B529: Homework 1

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Question 1

Please use the perceptron algorithm to manually find a weight vector **w** = [w0,w1,w2]$^{\mathrm{T}}$ *for linear classification of 3 data points* (**x**i,yi) *satisfying sign* (**w**$^{\mathrm{T}}$**x**i) = yi. *The input data is*

x1=(1,2,2)$^{\mathrm{T}}$ y1=+1

x2=(1,1,2)$^{\mathrm{T}}$ y2=+1

x3=(1,2,0)$^{\mathrm{T}}$ y3=-1

The first element in vector **x** is always x0=1 *(See slides in lecture 1).*

In the first step of the perceptron algorithm, the initial assignment of **w** is [0,6,6]$^{\mathrm{T}}$. *Please provide the intermediate steps of the computation. (30 points)*

Answer 1

- Step 1: **w** = [0,6,6]$^{\mathrm{T}}$
- Step 2:

$$sign(\boldsymbol{w}^T\boldsymbol{x}_1) = sign\left(\begin{bmatrix}0 & 6 & 6\end{bmatrix} \cdot \begin{bmatrix}1\\2\\2\end{bmatrix}\right) = sign(24) = 1$$

$$sign(\boldsymbol{w}^T\boldsymbol{x}_1) = sign\left(\begin{bmatrix}0 & 6 & 6\end{bmatrix} \cdot \begin{bmatrix}1\\1\\2\end{bmatrix}\right) = sign(18) = 1$$

$$sign(\boldsymbol{w}^T\boldsymbol{x}_1) = sign\left(\begin{bmatrix}0 & 6 & 6\end{bmatrix} \cdot \begin{bmatrix}1\\2\\0\end{bmatrix}\right) = sign(12) \neq -1$$

$$\boldsymbol{w} = \boldsymbol{w} + y_3\boldsymbol{x}_3 = \begin{bmatrix}0\\6\\6\end{bmatrix} + -1 \cdot \begin{bmatrix}1\\2\\0\end{bmatrix} = \begin{bmatrix}-1\\4\\6\end{bmatrix}$$

- Step 2: iteration

$$sign(\boldsymbol{w}^T\boldsymbol{x}_1) = sign\left(\begin{bmatrix}-1 & 4 & 6\end{bmatrix} \cdot \begin{bmatrix}1\\2\\2\end{bmatrix}\right) = sign(19) = 1$$

$$sign(\boldsymbol{w}^T\boldsymbol{x}_1) = sign\left(\begin{bmatrix}-1 & 4 & 6\end{bmatrix} \cdot \begin{bmatrix}1\\1\\2\end{bmatrix}\right) = sign(15) = 1$$

$$sign(\boldsymbol{w}^T\boldsymbol{x}_1) = sign\left(\begin{bmatrix}-1 & 4 & 6\end{bmatrix} \cdot \begin{bmatrix}1\\2\\0\end{bmatrix}\right) = sign(7) \neq -1$$

$$\boldsymbol{w} = \boldsymbol{w} + y_3\boldsymbol{x}_3 = \begin{bmatrix}-1\\4\\6\end{bmatrix} + -1 \cdot \begin{bmatrix}1\\2\\0\end{bmatrix} = \begin{bmatrix}-2\\2\\6\end{bmatrix}$$

- Step 2: iteration

$$sign(\boldsymbol{w}^T\boldsymbol{x}_1) = sign\left(\begin{bmatrix}-2 & 2 & 6\end{bmatrix} \cdot \begin{bmatrix}1\\2\\2\end{bmatrix}\right) = sign(14) = 1$$

$$sign(\boldsymbol{w}^T\boldsymbol{x}_1) = sign\left(\begin{bmatrix}-2 & 2 & 6\end{bmatrix} \cdot \begin{bmatrix}1\\1\\2\end{bmatrix}\right) = sign(12) = 1$$

$$sign(\boldsymbol{w}^T\boldsymbol{x}_1) = sign\left(\begin{bmatrix}-2 & 2 & 6\end{bmatrix} \cdot \begin{bmatrix}1\\2\\0\end{bmatrix}\right) = sign(2) \neq -1$$

$$\boldsymbol{w} = \boldsymbol{w} + y_3\boldsymbol{x}_3 = \begin{bmatrix}-2\\2\\6\end{bmatrix} + -1 \cdot \begin{bmatrix}1\\2\\0\end{bmatrix} = \begin{bmatrix}-4\\0\\6\end{bmatrix}$$

- Step 2: iteration

$$sign(\boldsymbol{w}^T\boldsymbol{x}_1) = sign\left(\begin{bmatrix}-4 & 0 & 6\end{bmatrix} \cdot \begin{bmatrix}1\\2\\2\end{bmatrix}\right) = sign(8) = 1$$

$$sign(\boldsymbol{w}^T\boldsymbol{x}_1) = sign\left(\begin{bmatrix}-4 & 0 & 6\end{bmatrix} \cdot \begin{bmatrix}1\\1\\2\end{bmatrix}\right) = sign(8) = 1$$

$$sign(\boldsymbol{w}^T\boldsymbol{x}_1) = sign\left(\begin{bmatrix}-4 & 0 & 6\end{bmatrix} \cdot \begin{bmatrix}1\\2\\0\end{bmatrix}\right) = sign(-4) = -1$$

Final **w** = $(-4, 0, 6)^{\mathrm{T}}$

Question 2

Please use R to manually compute the weight vector **w** = $[w0, w1, w2]^{\mathrm{T}}$ *for linear regression of N input data points* (xi,yi) *that minimizes the sum of squared error* $(\mathbf{w}^{\mathrm{T}}\mathbf{x}i - yi)^2$. *The input data is*

x1=$(1,2,3)^{\mathrm{T}}$ y1 = 2

x2=$(1,1,2)^{\mathrm{T}}$ y2 = 1

x3=$(1,1,0)^{\mathrm{T}}$ y3 = -1

x4=$(1,-1,-2)^{\mathrm{T}}$ y4= -2

The first element in vector **x** *is always* x0=1 *(See slides in lecture 1)*

Please provide the intermediate steps of the computation (20 points).

Answer 2

```
X <- matrix(c(1, 2, 3, 1, 1, 2, 1, 1, 0, 1, -1, -2), nrow = 4, byrow = TRUE)
X
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    3
## [2,]    1    1    2
## [3,]    1    1    0
## [4,]    1   -1   -2
```

```
y <- matrix(c(2, 1, -1, -2), ncol = 1)
y
```

```
##      [,1]
## [1,]    2
## [2,]    1
## [3,]   -1
## [4,]   -2
```

```
Xt <- t(X)
Xt
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    1    1    1
## [2,]    2    1    1   -1
## [3,]    3    2    0   -2
```

```
w <- solve(Xt %*% X) %*% Xt %*% y
w
```

```
##       [,1]
## [1,] -0.45
## [2,] -0.45
## [3,]  1.05
```