Question 1:

A. Convert the following numbers to their decimal representation. Show your work:

1. $1001\ 1011_2$
   i. $1\times2^0 = 1$
   ii. $1\times2^1 = 2$
   iii. $1\times2^3 = 8$
   iv. $1\times2^4 = 16$
   v. $1\times2^7 = 128$
   vi. **$\underline{1 + 2 + 8 + 16 + 128 = 155_{10}}$**
2. 456_7
   i. $6\times7^0 = 6$
   ii. $5\times7^1 = 35$
   iii. $4\times7^2 = 196$
   iv. **$\underline{6 + 35 + 196 = 237_{10}}$**
3. $38A_{16}$
   i. $10\times16^0 = 10$
   ii. $8\times16^1 = 128$
   iii. $3\times16^2 = 768$
   iv. **$\underline{10 + 128 + 768 = 906_{10}}$**
4. 2214_5
   i. $4\times5^0 = 4$
   ii. $1\times5^1 = 5$
   iii. $2\times5^2 = 50$
   iv. $2\times5^3 = 250$
   v. **$\underline{4 + 5 + 50 + 250 = 309_{10}}$**

B. Convert the following numbers to their binary representation:

1. 69_{10}
   i. $69 \div 2 = 34$ R 1
   ii. $34 \div 2 = 17$ R 0
   iii. $17 \div 2 = 8$ R 1
   iv. $8 \div 2 = 4$ R 0
   v. $4 \div 2 = 2$ R 0
   vi. $2 \div 2 = 1$ R 0
   vii. $1 \div 2 = 0$ R 1
   viii. Remainders in Reverse: [1, 0, 0, 0, 1, 0, 1]
   ix. **$\underline{69_{10} = 0100\ 0101_2}$**
2. 485_{10}
   i. $485 \div 2 = 242$ R 1
   ii. $242 \div 2 = 121$ R 0
   iii. $121 \div 2 = 60$ R 1
   iv. $60 \div 2 = 30$ R 0
   v. $30 \div 2 = 15$ R 0
   vi. $15 \div 2 = 7$ R 1
   vii. $7 \div 2 = 3$ R 1
   viii. $3 \div 2 = 1$ R 1
   ix. $1 \div 2 = 0$ R 1
   x. Remainders in Reverse: [1, 1, 1, 1, 0, 0, 1, 0, 1]
   xi. **$\underline{485_{10} = 1111\ 00101_2}$**

3. $6D1A_{16}$
    i. 6 = 0110
    ii. D = 1101
    iii. 1 = 0001
    iv. A = 1010
    v. **$6D1A_{16} = 0110\ 1101\ 0001\ 1010_2$**

C. Convert the following numbers to their hexadecimal representation:
  1. 11011011_2
      i. $1101\ 1011_2$ = 1101, 1011
      ii. $1101 = 13_{10} = D_{16}$
      iii. $1011 = 11_{10} = B_{16}$
      iv. **$11011011_2 = DB_{16}$**
  2. 895_{10}
      i. 895÷16 = 55 R 15
      ii. 55÷16 = 3 R 7
      iii. 3÷16 = 0 R 3
      iv. Remainders in Reverse: [3_{10}, 7_{10}, 15_{10}]
      v. **$895_{10} = 37F_{16}$**

Question 2:

Solve the following, do all calculations in the given base. Show your work.

