Theme

The history of every major civilisation tends to pass through three distinct and recognisable phases, those of Survival, Inquiry and Sophistication, otherwise known as the **How**, **Why**, *and* **Where** *phases...*

Theme

*For instance, the first phase is characterised by the question "**How** can we eat?", the second by the question "**Why** do we eat?", and the third by the question "**Where** shall we have dinner?"*

Douglas Adams
The Restaurant at the End of the Universe
Pan Books, 1980

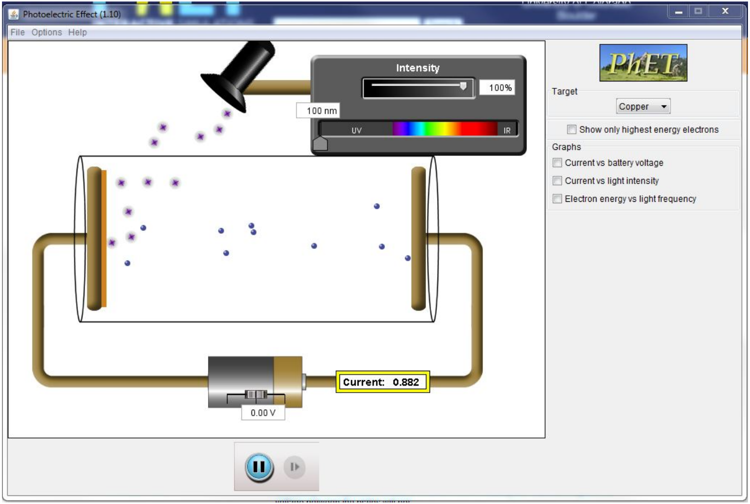
Quantum Systems

February 25, 2019

- Where?
- Why?
- How?

Where?

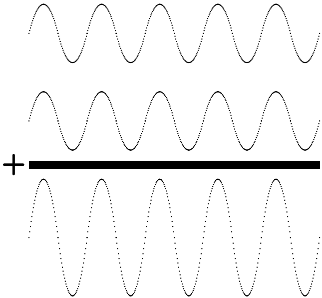
1: Where?
(Where are the Photons?)
1.1: The Photoelectric Effect



http://phet.colorado.edu/sims/photoelectric/photoelectric_e

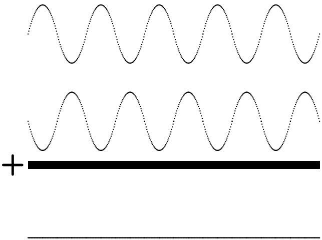
Where?

1.2: Double Slit Experiment
1.2.1: Interference
1.2.1 A: Waves in phase



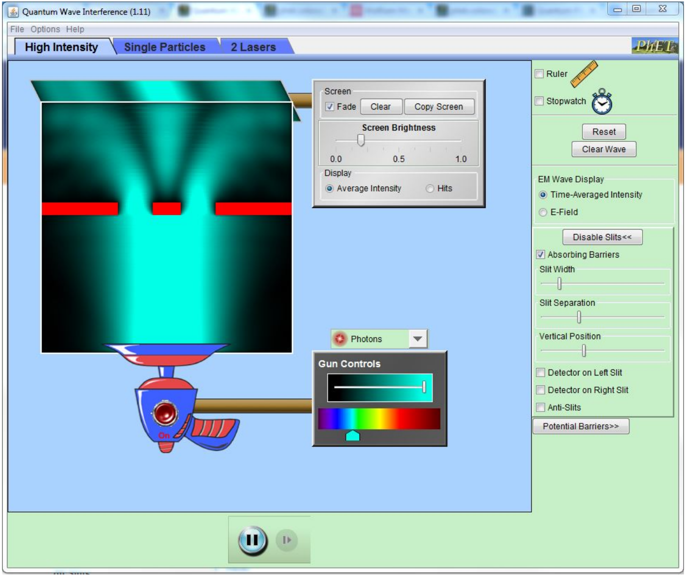
Where?

1.2.1 B: Waves out of phase



Where?

