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# H-Infinity control of the VEGA Launch Vehicle first stage in presence of roll

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**Abstract:** The VEGA Launch Vehicle of the European Space Agency performed its maiden flight in 2012 with outstanding success. The control laws flown are based on a mixed SISO/MIMO approach using PID with tuning filters plus a gyroscopic term to deal with coupling in presence of non-negligible roll rate. This paper presents a pure MIMO approach based on an H-Infinity controller. The generated controller is compared with the real implementation and performs better in presence of high roll rates.

*Keywords:* H-Infinity, MIMO, rocket, Mixed-sensitivity, optimization

## 1. INTRODUCTION

The VEGA Launch Vehicle (LV) is the new European Small Launcher developed by a set of industries leaded by ELV SpA. as prime contractor for the ESA (European Space Agency). It will be the reference European launcher for the ‘small launchers’ market segment in the next decade.

The VEGA Launch Vehicle is a 30 meters tall, four stage launcher able to deliver payloads of up to 1700 kg to low Earth orbits. The first 3 stages are SRM (Solid Rocket Motors). The fourth stage uses a LPS (Liquid Propulsion System) motor.

Chapter 1 introduces the VEGA GNC Avionics and algorithms. Chapter 2 introduces the problem of roll rate. Chapter 3 design the controller. Chapter 4 cross-compare the controllers. Chapter 5 present the conclusions.

### *1.1 The VEGA GNC Avionics and algorithms*

The VEGA GNC Avionics Subsystem is composed of the following HW equipment. The OBC (On Board Computer) that executes the FPS (Flight Program Software). The OBC communicates with the other equipment through a 1553 bus. The IMU (Inertial Management Unit) measures attitude, velocity and acceleration.

The TVC (Thrust Vector Control) are in charge of deflection of the nozzle according to the digital commands elaborated and sent by the FPS. There are four TVCs, one for each stage. Each TVC has 2 EMA (Electro Mechanical Actuators). The attitude of the LV is controlled by the linear elongation of each EMA that set the desired angle for the TVC nozzle wrt. the LV longitudinal axis.

The GNC algorithms are divided on navigation, guidance and control. Only the control is of interest here. The control algorithms in turn are divided on two types: TVC control and RACS control (Roll and Attitude Control Subsystem). The TVC control is in charge of controlling the attitude of the Launcher by commanding the vectoring of the nozzle of the active stage. The RACS control is in charge of roll rate control during atmospheric phase and fine attitude control during the orbital phases.

## 2. THE PROBLEM OF ROLL IN LAUNCHERS

### *2.1 Background of the Roll Problem*

The VEGA LV is a symmetrical body around the longitudinal axis. In principle, the LV plant is decoupled wrt. the TVC inputs: a deflection on the nozzle on a given plane only changes the orientation of the launcher on that plane.

In a real flight an amount of roll rate will be present. The causes of roll and their impact on the Launcher stability have been studied in detail in (Roux and Cruciani, 2007). The most important causes of roll torque are due to misalignment of the LV COG with respect of the LV longitudinal axis due to imperfections of the distribution of the solid propellant inside the SRM case and misalignment of the nozzle axis with the LV longitudinal axis. Other causes as non-symmetrical erosion of the nozzle and aerodynamic roll are of less importance. These geometrical imperfections depend on the manufacturing process of the launcher and can be limited by strict quality control but never removed.

Under the presence of roll rate, the LV dynamics becomes coupled. In presence of coupling the stability margins are reduced and instability could occur at high roll rates.

### 2.2 Management of the Roll Problem

The coupling roll rate was taken into account by the VEGA design team. It was estimated that controlling the roll angle would require additional RCTs (Reaction Control Thrusters) or higher duty cycle of the existing ones that in turn means higher propellant consumption.

