Linear Mixed-Effects Models

(aka Statistics III)

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Week 1: February 3, 2014

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Today: Introductions...

- **Who's your teacher?**
- **How does this course work?**
- **What are linear mixed-effects models?**
  - For what kind of data sets are they useful?
  - Pros and Cons
- **Installing the necessary R packages**
- **Homework**
- **Today's lab session**
  - Generate some data to be used in the course
  - Testing the lab computer situation

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Your teacher

- **M.A. and PhD in Zurich**
  - Cognitive and developmental psychology
  - Risky decision making in children, adolescents, and adults; role of affective vs. deliberative processes
- **5 years at Columbia University (Center for the Decision Sciences)**
  - Risky and intertemporal decisions
  - Decision Neuroscience/Neuroeconomics
- **2 years at UvA (Dev. Psych)**
- **Since Aug. 2012: Radboud; Clin. Psych. (EPAN)**

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Your teacher

- M.A. and PhD in Zurich **BEFORE**
  - Cognitive and developmental psychology
  - Risky decision making in children, adolescents, and adults; role of affective vs. deliberative processes
- 5 years at Columbia University (Center for the Decision Sciences) **AFTER**
  - Risky and intertemporal decisions
  - Decision Neuroscience/Neuroeconomics
- 2 years at University of Amsterdam (Dev. Psych)
- Since Aug. 2012: Radboud; Clinical Psych.

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My data: repeated measures

- Repeated-measures ANOVAs in SPSS
  - ANOVA influenced my research designs
- Binary choices → logistic models
- Ughh, repeated-measures binary choices??!?
- SPSS → crashed constantly
- (Generalized) Linear Mixed-Models in R
- Study eventually published → strong positive reinforcer...

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Now

- R/RStudio for all my statistical needs
- Great also for data handling, preprocessing, etc
- Even teaching this stuff to poor students...
- Mixed models: extremely flexible

→New research questions, task designs, etc
→Research questions determine study design

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Organizational Things

- **Lecture** (13:45-15:30; SP A 01.14)
  - **Ask questions, give feedback!**
- **Exercises** (15:45-17:30; SP A -1.55.A+B)
  - Lab computers or BYOL (bring your own laptop)
  - Work on assignments; Bill and I there for help
  - Today: data collection + testing lab computers

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BlackBoard

- **Syllabus**
  - Content description/topics
- **Things continuously updated and added**
  - Lecture slides
  - Homework instructions
  - R script examples
  - List with literature and other resources (papers, web pages, email lists, ...)
- **Discussion Board: 2 threads**
  - Administrative things (questions re course organization etc)
  - **Content-related things**
    → POST your mixed-model/R questions here
    → Answer each others' questions!! („incentive-compatible")

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Grades

- **Homework assignments: 20%**
  - Analyses: R scripts, figures, write-ups
  - Reading
- **In-class exam (April 7): 40%**
  - Multiple choice
- **Take-home exam: 40%**
  - Complete data analysis and write-up
  - Time preferences?
- **BlackBoard participation**
  - Asking and answering questions
  - For rounding of grades

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Please note

You can take a repeat exam („resit“) only if

- you failed the first attempt or
- you were sick and thus couldn‘t do the first attempt (medical attest)

Applies to take-home exam and in-class exam

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Course Objectives

Upon successful completion of this course you will be able to...

... use linear mixed-effects models in R

- recognize when they are appropriate
- do the analysis (including preparatory steps and model diagnostics)
- interpret and report the results (text and figures)
- describe the theoretical underpinnings

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Prerequisites

- Familiar with R and/or RStudio
- Multivariate statistics, in particular linear and logistic regression `lm()`, `glm()`

→ Content of Statistics I and II

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Course Content

- Theoretical background
- Hands-on „how to“ in R/RStudio

→**Syllabus on BlackBoard**

  – List of topics that are planned to be covered

  – Several changes compared to last year

  – Let‘s have a look...

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Planned Topics

- What are linear mixed-effects models and when are they useful?
- Pros and cons of mixed-effects models
- Many names for the same/similar models
- "Clustered errors," slopes and intercepts
- Fixed and random effects
- Gaussian and generalized (binary, Poisson) linear mixed-effects models

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- One, two, three, and more levels
- **How to analyze different types of data sets**
  - repeated-measures data
  - longitudinal data
  - nested/hierarchical data
  - questionnaire data; ...

→ Your Data!

