Question 7:

A. Exercise 3.1.1 Section A-G - *Use the definitions for the sets given below to determine whether each statement is true or false:*

$A = \{x \in Z: \ x \; is \; an \; integer \; multiple \; of \; 3\}$
$B = \{x \in Z: x \; is \; a \; perfect \; square\}$
$C = \{4, 5, 9, 10\}$
$D = \{2, 4, 11, 14\}$
$E = \{3, 6, 9\}$
$F = \{4, 6, 16\}$

a. 27 ∈ A = **True**
b. 27 ∈ B = **False**
c. 100 ∈ B = **True**
d. E ⊆ C or C ⊆ E = **False**
e. E ⊆ A = **True**
f. A ⊂ E = **False**
g. E ∈ A = **False**

B. Exercise 3.1.2, Sections A-E - Use the definitions for the sets given below to determine whether each statement is true or false:

$A = \{ x \in Z: \ x \; is \; an \; integer \; multiple \; of \; 3 \}$
$B = \{ x \in Z: \ x \; is \; a \; perfect \; square \}$
$C = \{ 4, \ 5, \ 9, \ 10 \}$
$D = \{ 2, \ 4, \ 11, \ 14 \}$
$E = \{ 3, \ 6, \ 9 \}$
$F = \{ 4, \ 6, \ 16 \}$

a. 15 ⊂ A = **False**
b. {15} ⊂ A = **True**
c. ∅ ⊂ C = **True**
d. D ⊆ D = **True**
e. ∅ ∈ B = **False**

C. Exercise 3.1.5, Sections B, D - Express each set using set builder notation. Then if the set is finite, give its cardinality. Otherwise, indicate that the set is infinite.

a. { 3, 6, 9, 12, ... } = $\{x \in Z: \ x \; is \; an \; integer \; multiple \; of \; 3\}$, Infinite.
d. { -3, -1, 1, 3, 5, 7, 9 } = $\{x \in Z: -3 \leq x \leq 9 \; and \; x \; is \; odd\}$, Cardinality = 7

D. Exercise 3.2.1 Sections A-K - *Let X = {1, {1}, {1, 2}, 2, {3}, 4 }. Which statements are true?*

   a. $2 \in X$ = **True**
   b. $\{2\} \subseteq X$ = **True**
   c. $\{2\} \in X$ = **False**
   d. $3 \in X$ = **False**
   e. $\{1, 2\} \in X$ = **True**
   f. $\{1, 2\} \subseteq X$ = **True**
   g. $\{2, 4\} \subseteq X$ = **True**
   h. $\{2, 4\} \in X$ = **False**
   i. $\{2, 3\} \subseteq X$ = **False**
   j. $\{2, 3\} \in X$ = **False**
   k. $|X| = 7$ = **False**

<u>Question 8:</u>

B. Exercise 3.2.4, Section B - Let A = {1, 2, 3}. What is {X ∈ P(A): 2 ∈ X}?

   i. P(A) = {∅, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}}.

   ii. {X ∈ P(A): 2 ∈ X} = {{2}, {1,2}, {2,3}. {1,2,3}}

Question 9:

A. Exercise 3.3.1 - Sections C-E

Define the sets A, B, C, and D as follows:

A = {-3, 0, 1, 4, 17}

B = {-12, -5, 1, 4, 6}

C = {x ∈ **Z**: x is odd}

D = {x ∈ **Z**: x is positive}

c. A ∩ C = {-3, 1}
d. A ∪ (B ∩ C) = {-12, -3, 0, 1, 4, 6, 17}
e. A ∩ B ∩ C = {1}

B. Exercise 3.3.3 - Sections A, B, E, F

Use the following definitions to express each union or intersection given. You can use roster or set builder notation in your responses, but no set operations. For each definition, i ∈ $\mathbf{Z}^+$.

$A_i = \{i^0, i^1, i^2\}$

$B_i = \{x \in R: -i \leq x \leq 1/i\}$

$C_i = \{x \in R: -1/i \leq x \leq 1/i\}$

a. $\bigcap_{i=2}^{5} = A_i$
   i. A2={1,2,4}
   ii. A3={1,3,9}
   iii. A4={1,4,16}
   iv. A5={1,5,25}
   v. {1}
b. $\bigcup_{i=1}^{5} = A_i$
   i. A_1 = {1}
   ii. A_2 = {1,2,4}
   iii. A_3 = {1,3,9}
   iv. A_4 = {1,4,16}
   v. A_5 = {1,5,25}
   vi. {1,2,3,4,5,9,16,25}
e. $\bigcap_{i=1}^{100} = C_i$
   i. $C_1 = \{x \in R: -1 \leq x \leq 1\}$
   ii. $C_{100} = \{x \in R: -.001 \leq x \leq .001\}$

iii. **As i increases the range of x narrows, so the intersection is the smallest set.** $\{x \in R: -.001 \leq x \leq .001\}$

f. $\bigcup_{i=1}^{100} = C_i$

i. $C_1 = \{x \in R: -1 \leq x \leq 1\}$

ii. $C_{100} = \{x \in R: -.001 \leq x \leq .001\}$

iii. **As i increases the range of x narrows, but the union will be equal to the broadest set.** $\{x \in R: -1 \leq x \leq 1\}$

