Linear Mixed-Effects Models

(aka Statistics III)

Bernd Figner

b.figner@psych.ru.nl

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Today: Digging Deeper

- · Questions: General, homework-related
- Some homework-related add-ons and leftovers from last class
- P values; multiple cores; post-hocs; writing up model and results; some plotting; linear and quadratic predictors; centering/scaling to reduce multicollinearity
- Advice from Barr et al and others
 - The R-sig-mixed-models debate
 - Testing parameters vs. effects: Bootstrap vs. LRT?
- Homework

Question 1 lab computers: R or RStudio?

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Question 2 YOUR data: who's willing to present?

March 24 ideal, but other dates possible

Homework

Questions? Problems? Comments?

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Some Comments

In the valuation ratings analysis: Item as a random intercept or not? Why (not)? More theoretical answer

- They are NOT random sample from all possible items
- They are carefully chosen and created by the experimenter ("repeatable")

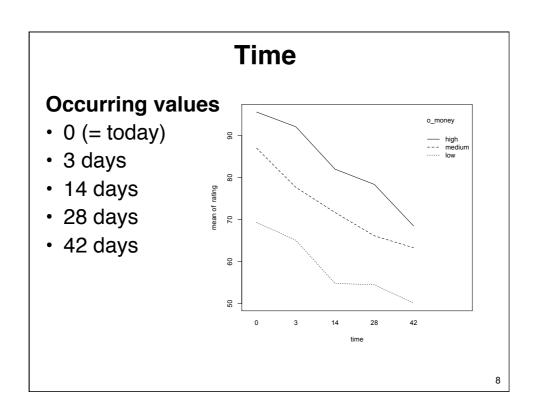
More pragmatic answer

- · We want to test the significance of money and time
- Adding the same predictor as fixed slope and random intercept is typically not a good idea

More Comments

Time and Money: What should they be?

- Continuous?
- Unordered factors?
- · Ordered factors?
- Linear?
- Linear and Quadratic?



As unordered Factor?

- Assumes that there is no inherent order among the levels
- → Seems odd, ignores a lot of information (Money: LOTS of levels, due to "jitter")

For most participants, there will be at least an ordinal relationship:

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As ordered factor?

Recodes the actual values as numeric levels

- today = 1
- 3 days = 2
- 14 days = 3
- 28 days = 4
- 42 days = 5
- → polynomial contrasts then test for linear, quadratic (cubic, ...) effects
- More fine-grained information (days): thrown away
- Ordered levels treated as continuous, assuming equal spacing between levels (0 vs. 3 = 3 vs. 14??)
- Good: captures linear and higher-order effects: quadratic effects will probably capture unequal spacing

As continuous predictor?

Keeps the fine grained information and unequal spacing between occurring values:

 E.g., difference between 0 and 3 days is smaller than difference between 3 days and 14 days

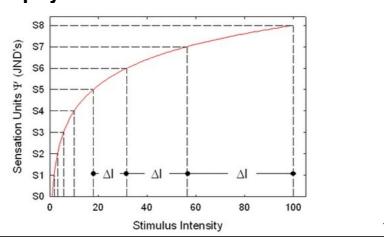
To capture non-linear effects

 You need to create a higher-order predictor yourself and add it to the model

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Non-Linearity

- Time: Sooner → more attractive
- More money → more attractive
- Psychophysics? Weber-Fechner



Non-Linearity

- Time: Sooner → more attractive
- More money → more attractive
- Psychophysics? Weber-Fechner
 - Effects might not be strictly linear
 - Time: e.g., 0 days vs. 14 days larger than 14 days vs. 28 days?
 - Money: €20 vs €50 larger than €50 vs €80?

How to capture such possible quadratic effects with continuous predictors?

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Linear and Quadratic IVs

For the Time IV (same for Money)

(1) Center the predictor (or scale)

v5\$c_time <- v5\$time - mean(v5\$time)

(2) Compute squared (quadratic) predictor

 $v5$q_time <- v5$c_time ^ 2$

Do NOT center again

(3) Include both in your model

(don't forget the random slopes for both the linear and quadratic term)

Centering (scaling) reduces collinearity

- Between linear and quadratic variants of the same continuous predictor
- · Between main effects and interactions

Time example

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Same with centered predictors

```
Correlation: .487
```

Still not perfect, but much better This is the standard approach

poly() creates uncorrelated linear, quadratic, cubic, ... terms

poly(v5\$time, 2) \rightarrow linear and quadratic terms

Best approach, as collinearity increases probability of non-convergence (and inflates standard errors, makes interpretation difficult, ...)