- Centering: grand-mean; participant-wise; group-wise
- Crossed/orthogonal random effects
- Within-level and cross-level interactions
- Contrast coding

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- Significance testing of "coefficients" and "effects"
  - Many ways to get *p* values
- How to build my model: theory-driven and data-driven approaches
- Non-convergence: what now?
- Speeding up computations: using multiple cores
- Report models and their results in text and figure

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- **The multilevel perspective:** ICCs, model-building, reporting, etc
- Indicators of goodness-of-fit: approaches to compute R^2 and other indicators
- Advanced R programming techniques

Time permitting

- Mediation in a mixed-effects framework
- Mixed-effects models with other packages (e.g., nlme, afex, MCMCglmm, etc)
- Mixed-effects models in SPSS
- Power analysis

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Many different names...

These are all (nearly) the same model

- Linear Mixed-Effects Models (LME)
- Linear Mixed Models (LMM)
- Multi-Level Models (MLM)
- Hierarchical Linear Models (HLM)
- Fixed and Random Effects Models
- Random Effects Models
- Random Coefficient Models
- Random Coefficient Regression Models
- ...

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To put it simply...

Mixed-model analysis is like ...

... linear regression for „grouped“ data

- **„Grouped“ data?!?**
  - aka „clustered“ or „grouped“ errors
  - observations (= data points in the DV) are not independent
- **Typical sources**
  - repeated-measures data
  - longitudinal data
  - „nested“ aka „hierarchical“ data

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Repeated Measures

- Each participant contributes more than 1 data point to the dependent variable (DV)
  - from 2 to hundreds or thousands
  - e.g., tasks like Stroop, Go/No-Go, AAT, ...
- Observations within a participant resemble each other → not independent
- „Grouping factor:“ participant
- Traditional approaches to deal with such data
  - repeated-measures ANOVA
  - averaging (creating „scores“)

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Longitudinal Data

- Also repeated measures
- Each participant is „observed“ repeatedly over time
- „Grouping factor:“ participant
- Time itself is a variable of interest
- E.g., how fast is reduction in depressive symptoms, as a function of
  - Intervention: psychotherapy vs. medication
  - Symptom severity at onset
- aka „Growth curve modeling“

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„Nested“ aka „Hierarchical“ Data

- Not repeated measures
- Specific data structures
- For example: School achievement of students
- Students are „nested“ in classes (same teacher: e.g., teacher motivation)
- Classes are nested in schools (e.g., teaching philosophy: Montessori vs. traditional)
- Schools are nested in school districts (e.g., rich/poor)

→ Several levels of „nesting“ → *multilevel* models

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Example: Repeated Measures 1

A Soccer Experiment: Influence of

- Goalkeeper posture (slumped/self-confident)
- On shooters‘ performance (goal vs. no goal)

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Experimental Design

IV

- goalkeeper „slumped“ or „self-confident“

10 participants (=shooters): each 10 shots

- 5x „slumped“ goalkeeper
- 5x „self-confident“ goalkeeper
- random order

DV

- goal vs. no-goal

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DV Characteristics

- *N* = 100 (10 participants x 10 shots)
- Binary data (goal, no-goal)
- Not independent (some participants are better shooters than others, etc)
- Observations are „grouped“ by participant

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Data Analysis?

Pre-mixed models („traditional“)

(1) Repeated-measures ANOVA?

- Shots 1-10 as repeated measures (1=goal; 0=no-goal)
- Goalie posture as within-subject factor

Problem

ANOVA needs continuous, not binary DV!

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(2) Aggregation?

- Sum up number of goals per participant per condition
- Paired samples t test

Problems

- **Scores (0-5 goals) not truly continuous**
- **What if we want to take into account additional factors, for example:**
  - Order effects (participants might get better with each shot)
  - Effect of previous outcome: Previous goal might improve performance

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(3) Logistic regression?

- Each observation as a function of
  - Condition (goalkeeper slumped/self-confident)
  - Shot number (1-10)
  - Outcome of previous trial (goal/no goal)
  - ...

Problems

- Treats observations as independent
- Estimated standard errors are too small
- Inflated Type I Error

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(4) Two-step approach?

(Step 1) Separate log regression per participant

- Condition (goalkeeper slumped/self-confident)
- Shot number (1-10)
- Outcome of previous trial, etc
- For each participant: Coefficients (and standard error)
  - Condition effect
  - Shot-number effect, etc

(Step 2) Do the coefficients differ from 0? (t test, ...)