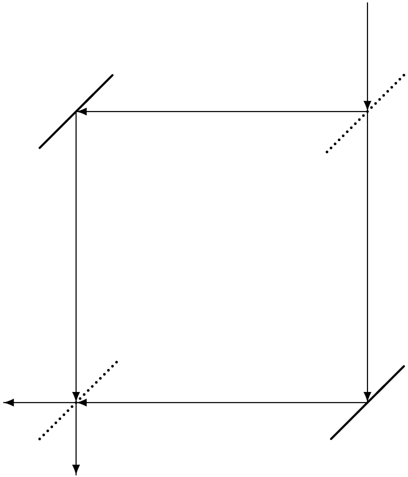
1.2.2: The Experiment



http://phet.colorado.edu/sims/quantum-wave-interference/qua

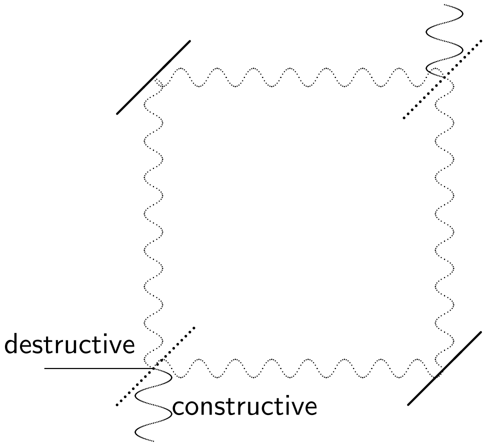
Where?

1.3: Beamsplitters
1.3.1: The Set-Up



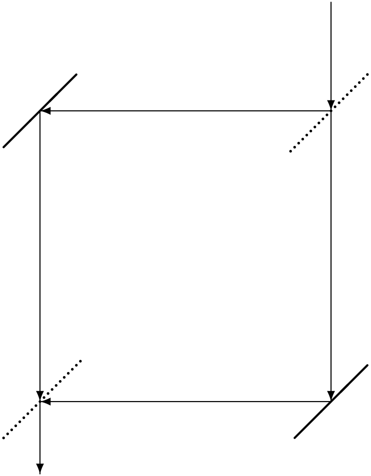
Where?

1.3.2: The Waves



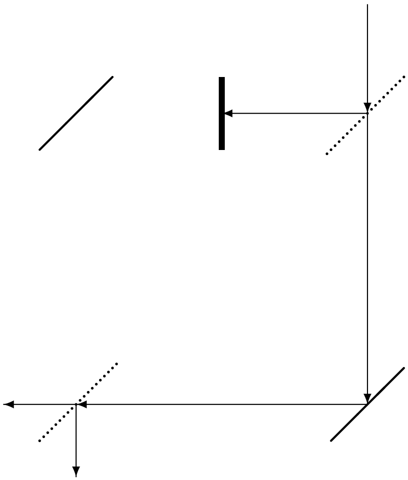
Where?

1.3.3: The result



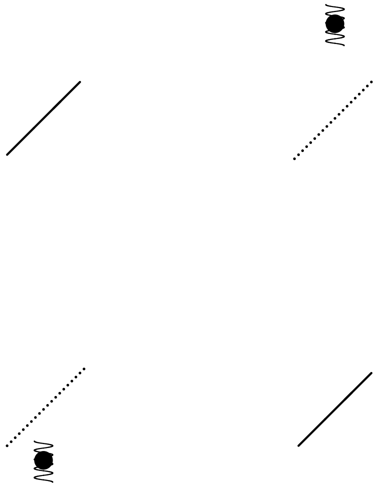
Where?

1.3.4: Adding a barrier



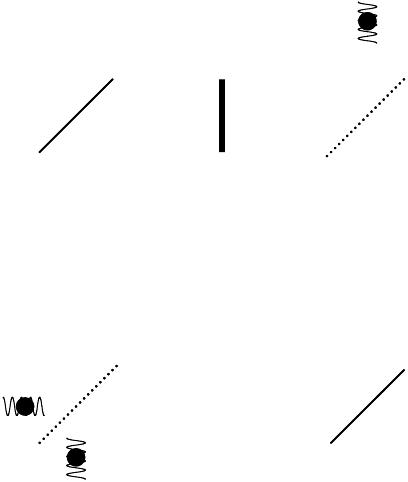
Where?