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- Model with linear and quadratic time and money generated with poly()
 - Gives warnings, but produces a model that looks reasonable
- Model with linear and quadratic time and money generated by squaring the centered predictors
 - Needs many more iterations to give reasonably looking model (~ 10,000,000 iterations)
 - More serious warnings even with that many iterations (but results quite similar to poly model)

Model with poly()

```
v_m1c_poly_lin_quad <- lmer(rating ~
poly_lin_Time + poly_quadr_Time +
poly_lin_Money + poly_quadr_Money +
(1 + poly_lin_Time + poly_quadr_Time +
poly_lin_Money + poly_quadr_Money | pp_code),
data = v5, control = lmerControl(optCtrl =
list(maxfun = 10000)))</pre>
```

Linear and quadratic predictors for time and money

- In fixed-effects part
- In random part (random slopes)

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summary(v_m1c_poly_lin_quad)

Fixed effects:

```
Estimate Std. Error t value
                  71.754
(Intercept)
                              2.013
                                      35.65
                -176.615
                                      -6.92
poly_lin_Time
                             25.538
poly_quadr_Time
                  40.375
                             10.945
                                       3.69
poly_lin_Money
                             25.261
                                       8.69
                 219.521
poly_quadr_Money -24.326
                             10.717
                                      -2.27
```

Quadratic term: opposite sign of linear term

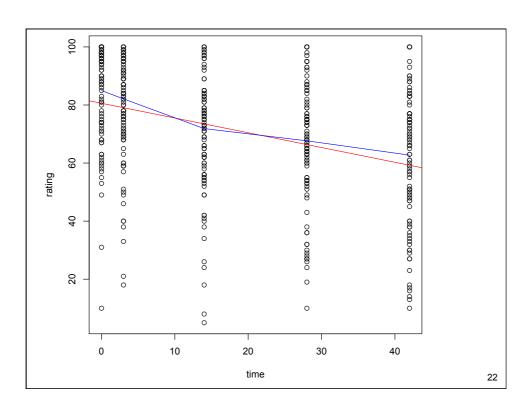
 Consistent with Weber-Fechner idea, but better check in plots

Idea: scatter plot with linear and smoothed (quadratic?) trend line

For time

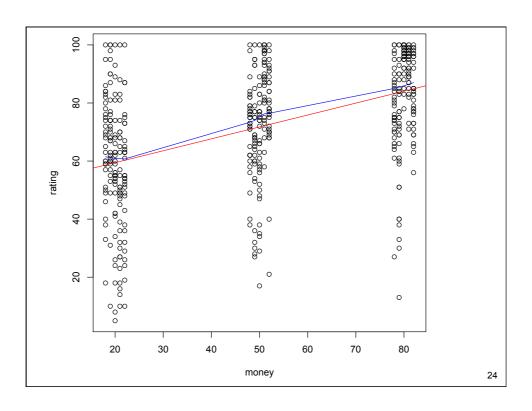
with(v5, plot(time, rating))
with(v5, abline(lm(rating ~ time), col = 'red'))
with(v5, lines(lowess(time, rating), col =
'blue'))

- scatter plot with time on x axis and DV on y axis
- · adds linear trend line in red
- · adds smoothed trend line in blue



Same for money

```
with(v5, plot(money, rating))
with(v5, abline(lm(rating ~ money), col = 'red'))
with(v5, lines(lowess(money, rating), col =
'blue'))
```



Significance?