The alternative chosen by the VEGA LV designers was that only the roll rate is to be limited (i.e. the RCT actuators only acts when the roll rate is over a threshold). There is no attempt to control the roll angle. A maximum acceptable roll rate of 45 deg/s was defined for the first stage (Roux, 2007). For the first stage the LV inertia is too big and the RCT too weak to set the roll rate back to zero. It is the role of the RACS control to guarantee that roll does not surpass this threshold. The TVC algorithms shall be designed with the constraint that a roll rate up to 45 deg/s can be present.

It shall be noted that during the first flight the roll during the first stage was minor than 10 deg/s. Of course this value can be surpassed in successive flights in presence of stronger geometrical disturbances or high winds.

### 2.3 Description of the TVC Control

The TVC control of the Launcher follows a classical approach. It is based on two separated channels (theta and psi channel) plus a gyroscopic term. Each channel has as inputs the error in angular position and its derivative, and the lateral error and its derivative. A set of filters H1, H2 and H4 are present to calculate the derivatives and shape the contribution in the desired frequency range.

The fact that roll rate can be present is taken into account by means of a gyroscopic compensation term: the angular rate is evaluated and compensations for the theta and psi channel contributions are computed. The gyroscopic compensation term has as input the quaternion in order to avoid numerical singularities associated to the Euler angles. A dedicated filter H5 is used for computing its derivative (angular rates).

A second family of filters (H3 filters) acts on the control contribution of each channel in order to control the bending modes. Additional blocks are needed for converting the angular contribution of each channel to EMA elongation and for compensating the vertical displacement of the pivot point of the nozzle due to the enormous pressure of the SRM thrust. These are not having into account in this study as do not have associated dynamics.

The set of H filters and PID gains are scheduled along the first stage based on the non-gravitational velocity. There are 12 different sets for the first stage. The tuning of these H filters and PID gains to cover the full flight domain (different trajectories, different payloads, different winds, etc.) is one of the most difficult tasks requiring a lot of GNC expert know-how and is very time consuming.

### 2.4 Limitations of the gyroscopic compensation approach

The gyroscopic compensation is based on a nominal inertia model and can produce errors in case of different inertia is present. Even if gyroscopic compensation is perfect it will cancel only the roll coupling in the rotational dynamics and not on the translational dynamics. This limitation is intrinsic to the method. The use of the gyroscopic compensation implies a new set of gains and filters that shall be tuned.

## 3. AN ALTERNATIVE H-INFINITE CONTROL

### 3.1 Equations of the VEGA Launcher

The classical reference of the non-linear and linear equations for a launcher is (Greensite, 1970). These equations were specialized for the VEGA Launcher by the VEGA design team. The complete derivation and specialization is given by (Cruciani, 2008).

We do not take into account the long term translational dynamic along the longitudinal X-axis but only the short term dynamics along the lateral axis Y and Z. As the launcher is ‘roll free’ we do not study the rotational dynamic about the longitudinal X-axis (roll) but only suppose that roll rate ($p_0$) is different from zero but constant (or changes slowly). Also we suppose that the mass variation is constant (or changes slowly) for the period of time considered. The equations are derived on a pseudo-inertial reference frame (yaw and pitch planes are parallel to the local trajectory frame that is inertial but roll axis rotates with the LV). We assume small angles for pitch and yaw deviations. The final linear equations are (1) and (2).

$$\begin{aligned} \dot{q} &= \left(-K_1 \beta_\psi\right) + A_6 \psi + \left(\frac{A_6}{V}\right)\dot{Z} + \lambda p_0 r \\ \dot{r} &= \left(K_1 \beta_\vartheta\right) + A_6 \vartheta - \left(\frac{A_6}{V}\right)\dot{Y} - \lambda p_0 q \end{aligned} \tag{1}$$

$$\begin{aligned} \ddot{Y} &= \frac{T}{m}\beta_\vartheta + \frac{T - p_{dyn} S_R (C_N - C_X)}{m}\vartheta + \frac{p_{dyn} S_R C_N}{mV}\dot{Y} - p\dot{Z} \\ \ddot{Z} &= -\frac{T}{m}\beta_\psi - \frac{T - p_{dyn} S_R (C_N - C_X)}{m}\psi - \frac{p_{dyn} S_R C_N}{mV}\dot{Z} - p\dot{Y} \end{aligned} \tag{2}$$