C. Exercise 3.3.4 - Sections B, D

Use the set definitions A = {a, b} and B = {b, c} to express each set below. Use roster notation in your solutions.

b. $P(A \cup B)$

i. $P(\{a, b, c\})$

ii. $P(A \cup B) = \{\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

d. P(A) ∪ P(B)

i. $P(A) = \{\{a\}, \{b\}, \{a, b\}, \emptyset\}$

ii. $P(B) = \{\{b\}, \{c\}, \{b, c\}, \emptyset\}$

iii. $P(A) \cup P(B) = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \emptyset\}$

Question 10:

A. Exercise 3.5.1, Sections B, C -

The sets A, B, and C are defined as follows:

- A = {tall, grande, venti}
- B = {foam, no-foam}
- C = {non-fat, whole}

Use the definitions for A, B, and C to answer the questions. Express the elements using n-tuple notation, not string notation.

b. Write an element from the set B × A × C.
   i. **{no-foam, tall, non-fat}**
c. Write the set B × C using roster notation.
   i. $\{foam,\ no\ foam\} \times \{nonfat,\ whole\}$ = **{(foam, non-fat), (foam, whole), (no-foam, non-fat), (no-foam, whole)}**

B. Exercise 3.5.3, Sections B, C, E - Indicate which of the following statements are true.
c. $\mathbf{Z}^2 \subseteq \mathbf{R}^2$ = **True**
d. $\mathbf{Z}^2 \cap \mathbf{Z}^3 = \varnothing$ is **True**
e. For any three sets, A, B, and C, if A ⊆ B, then A × C ⊆ B × C = **True**

C. Exercise 3.5.6, Sections D, E - Use the following set definitions to specify each set in roster notation. Except where noted, express elements of Cartesian products as strings.
c. {xy: where $x \in \{0\} \cup \{0\}^2$ and $y \in \{1\} \cup \{1\}^2$}
   i. **{“01”}**
d. {xy: $x \in$ {aa, ab} and $y \in \{a\} \cup \{a\}^2$}
   i. **{“aaa”,”aba”}**

D. Exercise 3.5.7, Sections C, F, G - Use the following set definitions to specify each set in roster notation. Except where noted, express elements of Cartesian products as strings.
- A = {a}
- B = {b, c}
- C = {a, b, d}

c. (A × B) ∪ (A × C)
   i. (A x B) = {ab, ac}

ii. (A x C) = {aa, ab, ad}

iii. (A × B) ∪ (A × C) = {ab, ac, aa, ad}

f. P(A × B)

   i. A x B = {ab, ac}

   ii. P(A x B) = {∅, {ab}, {ac}, {ab, ac}}

g. P(A) × P(B). Use ordered pair notation for elements of the Cartesian product.

   i. P(A) = {∅, {a}}

   ii. P(B) = {∅, {b}, {c}, {b, c}}

   iii. P(A) x P(B) = **{(∅, ∅), (∅, {b}), (∅, {c}), (∅, {b, c}), ({a}, ∅), ({a}, {b}), ({a}, {c}), ({a}, {b, c})}**

Question 11:

A. Exercise 3.6.2, Sections B, C - Use the set identities given in the table to prove the following new identities. Label each step in your proof with the set identity used to establish that step.
    b. (B ∪ A) ∩ (B ∪ A) = A
        i. B ∪ A = A (Idempotent, 1)
        ii. A = A (Substitute, 2)
    c. $(A \cap B')' = A' \cup B$
        i. $A' \cup B'' = A' \cup B$ (De Morgan's)
        ii. $A' \cup B = A' \cup B$ **(Double complement)**

B. Exercise 3.6.3, Sections B, D - Show that each set equation given below is not a set identity
    b. A - (B ∩ A) = A
        i. Any two sets with elements in common will negate this identity:
            1. A = {1, 2} & B = {1, 3}
            2. A - (B ∩ A) = {1, 2} - {1} = {2}
            3. {2} ≠ {1, 2}
    d. (B - A) ∪ A = A
        i. Any two sets A and B where B has at least one element that is not in A negates this identity.
            1. A = {1, 2} & B = {1, 3}
            2. (B - A) = {1, 3} - {1, 2} = {3}
            3. (B - A) ∪ A: {3} ∪ {1, 2} = {1, 2, 3}
            4. A = {1, 2}
            5. {1, 2, 3} ≠ {1, 2}
            6. (B - A) ∪ A ≠ A

C. Exercise 3.6.4, Sections B, C - The set subtraction law states that A - B = A ∩ B. Use the set subtraction law as well as the other set identities given in the table to prove each of the following new identities. Label each step in your proof with the set identity used to establish that step.
    b. A ∩ (B - A) = ∅
        i. A ∩ (B ∩ A') = ∅ (Set subtraction)
        ii. (A ∩ B) ∩ A' = ∅ (Associative, 1)
        iii. ∅ = ∅ (Complement, 2)
    c. A ∪ (B - A) = A ∪ B
        i. A ∪ (B ∩ A') = A ∪ B (Set subtraction)
        ii. A ∪ B ∩ A' = A ∪ B (Distributive, 1)
        iii. A ∪ B = A ∪ B (Simplify, 2)