Question 7:

a. $f(n) = n3 + 3n2 + 4$. Prove that $f = \Theta(n3)$.
   i. $c = 2$, $n_0 = 4$
   ii. $f(n) \leq 2n$ for $n \geq 4n$
   iii. $f(n) = n^3 + 3n^2 + 4$
   iv. $\leq n^3 + 3n^3 + 4$
   v. $= 4n^3 + 4$
   vi. $\leq 2n^3 + 2n^3$
   vii. $= 2n^3$
   viii. **$f(n)=O(n3)$ with $c=2$ and $n_0 = 4$**
   ix. $c = 1$, $n_0 = 1$
   x. $f(n) = n^3 + 3n^2 + 4$
   xi. $\geq n^3$
   xii. **$f(n) = \Omega(n3)$ with $c = 1$ and $n_0 = 1$.**

b. Use the definition of Θ to show that $\sqrt{7n^2 + 2n - 8} = \Theta(n)$
   i. $c=\sqrt{7}+2$ and $n_0=1$.
   ii. $g(n)\leq (7+2)n$ for $n\geq1$
   iii. $\sqrt{7n^2+2n-8}$
   iv. $\leq\sqrt{7n^2+2n}$
   v. $\leq(\sqrt{7}+2)n$
   vi. **$g(n) = O(n)$ with $c = 7+2$ and $n_0= 1$.**
   vii. $c = 2$ and $n_0=2$.
   viii. $g(n) \geq 2n$ for $n \geq 2$
   ix. $g(n)=7n2+2n-8$
   x. $\geq2n-8$
   xi. $\geq2n$
   xii. **$g(n)=\Omega(n)$ with $c=2$ and $n_0=2$.**

c. Exercise 8.3.5, A-E
   a. The algorithm partitions the input sequence based on the value of pp. It places all elements that are less than pp to the left side and all elements that are greater than or equal to p to the right side of the sequence. This is done by initializing two pointers, i and j, at the start and end of the sequence, respectively, and then swapping elements that are on the wrong side of the partition until i is no longer less than j.

   b. The total number of times the lines "i := i + 1" or "j := j - 1" are executed is dependent on the actual values of the numbers in the sequence, not just the length of the sequence. The maximum number of times these lines are executed is when the sequence is arranged such that all elements are less than p or greater than or equal to p, leading to n executions. The minimum number of times is when the sequence is already partitioned according to p, leading to 0 executions.

c. The total number of times the swap operation is executed also depends on the actual values of the numbers in the sequence. The maximum number of swaps occurs when the sequence is arranged such that the first half of the elements are greater than or equal to p and the second half are less than p, leading to n/2 swaps. The minimum number of swaps is 0, which occurs when the sequence is already partitioned according to p.

d. The time complexity of the algorithm is determined by the number of times that i is incremented or j is decremented. Since the number of swaps is at most the number of times that i is incremented or j is decremented, an asymptotic lower bound for the time complexity of the algorithm is $\Omega(n)$. In this case, considering the worst-case input is useful to establish the lower bound.

e. Since the incrementing and decrementing operations are executed at most n times, and each operation (including the swap) takes constant time, the matching upper bound for the time complexity of the algorithm is $O(n)$.

Question 8

A. Exercise 5.1.2 Section A-B
    a. 40^6 = **4,096,000**
    b. $40^7 + 40^8 + 40^9$ = **268,861,440,000,000**
B. Exercise 5.3.2, Section A
    a. $3×2×2×2×2×2×2×2×2×2=3×2^9=3×512=$**1536**
C. Exercise 5.3.3 Sections B-C
    a. $10 × 26^4 × 9 × 8$ = **32,514,048,000**
    b. 10 × 26 × 25 × 24 × 23 × 9 × 8 = **78,364,800,000**
D. Exercise 5.2.3 Section A, B
    a. $f:B9 \to E10$ such that for any 9-bit string $x \in B9$, $f(x)$ is obtained by taking the 9-bit string and appending a bit that makes the total number of 1's even.
        - If x has an even number of 1's, append a 0.
        - If x has an odd number of 1's, append a 1.
        - No two different 9-bit strings will be mapped to the same 10-bit string with even parity.
        - Any 10-bit string with even parity can be reached by taking the first 9 bits (which will form a valid element in B9) and applying the function.
    b. Since we've established a bijection between B9 and E10, the cardinalities of these two sets must be equal.
        - The cardinality of B9, which consists of all 9-bit binary strings, is $2^9 = 512$

Question 9

A. Exercise 5.4.2 Section A, B
    a. How many different phone numbers are possible?
        i. $2 \times 10^4 = 20{,}000$
    b. How many different phone numbers are there in which the last four digits are all different?
        i. $2 \times 10 \times 9 \times 8 \times 7 = 10080$

B. Exercise 5.5.3 Section A-G
    a. $2^{10} = 1024$
    b. $2^7 = 128$
    c. $2^8 = 256 + 128 = 384$
    d. $4 \times 2^6 = 256 \times 4 = 1024$
    e. $C(10,6) = 210$
    f. $C(9,3) = 84$
    g. $C(5,1) \times C(5,3) = 50$

C. Exercise 5.5.5 Section A
    a. $C(35,10)$ x $C(30,10) = 5{,}514{,}406{,}172{,}635$

D. Exercise 5.5.8 Section C-F
    a. $C(26,5) = 65{,}780$
    b. 13 x 48 = 624
    c. $C(13,1) \times C(4,3) \times C(12,1) \times C(4,2) = 13 \times 4 \times 12 \times 6 = 3{,}744$
    d. $C(13,5) \times 4^5 = 1{,}287 \times 1{,}024 = 1{,}317{,}504$

E. Exercise 5.6.6 Section A, B
    a. $C(44,5) \times C(56,5) = 4{,}482{,}231{,}613{,}696$
    b. $C(44,1) \times C(43,1) \times C(56,1) \times C(55,1) = 5{,}742{,}080$

Question 10

A. Exercise 5.7.2 Sections A, B
    a. C(52,5) - C(39,5) = 2,598,960 − 575,757 = 2,023,203
    b. C(52,5) - C(13,5) = 2,598,960 − 1,317,888 = 1,281,072

B. Exercise 5.8.4 Section A, B
    a. 5^{20} = 95,367,431,640,625
    b. C(20,4)×C(16,4)×C(12,4)×C(8,4)×C(4,4)=484,545,408,000

Question 11

A. How many one to one functions are there from a set with five elements to sets with the following number of elements:
   a. 4
      i. 0
   b. 5
      i. 5! = 120
   c. 6
      i. C(6,5) = 6! = 720
   d. 7
      i. C(7,5) = 7!/2! = 5040