Being:

$$A_6 = \frac{p_{dyn} S_R (x_{CP} - x_{COG})}{I_T} C_N; \quad K_1 = -\frac{T(x_{COG} - x_{PP})}{I_T}$$

$$I_T = I_{YY} = I_{ZZ}; \quad \lambda = \frac{I_T - I_{XX}}{I_T} = 1 - \frac{I_{XX}}{I_T}; \quad p_{dyn} = 0.5\,\rho V^2$$

*Notation* :

$\vartheta, \psi$ : pitch and yaw angles

p,q,r : angular velocity of body frame wrt inertial (roll, yaw, pitch)

$K_1$ : controllability coefficient; $A_6$ : aerodynamic instability coefficient

V : modulus of the velocity; Y, Z : lateral drift

$C_N, C_X$ : normal and axial aerodynamic coeffcients

$X_{CP}$ : x coordinate of aerodynamic centre of pressure

$X_{COG}$ : x coordinate of center of gravity

$X_{PP}$ : x coordinate of the nozzle pivot point
$\rho$ : air density; $p_{dyn}$ : dynamic pressure
$S_R$ : transversal ref surface; T : Thrust
$\beta_\vartheta, \beta_\psi$ : angle of TVC actuator on pitch and yaw planes

### *3.3 State space model*

The space state model of the 6DOF model is given by the set of matrices (3), where we have chosen theta and psi as available measures (see justification later). Then we compose the complete open loop plant as the composition of the 6DOF model plus the TVC model (ideal second order actuators).

$$\vec{x} = \begin{bmatrix} \dot{y} & \vartheta & r & \dot{z} & \psi & q \end{bmatrix}, \quad \vec{u} = \begin{bmatrix} \beta_\vartheta & \beta_\psi \end{bmatrix}, \quad \vec{y} = \begin{bmatrix} \vartheta & \psi \end{bmatrix}$$

$$A = \begin{bmatrix} -A_{vlat} & A_{theta} & 0 & p_0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -p_0 & 0 \\ -\dfrac{A_6}{V} & A_6 & 0 & 0 & 0 & -\lambda p_0 \\ -p_0 & 0 & 0 & -A_{vlat} & -A_{psi} & 0 \\ 0 & p_0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \lambda p_0 & \dfrac{A_6}{V} & A_6 & 0 \end{bmatrix}, B = \begin{bmatrix} \dfrac{T}{m} & 0 \\ 0 & 0 \\ K_1 & 0 \\ 0 & -\dfrac{T}{m} \\ 0 & 0 \\ 0 & -K_1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad (3)$$

Being:

$$A_{theta} = A_{psi} = \frac{T - p_{dyn} S_{ref} (C_N - C_x)}{m}, \quad A_{vlat} = \frac{p_{dyn} S_{ref} C_N}{mV}$$

We add delays (due to digital processing) of IMU (10 ms), OBC(12 ms), TVC(15 ms) with a Pade transform of second order at plant input. The figure (2) shows the diagram of singular values of the uncertain open loop plant with scattering (5%) on mass, thrust, XCOG, dynamic pressure and total velocity modulus V. In addition the roll rate is scattered between [-45, + 45] deg/s.