- Condition effect significant?
- Shot-number effect significant? etc

→ Common approach in fMRI analysis

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Problems

- Participants‘ coefficients normally distributed?
- Noise-sensitive/low power
  - Each participant is equally weighted
  - Many 1st-level analyses, each with a small *n*
  - Many parameters being estimated

Good idea, but worthy of improvement

- Untrustworthy coefficients (large SE) → less weight
- Fewer estimated parameters
- Use information of full data set

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Example: Repeated Measures 2

Multi-Trial Response Time Tasks

Stroop: Name the color (don‘t read the word)

- 24 congruent trials: **blue**, **red**, **yellow**, **green**
- 24 incongruent: **blue**, **red**, **yellow**, **green**

- similar: IAT, AAT, psycholinguistic lexicographic tasks, ...

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Hypotheses

- **H1: Congruency**
  - Longer response times in incongruent than congruent trials
- **H2: Gender differences**
  - Main effect: Females are faster than males
  - Interaction: Congruency effect smaller in females than males

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„Traditional:“ Aggregation

Procedure

- Throw out trials with incorrect responses
- Compute average RTs for each participant
  - 24 congruent trials
  - 24 incongruent trials

rm ANOVA

DV: Average RT (2 data points per participant)

- Congruency: within-subject
- Gender: between-subject
- Interaction Congruency x Gender

→ Congruency and Gender could also be tested with t tests

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So what's wrong with that??!

- **Nothing or, well, not necessarily *that* much**
- **But…**

It's a shame to throw away so much information!

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Limitations of Aggregation

(ANOVA/Regression)

- **Retain only the mean of the DV**
  - Throwing away the variance and other parameters
- **Weight each participant equally**
  - No weighting of participants with more/less data, more/less variable or noisy data

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How "trustworthy" is an estimate?

RTs (in msec)

- Participant A: 500, 512, 490, 499, 504, ...
- Participant B: 130, 890, 500, 755, 230, ...
- Both have the same mean RT (501)

Whose participant's mean RT do you "trust" more?

- **Aggregation** (mean RTs): equal weight to both, information about variability is lost
- **Wouldn't it be nice? More weight to...**
  - More trustworthy (=less variable) data
  - Participants with more observations

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Limitations of Aggregation

(ANOVA/Regression)

- **Cannot have *predictor variables* at trial level**
  - Cannot explain variance in the dependent variable *before averaging*
  - E.g.: How does making an error affect the following trial? Does that differ between genders?
  - Compelled to convert continuous trial-level predictors into categorical ones (e.g., pps becoming faster during task)
  - Cannot model between-participant variation in within-participant slopes (e.g., some pps get faster, others slower)
  - Cannot model per-participant and per-item variation simultaneously

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What about rm ANOVA?

Disadvantages (compared to mixed models)

- **Problems with unbalanced designs**
  - Requires factorial design and/or same number of observations in the relevant cells
- **Problems with missing data**
  - Participants with missing data are often removed from analysis
- **Works only with categorial predictors**
  - There‘s ANCOVA, but it‘s not very flexible
- **Requires equal variances in all groups**
  - Cannot handle heteroscedasticity
- **Cannot handle non-normal dependent variable**
  - No „generalized“ ANOVA (e.g., binary DV or counts)

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The Bottom Line

Aggregation and/or rm ANOVA

- Necessary „crutches“ in a time when we were missing:
  - the appropriate statistical models
  - fast computers
- **These days, we have both**

→ Time to throw away the crutches!

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Benefits of Mixed Models

- No need to aggregate; no loss of information
- Developed to handle
  - unbalanced designs
  - heteroscedasticity
  - missing data (up to a point)
- Regression framework: Continuous and categorical predictors
- Analyze data at different "levels" (→ multi-level)
- Handle "multiple nesting"
  (students in classes in schools in school districts...)

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Benefits cont.

- Variance in DV explained using predictors at *each level of analysis*
- Variability in predictor-DV relationships at trial-level explained by participant-level predictors
  - E.g., some participants show a stronger congruency effect, some show a weaker congruency effect
  - These differences can be explained by participant characteristics, e.g., gender, IQ, color blindness, ...
- More weight given to more "trustworthy" participants
  - more data
  - more reliable data (smaller SEs; less variability; ...)

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Benefits cont.

Typically more power to detect participant-level effects

(compared to ANOVA/Regression/Aggregation)

- More precise measure of the DV after explaining variance at lower-levels
- Larger degrees of freedom for predictor variables that vary across trials
  → simply: more data points in the analysis
- Weighting participants in inverse proportion to their error variance

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To Summarize

Mixed-models: able to model more of the task-relevant processes

→ less unexplained variance in the data (less "noise")

More flexibility and freedom in data analysis

→ Effects that are otherwise difficult to investigate

- Effect of incorrect trial on following trial (slow down)?
- How does that change over the duration of the task?
- How does that differ between genders?
- ...

