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Ans-6 → 1) Additive

$$T(u+v) \Rightarrow T(u) + T(v)$$

$$u = a_1 + b_1 x + c_1 x^2$$

$$v = a_2 + b_2 x + c_2 x^2$$

$$T(u+v) \Rightarrow T((a_1+a_2) + (b_1+b_2)x + (c_1+c_2)x^2)$$

$$\Rightarrow (a_1+a_2+1) + (b_1+b_2+1)x + (c_1+c_2+1)x^2$$

$$\Rightarrow (a_1+1) + (b_1+1)x + (c_1+1)x^2 + (c_2+1) + (b_2+1)x + (c_2+1)x^2$$

$$\Rightarrow T(u) + T(v)$$

Hence proved.......

2. Homojenity

$$T(Ku) \Rightarrow K\,T(u)$$

$$T(K(a+bx+cx^2))$$

$$T(Ka + Kbx + Kcx^2)$$

$$\Rightarrow (Ka+Kb+Kc+1) + (Ka+Kb+Kc+1)x + (Ka+Kb+Kc+1)x^2$$

$$\Rightarrow K(a+1) + K(b+1) + K(c+1)x^2$$

$$\rightarrow K\,T(u)$$

Proven

So it is linear Transformation

Ans 7 → $a(1,2,3) + b(3,1,0) + c(-2,1,3) = (0,0,0)$

$$a + 3b - 2c = 0$$

$$2a + b + c = 0 \qquad \Rightarrow c = -a,\ b = -a$$

$$3a + 3c + 0b = 0$$

So one sol^n posible $a=b=c=0$ Hence it is l.I

dim of $V_3(R) = 3$ & S also contain 3 vector and also S = L.I so it spans $V_3(R)$ making it a basis for $V_3(R)$.

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9 → Suppose we have 2-D image represented as grid or pixels. We can use AT matrix to rotate around Centre

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation of image by θ

1) Translation ~~of~~ to origin

→ Translate the image so that it Centre aligns with origin.

2) Rotation

→ apply Rotation matrix.

3) Translation Back

→ translate it back with its original position by adding Co-ordinates of Centre.

5 → Consistent → having atleast 1 sol^n

- → dependent (infinite sol^n)
- → Independent (unique sol^n)

Inconsistent → no sol^n

$$A \Rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - R_1$

$$\Rightarrow \left[\begin{array}{cccc} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{array}\right] \xrightarrow[R_4 \to R_4 + 2R_2]{R_3 \to R_3 - 2R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

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$\rho(A) \Rightarrow 2$

$\rho(A:B) \Rightarrow 2$ → Consistent (infinite sol^n)

$n \Rightarrow 3$

$\rho(A) = \rho(A:B) \neq n$

3→ $(A - \lambda I) = 0$

$$\det \begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} = 0$$

$(2-\lambda)^2 - 1 = 0$

$(2-\lambda) = \pm 1$

$\lambda = 1, 3$

for $\lambda = 1$ $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$x - y = 0$ Let $x = t$

$x = y$ $y = t$

Eigen vector $V_1 = t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

for $\lambda = 3$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-x - y = 0$ let $x = K$

$x = -y$ $y = -K$

So eigen vector $V_2 \Rightarrow K \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

For A^{-1}

⇒ eigen values of A^{-1} will be

$\frac{1}{\lambda_1}$ & $\frac{1}{\lambda_2}$ ⇒ $1, \frac{1}{3}$ & Eigen vector will be same as A

For $A + 4I$ ⇒ Eigen values for $A + 4I$ will be $\lambda_1 + 4, \lambda_2 + 4$ ⇒ 5, 7 & Eigen vectors are same as of A

1. $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - 6R_1$

$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$

$R_2 \longleftrightarrow R_3$ $\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$ $R_2 \Rightarrow \frac{-R_2}{4}$

$A \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & 4 & -11 & 5 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 0 & 7/12 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Rank of matrix is '3'

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Q → max deg of polynomial $T = 2$

$\dim(P_2) = 3$

Kernel

So a subset of Kernel T is $T(A) = 0$

$(a-b) + (b-c)x + (c-d)x^2 = 0$

$a = b = c = t$ (let)

new matrix ⇒ $\begin{bmatrix} t & t \\ t & d \end{bmatrix}$

dimension of Kernel = 1 bcoz there is only one independent parameter (t)

Acc. to rank nullity theorem rank(T) + nullity(T) ⇒ dim(W)

rank(T) + 1 = 4

So rank of T = 3 and nullity is 1