Larmor Equation :

A dipole μ experiences a torque $\vec{T}$ due to a magnetic field B_0.

$$\vec{T} = \vec{\mu} \times \vec{B}_0$$

torque is the rate of change of angular momentum J.

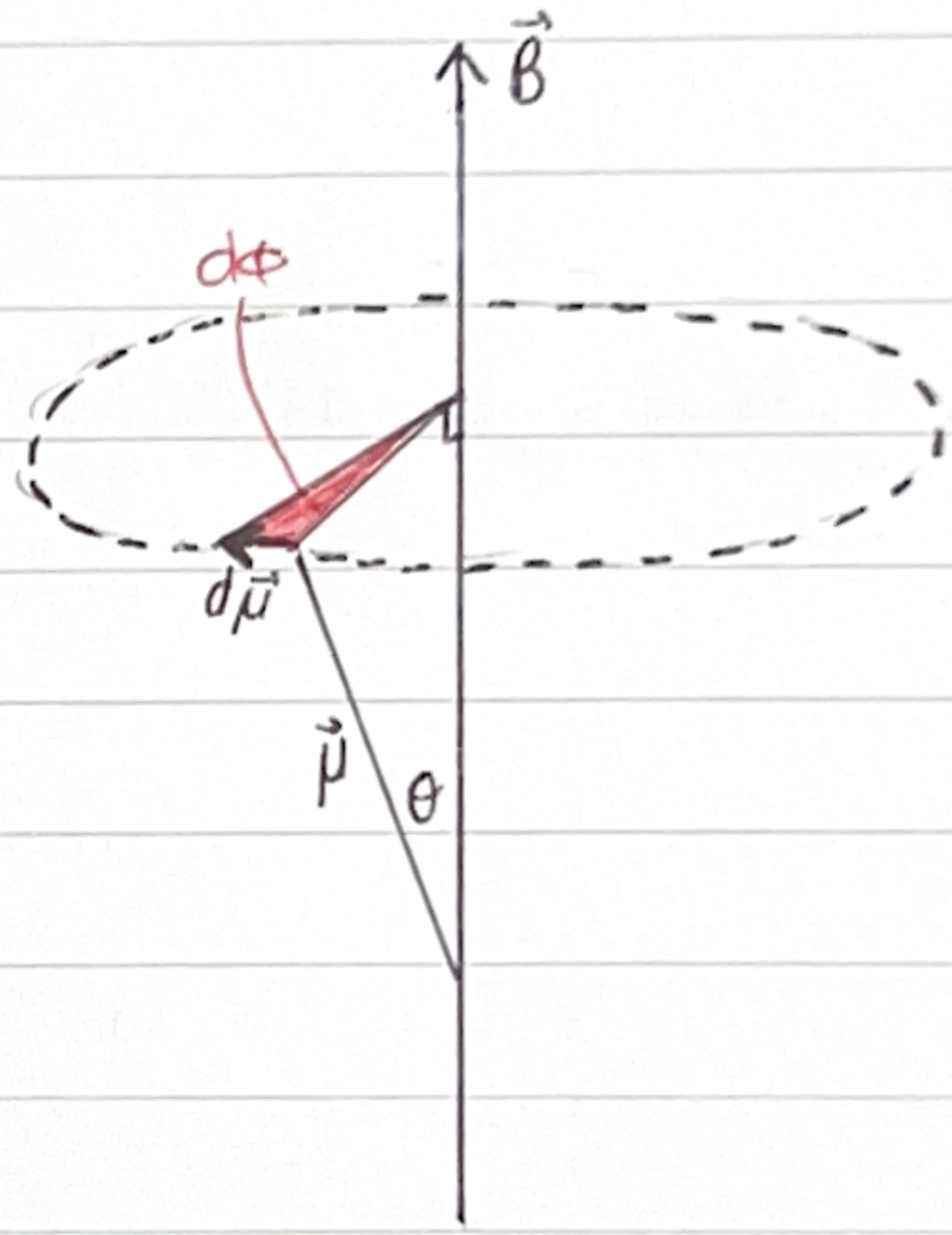
$$\vec{T} = \frac{d\vec{J}}{dt} = \vec{\mu} \times \vec{B}_0$$

From definition of gyromagnetic ratio $\mu = \gamma J$

the larmor equation is obtained $\frac{d\vec{\mu}}{dt} = \gamma\vec{\mu} \times \vec{B}_0$

since $d\vec{\mu}/dt$ is $\perp$ to both $\vec{\mu}$ and $\vec{B}$, then in the event that $\vec{\mu}$ and $\vec{B}_0$ are not aligned, $\vec{\mu}$ must move in a circular path

↳ precession.



From the geometry of the situation, the arc length,

$$|d\vec{\mu}| = \text{radius} \cdot |d\phi|$$

i.e. $|d\vec{\mu}| = \mu \sin\theta \, |d\phi|$

from $\frac{d\vec{\mu}}{dt} = \gamma\vec{\mu} \times \vec{B}$

$$\Rightarrow |d\vec{\mu}| = \gamma \, |\vec{\mu} \times \vec{B}| \, dt = \gamma\mu B \sin\theta \, dt$$

$$\Rightarrow |d\vec{\mu}| = \gamma\mu B \sin\theta \, dt = \mu \sin\theta \, |d\phi|$$

$$\Rightarrow \gamma B \, dt = d\phi$$

$$\Rightarrow \frac{d\phi}{dt} = \gamma B$$

Since the rate of change of ϕ is the angular precessional frequency ($\omega = -d\phi/dt$ (the rotation is left-handed in the direction of $\vec{B}$).

$$\omega = -\gamma B$$

If the external magnetic field is constant (B_0) then $\varphi = -\omega_0 t + \varphi_0$

where φ_0 is the initial angle, and since $\omega = -\frac{d\varphi}{dt} = -(-\omega_0)$

we may write the larmor equation:

$$\omega_0 = \gamma B_0$$