Quantum Computing — Some Maths

Last updated: February 11$^{\text{th}}$ 2019, at 12.50pm

Matlab

Introduction

I recommend doing these exercises both as pen and paper exercises and as Matlab exercises. Matlab (short for "matrix laboratory") is a mathematical tool for manipulating matrices (and a lot more, but we will be using it primarily for working with matrices).

Matlab is available on University machines (certainly in the Spärck Jones building). The University also has a student licence so that you can download and install it on *one* machine. Instructions on how to do this are available on Brightspace, in AP& D's week 17 folder.

Some Matlab functions

For this week's exercises you may find some of the following Matlab functions useful:

- $\boldsymbol{abs(c)}$ for the magnitude of a complex number $\boldsymbol{c}$ (the name comes from "absolute value" — a related concept).
- $\boldsymbol{transpose(a)}$ for the transpose of an array $\boldsymbol{a}$.
- $\boldsymbol{conj(a)}$ for the complex conjugate ($\boldsymbol{a}$ can be an array or a number).
- $\boldsymbol{kron(a, b)}$ for the tensor product of two matrices $\boldsymbol{a}$ and $\boldsymbol{b}$ (the name comes from "Kronecker tensor product").

This week's "code"

The `practical17.m` file available on Brightspace contains a Matlab script defining some of the objects occuring in this week's exercises. These are:

- The complex numbers $\boldsymbol{a}$ and $\boldsymbol{b}$ from section 1.
- The matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ from section 2.

- For section 3:
  - the matrices for the ***not*** and ***and*** gates;
  - the matrices for the truth values ***true*** and ***false***;
  - the matrices for the **0** and **1** bit values.

To use the definitions in this script you will need not only to load it into Matlab, but also to run it.

1 Complex Numbers

Let $a = 4 + i$ and $b = -2 + 6i$. What is:

i) $3 \times a$?

ii) $-4i \times b$?

iii) $a + b$?

iv) $a - b$?

v) $a \times b$?

vi) $\frac{a}{b}$?

vii) $|a|$?

viii) $|b|$?

2 Matrices

In the following let:

$$A = \begin{bmatrix} 0 & -4 & 3 & 0 \\ 1 & 2 & 0 & -2 \\ 0 & -3 & 1 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & -4 & 0 & 0 & 2 \\ 0 & -1 & 3 & -2 & 0 \\ 1 & 1 & 1 & 0 & -1 \\ 0 & 2 & -3 & 3 & 4 \end{bmatrix}$$

What is:

ix) $3 \cdot A$?

x) $-5 \cdot B$?

xi) $A * B$?

xii) $A \otimes B$?

In the following let

$$\begin{bmatrix} 0 & 3+2i & 2-4i \\ 2i & -3+i & -4-3i \end{bmatrix}$$

xiii) C^T?

xiv) $\overline{C}$?

xv) $C^\dagger$?

3 Circuits

All of these exercises should be completed using matrices to represent bits, bit sequences, gates and circuits.

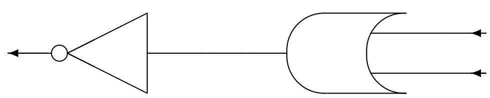
xvi) What is $|11\rangle$?

xvii) Show that **not false** $\equiv$ **true**.

xviii) Show that **and** $|11\rangle \equiv |1\rangle$.

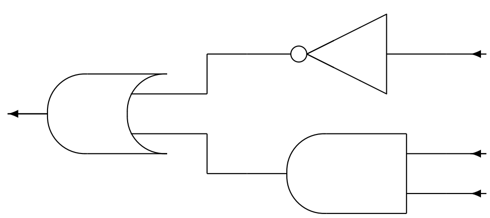
xix) Design, and verify, a matrix for an **or** gate.

xx) The **nor** gate is a sequence of a **or** gate and a **not** gate:



Construct the **nor** matrix.

xxi) **Logbook question.** What is the matrix for the following circuit?



xxii) **Model question.** The matrix given in the lecture for the half-adder circuit (shown in figure 1 on page 5), which is shown in figure 2, is unnecessarily large/complex. This is actually the matrix for the circuit shown in figure 3 which has four inputs, rather than two. In the original half-adder circuit the subcircuit shown in figure 4 can be considered to be a "gate" that takes input $|xy\rangle$ to output $|xyxy\rangle$. Let us call this a **2,2dup** gate (two input, two duplicator).

(a) Give a matrix for the **2,2,dup** gate.

(b) Hence calculate the proper matrix for the half-adder circuit (the half-adder is a sequential composition of a **2,2dup** gate and the circuit in figure 3.

(c) Check your matrix by also constructing the matrix from the truth table for a half-adder.

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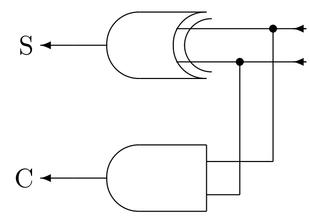

Figure 1: A half-adder circuit

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure 2: An incorrect matrix for the half-adder in figure 1

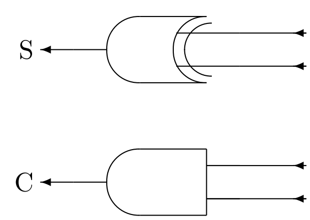

Figure 3: The circuit modelled by the matrix in figure 2

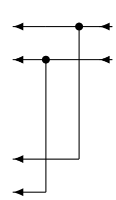

Figure 4: A two input double duplicator “gate”