1. $7566_8 + 4515_8$
   a. $6_8 + 5_8 = \cancel{1}\underline{3_8}$
   b. $6_8 + 1_8 + 1$ (carried) $= \cancel{1}\underline{0_8}$
   c. $5_8 + 5_8 + 1$ (carried) $= \cancel{1}\underline{3_8}$
   d. $7_8 + 4_{8\,+}\ 1$ (carried) $= \cancel{1}\underline{4_8}$
   e. 1 (carried) $= 1_8$
   f. $\underline{\mathbf{7566_8 + 4515_8 = 14303_8}}$
2. $10110011_2 + (0000)1101_2$
   a. $1_2 + 1_2 = \cancel{1}\underline{0_2}$
   b. $1_2 + 0_2 + 1$ (carried) $= \cancel{1}\underline{0_2}$
   c. $0_2 + 1_2 + 1$ (carried) $= \cancel{1}\underline{0_2}$
   d. $0_2 + 1_2 + 1$ (carried) $= \cancel{1}\underline{0_2}$
   e. $1_2 + 0 + 1$ (carried) $= \cancel{1}\underline{0_2}$
   f. $1_2 + 0_2 + 1$ (carried) $= \cancel{1}\underline{0_2}$
   g. $0_2 + 0_2 + 1$ (carried) $= \underline{1_2}$
   h. $1_2 + 0_2 = \underline{1_2}$
   i. $\underline{\mathbf{1011\ 0011_2 + (0000)1101_2 = 1100\ 0000_2}}$
3. $7A66_{16} + 45C5_{16}$
   a. $6_{16} + 5_{16} = B_{16}$
   b. $6_{16} + C_{16} = \cancel{1}\underline{2_{16}}$
   c. $A_{16} + 5_{16} + 1$ (carried) $= \cancel{1}\underline{0_{16}}$
   d. $7_{16} + 4_{16} + 1$ (carried) $= C_{16}$
   e. $\underline{\mathbf{7A66_{16} + 45C5_{16} = C02B_{16}}}$
4. $3022_5 - 2433_5$
   a. $2_5 - 3_5$
   b. $12_5 - 3_5 = \underline{4_5}$
   c. $1_5 - 3_5$
   d. $11_5 - 3_5 = \underline{3_5}$
   e. $4_5 - 4_5 = \cancel{0_5}$
   f. $2_5 - 2_5 = \cancel{0_5}$
   g. $\underline{\mathbf{3022_5 - 2433_5 = 34_5}}$

Question 3

A. Convert the following numbers to their 8-bit two's complement representation. Show your work.

1. 124_{10}
   i. $|124_{10}| \div 2 = 62$ R 0
   ii. $62 \div 2 = 31$ R 0
   iii. $31 \div 2 = 15$ R 1
   iv. $15 \div 2 = 7$ R 1
   v. $7 \div 2 = 3$ R 1
   vi. $3 \div 2 = 1$ R 1
   vii. $1 \div 2 = 0$ R 1
   viii. Remainders in Reverse: [1, 1, 1, 1, 1, 0, 0]
   ix. Leading zero for positive number: $0111\ 1100_2$
   x. **$\underline{124_{10} = 0111\ 1100_2}$**
2. -124_{10}
   i. $|124_{10}| \div 2 = 62$ R 0
   ii. $62 \div 2 = 31$ R 0
   iii. $31 \div 2 = 15$ R 1
   iv. $15 \div 2 = 7$ R 1
   v. $7 \div 2 = 3$ R 1
   vi. $3 \div 2 = 1$ R 1
   vii. $1 \div 2 = 0$ R 1
   viii. Remainders in Reverse: [1, 1, 1, 1, 1, 0, 0]
   ix. Leading zero for positive number: (0)111 1100
   x. $|-124_{10}| = 0111\ 1100\ _2$
   xi. $0111\ 1100_2$ inverted = $1000\ 0011_2$
   xii. $1000\ 0011_2 + 0000\ 0001_2 = 1000\ 0100_2$
   xiii. Leading 1 for negative number: $1000\ 0100_2$
   xiv. **$-\underline{124_{10} = 1000\ 0100_2}$**
3. 109_{10}
   i. $|109_{10}| \div 2 = 54$ R 1
   ii. $54 \div 2 = 27$ R 0
   iii. $27 \div 2 = 13$ R 1
   iv. $13 \div 2 = 6$ R 1
   v. $6 \div 2 = 3$ R 0
   vi. $3 \div 2 = 1$ R 1
   vii. $1 \div 2 = 0$ R 1
   viii. Remainders in Reverse: [1, 1, 0, 1, 1, 0, 1]
   ix. Leading zero for positive number: $0110\ 1101_2$
   x. **$\underline{109_{10} = 0110\ 1101_2}$**
4. -79_{10}
   i. $|-79_{10}| \div 2 = 39$ R 1
   ii. $39 \div 2 = 19$ R 1
   iii. $19 \div 2 = 9$ R 1
   iv. $9 \div 2 = 4$ R 1
   v. $4 \div 2 = 2$ R 0
   vi. $2 \div 2 = 1$ R 0
   vii. $1 \div 2 = 0$ R 1
   viii. Remainders in Reverse: [1, 0, 0, 1, 1, 1, 1]
   ix. Inverted: [0, 1, 1, 0, 0, 0, 0]

x. $0110000_2 + 0000001_2 = (0)011\ 0001_2$
xi. Leading 1 for negative number: $1011\ 0001_2$
xii. $-79_{10} = 1011\ 0001_2$