Anova($v_m1c_poly_lin_quad$, type = 3, test = 'F')

```
F Df Df.res
                                          Pr(>F)
(Intercept)
               1270.9801 1
                               31
                                       < 2.2e-16 ***
poly_lin_Time
                               31 0.0000000934307 ***
                47.8298 1
                 13.6077 1
poly_quadr_Time
                               31
                                       0.0008602 ***
poly_lin_Money
                 75.5162 1
                               31 0.0000000008191 ***
poly_quadr_Money
                  5.1521 1
                               31
                                       0.0303224 *
```

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Recap and Leftovers from last class

Significance tests

- · Typically only for fixed effects of interest
- · Several different options
- · Tests of coefficents vs. test of effects
- Coefficients → like regression: coefficient significantly different from 0?
- Effects → like ANOVA: Is whole predictor significant?
 - factor with more than 2 levels
 - whole interaction term
- · Post-hoc tests

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Tests of Coefficients and "Effects"

- Example: 3 experimental conditions
 - neutral
 - positive mood induction
 - negative mood induction
- Effects: familiar from ANOVA framework
 - Is the whole factor significant?
 - → "Effect of mood induction"
 - Similarly: interaction between categorical factors: Is whole interaction significant?

- Coefficients: familiar from regression framework
 - Is the coefficient significantly different from 0?
 - Continuous predictors (→ 1 df)
 - Interactions between continuous predictors (→ 1 df)
 - Factors with 2 levels (→ 1df)
 - Factors with more levels (mood induction example)
 - Neutral condition vs average of positive and negative mood significant?
 - · Positive vs negative mood condition significant?
 - ..
 - → How you set up contrasts matters!

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Getting p values

→ Different methods (usually similar results)
(Douglas Bates: https://stat.ethz.ch/pipermail/r-help/2006-May/094765.html)

List ordered according to recommendations

http://glmm.wikidot.com/faq

Three "most recommended" ones

(1) (non)Parametric bootstrap

- coefficients: bootMer() → boot.ci()
- effects: PBmodcomp() (package pbkrtest)

(2) Conditional F-tests with df correction

- Anova(..., test="F") (package car)
- KRmodcomp() (package pbkrtest)

(3) Likelihood Ratio Tests

anova() or drop1()

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Conditional F test with df correction (Kenward-Roger)

library(car)

```
Anova(mymodel, type = 3, test = "F")
```

To get type 2 test: type = 2

Anova() calls KRmodcomp from pbkrtest

KRmodcomp(LargeModel, SmallModel)

SmallModel: same as LargeModel, but without the fixed effect of interest

Notes

- Typically fast; widely accepted
- NOT available for generalized mixed models

Valuation Rating Example

```
Anova(v_m1c, type = 3, test = 'F')
```

Analysis of Deviance Table (Type III Wald F tests with Kenward-Roger df)

Response: rating

```
F Df Df.res Pr(>F) (Intercept) 1270.924 1 31 < 2.2e-16 *** s_time 47.587 1 31 0.0000000980867 *** s_money 75.760 1 31 0.0000000007902 ***
```

VERY detailed example R script

BlackBoard → Course Documents → Week 4 → Homework Week 4 Example Solution

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Bootstrap methods: bootMer()

library(lme4)

- (1) Run your Imer model (mymodel)
- (2) Define a function (only once per R session)

```
FUN_bootMer <- function(fit) {
    return(fixef(fit))
}</pre>
```

(3) Run bootstrap (can take a while!)

```
boot_mymodel <- bootMer(mymodel, FUN_bootMer,
nsim = 1000, type = "parametric", .progress =
"txt", PBargs = list(style = 3))</pre>
```

- → Look up ?bootMer
- → Possible to use several cores → much faster!

Bootstrapping can take a long time!

- → Try it first out doing only a few iterations
- → use Sys.time() as a stop watch

```
t1 <- Sys.time()
boot_v_m1c <- bootMer(v_m1c, FUN_bootMer, nsim
= 3, type = "parametric")
t2 <- Sys.time()
t2 - t1</pre>
```

- → Tells you how long the 3 iterations took
- → Gives you a feel how long 1000 will take...
- → Then run the 1000 iterations (e.g., before going to bed)

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Progress bar

```
boot_mymodel <- bootMer(mymodel, FUN_bootMer,
nsim = 1000, type = "parametric", .progress =
"txt", PBargs = list(style = 3))</pre>
```

- Shows a simple progress bar plus percentage how many of the simulations R has already finished
- Not possible if you use more than 1 core

Multiple Cores with bootMer()

- detectCores() from library(pbkrtest) → tells you the number of CPUs (cores) of your computer
- bootMer() can use more than 1 → MUCH faster often!

