written as a combination (tensor product) of independent states of the two bits.

Assume that $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ can be written as a tensor product of independent states of the two bits:

$$\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\1\\0\end{bmatrix} = \begin{bmatrix}a\\b\end{bmatrix} \otimes \begin{bmatrix}c\\d\end{bmatrix} = \begin{bmatrix}ac\\ad\\bc\\bd\end{bmatrix}$$

From this we have

$$\begin{aligned} ac = 0 &\text{ so } a = 0 \vee c = 0\\ ad \neq 0 &\text{ so } a \neq 0 \wedge d \neq 0\\ bc \neq 0 &\text{ so } b \neq 0 \wedge c \neq 0\\ bd = 0 &\text{ so } b = 0 \vee d = 0 \end{aligned}$$

It is clear that there is no solution, therefore $(|01\rangle + |10\rangle)$ cannot be written as a combination of independent states of the two bits. The bits are entangled.

3.1.3 Decoherence

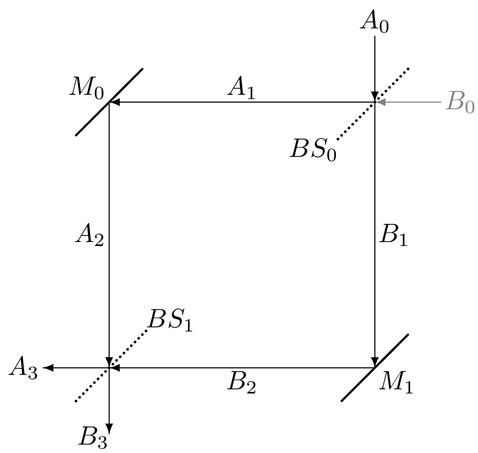
E.g.

$$\begin{bmatrix}-3 - & i\\ - & 2i\end{bmatrix} \rightsquigarrow \begin{bmatrix}1\\0\end{bmatrix}$$

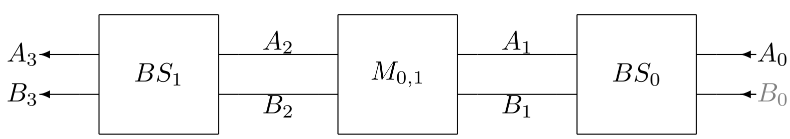
Note: could also decohere to $|1\rangle$ (with probability 28.6%).

3.1.4 Example — Beamsplitter

We will label the beamsplitter set up, identifying two paths, A and B, through the system. Note that these are not the only paths a photon can take — A_0, B_1, B_2, A_3 and B_0, A_1, A_2, A_3 are, for example, also possible paths. We chose, in the set up, to always send photons into the system along A_0, but starting at B_0 is also a possibility.



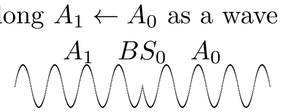
We have also labelled the mirrors: BS_0 and BS_1 (beam splitters 0 and 1), and M_0 and M_1 (mirrors 0 and 1). We can model this system as:



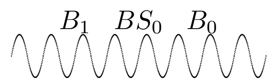
$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \qquad \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \qquad \frac{1}{\sqrt{2}}\begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix}$$

The matrices for the various components are also shown. The $\frac{1}{\sqrt{2}}$ in the matrices for the beam splitters is because these matrices create superpositions, and the total probability of the two components of these superpositions has to be 1.

