rotating frame Bloch equations

$$\frac{d\vec{M}}{dt} = -\gamma(\vec{B} \times \vec{M}) - \frac{(\vec{M} - \vec{M}_0)}{T}$$

transform to a frame (x', y', z') but $z' = z$
rotating at ω around the z axis. The time derivative transforms as:

$$\left(\frac{d\vec{M}}{dt}\right)_{lab} = \left(\frac{d\vec{M}}{dt}\right)_{rot} + \vec{\omega} \times \vec{M}$$

subbing into primary Bloch equation:

$$\left(\frac{d\vec{M}}{dt}\right)_{rot} = \gamma(\vec{M} \times \vec{B}) - (\vec{\omega} \times \vec{M}) - \text{relaxation terms}$$

$$\left(\frac{d\vec{M}}{dt}\right)_{rot} = \gamma \vec{M} \times \left(B + \frac{\omega}{\gamma}\right) - \text{relaxation terms}$$

define effective magnetic field : $\vec{B}_{eff} = (B_1, 0, B_0 - \frac{\omega}{\gamma})$

component equations in rotating frame:

x' (dispersion) : $\frac{dM_{x'}}{dt} = (\omega_0 - \omega) M_{y'} - \frac{M_{x'}}{T}$

y' (absorption) : $\frac{dM_{y'}}{dt} = -(\omega_0 - \omega) M_{x'} + \gamma B_1 M_z - \frac{M_{y'}}{T}$

z (longitudinal) : $\frac{dM_z}{dt} = -\gamma B_1 M_{y'} - \frac{M_z - M_0}{T}$

steady-state ratios (M/M_0)
all derivatives are zero. we can divide all equations by M_0.

$\Rightarrow$ ① $m_{x'} = (\omega_0 - \omega) T m_{y'}$

② $\frac{m_{y'}}{T} = \gamma B_1 m_z - (\omega_0 - \omega) m_{x'}$

③ $m_z = 1 - \gamma B_1 T m_{y'}$

subbing ① into ②:

$$\frac{M_{y'}}{T} = \gamma B_1 M_z - (\omega_0 - \omega)^2 T M_{y'}$$

$$M_{y'}\left(\frac{1 + (\omega_0 - \omega)^2 T^2}{T}\right) = \gamma B_1 M_z$$

sub ③ into this:

$$M_{y'}\left(\frac{1 + (\omega_0 - \omega)^2 T^2}{T}\right) = \gamma B_1 (1 - \gamma B_1 T M_{y'})$$

$$M_{y'}\left(\frac{1 + (\omega_0 - \omega)^2 T^2}{T} + \gamma^2 B_1^{\,2} T\right) = \gamma B_1$$

multiply by T and solve for $M_{y'}$:

$$\frac{M_{y'}}{M_0} = \frac{\gamma B_1 T}{1 + (\omega_0 - \omega)^2 T^2 + (\gamma B_1 1)^2}$$

$$\frac{M_{x'}}{M_0} = \frac{\gamma B_1 (\omega_0 - \omega) T^2}{1 + (\omega_0 - \omega)^2 T^2 + (\gamma B_1 T)^2}$$

$$\frac{M_z}{M_0} = \frac{1 + (\omega_0 - \omega)^2 T^2}{1 + (\omega_0 - \omega)^2 T^2 + (\gamma B_1 T)^2}$$