NOTES

- Don't use ALL your cores for R!!!
- Leave at least 1 for other tasks

```
boot_mymodel <- bootMer(mymodel, FUN_bootMer,
nsim = 1000, parallel = "multicore", ncpus =
3)</pre>
```

boot_mymodel <- bootMer(mymodel, FUN_bootMer,
nsim = 1000, parallel = "snow", ncpus = 3)</pre>

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Get Confidence Intervals (for *p* **values)**

Have a look at boot_mymodel

head(as.data.frame(boot_mymodel))

Get CIs

Intercept

```
boot.ci(boot_mymodel, index = 1, conf =
0.95, type=c("norm", "basic", "perc"))
```

99% CI

```
boot.ci(boot_mymodel, index = 1, conf =
0.99, type=c("norm", "basic", "perc")
```

For second column (= first coefficient after intercept)

```
boot.ci(boot_mymodel, index = 2, conf =
0.95, type=c("norm", "basic", "perc"))
```

etc for 3rd column, 4th column,

When a CI does NOT include $0 \rightarrow$ significant!

- 95% CI does NOT include $0 \rightarrow p < .05$
- 99% CI does NOT include 0 → p < .01
- 95% CI DOES include $0 \rightarrow ns$, p > .05
- 90% CI DOES include $0 \rightarrow ns$, p > .10

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Bootstrap method to test EFFECTS

 - PBmodcomp(LargeModel, SmallModel) from pbkrtest → test of effects

Same idea as KRmodcomp

- · First, fit model with and without effect of interest
- Then, compare the fit of the 2 models via PBmodcomp()
 PBmodcomp(LargeModel, SmallModel)

Notes

- Can take quite some time

Multiple Cores with PBmodcomp()

n_cores <- detectCores()</pre>

NOTE: Don't use all cores; leave at least 1 (i.e., n_cores - 1)

Create the clusters

```
clusters <- makeCluster(rep("localhost",
n_cores - 1))</pre>
```

Run your model comparison

PB_1 <- PBmodcomp(mylarge_m, mysmall_m, nsim = 1000, cl = clusters)

Look at the result

PB 1

Stop the cluster

StopCluster(clusters)

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LRTs

Again model comparison approach

- · First, fit model with and without effect of interest
- Then, compare the fit of the 2 models via anova()

anova(SmallModel, LargeModel)

Notes

- Models need to be nested!!
- LRTs often NOT recommended (for smaller data sets)
- BUT: Barr et al. (2013) → worked very well!
- Recent discussion on R-sig-mixed-models:
- LRTs does not rely on a precise estimate of the coefficients, thus may be better than other methods when model is "overparametrized"

- LRTs: models must be fit with ML, not REML
- → Fit Imer models with REML = FALSE

BUT: R likes you!

- → R refits the models in the anova() command using ML
- → Then does model comparison with the ML models

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drop1()

→ If you need to compare many models

→ Does all the relevant anova() model comparisons (i.e., LRTs) for you.

Post-hoc tests

- If you have a factor with more than 2 levels
- Age Group
 - Children
 - Adolescents
 - Adults
- You know how to test whether Age Group as a whole is significant
- How to do pairwise post-hoc comparisons?
 - Children vs. Adolescents
 - Children vs. Adults
 - Adolescents vs. Adults

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```
myAgeModel <- lmer(DV \sim f_Agegroup + IV + (1 + IV | f_pp), data = mydata)
```

Simple way to do post-hoc tests for f_Agegroup

```
library(lsmeans)
posthoc_Agegroup <- lsmeans(myAgeModel,
list(pairwise ~ f_Agegroup))
posthoc_Agegroup</pre>
```

Same, but glht() from multcomp

```
library(multcomp)
posthoc_Agegroup <- glht(myAgeModel, linfct =
mcp(f_Agegroup = "Tukey"))
summary(posthoc_Agegroup)</pre>
```

An output example (from one of my studies)

```
$`f_Agegroup lsmeans`
f_Agegroup lsmean SE df lower.CL upper.CL
    child 59.42722 1.611361 193.7102 56.24916 62.60529
adolescent 65.61498 1.413526 198.4173 62.82752 68.40245
    adult 66.37734 1.613881 194.8105 63.19442 69.56027
```

\$`f_Agegroup pairwise differences`

