SGD Algorithm and SVD decomposition to predict movie ratings

Problem statement

- we need to predict the rating given by the user_i to the movie_j for the given dataset

Predicted rating $\hat{y}_{ij}$ for user i, movied j pair is calcuated as $\hat{y}_{ij} = \mu + b_i + c_j + u_i^T v_j$, here we will be finding the best values of b_i and c_j using SGD algorithm with the optimization problem for N users and M movies is defined as

$$L = \min_{b,c,\{u_i\}_{i=1}^N,\{v_j\}_{j=1}^M} \quad \alpha\Big(\sum_j \sum_k v_{jk}^2 + \sum_i \sum_k u_{ik}^2 + \sum_i b_i^2 + \sum_j c_i^2\Big) + \sum_{i,j\in\mathcal{I}^{\text{train}}} (y_{ij} - \mu - b_i - c_j - u_i^T v_j)^2$$

- μ : scalar mean rating
- b_i : scalar bias term for user i
- c_j : scalar bias term for movie j
- u_i : K-dimensional vector for user i
- v_j : K-dimensional vector for movie j

In [121]:

```
#importing libraries
import numpy as np
from scipy.sparse import csr_matrix
import pandas as pd
from sklearn.metrics import mean_squared_error
from sklearn.preprocessing import MinMaxScaler
from sklearn.model_selection import GridSearchCV
from sklearn.metrics import plot_confusion_matrix
from tqdm import tqdm
from sklearn.metrics import roc_curve, auc
from  sklearn.linear_model import LogisticRegression
```

Reading the csv file

In [122]:

```
data=pd.read_csv('ratings_train.csv')
data.head()
```

Out[122]:

	user_id	item_id	rating
0	772	36	3
1	471	228	5
2	641	401	4
3	312	98	4
4	58	504	5

In [123]:

```
data.shape
```

Out[123]:

```
(89992, 3)
```

Create adjacency matrix

In [124]:

```
row=data['user_id'].tolist()
column=data['item_id'].tolist()
ratings_data=data['rating'].tolist()

adjacency_matrix=csr_matrix((ratings_data, (row, column))).toarray()   #https://docs.scipy.org/doc/scipy/reference/generated/scip
```

In [125]:

```
adjacency_matrix.shape
```

Out[125]:

```
(943, 1681)
```

The unique items in the given csv file are 1662 only . But the id's vary from 0-1681 but they are not continuous and hence you'll get matrix of size 943x1681.

SVD decompostion

code for SVD decompostion

In [127]:

```
#SVD decomposition with n_componenet as 5

from sklearn.utils.extmath import randomized_svd
import numpy as np

U, Sigma, VT = randomized_svd(adjacency_matrix, n_components=50,n_iter=5, random_state=16)
print(U.shape)
print(Sigma.shape)
print(VT.T.shape)
```

```
(943, 50)
(50,)
(1681, 50)
```

Compute mean of ratings

In [128]:

```
def m_u(ratings):
    '''In this function, we will compute mean for all the ratings'''
    # you can use mean() function to do this
    # check this (https://pandas.pydata.org/pandas-docs/stable/reference/api/pandas.DataFrame.mean.html) link for more details.

    return  data['rating'].mean()
```

In [129]:

```
mu=m_u(data['rating'])
print(mu)
```

```
3.529480398257623
```

Initialize B_i and C_j

In [131]:

```
def initialize(dim):
    '''In this function, we will initialize bias value 'B' and 'C'.'''
    # initalize the value to zeros
    # return output as a list of zeros
    v=np.zeros(dim)

    return v
```

In [132]:

```
dim= adjacency_matrix.shape[0]      # give the number of dimensions for b_i (Here b_i corresponds to users)
b_i=initialize(dim)
```

In [133]:

```
dim=adjacency_matrix.shape[1]     # give the number of dimensions for c_j (Here c_j corresponds to movies)
c_j=initialize(dim)
```