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Downsides?

Increased flexibility comes at a price... Complexity

- No formula-based solution (unlike regression, ANOVA, t tests, ...)
- Instead: Numeric estimating procedure that iterates until convergence
  - Takes time (sometimes a lot)
  - Sometimes no convergence → need to simplify model
  - Mathematical details: not easy to understand

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- Relatively new methods; still under development
- Less agreement, more opinionated, even among experts
  - Different "traditions" or "schools of though" → mixed-models versus multilevel models
- What is seen as good practice today may be discouraged in a few months (rare, but happens)
- No ideally suited handbook for this course
  - e.g., FMF chapter: somewhat outdated; only multilevel perspective
  - Instead: Scientific articles (most recent, but more difficult)

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Worth Paying the Price?

Hell yeah, absolutely!

- Mathematical background: We're behavioral scientists, not statisticians...
- If I was able to learn it, you are, too!
- Mixed-models: Growing in popularity A LOT
- Soon no way around them anyway → t tests, ANOVAs, etc soon not defensible anymore
- Basic idea is very simple: linear regression with "grouped" data (e.g., repeated measures)

Expanded horizon when designing your studies!

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Goals of the Course

- Demonstrate how to formulate mixed-model hypotheses
- Demonstrate how to analyze different types of data with mixed models
- Demonstrate how to report the results
- Expand the scope of possible study designs thanks to mixed-model framework

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OK, after all this PR...

Next week

- Some formulas and math

Today

- A tiny bit of hands-on

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Installing R/RStudio

You need the most recent R version!

→ `3.0.2 "Frisbee Sailing"` **(or higher)**

If you have an older version:

→ Download and install newest version

→ Amsterdam mirror might have only 3.0.1.; Utrecht had 3.0.2

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Installing Packages

Standard way

```
install.packages("MyPackage")
```

→**Installs packages from the main repository**

→**Perfectly fine for most purposes**

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Installing Packages

For mixed-models

- Most recent “development” versions
- Not yet available via the default repository
- Slightly different commands necessary
- Best to install the following packages in the following order

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Matrix

```
install.packages('Matrix', repos='http://
cran.us.r-project.org')
```

- Needed for mixed-model computations
- Once installed...
  - loading lme4 automatically loads Matrix

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pbkrtest

```
install.packages('pbkrtest',
repos='http://cran.us.r-project.org')
```

- One way to do significance tests
- Used a bit later in this course

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lme4

```
install.packages('lme4', repos =
c("http://lme4.r-forge.r-project.org/
repos", getOption("repos")))
```

- Our main mixed-model package
- It will install several other required things

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Checking Versions

```
packageVersion('Matrix')
[1] '1.1.1.1' (or higher)

packageVersion('pbkrtest')
[1] '0.3.8' (or higher)

packageVersion('lme4')
[1] '1.1.2' (or higher)
```

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Loading lme4

```
library(lme4)
Loading required package: Matrix
```

→ Matrix is automatically loaded

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Today's Lab Session

(A) Data collection

- Generate data sets we'll use in the course

(B) Test lab computer solutions

(1) IT people generated a solution (portable **R**, not RStudio)
(2) ‚Plan B' solution (RStudio web server, similar to Stats II)

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(A) Data Collection

- Generate data sets we'll use in the course
- 3 decision-making tasks
  - Risky choice („Columbia Card Task"): Version 1, Version 2
  - Intertemporal choice: DITC
- **Web-based:**
  - **Risky choice tasks** (English): https://vlab.decisionsciences.columbia.edu/collaboration/cct_hosted/admin/index.php
  - **Intertemporal choice tasks** (Dutch): https://vlab.decisionsciences.columbia.edu/collaboration/AOD_hosted_2/main_login.php
  - This document and links also on BlackBoard: Course Documents → Week 1

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- Each link takes you to a login page asking for username and password
- These are the same for both pages
  - **Username: mixmod2014**
  - **Password: stats3**

- After the login you get to a page where:
- You can choose the task version
- You enter a "Serial/Subject ID" (participant code)

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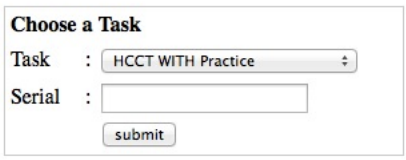
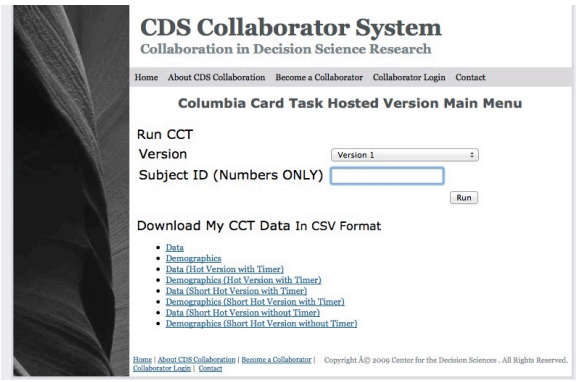
For the intertemporal choices (DITC)

ONLY NUMBERS CAN BE USED FOR THE SERIAL (NO LETTERS, NO SPACES, ETC.)



