Given A = {1, {2}, {{3, 4}}}

For each of the following statements, state whether they are **true** or **false**.

a. $1 \in A$ - True
b. $1 \subseteq A$ - False
c. $\{2\} \in A$ - True
d. $\{2\} \subseteq A$ - False
e. $\{3, 4\} \in A$ - False
f. $\{3,\ 4\} \subseteq A$ - False
g. $\{\{3,\ 4\}\} \in A$ - True
h. $\{\{3,\ 4\}\} \subseteq A$ - False
i. $\emptyset \in A$ - False
j. $\emptyset \subseteq A$ - True

Let A = {1, 2, 3, 4}

Select the statement that is false. - C

a. $\emptyset \in P(A)$
b. $\emptyset \subseteq P(A)$
c. $\{2, 3\} \in A$
d. $\{2, 3\} \subseteq A$

Choose the property for which the function satisfies if well defined.

a. Neither one-to-one, nor onto
b. One-to-one, but not onto
c. Onto, but not one-to-one
d. One-to-one and onto
e. Not well defined

Given a function whose domain is the **set of all integers and whose target is the set of all positive integers:**

i. $f(x) = 2x + 1$ - B
ii. $f(x) = |x| + 1$ - A
iii. $f(x) = x^2 + 1$ - A
iv. $f(x) = \{(x > 0 : 2x + 1) \wedge (x \leq 0 : -2x)\}$ - D
v. $f(x) = \{(x \geq 0: 2x + 1) \wedge (x < 0: -2x + 2)\}$ - B

4.1 Direct Proofs

Prove that the product of two odd integers is an odd integer.

1. $m = 2a + 1$
2. $n = 2b + 1$
3. $m \times n = (2b + 1)(2a + 1)$
4. $m \times n = 4ab + 2a + 2b + 1$
5. $m \times n = 2(2ab + a + b) + 1$
6. $k = 2ab + a + b$
7. $m \times n = 2k + 1$

4.2 Proof by Contrapositive

Prove that if n^2 is even, then n is even.

1. Assume n is odd: $n = 2k + 1$
2. Express n^2 as $(2k+1)^2$
3. $4k^2+4k+1=2(2k^2+2k)+1$
4. Since k is an integer, m must be an integer, therefore m=2k2+2k.
5. $n^2 = 2m+1$
6. Since m is an integer, 2m+1 is an odd integer, so n^2 is odd.

4.2 Proof by Contradiction

Prove by contradiction that if 3n + 5 is odd, then n is even.

1. 3n + 5 is odd and n is not even
2. If n is odd, then it can be written as 2k + 1 for some integer k.
3. $3n + 5 = 3(2k + 1) + 5 = 6k + 3 + 5 = 6k + 8 = 2(3k + 4)$
4. 3n + 5 is even, because it's of the form 2m, where m is an integer (m = 3k + 4).
5. However, we initially assumed that 3n + 5 is odd. This is a contradiction, as a number can't be both even and odd.

5.1 Decimal to 8-bit Two's Complement

1. $(-43)_{10}$
   a. $|-43_{10}| = 00101011$
   b. Flip the bits: 11010100
   c. Add one: 11010101

5.2 Binary to Hexadecimal

1. $(110011100)_2 =$
   a. 0001 1001 1100
   b. 0001 = 1
   c. 1001 = 9
   d. 1100 = C
   e. 19C