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

1. 00011110_2
   i. Remove leading zero for positive number: 0011110_2.
   ii. $0 \times 2^0 = 0$
   iii. $1 \times 2^1 = 2$
   iv. $1 \times 2^2 = 4$
   v. $1 \times 2^3 = 8$
   vi. $1 \times 2^4 = 16$
   vii. $0 \times 2^5 = 0$
   viii. $0 \times 2^6 = 0$
   ix. $0 + 2 + 4 + 8 + 16 + 0 + 0 = 30$
   x. $00011110_2 = 30_{10}$
2. 11100110_2
   i. Invert bits: $0001\ 1001_2$.
   ii. Add $0001\ 1001_2 + 0000\ 0001_2 = 0001\ 1010_2$
   iii. $0001\ 1010_2 = 26_{10}$
   iv. $11100110_2 = -26_{10}$
3. 00101101_2
   i. Remove leading zero for positive number: 0101101_2.
   ii. $1 \times 2^0 = 1$
   iii. $0 \times 2^1 = 0$
   iv. $1 \times 2^2 = 4$
   v. $1 \times 2^3 = 8$
   vi. $0 \times 2^4 = 0$
   vii. $1 \times 2^5 = 32$
   viii. $0 \times 2^6 = 0$
   ix. $1+0+4+8+0+32+0 = 45_{10}$
   x. $00101101_2 = 45_{10}$
4. 10011110_2
   i. Invert bits: 01100001_2
   ii. Add $0110\ 0001_2 + 0000\ 0001_2 = 0110\ 0010_2$
   iii. $0110\ 0010_2 = 98_{10}$
   iv. $10011110_2 = -98_{10}$

Question 4

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.4, Sections B, C
   b. Write a truth table for $\neg (p \lor q)$:

p	q	$\neg (p \lor q)$
0	0	1
0	1	0
1	0	0
1	1	0

   c. Write a truth table for the expression $r \lor (p \land \neg q)$:

p	q	r	$r \lor (p \land \neg q)$
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	0

2. Exercise 1.3.4, Sections B, D
   b. Give a truth table for $(p \rightarrow q) \rightarrow (q \rightarrow p)$:

p	q	$(p \rightarrow q) \rightarrow (q \rightarrow p)$
0	0	1
0	1	0
1	0	1
1	1	1

   d. Give a truth table for the expression $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
0	0	1
0	1	0
1	0	1
1	1	0

Question 5

1. Exercise 1.2.7, Sections B, C
    b. Write a logical expression for the requirements under the following conditions: *The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.*

       $(B \land D) \lor (B \land M) \lor (D \land M)$

    c. Write a logical expression for the requirements under the following conditions: *Applicant must present either a birth certificate or both a driver's license and a marriage license.*

       $B \lor (D \land M)$

2. Exercise 1.3.7, Sections B, C, D, E
    - *S: a person is a senior*
    - *Y: a person is at least 17 years of age.*
    - *P: A person is allowed to park in the school parking lot.*

    b. Express the following English sentence with a logical expression: *A person can park in the school parking lot if they are a senior or at least seventeen years of age.*

       $P \rightarrow (Y \lor S)$

    c. Express each of the following English sentences with a logical expression: *Being 17 years of age is a necessary condition for being able to park in the school parking lot.*

       $P \rightarrow Y$

    d. Express each of the following English sentences with a logical expression: *A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.*

       $P \leftrightarrow (S \land Y)$

    e. Express each of the following English sentences with a logical expression: *Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.*

       $(P \rightarrow S) \lor (P \rightarrow Y)$

3. Exercise 1.3.9, Sections C, D

Y: the applicant is at least eighteen years old.

P: The applicant has parental permission.