1.3.5: Single photon source



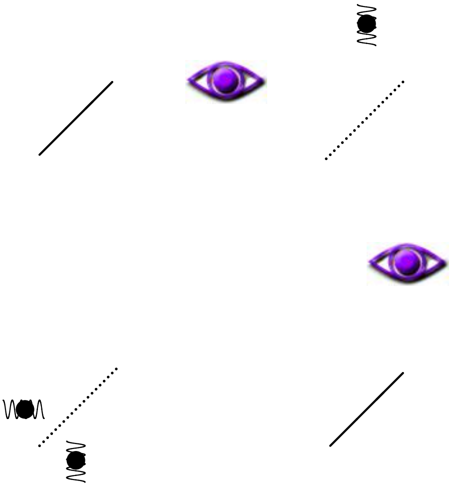
Where?

1.3.6: Single photon source with a barrier



Where?

1.3.7: Single photon source with observation



Interlude

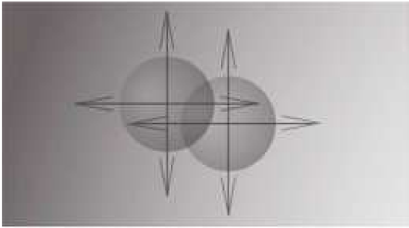
If you think you understand quantum mechanics, you don't understand quantum mechanics.

Richard Feynman
The Character of Physical Law
Penguin Press Science, 1992

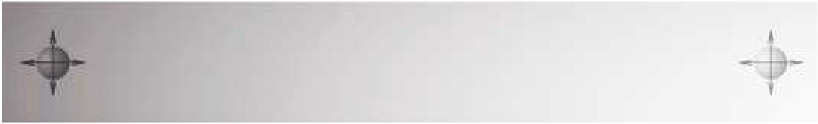
Why?

2: Why?
(Why Does This Happen?)
2.1: Superposition
Photon is "on both paths at the same time"
2.2: Decoherence
Superposition "collapses" to single state (with a certain probability)
2.3: Entanglement
Two (or more) superpositions are connected

Why?



Two entangled photons. . .



. . . move apart. . .

. . . then decohere

Why?

WANTED

and

Dead ~~or~~ Alive

Schrödinger's Cat

How?

3: How?
(How Can We Model It?)
3.1: Maths
3.1.1: Superposition

α	β	γ	δ
$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$\begin{bmatrix} -3 & - & i \\ & - & 2i \end{bmatrix}$
100% zero	100% one	50% zero, 50%one	71.4% zero, 28.6% one

How?

3.1.2: Entanglement

For example

$$\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

Note

$$\frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$$

is an entanglement, but

$$\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

is just a superposition.

How?

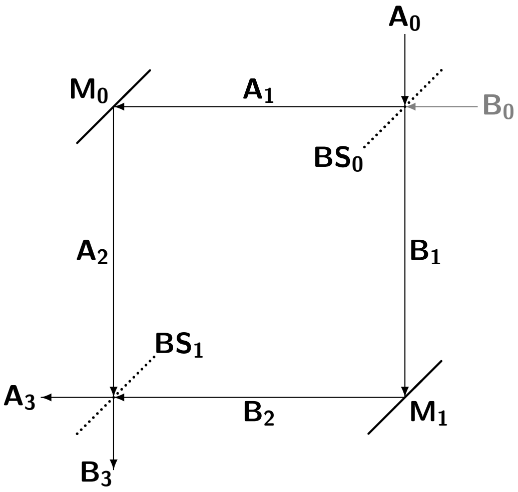
3.1.3: Decoherence

E.g.

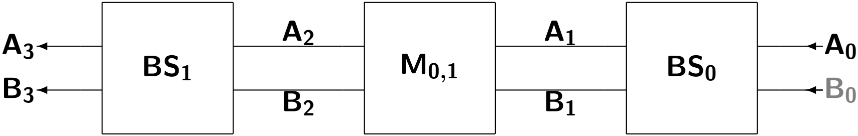
$$\begin{bmatrix} -3 & - & \mathbf{i} \\ & - & 2\mathbf{i} \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

How?