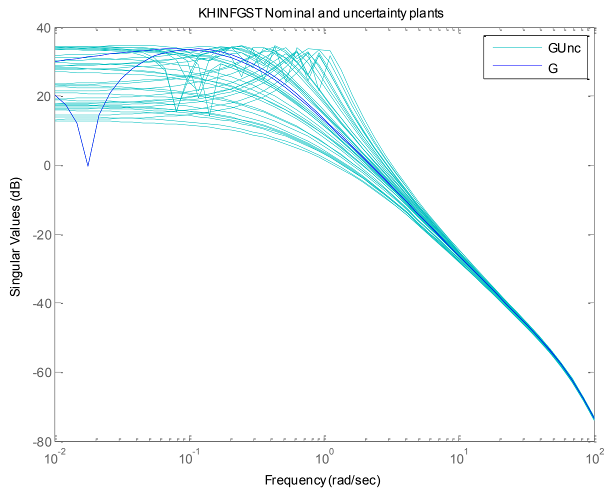


**Figure 1: nominal and scattered open loop plant**

### *3.4 Design of the H-Infinity controller*

Several authors, for example (Skogestad, 1996) have remarked that H-Infinity is not appropriate for controlling ill conditioning plants (i.e. with strong coupling). The Control of a Distillation Column (CDC) benchmark problem (Limebeer, 1991) is an example of such plants.

In effect has been demonstrated (Christen, 1997) that H-Infinity Mixed Sensitivity controllers trends to invert the plant in the controller creating pole-zero cancellations. For plants with strong coupling the real disturbed plant will be different from the ideal design plant and the cancellation will be imperfect, resulting in poor performance and robustness. The paper (Christen, 1997) also proposes an optimization of the H-Infinity method for dealing with these ill conditioned plants. Let's name this variant 'H-Infinity CHGE'. We demonstrate in another work (Sanchez, unpublished) that the CHGE alternative is able to control successfully the distillation plant of the CDC Benchmark.

Another alternative of H-Infinity, the 'Structured approach' proposed by (Apkarian, 2006) can also control successfully the distillation plant of the CDC Benchmark.

The key point in both H-Infinity improved methods (CHGE and Structured) is that they include in the control problem some knowledge about the disturbances affecting to P.

In this paper we use the H-Infinity CHGE variant. In order to consider the disturbances affecting to P, the trick is simply to consider the transfer function $S_oP$ between the transfer functions to weight. We can do that by adding the signal $d_u$ (perturbations at plant input). See (figure 2). The transfer matrix of the augmented plant is given by (4).

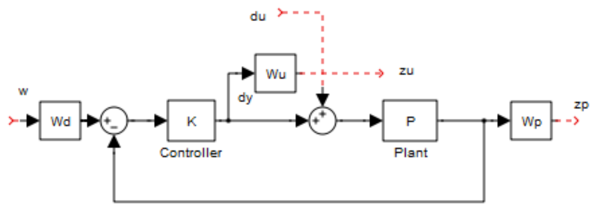


**Figure 2 Augmented plant of the CHGE formulation**

$$\begin{bmatrix} dy \\ z \end{bmatrix} = \begin{bmatrix} -W_u T_I & -W_u K S_o W_d \\ W_P S_o P & W_P T_o W_d \end{bmatrix} \begin{bmatrix} du \\ dw \end{bmatrix} \qquad (4)$$

In atmospheric phase the most important objective is to minimize the attack angle. (The real requirement is imposed as a gabarit vs. mach number of the product (pDyn*alpha) but as pDyn cannot be controlled the requirement translates to the angle of attack (alpha). (Note however that VEGA does not have an angle of attack sensor).

As the launcher trajectory is designed for having an almost zero angle of attack, keeping low the attitude error helps to achieve a low attack angle. For this we select pitch and yaw error as input to the controller. Other alternatives can be found on (Renault, 2008) The control of the lateral deviations and lateral velocities errors is also required. (These deviations are recovered in later stages where the aerodynamic drag is not present. However excessive deviation could lead to exit of the allowed corridor and commanded neutralization by Ground Safety). Being the inputs and outputs angles of similar magnitude we do not need to scale the plant (a good scaling is fundamental in the H-Infinity theory).

A design decision to be taken is the roll rate (assumed constant) at which we want to linearize the model. As the roll rate can be positive or negative, we have chosen a design roll rate of 1 deg/s (near to the centre of interval but non zero to avoid losing the information about the roll coupling in the state space model).

The weight $W_d$ is chosen to be small (so the second column of the augmented plant does not play a role in the weighted optimization).

$W_p$ weights $S_oP$. It shall be designed on 2 parts: one for $S_o$ and one for P.