```
estimate SE df t.ratio p.value child - adolescent -6.1877614 2.143488 195.7366 -2.88677 0.01200 child - adult -6.9501220 2.280591 194.2601 -3.04751 0.00738 adolescent - adult -0.7623606 2.145383 196.3642 -0.35535 0.93279 p values are adjusted using the tukey method for 3 means
```

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Post-hocs for factor interactions

$$mymodel \leftarrow lmer(DV \sim f_1 * f_2 +)$$

(1) Create one single new factor that combines the two predictors IV1 and IV2

- (2) Run the your model again, but use the newly created factor instead of $f_1 * f_2$ mymodel_2 <- lmer(DV ~ int_f_12 +)
- (3) Use lsmeans() or glht() to do all pairwise post-hoc comparisons

```
lsmeans(mymodel_2, list(pairwise ~
int_f_12))
```

```
glht(mymodel_2, linfct = mcp(int_f_12 =
"Tukey"))
```

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How to write up a model and the results

Mixed-models are relatively new methods (e.g., "mixed-model" vs "multilevel")

- No established standards how to write it up (no APA guidelines for mixed-models)
- Some examples: BlackBoard → Course Documents → Examples of papers using Ime4
- Example how I tend to write up my results (made-up)
 → It's also on BlackBoard

NOTE: cite R and and packages you use!
citation() → Citation info for R
citation("lme4") → Citation info for Ime4
citation("car") → Citation info for car

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Model Setup

The valuation rating data were analyzed with a linear mixed-effects model approach, using the lmer function of the lme4 package (version 1.1.-4; Bates, Maechler, Bolker, & Walker, 2014) in R (R Core Team, 2013). The model included a fixed intercept and each a fixed slope for the continuous predictors Time and Money. These continuous predictors were scaled before we entered them into the model (i.e., they had a mean of 0 and a standard deviation of 1).

cont.

We followed Barr, Levy, Scheepers, and Tily's (2013) advice to use a maximal random-effects structure: The repeated-measures nature of the data was accordingly modeled by including a perparticipant random adjustment to the fixed intercept ("random intercept"), as well as per-participant random adjustments to the Time and Money slopes ("random slopes"); in addition, we included all possible random correlation terms among the random effects.

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P Values

P values were determined using conditional F tests with Kenward-Roger correction of degrees-of-freedom, as implemented in the Anova function (with Type III F tests) from the package car (version 2.0.19; Fox & Sanford, 2011; this function calls the KRmodcomp function from the package pbkrtest: Halekoh & Højsgaard, 2013).

OR: *P* values were determined using Likelihood Ratio Tests as implemented in the anova function of the R base package.

OR: *P* values were determined using parametric bootstrapping as implemented in lme4's bootMer function, with 1000 simulations and deriving confidence intervals using the function boot.ci of the package boot (version 3.1.9.; Canty & Ripley, 2013; Davison & Hinkley, 1997.

OR: ... whatever you used!

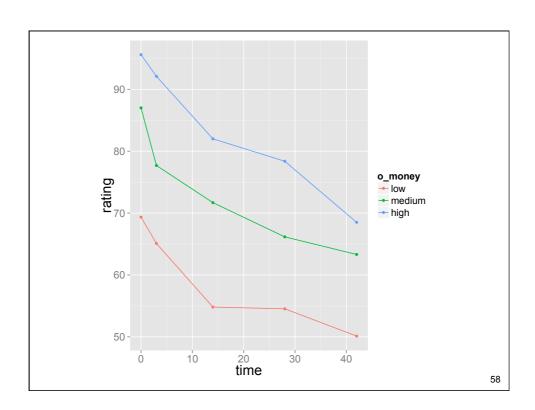
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Results

As expected, we found significant effects of Time (coef = -7.965; p < .001) and Money (coef = 10.038; p < .001): As time-of-delivery increased, the valuation ratings decreased and as Money increased, the valuation ratings increased. Figure 1 shows the effects of Time and Money on the valuation ratings. ETC ETC...

Figures: make them pretty...

