B529: Homework 1

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Question 1

Please use the perceptron algorithm to manually find a weight vector **w** = [w0,w1,w2]$^{\mathrm{T}}$ *for linear classification of 3 data points* (**x**i,yi) *satisfying sign* ($\mathbf{w}^{\mathrm{T}}\mathbf{x}$i) = yi. *The input data is*

x1=(1,2,2)$^{\mathrm{T}}$ y1=+1

x2=(1,1,2)$^{\mathrm{T}}$ y2=+1

x3=(1,2,0)$^{\mathrm{T}}$ y3=-1

The first element in vector **x** is always x0=1 *(See slides in lecture 1).*

In the first step of the perceptron algorithm, the initial assignment of **w** is [0,6,6]$^{\mathrm{T}}$. *Please provide the intermediate steps of the computation. (30 points)*

Answer 1

- Step 1: **w** = [0,6,6]$^{\mathrm{T}}$
- Step 2:

$$sign(\boldsymbol{w}^T\boldsymbol{x}_1) = sign\left(\begin{bmatrix}0 & 6 & 6\end{bmatrix} \cdot \begin{bmatrix}1\\2\\2\end{bmatrix}\right) = sign(24) = 1$$

$$sign(\boldsymbol{w}^T\boldsymbol{x}_1) = sign\left(\begin{bmatrix}0 & 6 & 6\end{bmatrix} \cdot \begin{bmatrix}1\\1\\2\end{bmatrix}\right) = sign(18) = 1$$

$$sign(\boldsymbol{w}^T\boldsymbol{x}_1) = sign\left(\begin{bmatrix}0 & 6 & 6\end{bmatrix} \cdot \begin{bmatrix}1\\2\\0\end{bmatrix}\right) = sign(12) \neq -1$$

$$\boldsymbol{w} = \boldsymbol{w} + y_3\boldsymbol{x}_3 = \begin{bmatrix}0\\6\\6\end{bmatrix} + -1 \cdot \begin{bmatrix}1\\2\\0\end{bmatrix} = \begin{bmatrix}-1\\4\\6\end{bmatrix}$$

- Step 2: iteration

$$sign(\boldsymbol{w}^T\boldsymbol{x}_1) = sign\left(\begin{bmatrix}-1 & 4 & 6\end{bmatrix} \cdot \begin{bmatrix}1\\2\\2\end{bmatrix}\right) = sign(19) = 1$$

$$sign(\boldsymbol{w}^T\boldsymbol{x}_1) = sign\left(\begin{bmatrix}-1 & 4 & 6\end{bmatrix} \cdot \begin{bmatrix}1\\1\\2\end{bmatrix}\right) = sign(15) = 1$$

$$sign(\boldsymbol{w}^T\boldsymbol{x}_1) = sign\left(\begin{bmatrix}-1 & 4 & 6\end{bmatrix} \cdot \begin{bmatrix}1\\2\\0\end{bmatrix}\right) = sign(7) \neq -1$$

$$\boldsymbol{w} = \boldsymbol{w} + y_3\boldsymbol{x}_3 = \begin{bmatrix}-1\\4\\6\end{bmatrix} + -1 \cdot \begin{bmatrix}1\\2\\0\end{bmatrix} = \begin{bmatrix}-2\\2\\6\end{bmatrix}$$