

# Laboratory 1

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```
<<hello.hs>>=  
<<printMessage>>  
@  
  
<<message>>=  
"HelloWorld!"  
@  
  
<<printMessage>>=  
print(<<message>>)  
@
```

## Euclidean algorithm

The purpose of this program is to give an implementation, and a correctness proof, of the Euclidean algorithm for computing the greatest common divisor of a list of positive integers chosen by the user.

But the goals of this text are more broader.

Algorithm description:

As long as b, which is the second element, is not equal to 0, we call recursively `euclidean(b, a % b)`.

We can observe that always the second element becomes the first element in the new recursive call. If we would not inverse the position of the elements, we would get stuck. Ex.:  $a = 1000, b = 100$  `euclidean(1000, 100) = euclidean(1000, 100) = \dots = euclidean(1000, 100) = \dots`

The mathematical model for our function:

$$euclidean(a, b) = \begin{cases} euclidean(b, a \% b), & \text{if } b > 0 \\ a, & \text{else} \end{cases}$$

The proof of correctness is based on the following lemma:

If  $a = c \pmod{b}$  (1), then  $(a, b) = (c, b)$  (3)

Proof of this lemma:

From (1) we have  $b|a-c$ , so there is a  $y$  such that  $by=a-c$ . If there is a  $d$  such that  $d$  divides  $a$  and  $b$ , then it will also divide  $c=a-by$ .  $\Rightarrow$  any divisor of  $a$  and  $b$  is a divisor of  $c$  and  $b$ . (2) Suppose  $(a,b)=x$  and  $(c,b)=y$ . Using (2) we have that  $x|y$  and  $y|x$ , so we have that  $(a,b)=(c,b)$ .

If we put  $a=a \% b \pmod{b} \Rightarrow$  using lemma (3) we have that  $(a,b)=(a \% b, b)$

```
<<euclidean>>=
euclidean a b =
  if (b>0)
    then (euclidean b (mod a b) )
    else a
```

@