C: the applicant can enroll in the course.

c. Translate this English statement into an equivalent logical expression: *The applicant can enroll in the course only if the applicant has parental permission.*

   $C \rightarrow P$

d. Translate this English statement into an equivalent logical expression: *Having parental permission is a necessary condition for enrolling in the course.*

   $C \rightarrow P$

Question 6

1. Exercise 1.3.6, Sections B, C, D
    b. Give an English sentence in the form "If...then..." that is equivalent to the following: *Maintaining a B average is necessary for Joe to be eligible for the honors program.*
        i. **If Joe maintains a B average, then he will be eligible for the honors program.**
    c. Give an English sentence in the form "If...then..." that is equivalent to the following: *Rajiv can go on the roller coaster only if he is at least four feet tall.*
        i. **If Rajiv is not at least four feet tall, then he cannot go on the roller coaster.**
    c. Give an English sentence in the form "If...then..." that is equivalent to the following: *Rajiv can go on the roller coaster if he is at least four feet tall.*
        i. **If Rajiv is at least four feet tall, then he can go on the roller coaster.**
2. Exercise 1.3.10, Sections C, D, E, F

    The variable p is true, q is false, and the truth value for variable r is unknown. Indicate whether the truth value of each logical expression is true, false, or unknown.

    c. $(p \lor r) \leftrightarrow (q \land r)$
        i. $(p \lor r)$ - OR operation, if any of p or r is True, the whole expression is True. p is True, so $(p \lor r)$ is True.
        ii. $(q \land r)$ - AND operation, if any of q or r is False, the whole expression is False. q is False, so $(q \land r)$ is False.
        iii. $(p \lor r) \leftrightarrow (q \land r)$ is True only if both $(p \lor r)$ and $(q \land r)$ have the same truth value. We've found that $(p \lor r)$ is True and $(q \land r)$ is False. Therefore, **$(p \lor r) \leftrightarrow (q \land r)$ is False**.
    d. $(p \land r) \leftrightarrow (q \land r)$
        i. $(p \land r)$ - AND operation, so for the whole expression to be True, both p and r must be True. the truth value of r is Unknown. Therefore, the truth value of $(p \land r)$ is also Unknown.
        ii. $(q \land r)$ - AND operation, so for the whole expression to be True, both q and r must be True. However, q is False, so regardless of the value of r, $(q \land r)$ is False.
        iii. $(p \land r) \leftrightarrow (q \land r)$ is True only if both $(p \land r)$ and $(q \land r)$ have the same truth value. the truth value of $(p \land r)$ is Unknown and the truth value of $(q \land r)$ is False. We can't say if the expression is true or false without knowing the truth value of r. **$(p \land r) \leftrightarrow (q \land r)$, is Unknown.**
    e. $p \rightarrow (r \lor q)$
        i. If p, then r or q.
        ii. Here, p is True, but the truth value of q is False, and the value of r is Unknown.
        iii. We can't definitively say it's true or false without knowing the truth value of r. Therefore, the truth value of the entire statement **$p \rightarrow (r \lor q)$ is Unknown.**
    f. $(p \land q) \rightarrow r$
        i. AND operation. Since p is True and q is False $(p \land q)$ evaluates False.
        ii. An implication is considered false only when the first part is True, and the second part is False. In all other situations, the implication is considered true.
        iii. Therefore, **$(p \land q) \rightarrow r$ is (vacuously) true**.

Question 7

Solve Exercise 1.4.5 Sections B, C, D

1. Express each pair of sentences using logical expressions. Then prove whether the two expressions are logically equivalent.

   - *j: Sally got the job.*
   - *l: Sally was late for her interview.*
   - *r: Sally updated her resume.*

   b. If Sally did not get the job, then she was late for her interview or did not update her resume.
   If Sally updated her resume and was not late for her interview, then she got the job.

   $\neg j \rightarrow (l \land \neg r)$ OR "if not j, then both l and not r"
   $(r \land \neg l) \rightarrow j$ OR "if both r and not l, then j"

j	l	r	$\neg j \rightarrow (l \land \neg r)$
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

j	l	r	$(r \land \neg l) \rightarrow j$
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Not Logically Equivalent

c. If Sally got the job, then she was not late for her interview.
If Sally did not get the job, then she was late for her interview.
$j \rightarrow \neg l$
$\neg j \rightarrow l$

j	l	$j \rightarrow \neg l$
0	0	1
0	1	1
1	0	1
1	1	0

j	l	$\neg j \rightarrow l$
0	0	0
0	1	1
1	0	1
1	1	1

Not Logically Equivalent

d. If Sally updated her resume or she was not late for her interview, then she got the job.
If Sally got the job, then she updated her resume and was not late for her interview.