Compute dL/db_i

In [135]:

```
def derivative_db(user_id,item_id,rating,U,V,mu,alpha):
    '''In this function, we will compute dL/db_i'''

    l_b=2*alpha*(b_i[user_id])-2*(rating-mu-b_i[user_id]-c_j[item_id]-np.dot(U[user_id],V.T[item_id]))

    return l_b
```

Compute dL/dc_j

In [137]:

```
def derivative_dc(user_id,item_id,rating,U,V,mu, alpha):
    '''In this function, we will compute dL/dc_j'''

    l_c=2*alpha*(c_j[item_id])-2*(rating-mu-b_i[user_id]-c_j[item_id]-np.dot(U[user_id],V.T[item_id]))

    return l_c
```

Compute MSE (mean squared error) for predicted ratings

In [139]:

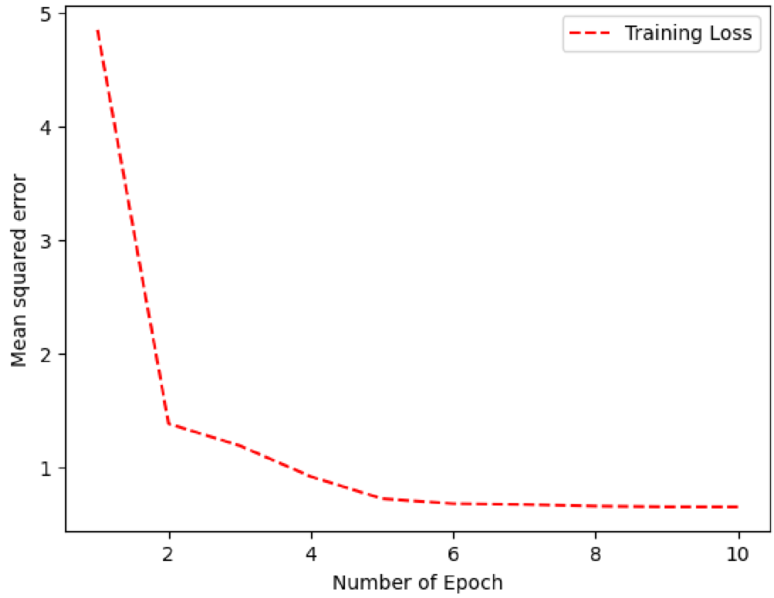
```
alpha=0.01
epoch=10      #number of epoch
eta0=0.01     #learing rate
db=0
dc=0
b_i=initialize(adjacency_matrix.shape[0])       # initialising b_i and c_j
c_j=initialize(adjacency_matrix.shape[1])
MSE=[]         #to store MSE for each epoch
EPOCh=[]       #to store the number of epoch
for k in range(epoch):                            #for each epoch
    for i in range(U.shape[0]):                   #for each user id
        for j in range(VT.shape[1]):              #for each movie id
            db=derivative_db(i,j,adjacency_matrix[i][j],U,VT,mu,alpha)  #computing gradient w.r.to user bias
            dc=derivative_dc(i,j,adjacency_matrix[i][j],U,VT,mu,alpha)  #computing gradient w.r.to movie bias
            b_i[i]-=eta0*db                                  #update bias
            c_j[j]-=eta0*dc

    pre=[]      #to store predicted value
    act=[]      #to store actual value
    for i in range(U.shape[0]):                   #for each user id
        for j in range(VT.shape[1]):              #for each movie id

            predicted=mu+b_i[i]+c_j[j]+np.dot(U[i],VT.T[j])   #predicted value of rating
            pre.append(predicted)
            actual=adjacency_matrix[i][j]                     #actual value of rating
            act.append(actual)

    mse=mean_squared_error(act,pre)                          #finding mean square error
    MSE.append(mse)
    EPOCh.append(k+1)
    print(f"MSE for Epoch {k+1} ",mse)
    print("*"*50)
```

```
MSE for Epoch 1  4.847428705992636
**************************************************
MSE for Epoch 2  1.3832934234373997
**************************************************
MSE for Epoch 3  1.1906714999112504
**************************************************
MSE for Epoch 4  0.9188556703360091
**************************************************
MSE for Epoch 5  0.7247999293142194
**************************************************
MSE for Epoch 6  0.6807862130327451
**************************************************
MSE for Epoch 7  0.6735121787944652
**************************************************
MSE for Epoch 8  0.6584205547939809
**************************************************
MSE for Epoch 9  0.6520789412444189
**************************************************
MSE for Epoch 10  0.6518578911078403
**************************************************
```