The weight for $S_o$ shall follow the common guidelines given in literature (a high pass filter that will force $S_o$ to be small at low frequencies resulting on small tracking errors). We have proceed as follow:

- Take $T_{o\_id} = \omega_n^2 / (s^2 + 2 \text{ delta } \omega_n s + \omega_n^2)$
- Make $S_{o\_id} = 1 - T_{o\_id}$
- Define the weight as the inverse of $S_{o\_id}$

We select $T_{o\_id}$ with no overshoot (delta = 1) and setting time of about 0.400 ms ($\omega_n = 20$).

As said in (Christen, 1997) the part for P shall be selected as (the inverse) of the maximum singular value of P (for a set of perturbed plants). Let's name this value *A* and take:

$$A = \max(\text{sigma}(P))$$

The $W_p$ weight is given by:

$$W_p = inv(A)\ inv\begin{pmatrix} 1-\dfrac{20^2}{s^2+20s+20^2} & 0 \\ 0 & 1-\dfrac{20^2}{s^2+20s+20^2} \end{pmatrix}$$

$W_u$ weights $T_i$ or $KS_o$. $KS_o$ is related to the control effort. In this particular problem we leave this weight free (equal to 1). However, a technical problem point shall be taken into account: the H-Infinity optimization algorithm assumes that the terms being weighted have a similar magnitude (if not, only the term with the greater magnitude contributes significantly to the overall weighted value). As we have introduced a factor of about $1/P_0$ in the weight $W_p$. we should introduce a similar factor in the weight for $W_u$. This achieves that both weighted transfer functions contribute equivalently to the norm.

$$W_u = inv\left(A\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)$$

The synthetized continuous controller has 18 states.

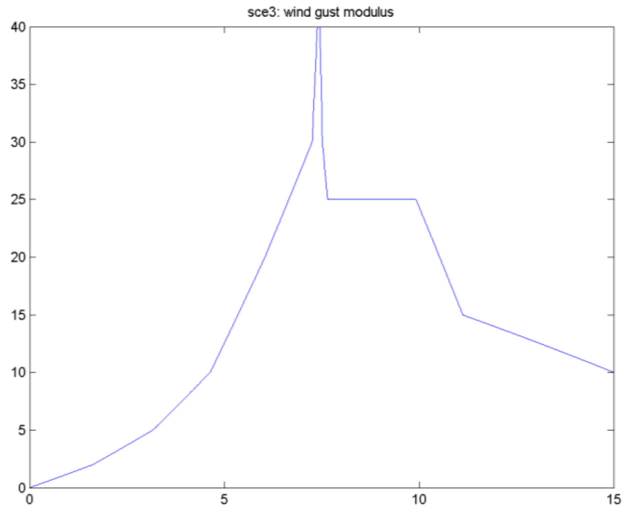


**Figure 3: Wind gust for scenario 3**

*3.5 Design of 'miniVEGA': a simplified simulator of the VEGA LV*

The reference simulators of the VEGA Launcher Program are VEGAMATH (property of ELV SpA.) and VESAT (developed by ESA). It is not possible to use these simulators in this paper by reasons of industrial confidentially. Instead a simplified simulator called 'miniVEGA' has been developed from scratch.

Our 'miniVEGA' simulator contains a number of simplifications. Only the first stage is modelled (no lift-off, no separation). Bending and sloshing modes are not modelled. An ideal IMU is assumed (measures are perfect). The TVC model is a second order model with non-linear saturation of position and velocity of the EMAs. The navigation and guidance are supposed perfect. The simulation is representative only during short times. Note however that 'miniVEGA' is full 6DOF and includes wind effects and possibility of introducing scatterings.

## 4. COMPARISON OF CONTROLLERS

*4.1 Simulation scenarios*

Three scenarios have been defined for comparison of the controllers. Scenario 1: Response to ramps commands in theta and psi in absence of winds. Scenario 2: Response to ramps commands in theta and psi in presence of light winds. Scenario 3: Disturbance rejection in presence of strong winds and wind gusts see figure (3). This is the more realistic scenario.