For the risky choices



Serial/Subject ID is your participant code

- We create this as follows (let's assume, you are 23)
- Add 100 → 123
- Add a last digit
  - 1 for the 1st task you do → 1231
  - 2 for the 2nd task you do → 1232
  - 3 for the 3rd task you do → 1233

We'll have 2 different task orders (between-subject)

- Students 1-20
- Students 21 and higher

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Students 1-20 (let's assume you are 9)

- 1st task: Risky choice, Version 1 (pp-code: 1091)
- 2nd task: Intertemp. choice, DITC (pp-code: 1092)
- 3rd task: Risky choice, Version 2 (pp-code: 1093)

Students 21 and (let's assume you are 30)

- 1st task: Risky choice, Version 2 (pp-code: 1301)
- 2nd task: Intertemp. choice, DITC (pp-code: 1302)
- 3rd task: Risky choice, Version 1 (pp-code: 1303)

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Important Notes

- **Please write down YOUR order of tasks and the respective participant codes NOW**
- Take breaks between, not during tasks
- Although it says we will pay you, we won't (sorry!)
- Please still try to imagine it's for real money
- At the end of the intertemporal choice tasks, it says you should NOT press the button „verder"
- → Ignore that, go ahead and press that button, it will get you back to the login window
- If you have problems, you can try to „clear browsing data" (i.e., empty cache) and login again

So: It's risky → intertemporal → risky choice; never do the same type of task twice in a row

Risky https://vlab.decisionsciences.columbia.edu/collaboration/cct_hosted/admin/index.php

Intertemporal https://vlab.decisionsciences.columbia.edu/collaboration/AOD_hosted_2/main_login.php

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(B) Testing Lab Computers

(1) IT solution (preferred)

- Start up computer, log in with your usual student (s...) account
- Try to start RStudio (might not work)
- If error → Start R (should work for sure)
- Create new empty script file and paste the following commands into the file...

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(1) Testing Lab Computers cont.

```
install.packages('Matrix', repos='http://cran.us.r-project.org')

install.packages('pbkrtest', repos='http://cran.us.r-project.org')

install.packages('lme4', repos = c("http://lme4.r-forge.r-project.org/repos", getOption("repos")))

packageVersion('Matrix')

packageVersion('pbkrtest')

packageVersion('lme4')

library(lme4)

fm1 <- lmer(Reaction ~ Days + (Days | Subject), sleepstudy)

summary(fm1)
```

→ **Run one after the other, letting each first finish**

→ **Errors? Warnings? Extremely slow?**

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(2) Plan B: Thanks  on!!!

Open browser, go to: http://131.174.204.182:8787

- login: s + your data collection number (s1, s2, s3, ...)
- password: rstudiorulez
- Create new empty script file and paste the following commands into the file...

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(2) Plan B cont.

```
install.packages('Matrix', repos='http://cran.us.r-project.org')
install.packages('pbkrtest', repos='http://cran.us.r-project.org')
install.packages('lme4', repos = c("http://lme4.r-forge.r-project.org/repos", getOption("repos")))
packageVersion('Matrix')
packageVersion('pbkrtest')
packageVersion('lme4')
library(lme4)
fm1 <- lmer(Reaction ~ Days + (Days | Subject), sleepstudy)
summary(fm1)
```

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Getting files to and from RStudio server

Instructions from here

http://www.rstudio.com/ide/docs/advanced/uploading_and_downloading_files

Uploading Files

- Switch to the **Files** pane
- Navigate to the directory you wish to upload files into
- Click the **Upload** toolbar button
- Choose the file you wish to upload and press OK

Downloading Files

- Switch to directory you want to download files from within the **Files** pane
- Select the file(s) and/or folder(s) you want to download
- Click **More -> Export** on the toolbar
- You'll then be prompted with a default file name for the download. Either accept the default or specify a custom name then press OK.

Many files? → zip into a single one before up/download 73

That's it for today's lecture!
Questions or comments?

See you in the basement!
-1.55A/B