Plot epoch number vs MSE

In [140]:

```
from matplotlib import pyplot as plt
plt.plot(EPOCh, MSE, 'r--')
plt.legend(['Training Loss'])
plt.xlabel('Number of Epoch')
plt.ylabel('Mean squared error')
plt.show()
```



Task 2

- For this task we have to consider the user_matrix U and the user_info.csv file.
- we have to consider is_male columns as output features and rest as input features. Now we have to fit a model by posing this problem as binary classification task.
- we can apply any model like Logistic regression or Decision tree and check the performance of the model.
- Do plot confusion matrix after fitting our model and write our observations how our model is performing in this task.
- Optional work- we can try scaling your U matrix.Scaling means changing the values of n_componenets while performing svd and then check our results.

In [141]:

```
#reading data
data_2=pd.read_csv('user_info.csv.txt')
data_2.head()
```

Out[141]:

	user_id	age	is_male	orig_user_id
0	0	24	1	1
1	1	53	0	2
2	2	23	1	3
3	3	24	1	4
4	4	33	0	5

In [142]:

```
#extracting y_true label
Y_actual=data_2['is_male'].tolist()
```

Creating age feature and merge with user vector

In [143]:

```
#minmax scaling age feature

scaler = MinMaxScaler()
data_2['age'] = scaler.fit_transform(data_2['age'].values.reshape(-1,1))
age=data_2['age'].tolist()
age=np.array(age).reshape(-1,1)       #reshaping array
```

In [144]:

```
age.shape     #shape of age feature
```

Out[144]:

```
(943, 1)
```

In [145]:

```
U.shape         #shape of user vector
```

Out[145]:

```
(943, 50)
```

In [146]:

```
# Adding "age" feature to User vector (array) using append() method
U_with_age = np.append(U, age, axis=1)
```

In [147]:

```
U_with_age.shape        #dim of user vector with age feature in it.
```

Out[147]:

