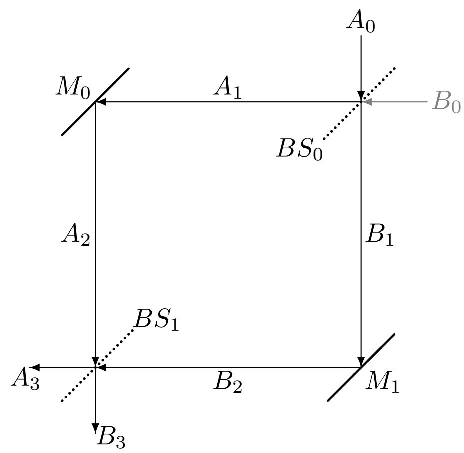
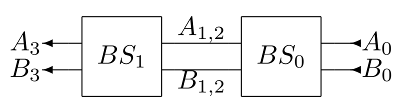
Quantum Systems

Last updated: February 25$^{\text{th}}$ 2019, at 3.23pm
Consider the beamsplitter experiment from the lecture:



which we modelled as:



$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad \frac{1}{\sqrt{2}}\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

and analysed for a photon enetering along path A_0.

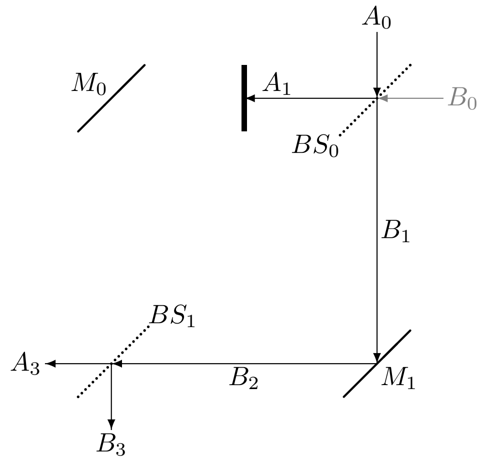
1. Perform a similar analysis when the photon enters along path B_0. Is there any difference in the outcome?

2. Now what is the outcome if the input is a 50/50 (in phase) superposition of inputs along A_0 and B_0?

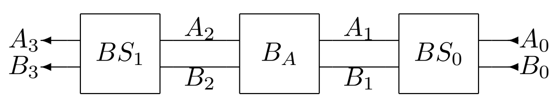
3. What if the superposition is out of phase?

4. How might you construct an actual experiment to achieve the input in questions 2 and 3?

5. Extend the model of the beamsplitter to include a barrier on path segment A_1:



I.e. the model should now become

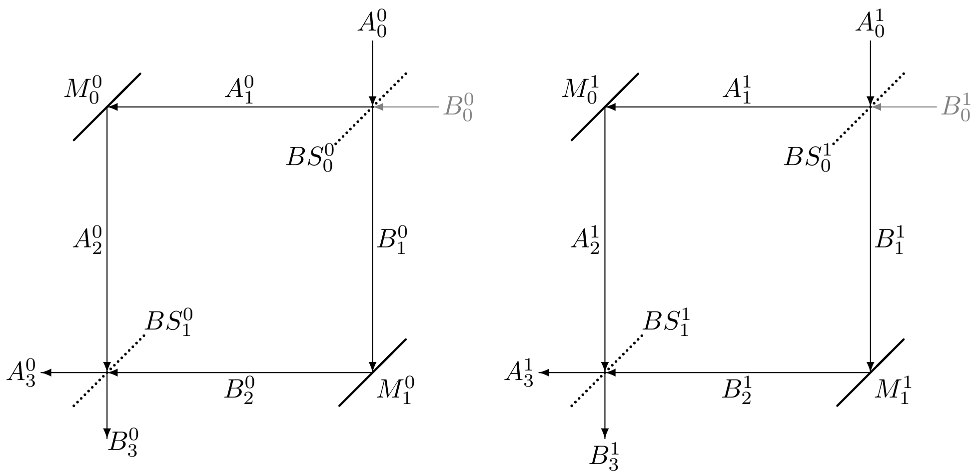


$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad ? \qquad \frac{1}{\sqrt{2}}\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

where component B_A models a barrier between A_1 and A_2 (but not between B_1 and B_2).

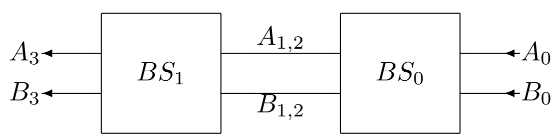
- Derive a matrix for component B_A.
  Hint: It will be a 2×2 matrix. Consider a photon that is some superposition along A_1 and B_1 — i.e. its state is $\begin{bmatrix} p & q \end{bmatrix}^T$ for some p and q. If there is a barrier between A_1 and A_2 then the A_1 component of this superposition should disappear.
- Now apply this model to:
  - a photon input along A_0
  - a photon input along B_0
  - a 50/50 in phase superposed input
  - a 50/50 out of phase superposed input

Now consider a double beamsplitter experiment:

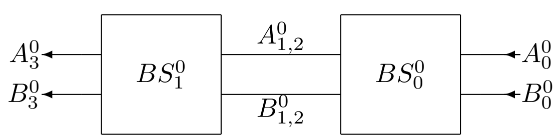


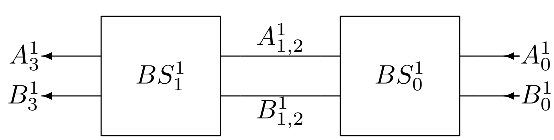
6. Model this system.

   Hint: refer back to last week's lecture on constructing circuits. Think of our model of the single experiment:



   as a circuit, containing two "gates" BS_0 and BS_1. Now construct a parallel circuit:





   Try your model on various inputs.

7. Now model it where beamsplitters BS_0^0 and BS_0^1 somehow entangle the photons such that if the left hand photon is reflected, the right hand one is not, and vice versa. Try this system on various inputs. *Hint:* Derive the matrix for the entangling beamsplitter by

- considering what paths the photons can follow on segment 1 for the four possible input pairs $\left|A_0^0, A_0^1\right\rangle$, $\left|A_0^0, B_0^1\right\rangle$, $\left|B_0^0, A_0^1\right\rangle$ and $\left|B_0^0, B_0^1\right\rangle$,
- expressing these as entanglements — e.g. $\frac{1}{\sqrt{2}}\left(\left|-A_1^0, B_1^1\right\rangle + \left|B_1^0, -A_1^1\right\rangle\right)$
- and giving the matrix representations thereof
- and hence deriving the matrix for the entangling beamsplitter.

8. Try expanding the system so that the left hand system has a barrier between A_1^0 and A_2^0 (but the right hand system does not have a barrier between A_1^1 and A_2^1) and repeat questions 6 and 7.

And, finally, one additional question:

9. Is Schrödinger's cat alive or dead?