Each scenario is run with combinations of the compared controllers and a set of roll rates: [-45, 0, 10, 30, 45] deg/s. (The controllers assume that the RACS has limited the roll rate to be minor than 45 deg/s). The scenarios are run at from t = 50 s to t = 65 s (being t = 0 lift-off). At that time the maximum dynamic pressure is reached. This phase of the first stage is considered the most difficult to control.

All the controllers are continuous (ELV controllers have been passed from discrete to continuous). The control objectives are: to minimize angle of attack, to minimize attitude error, lateral deviation (<500 m) and lateral velocity error (<15 m/s). (Note for second flight these were relaxed to respectively < 1000 m and < 35 m/s).

The TVC constraints (max ±4.9 deg deflection and max 10 deg/s deflection rate) shall be satisfied.

We use the following notation for the controllers.

| Name | Description |
|---|---|
| KHINFGST | The proposed H-Infinity controller |
| KELVI | The PID controller without gyro compensation (outer loop disconnected) |
| KELVF | The PID controller without gyro compensation (outer loop connected) |
| KELVIG | The PID controller with gyro compensation (outer loop disconnected) |
| KELVGF | The PID controller with gyro compensation (outer loop connected) |

*4.2 Simulation Results and Analysis*

*Scenario 1: response to ramps in absence of winds*

Ramp inputs are commanded in theta and psi at different roll rates in absence of winds. In this scenario the outer loop in position and velocity is disconnected for the ELV controllers. The justification is simplicity: if a constant pitch angle is required, we should also command a change on the $y_{dot}$ velocity and y position. If $y_{dot}$ and y references were kept to zero, we are sending contradictory orders to the controller.

The figure (4) is the simulation at zero roll rate. The results are very similar between the 3 controllers, being the HINFGST controller a bit more agressive. Angle of attack and trajectory are almost identical.

**Note: in all the figures the continuous line is the variable for the pitch plane (theta, y, yDot) and the dotted line the variable on the yaw plane (psi, z, zDot)**

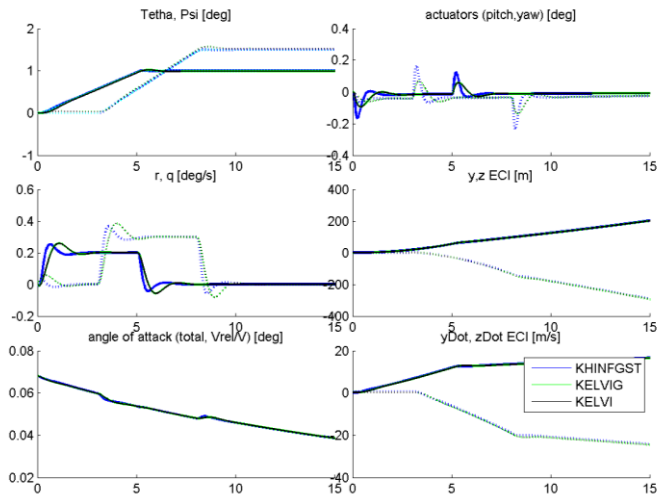


**Figure 4: Sce1 – ramp, no wind and roll rate = 0 deg/s**

In the simulation at roll rate 45 deg/s (figure 5) we see that the KELVI and KELVIG controller have a stationary error in the tracking of theta and psi (zoomed). In KHINFGST this stationary tracking error is not present. The angle of attack is identical for the 3 controllers.