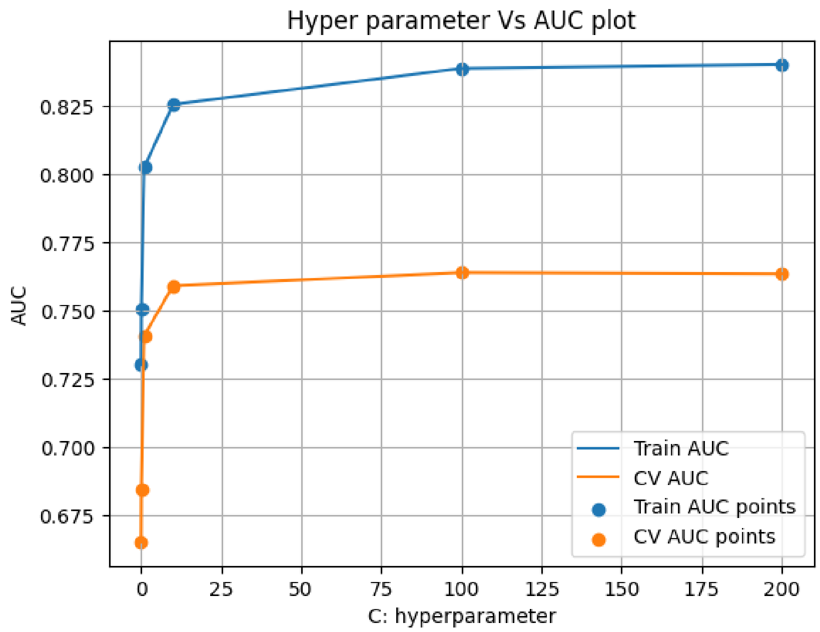
```
(943, 51)
```

Logistic regression

Hyperparameter tuning

In [148]:

```
# https://scikit-learn.org/stable/modules/generated/sklearn.model_selection.GridSearchCV.html

lr=LogisticRegression()
parameters={"C":[200,100, 10, 1.0, 0.1, 0.01],"penalty":['l2']} #hyperparameter list

clf = GridSearchCV(lr, parameters, cv=3, scoring='roc_auc',return_train_score=True) #applying gridsearch to find best hyperparame
clf.fit(U_with_age, Y_actual)    #fitting the LR model with train data

results = pd.DataFrame.from_dict(clf.cv_results_)   #storing Gridsearch results

train_auc= results['mean_train_score']       #storing required Gridsearch results in required variable
cv_auc = results['mean_test_score']
C =  results['param_C'].tolist()
plt.plot(C, train_auc, label='Train AUC')
plt.plot(C, cv_auc, label='CV AUC')
plt.scatter(C, train_auc, label='Train AUC points')
plt.scatter(C, cv_auc, label='CV AUC points')
plt.legend()
plt.xlabel("C: hyperparameter")
plt.ylabel("AUC")
plt.title("Hyper parameter Vs AUC plot")
plt.grid()
plt.show()

results.head()
```



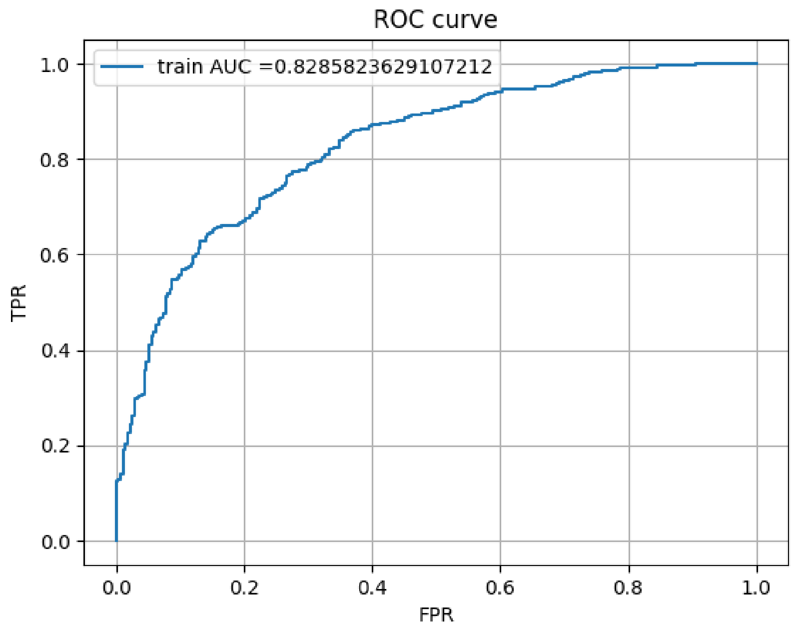
Out[148]:

	mean_fit_time	std_fit_time	mean_score_time	std_score_time	param_C	param_penalty	params	split0_test_score	split1_test_score	split2_t
0	0.034177	0.002984	0.005213	0.007372	200	l2	{'C': 200, 'penalty': 'l2'}	0.749068	0.757946	
1	0.039574	0.003901	0.001335	0.000472	100	l2	{'C': 100, 'penalty': 'l2'}	0.746762	0.758784	
2	0.019951	0.001464	0.001334	0.000473	10	l2	{'C': 10, 'penalty': 'l2'}	0.735479	0.755827	
3	0.010005	0.001639	0.001001	0.000002	1.0	l2	{'C': 1.0, 'penalty': 'l2'}	0.724784	0.751540	
4	0.005904	0.000146	0.000994	0.000814	0.1	l2	{'C': 0.1, 'penalty': 'l2'}	0.691327	0.732321	