$(r \vee \neg l) \rightarrow j$
$J \rightarrow (r \wedge \neg l)$

r	l	j	$(r \vee \neg l) \rightarrow j$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

j	r	l	$J \rightarrow (r \wedge \neg l)$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Not logically equivalent

Question 8

1. Solve Exercise 1.5.2, Sections C, F, I
   Use the laws of propositional logic to prove the following:
   c. $(p \to q) \land (p \to r) \equiv p \to (q \land r)$
      i. Conditional identities: $(p \to q) \equiv (\neg p \lor q)$, $(p \to r) \equiv (\neg p \lor r)$
      ii. Distributive laws: $(\neg p \lor q) \land (\neg p \lor r) \equiv p \to (q \land r)$ Changed to: $\neg p \lor (q \land r) \equiv p \to (q \land r)$
      iii. Revert from Conditional identities to implication: **$\underline{p \to (q \land r) \equiv p \to (q \land r)}$**

   f. $\neg (p \lor (\neg p \land q)) \equiv \neg p \land \neg q$
      i. De Morgan's law: $\neg p \land \neg (\neg p \land q) \equiv \neg p \land \neg q$
      ii. De Morgan's law & simplify double negation: $\neg p \land (p \lor \neg q) \equiv \neg p \land \neg q$
      iii. Distributive law: $(\neg p \land p) \lor (\neg p \land \neg q) \equiv \neg p \land \neg q$
      iv. Complement law: $F \land q\ (\neg p \land \neg q) \equiv \neg p \land \neg q$
      v. Identity law: **$\underline{\neg p \land \neg q \equiv \neg p \land \neg q}$**

   i. $(p \land q) \to r \equiv (p \land \neg r) \to \neg q$
      i. Conditional identities: $\neg (p \land q) \lor r \equiv \neg (p \land \neg r) \lor \neg q$
      ii. De Morgan's law: $(\neg p \lor \neg q) \lor r \equiv (\neg p \lor r) \lor \neg q$
      iii. Associative law: $\neg p \lor (\neg q \lor r) \equiv \neg p \lor (r \lor \neg q)$
      iv. Commutative law: **$\underline{\neg p \lor (r \lor \neg q) \equiv \neg p \lor (\neg q \lor r)}$ <u>(identical expressions)</u>**

2. Solve Exercise 1.5.3, sections C, D
   c. $\neg r \lor (\neg r \to p)$
      i. Conditional identities: $\neg r \lor (\neg(\neg r) \lor p)$
      ii. Simplify double negation: $\neg r \lor (r \lor p)$
      iii. Associative law: $(\neg r \lor r) \lor p$
      iv. Complement law: True $\lor\ p$
      v. Domination law: **<u>True</u>**
   d. $\neg (p \to q) \to \neg q$
      i. Conditional identity: $\neg (\neg p \lor q) \to \neg q$
      ii. De Morgan's law & simplify double negation: $(p \land \neg q) \to \neg q$
      iii. Conditional identity: $(\neg p \lor q) \lor \neg q$
      iv. Complement & identity law: **<u>True</u>**

Question 9

1. Solve Exercise 1.6.3, Sections C, D
   Consider the following statements in English. Write a logical expression with the same meaning. The domain is the set of all real numbers.
   c. There is a number that is equal to its square.
      i. **$\exists x\ (x = x^2)$**
   d. Every number is less than or equal to its square plus 1.
      i. **$\forall x\ (x \le x^2 + 1)$**

2. Solve Exercise 1.7.4, Sections B, C, D
   In the following questions, the domain is a set of employees who work at a company. Ingrid is one of the employees at the company. Define the following predicates:
   - S(x): x was sick yesterday
   - W(x): x went to work yesterday
   - V(x): x was on vacation yesterday

   b. Everyone was well and went to work yesterday.
      i. **$\forall x\ (\neg\ S(x) \land W(x))$**
   c. Everyone who was sick yesterday did not go to work.
      i. **$\forall x\ (S(x) \rightarrow \neg W(x))$**
   d. Yesterday someone was sick and went to work.
      i. **$\exists x\ (S(x) \land W(x))$**