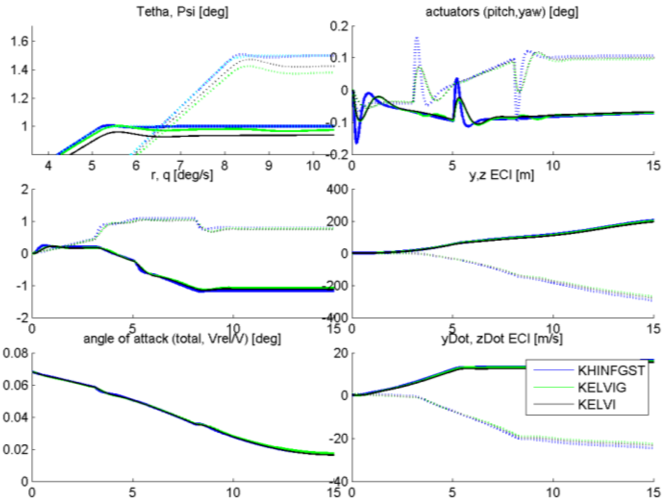


**Figure 5: Sce 1 – ramp, no wind and roll rate = 45 deg/s**

*Scenario 2: response to ramps in presence of winds*

Figure (6) is the simulation at zero roll rate. The presence of lateral winds introduces stationary errors in for KELVI and KELVIG even when the roll rate is zero. The error is about 0.25 deg for yaw. The KHINFGST controller does not have stationary error.

The situation changes as the roll rate increases. As the LV is rolling the wind is not received always on the same side. At 45 deg/s the LV completes a full rotation in 8 s.

This creates oscillations in theta, psi and the actuators that try to counteract the changing wind. See figure (7). Only the KHINFGST controller is able to keep a low stationary error and to decrease the amplitude of the oscillations. This is without penalizing the angle of attack.

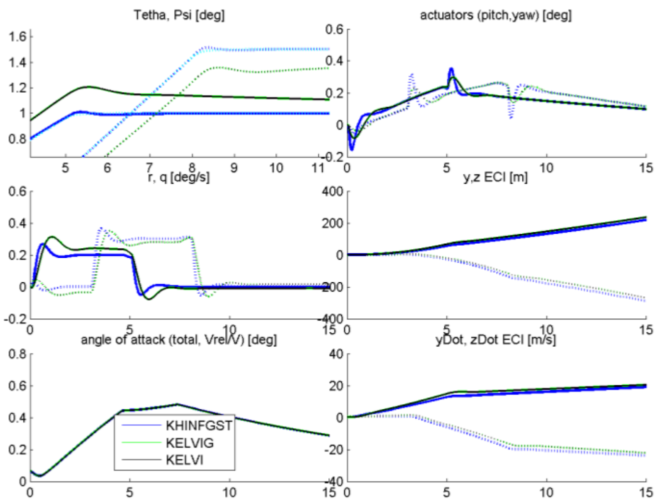


**Figure 6: Sce 2 - ramp with wind and roll rate = 0 deg/s**

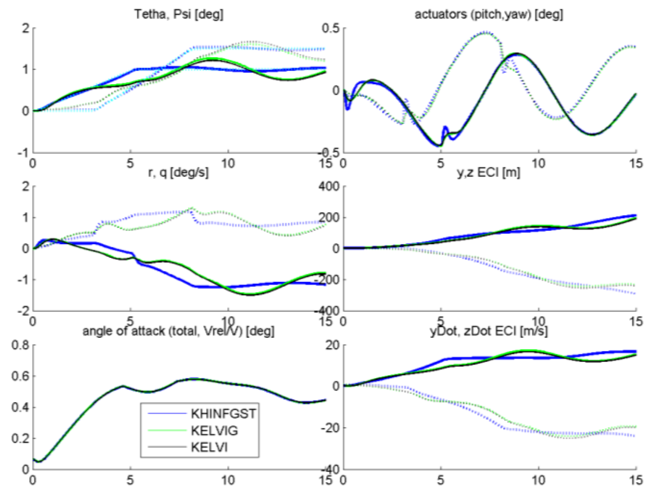


**Figure 7: Sce 2 - ramp with wind and roll rate = 45 deg/s**

*Scenario 3: disturbance rejection with wind gusts*

The outer loop in position and velocity is *connected* on this scenario for the ELV controllers. Only KHINFGST and KELVGF are shown as KELVF results are quite similar.