Fitting with best C=100

In [149]:

```
# https://scikit-learn.org/stable/modules/generated/sklearn.metrics.roc_curve.html#sklearn.metrics.roc_curve

lr=LogisticRegression(C=100,penalty='l2')

lr.fit(U_with_age, Y_actual)              #fitting the LR model with best alpha
# roc_auc_score(y_true, y_score) the 2nd parameter should be probability estimates of the positive class,not the predicted output

y_train_pred_proba = lr.predict_proba(U_with_age)
y_test_pred_proba = lr.predict_proba(U_with_age)

train_fpr, train_tpr, tr_thresholds = roc_curve(Y_actual, y_train_pred_proba[:,[1]])        #find FPR and TPR for plotting roc cu
plt.plot(train_fpr, train_tpr, label="train AUC ="+str(auc(train_fpr, train_tpr)))   #finding area under the ROC curve using FPR
plt.legend()
plt.xlabel("FPR")
plt.ylabel("TPR")
plt.title("ROC curve")
plt.grid()
plt.show()
```



Plotting confustion matrix

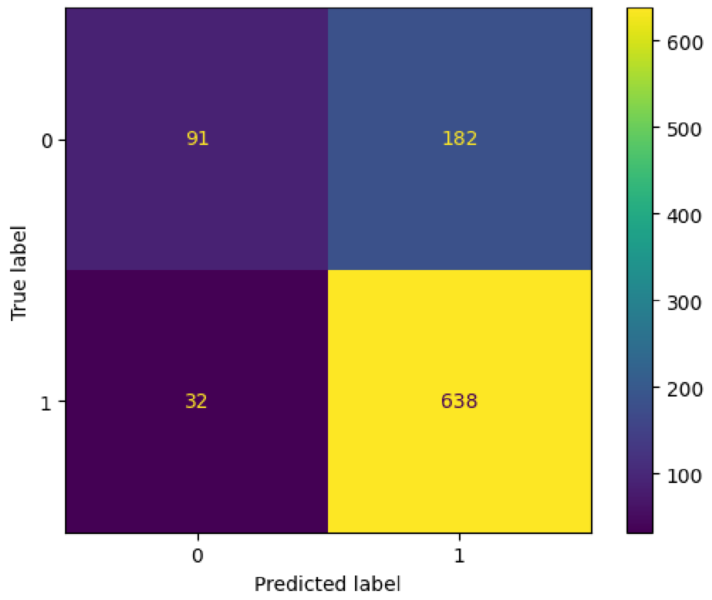
In [150]:

```
plot_confusion_matrix(lr, U_with_age, Y_actual)
```

```
C:\Users\natar\anaconda3\lib\site-packages\sklearn\utils\deprecation.py:87: FutureWarning: Function plot_confusion_
matrix is deprecated; Function `plot_confusion_matrix` is deprecated in 1.0 and will be removed in 1.2. Use one of
the class methods: ConfusionMatrixDisplay.from_predictions or ConfusionMatrixDisplay.from_estimator.
  warnings.warn(msg, category=FutureWarning)
```

Out[150]:

```
<sklearn.metrics._plot.confusion_matrix.ConfusionMatrixDisplay at 0x1ae0ee246a0>
```



Observation

- We can see that we get Very high AUC score of 0.8285 while trying to predict the gender of the user with the given SVD decomposed user vector and age feature.
- So we can say that the user vector that we got by decompoding adjacency_matrix in SVD, have captured some information about the user's gender also.