The "-1"s in the matrices for BS_0 and BS_1 reflect (pun intentional) the fact that the "phase" of the photons travelling along the paths $A_1 \leftarrow A_0$ and $B_3 \leftarrow B_2$ flips, while that of photons travelling along other paths ($B_1 \leftarrow B_0$, $B_1 \leftarrow A_0$, $A_1 \leftarrow B_0$, and $A_4 \leftarrow A_3$, $A_4 \leftarrow B_3$, $B_4 \leftarrow A_3$) does not change. I.e. if we draw the light path along $A_1 \leftarrow A_0$ as a wave we get



while the path along $B_1 \leftarrow B_0$ (or $B_1 \leftarrow A_0$, or $A_1 \leftarrow B_0$) is



This phase shift is represented by having the A_1 component of the superposition as $-1 \times \frac{1}{\sqrt{2}}$ when the photon comes in along A_0, but $+1 \times \frac{1}{\sqrt{2}}$ when the photon comes in along B_0. We therefore need a matrix for BS_0 such that

$$\frac{1}{\sqrt{2}}\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 \\ 1 \end{bmatrix} \tag{1}$$

and

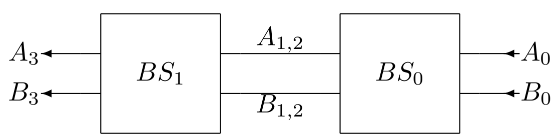
$$\frac{1}{\sqrt{2}}\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{2}$$

From 1 we get $a = -1$ and $c = 1$, and from 2 we get $b = 1$ and $d = 1$, giving us

$$\frac{1}{\sqrt{2}}\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

as the matrix for BS_0.

The matrix for the mirrors $M_{0,1}$ is a 2×2 identity matrix[1]. This reflects the fact that the mirrors do not effect any change to the system, so we can leave them out of our model, and simplify to



$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \frac{1}{\sqrt{2}}\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

So, a photon coming in at A_0, given by $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ gives output

$$\begin{aligned} & \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} * \frac{1}{\sqrt{2}}\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} * \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ = & \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} * \frac{1}{\sqrt{2}}\begin{bmatrix} -1 \\ 1 \end{bmatrix} \end{aligned} \tag{3}$$

[1]Matrix multiplication of any matrix by an identity matrix does not change that matrix — i.e. for any 2×2 matrix M we have

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} * M = M \text{ and } M * \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} M = M.$$

$$
\begin{aligned}
&= \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} * \begin{bmatrix} -1 \\ 1 \end{bmatrix} \\
&= \frac{1}{2} \begin{bmatrix} 0 \\ -2 \end{bmatrix} \\
&= \begin{bmatrix} 0 \\ -1 \end{bmatrix}
\end{aligned}
$$

Line 3 shows the superposition.

3.2 Models

3.2.1 Copenhagen Model

- Quantum world and classical world
- Schrödinger's cat is not "alive and dead"
- The superposition is a measure of probability
- Superpositions collapse when interacting with classical systems

$$
\left|\mathbf{alive},\ \text{☺}\right\rangle \to \left|\frac{\mathbf{alive} + \mathbf{dead}}{\sqrt{2}},\ \overset{?}{\text{😐}}\right\rangle \rightsquigarrow \left|\mathbf{alive},\ \text{☺}\right\rangle
$$

or

$$
\left|\mathbf{alive},\ \text{☺}\right\rangle \to \left|\frac{\mathbf{alive} + \mathbf{dead}}{\sqrt{2}},\ \overset{?}{\text{😐}}\right\rangle \rightsquigarrow \left|\mathbf{dead},\ \text{☹}\right\rangle
$$

3.2.2 Many Worlds Model

Hugh Everett, 1957

- No distinction between quantum and classical systems
- Schrödinger's cat *is* alive *and* dead
- Every possible outcome is real
- The superposition is a superposition of realities
- Observer and observed become entangled

$$
\begin{aligned}
\left|\mathbf{alive},\ \text{☺}\right\rangle &\to \left|\frac{\mathbf{alive} + \mathbf{dead}}{\sqrt{2}},\ \overset{?}{\text{😐}}\right\rangle \\
&\to \frac{\left|\mathbf{alive},\ \text{☺}\right\rangle + \left|\mathbf{dead},\ \text{☹}\right\rangle}{\sqrt{2}}
\end{aligned}
$$