Figure (8) is the simulation at zero roll rate. The KELVGF performances are 1.5 deg on theta error (pitch plane), the lateral deviation is 180 m in y and of 25 m/s in $y_{dot}$ (exceeding the reqt of 15 m/s for 1st flight but not the relaxed of 35 m/s for 2nd flight). The KHINFGST performances are: maximum error on theta is 0.12 deg. max lateral deviation is 60 m in y and of 8 m/s in $y_{dot}$.

Figure (9) is at roll rate 45 deg/s. For KELVGF controller the maximum theta error is of 1.2 deg at the time of the wind gust. The maximum lateral deviation is about 115 m and the lateral velocity deviation of about 17 m/s (slight violation for 1st flight reqt). The KHINFGST performances are: max deviation in theta or psi is about 0.25 deg. The max lateral deviation is about 25 m and the max lateral velocity deviation of about 4 m/s. Angle of attack is almost identical.

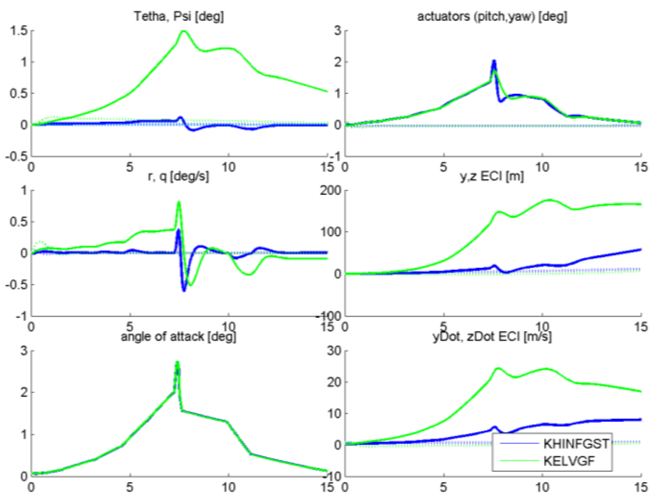


**Figure 8: Sce 3 - wind gust and roll rate = 0 deg/s**

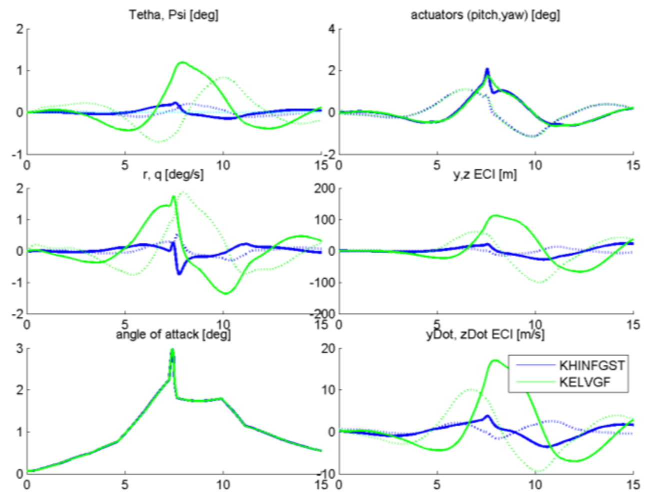


**Figure 9: Sce3 – wind gust and roll rate = 45 deg/s**

## 5. CONCLUSIONS

A satisfactory H-Infinity MIMO controller has been developed for the first stage of the VEGA LV. The controller performs better that the real controllers in presence of winds and roll rate without penalizing the angle of attack. The cause is that the 'knowledge' about how the roll rate affects the translational and rotational dynamics is present on the MIMO open loop plant. The synthetized controller is not a 'qualified' controller in the aerospace sense. For achieving this goal the controller shall be tested on the full flight domain (other stages, different payloads, etc). The bending modes cannot be neglected. This is a work to be done. A complementary direction of investigation is the H-Infinity structured approach. It can impose a predefined structure to an H-Infinity controller (for example as PID plus a set of filters) and tune in an automated way these filters (Apkarian, 2006).

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