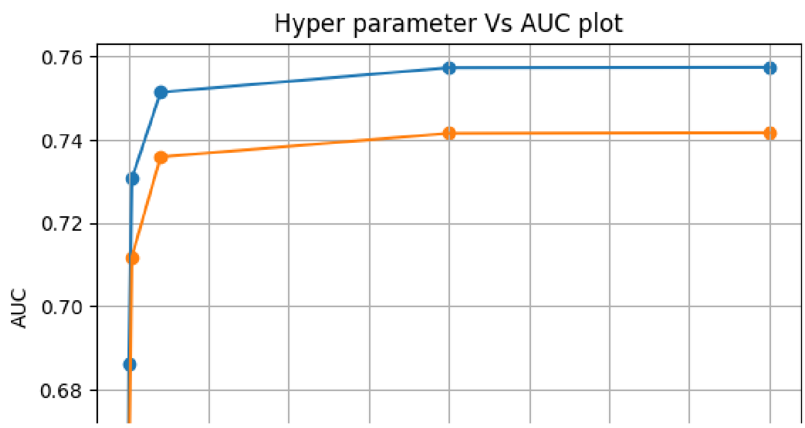
Optional work

Finding best hyperparameter for each User vector of different sizes

In [151]:

```
for i in range(10,81,10):      #for diffent n_components size,create U and VT

    U, Sigma, VT = randomized_svd(adjacency_matrix, n_components=i,n_iter=5, random_state=16)
    print("User vector dimension before adding age feature is ",U.shape[1])


    U_with_age = np.append(U, age, axis=1)  #adding age feature to user vector

    # https://scikit-learn.org/stable/modules/generated/sklearn.metrics.roc_curve.html#sklearn.metrics.roc_curve
    # https://scikit-learn.org/stable/modules/generated/sklearn.model_selection.GridSearchCV.html

    #hyper paramater tuning
    lr=LogisticRegression()
    parameters={"C":[200,100, 10, 1.0, 0.1, 0.01],"penalty":['l2']} #hyperparameter list

    clf = GridSearchCV(lr, parameters, cv=3, scoring='roc_auc',return_train_score=True) #applying gridsearch to find best hyperpa
    clf.fit(U_with_age, Y_actual)    #fitting the LR model with train data

    results = pd.DataFrame.from_dict(clf.cv_results_)   #storing Gridsearch results
    train_auc= results['mean_train_score']       #storing required Gridsearch results in required variable
    cv_auc = results['mean_test_score']
    C =  results['param_C'].tolist()
    plt.plot(C, train_auc, label='Train AUC')
    plt.plot(C, cv_auc, label='CV AUC')
    plt.scatter(C, train_auc, label='Train AUC points')
    plt.scatter(C, cv_auc, label='CV AUC points')
    plt.legend()
    plt.xlabel("C: hyperparameter")
    plt.ylabel("AUC")
    plt.title("Hyper parameter Vs AUC plot")
    plt.grid()
    plt.show()
```

```
User vector dimension before adding age feature is  10
```



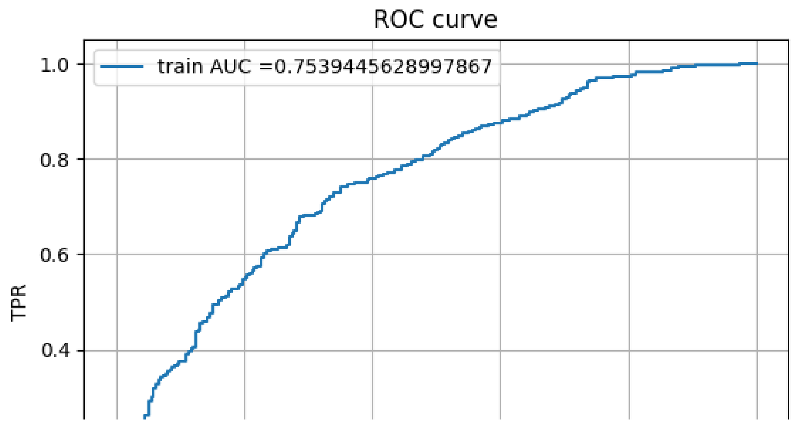
We can see that for the C=100 we get max cv AUC score for all the tested size of the feature vectors