3.1.4: Example — Beamsplitter



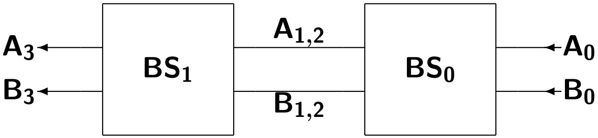
How?



$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \qquad \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \qquad \frac{1}{\sqrt{2}}\begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix}$$

How?

Simplify to



$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \frac{1}{\sqrt{2}}\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

How?

So, a photon coming in at $\mathbf{A_0}$, given by $\begin{bmatrix}\mathbf{1}\\ \mathbf{0}\end{bmatrix}$ gives output

$$\begin{aligned}
& \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\begin{bmatrix}\mathbf{1} & \mathbf{1}\\ \mathbf{1} & \mathbf{-1}\end{bmatrix} * \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\begin{bmatrix}\mathbf{-1} & \mathbf{1}\\ \mathbf{1} & \mathbf{1}\end{bmatrix} * \begin{bmatrix}\mathbf{1}\\ \mathbf{0}\end{bmatrix} \\
= \; & \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\begin{bmatrix}\mathbf{1} & \mathbf{1}\\ \mathbf{1} & \mathbf{-1}\end{bmatrix} * \color{red}{\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\begin{bmatrix}\mathbf{-1}\\ \mathbf{1}\end{bmatrix}} \\
= \; & \frac{\mathbf{1}}{\sqrt{\mathbf{2}}} \cdot \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\begin{bmatrix}\mathbf{1} & \mathbf{1}\\ \mathbf{1} & \mathbf{-1}\end{bmatrix} * \begin{bmatrix}\mathbf{-1}\\ \mathbf{1}\end{bmatrix} \\
= \; & \frac{\mathbf{1}}{\mathbf{2}}\begin{bmatrix}\mathbf{0}\\ \mathbf{-2}\end{bmatrix} \\
= \; & \begin{bmatrix}\mathbf{0}\\ \mathbf{-1}\end{bmatrix}
\end{aligned} \tag{1}$$

Line 1 shows the superposition.

How?

3.2: Models
3.2.1: Copenhagen Model

Wovon man nicht sprechen kann, darüber muß man schweigen.

Ludwig Wittgenstein
Tractatus Logico-Philosophicus
1921

How?

- Quantum world and classical world
- Schrödinger's cat is not "alive and dead"
- The superposition is a measure of probability
- Superpositions collapse when interacting with classical systems

$$\left|\textbf{alive}, \ \smiley \ \right\rangle \to \left|\frac{\textbf{alive} + \textbf{dead}}{\sqrt{2}}, \ \overset{?}{\text{😐}} \ \right\rangle \rightsquigarrow \left|\textbf{alive}, \ \smiley \ \right\rangle$$

or

$$\left|\textbf{alive}, \ \smiley \ \right\rangle \to \left|\frac{\textbf{alive} + \textbf{dead}}{\sqrt{2}}, \ \overset{?}{\text{😐}} \ \right\rangle \rightsquigarrow \left|\textbf{dead}, \ \frownie \ \right\rangle$$

How?

3.2.2: Many Worlds Model

Hugh Everett, 1957

- No distinction between quantum and classical systems
- Schrödinger's cat *is* alive *and* dead
- Every possible outcome is real
- The superposition is a superposition of realities
- Observer and observed become entangled

$$\left|\textbf{alive}, \quad \text{☺} \right\rangle \rightarrow \left|\frac{\textbf{alive} + \textbf{dead}}{\sqrt{2}}, \quad \overset{?}{\text{😐}} \right\rangle$$

$$\rightarrow \frac{\left|\textbf{alive}, \quad \text{☺} \right\rangle + \left|\textbf{dead}, \quad \text{☹} \right\rangle}{\sqrt{2}}$$

Disclaimer

No cats were harmed in the making of this lecture.