```
ggplot(data = v5, aes(x = time, y = rating,
colour = o_money, group = o_money)) +
stat_summary(fun.y = mean, geom = "point") +
stat_summary(fun.y = mean, geom = "line") +
theme(axis.text.x = element_text(size = 15),
axis.title.x = element_text(size = 20),
axis.text.y = element_text(size = 15),
axis.title.y = element_text(size = 20),
legend.title = element_text(size = 14),
legend.text = element_text(size = 14))
```



Make them prettier...

- While this is better than my usual ugly ones, there's room for further improvement:
- Categories on the x axis
- · Title of the legend (o_money) and its position
- •

Error bars?

- Long and complicated story (mixed designs!)
- · I don't add error bars usually
- Recent advice → if you want to show uncertainty, plot the coefficients and their error bars

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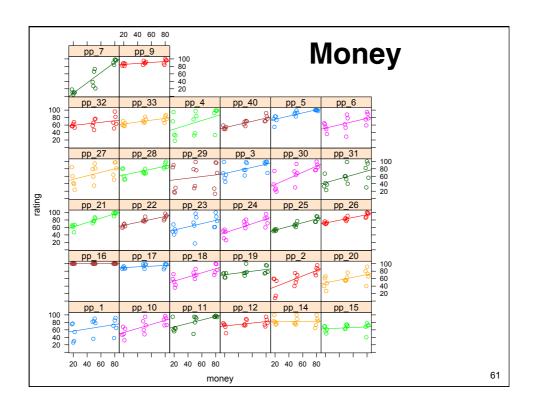
xyplot() for individual participant patterns

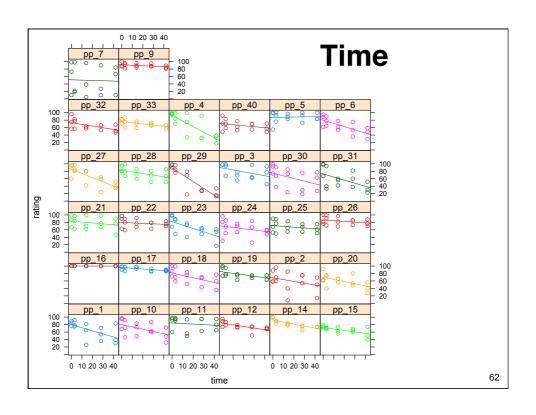
For Money

```
xyplot(rating ~ money | pp_code, groups
= pp_code, data = v5, type = c('p',
'r'))
```

For Time

```
xyplot(rating ~ time | pp_code, groups =
pp_code, data = v5, type = c('p', 'r'))
```



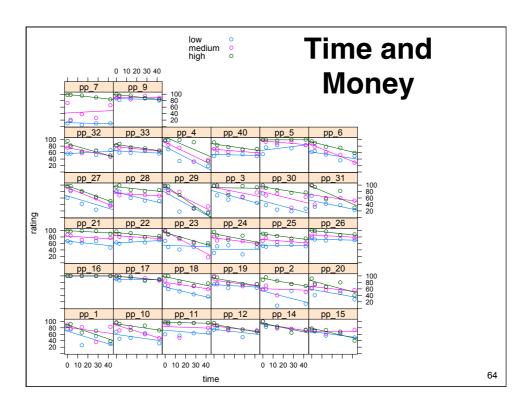


Time and Money

Time on x axis; o_money as separate lines

```
xyplot(rating ~ time | pp_code, groups =
o_money, data = v5, type = c('p', 'r'),
auto.key = TRUE)
```

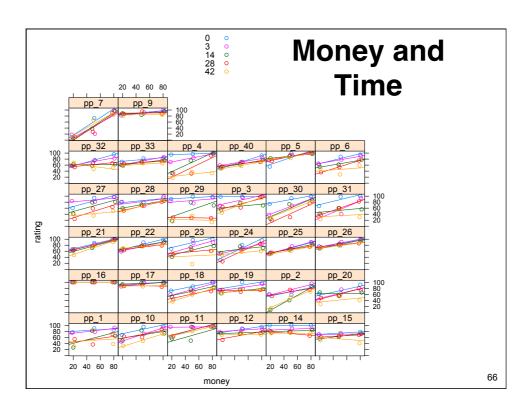
auto.key = TRUE → adds figure legend



Money and Time

Money on x axis; Time as separate lines

xyplot(rating ~ money | pp_code, groups
= time, data = v5, type = c('p', 'r'),
auto.key = TRUE)



Individuals differ a lot in effects of Time and Money

It would be nice to get the numbers how the participants differ from each other (i.e., the random intercepts and random slopes)

fixef()

→fixed effect estimates of your model

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ranef()

> ranef(v_m1c)

→ per-participant adjustments to fixed effects

coef()

→ random and fixed effects together i.e., best estimate for each pp's coefficients

