CIT2213 Game Engine Architecture

Lecture: Basic Physics Concepts

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Informatics

Overview

- Rigid bodies
- Kinematics
- Forces
- Angular motion

Basic Operations in Game Physics

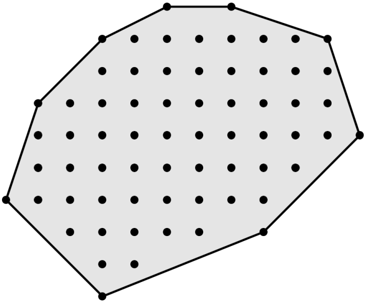
- Move game entities
  - Apply forces
  - Compute motions
  - Integrate velocity, acceleration
- Collision detection
- Resolve collisions

Rigid Bodies

Definition

For a body to be characterised as rigid, its internal mass elements must be fixed relatively to each other.

- All mass elements undergo the same motion
- Simplify the notion of linear motion by treating a rigid body as a particle



A rigid body viewed as a particle system

Basic Properties

If a rigid body consists of n discrete mass elements, its mass and centre of mass are:

Mass:

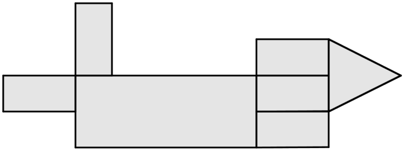
The mass M is

$$M = \sum_{i=1}^{n} m_i$$

Centre of Mass:

The centre of mass $\vec{c}$ is

$$\vec{c} = \frac{\sum_{i=1}^{n} m_i \vec{p_i}}{\sum_{i=1}^{n} m_i}$$



The body of a box car is constructed by “gluing” together multiple rigid bodies with uniform mass.

Kinematics

Definition

The study of motion without considering the influence of external forces is referred to as *kinematics*. For examples, we can describe the motion of our orbiting moon without knowing the gravitational forces; in TSP, an optimal path can be found without taking into account on how the salesman "travels"; motions from devices such as Kinect.

Describe the position of a particle as a function of time

$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$$

The Derivatives

- Velocity
  - $\vec{v}(t) = \frac{d\vec{r}}{dt}$
  - $\frac{d\vec{r}}{dt} \approx \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}(t_1) - \vec{r}(t_0)}{t_1 - t_0}$
- Acceleration
  - $\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$
  - $\frac{d\vec{v}}{dt} \approx \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}(t_1) - \vec{v}(t_0)}{t_1 - t_0}$

Core quantities used in kinematics

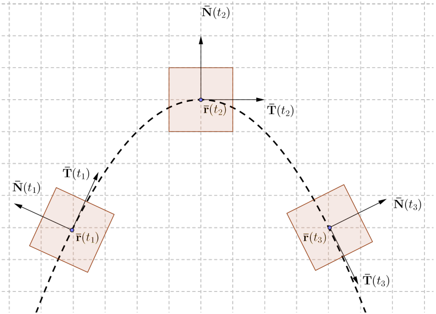
- Displacement/position
- Velocity
- Acceleration

An Example 2D Motion

- $\vec{r}(t) = \vec{r_0} + \vec{v}t + \frac{1}{2}\vec{a}t^2$
- Motion in each dimension

$$x(t) = x_0 + v_x t + \frac{1}{2}a_x t^2$$

$$y(t) = y_0 + v_y t + \frac{1}{2}a_y t^2$$



Newtonian Laws of Motion

First law:

The uniform motion of an object is maintained unless an external force is applied to it.

Second law:

$$\mathbf{F} = m\mathbf{a}$$

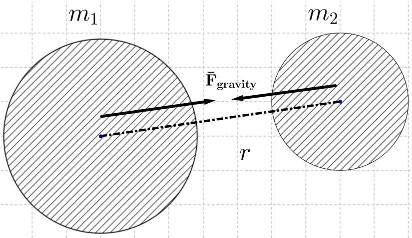
Third law:

For every action there is an equal and opposite reaction.

Forces - Newton's Gravity

The gravitational force between two bodies with mass m_1 and m_2 is,

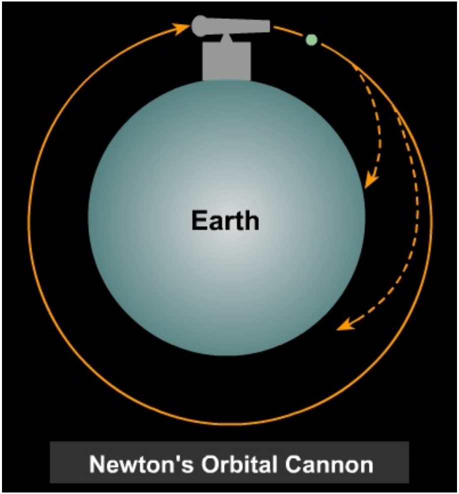
- $\mathbf{F} = G\frac{m_1 m_2}{r^2}$;
- where $G \approx 6.67 \times 10^{-11}(Nm^2kg^{-2})$,
- r is the distance between two bodies.