Fitting best hyperparameter c=100 for each User vector of different sizes and Calculate AUC score for different dimentional user vectors

In [152]:

```
AUC=[]
dim=[]
for i in range(10,81,10):                #for diffent n_components size,create U and VT
    U, Sigma, VT = randomized_svd(adjacency_matrix, n_components=i,n_iter=5, random_state=16)
    U_with_age = np.append(U, age, axis=1)   #adding feature vector with user vector

    lr=LogisticRegression(C=100,penalty='l2')
    lr.fit(U_with_age, Y_actual)              #fitting the LR model with best alpha
    # roc_auc_score(y_true, y_score) the 2nd parameter should be probability estimates of the positive class,not the predicted ou

    y_train_pred_proba = lr.predict_proba(U_with_age)
    y_test_pred_proba = lr.predict_proba(U_with_age)
    train_fpr, train_tpr, tr_thresholds = roc_curve(Y_actual, y_train_pred_proba[:,[1]])        #find FPR and TPR for plotting ro
    print(f"User vector dimension before adding age feature is {U.shape[1]} and respective AUC score is {auc(train_fpr, train_tpr
    plt.plot(train_fpr, train_tpr, label="train AUC ="+str(auc(train_fpr, train_tpr)))   #finding area under the ROC curve using
    plt.legend()
    plt.xlabel("FPR")
    plt.ylabel("TPR")
    plt.title("ROC curve")
    plt.grid()
    plt.show()

    AUC.append(auc(train_fpr, train_tpr))
    dim.append(i)
```

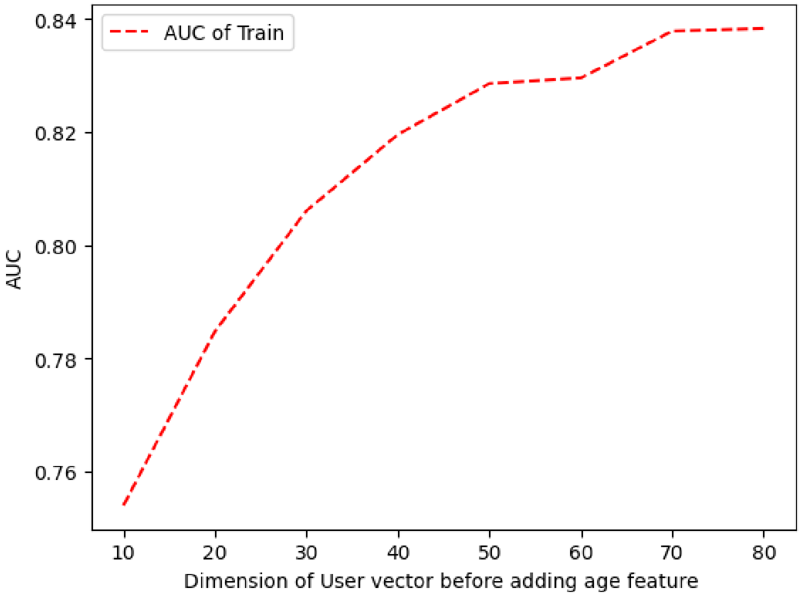
User vector dimension before adding age feature is 10 and respective AUC score is 0.7539445628997867



Dimentional of user vector and their respective AUC score

In [153]:

```
from matplotlib import pyplot as plt
plt.plot(dim, AUC, 'r--')
plt.legend(['AUC of Train'])
plt.xlabel('Dimension of User vector before adding age feature ')
plt.ylabel('AUC')
plt.show()
```



Observation

- as number of dimention of user vector increases, the AUC score also increases which means as dim increases, the user vector can hold more information about that user which results in better performance.