```
> coef(v_m1c)
$pp_code
      (Intercept)
                       s_time
                                 s_money
pp_1
         66.13815 -12.74340079 8.3864244
         68.82252 -9.04918823 12.3934909
pp_10
pp_11
        80.86839 -2.43812376 10.2738132
pp_12
        76.52676 -7.26991482 5.9872177
        82.65129 -8.37780546 1.4289180
pp_14
pp_15
         67.73749 -8.60869805 5.7816330
```

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If you have more than 1 grouping variable

```
coef(mymodel)$pp_code
coef(mymodel)$f_item
```

To save the coefs in a csv file

```
v_m1c_coefs <- coef(v_m1c)$pp_code
write.csv(v_m1c_coefs, row.names = TRUE, file
= 'v_m1c_coefs.csv')
row.names = TRUE → pp code is included as 1st column</pre>
```

Why useful to save them?

- Descriptives, plots, ...
- Use in some other analyses (e.g., fMRI)

	Α	В	С	D	
1		(Intercept)		s_money	
2	pp_1	66.1381472572	-12.7434007914	8.3864244114	
3	pp_10	68.82252237	-9.0491882322	12.3934908987	
4	pp_11	80.8683855158	-2.438123758	10.2738132174	
5	pp_12	76.5267557415	-7.2699148168	5.9872177209	
6	pp_14	82.6512887612	-8.3778054582	1.4289179751	
7	pp_15	67.7374862816	-8.6086980507	5.7816329675	
8	pp_16	98.4146427397	0.0584053472	0.1251181152	
9	pp_17	91.2446027856	-2.8344483949	2.3280414432	
10	pp_18	69.4885297045	-9.0795906928	12.4211233642	
11	pp_19	77.5661384166	-6.4604392642	6.533258789	
12	pp_2	60.646298243	-8.2973887227	17.0735410335	
13	pp_20	61.60410219	-10.4891250255	10.9044332116	
14	pp_21	77.8488171908	-3.3717236554	12.5630517301	
15	pp_22	77.598557432	-2.9070871406	9.754576843	
16	pp_23	65.8868752597	-13.1229341827	10.3932670512	
17	pp_24	64.5482595516	-6.5455323517	14.940972429	
18	pp 25	68.5583559368	-4.8510239655	12.7445090569	
19	pp_26	82.2097265985	-3.1033641951	8.3688932814	
20	pp_27	65.1912963837	-15.4904477586	10.5387662904	
21	pp_28	74.7331485863	-6.7622635605	9.4681826219	
22	nn 20	57 N7762275	_2E 200063647E	£ 2157022028	

Today: Digging Deeper

- · Questions: General, homework-related
- · Some homework-related add-ons
- Leftovers from last class: p values; multiple cores
- Post-hocs; writing up model and results; some plotting; linear and quadratic predictors; centering/ scaling to reduce multicollinearity
- Advice from Barr et al and others
 - The R-sig-mixed-models debate
 - Testing parameters vs. effects: Bootstrap vs. LRT?
- Homework

R-sig-mixed models

Who has read these emails recently?

If not, do it as homework (available online as archives)
 → debate started on Feb 28, 2014

Short summary

- E. Ellsiepen: Followed Barr et al advice: maximal randomeffects structure + LRTs for p values; odd results
- D. Bates: model is overparametrized (too complex for data);
 "Barr et al advice is dangerous"
- R. Levy (from Barr et al): perhaps no overparametrization; Bates misunderstood advice in paper; LRTs are fine even when overparametrized (testing significance of coefficient more problematic!)
- · D. Bates: I should have re-read the paper

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- R. Levy: More detailed advice about strategies; suggestions: try different optimizer; use LRTs, not tests that rely on precise estimate of coefficient
- · Maechler, Bolker, ... thread is still alive

Important thoughts

If model overparametrized/with convergence problems:

- Try different optimizer(s): glmer: nlminb from package optimx optimizer="optimx", optCtrl=list(method="nlminb")
- Use LRTs for p values → For model comparison approach, not important whether the really-really best value for a coefficient is found!
- → Different coefficient values give virtually equally good fit
- → Actual value of coeff irrelevant for comparison of models with/ without the predictor → more trustworthy p values!