The polar radius of the Earth is approx. 6.371 km and its mass 5.972×10^{24} kg

Shoot Thyself in the Back

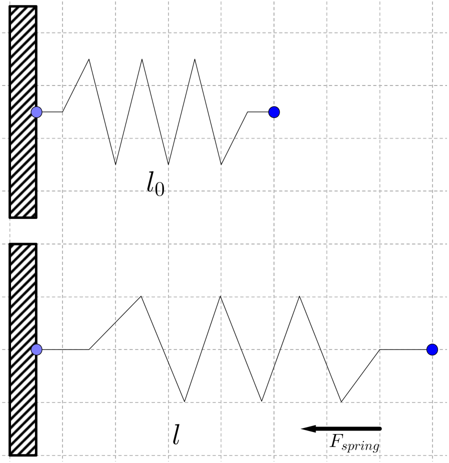
What is the muzzle velocity required?



Forces - Hooke's Spring

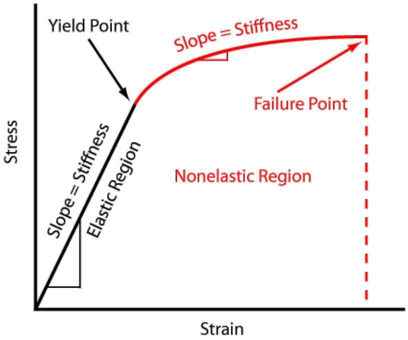
Given a spring with rest length l_0, the spring force $\mathbf{F}$ it generates when depressed or stretched is given by Hooke's law

- $\mathbf{F} = -k(l - l_0)$
- l is depressed or stretched length
- k is the stiffness



More Realistic Springs

- Hooke's law only considers the linear property of a spring
- Unrealistic when large displacement occur
- A possible solution: model the spring constant k as a function of l



The property of material described using Young's modulus.

Forces - Dissipative Forces

Definition

Dissipative forces reduce the energy of a system when motions take place.

Given a body moving at velocity $\vec{v}$, the general dissipative model is:

$$|\vec{F}_{dissipative}| = c|\vec{v}|^n$$

- $|\vec{F}|$ is the magnitude of the force
- c is a constant related to the environment
- In general, a dissipative force counters the object's motion, thus its direction is $-\vec{v}$

Static Friction

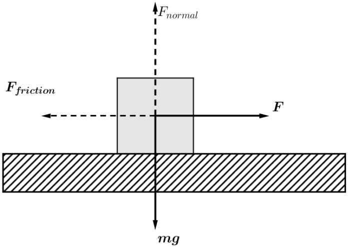
$$\vec{F} = c_s \vec{F}_{normal}$$

where
$c_s = \frac{max(F_{friction})}{F_{normal}}$

Dynamic/Kinetic Friction

$$\vec{F} = -c_k \frac{\vec{v}}{|\vec{v}|}$$

where $c_k = \frac{F_{friction}}{F_{normal}}$



To move the object, F has to first overcome static friction; when in motion the friction becomes kinetic friction.

How can we work out the friction coefficient of a specific surface?

Some Useful Friction Coefficients for Cars

The rolling resistance of a car is caused by friction between its tires and the surface it travels on.

On concrete: $c_s = 1.0, c_k = 0.8$

On wet road: $c_s = 0.6, c_k = 0.4$

On snow: $c_s = 0.3, c_k = 0.2$

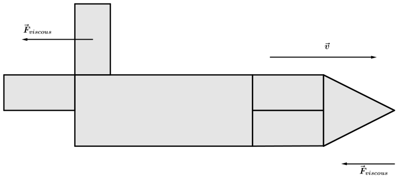
Dissipative Forces - Viscous Force

- When a body travels through fluid (e.g. air, water),
- The viscous force is modelled by:

$$\vec{F} = -c|\vec{v}|^n \frac{\vec{v}}{|\vec{v}|}$$

- The aerodynamic drag of a car is a specific instance of viscous force:

$$\vec{F} = -c|\vec{v}|^2 \frac{\vec{v}}{|\vec{v}|} = -c\vec{v}|\vec{v}|$$

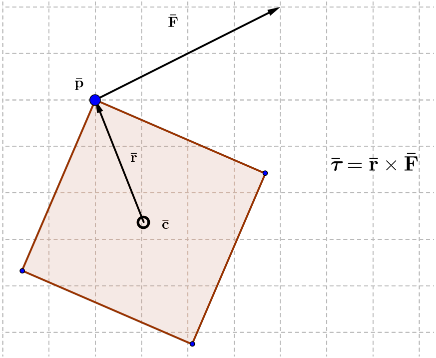


Forces - Torque

Definition

Torque is a physical quantity that describes the tendency for a force to cause rotation on a body.

- A force $\vec{F}$ is exerted on point $\vec{p}$, the torque $\vec{\tau}$ is
- $\vec{\tau} = \vec{r} \times \vec{F}$
- where $\vec{r} = \vec{p} - \vec{c}$
- $\vec{c}$ is the centre of mass



Angular Motion in 2D

- In 2D, objects only have 1 DOF in angular motion
- Given a torque τ, the angular acceleration α is given by

$$\alpha = \frac{\tau}{I}$$

$$\tau = I\alpha$$

- I is the moment of inertia, defined as

$$I = \sum_{i=1}^{n} m_i |\vec{p}_i - \vec{c}_i|^2$$

Angular Motion in 3D

- In 3D, objects have 3 DoFs in angular motion
- $\vec{\tau}$ and $\vec{\alpha}$ are related by the inertia tensor $\mathbf{I}$

$$\begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{bmatrix}$$

$$\vec{\tau} = \mathbf{I}\vec{\alpha}$$

- To find angular acceleration from torque (require matrix inversion)

$$\vec{\alpha} = \mathbf{I}^{-1}\vec{\tau}$$