Question 10

1. Exercise 1.7.9, Sections C, D, E, F, G, H, I

	P(x)	**Q(x)**	**R(x)**
a	T	T	F
b	T	F	F
c	F	T	F
d	T	T	F
e	T	T	T

c. $\exists x\ ((x = c) \rightarrow P(x))$
   ii. "There exists an x such that if x is equal to c, then P(x) is true."
   iii. P(c) exists, therefore **True.**

d. $\exists x\ (Q(x) \wedge R(x))$
   iv. "There exists an x such that Q(x) and R(x) are both true."
   v. x = e evaluates Q(e) and R(e) to **True.**

e. $Q(a) \wedge P(d)$
   vi. "Q(a) and P(d) are true."
   vii. Check table. **True.**

f. $\forall x\ ((x \neq b) \rightarrow Q(x))$
   viii. "For every x, if x is not equal to b, then Q(x) is true."
   ix. Evaluates to true for all values of x. Therefore **True**.

g. $\forall x\ (P(x) \vee R(x))$
   x. "For every x, either P(x) or R(x) is true."
   xi. x = c evaluates to be false for both P(c) and R(c). Therefore **False.**

h. $\forall x\ (R(x) \rightarrow P(x))$
   xii. "For every x, if R(x) is true then P(x) is true."
   xiii. No instances where R(x) evaluates true and P(x) evaluates false, therefore **True.**

i. $\exists x\ (Q(x) \vee R(x))$
   xiv. "There exists and x within the domain where either Q(x) or R(x) is true."
   xv. There is an x value where Q(x) or R(x) is true. Therefore, **True.**

2. Exercise 1.9.2, Sections B, C, D, E, F, G, H, I

P	**1**	**2**	**3**
1	T	F	T
2	T	F	T
3	T	T	F

Q	**1**	**2**	**3**
1	F	F	F
2	T	T	T
3	T	F	F

S	**1**	**2**	**3**
1	F	F	F
2	F	F	F
3	F	F	F

b. $\exists x\ \forall y\ Q(x, y)$
   i. "There exists an x such that for all y's, Q (x, y) is true."
   ii. x = 2 evaluates to true for all values of y. Therefore, **True.**

c. $\exists y\ \forall x\ P(x, y)$
   i. "There exists a value of y where all values of x for P (x, y) evaluate to true."
   ii. P (x, y) evaluates to true for all x-values at y = 1. Therefore, **True.**

d. ∃x ∃y S (x, y)
    i. “There is a value of x for which there is a value of y that S (x, y) is true.”
    ii. No ‘true’ in the S (x, y) truth table above. Therefore, **False.**
e. ∀x ∃y Q (x, y)
    i. “For all values of x, there exists a value of y for which Q (x, y) is true.”
    ii. No y value for x = 1 that Q (x, y) evaluates to true. Therefore, **False.**
f. ∀x ∃y P (x, y)
    i. "For every x, there exists a y such that P (x, y) is true."
    ii. Every x value in the P (x, y) truth table has a y value that is true. Therefore, **True.**
g. ∀x ∀y P (x, y)
    i. "For every x and every y, P (x, y) is true."
    ii. All x, y permutations in the P-table are not true, therefore **False.**
h. ∃x ∃y Q (x, y)
    i. "There exists an x and there exists a y such that Q (x, y) is true."
    ii. There is at least one x-value for which there is at least one y-value where Q (x, y) is true. Therefore, **True.**
i. ∀x ∀y ¬S (x, y)
    i. “"For every x and every y, S (x, y) is not true."
    ii. All permutations in the S truth table are false, therefore **True.**

Question 11

1. Solve Exercise 1.10.4, Sections C, D, E, F, G
   Translate each of the following English statements into logical expressions. The domain is the set of all real numbers.
   c. There are two numbers whose sum is equal to their product.
      i. **$\exists x \exists y (x + y = x * y)$**
   d. The ratio of every two positive numbers is also positive.
      i. P () indicates number is positive
      ii. **$\forall x \forall y (P(x) \land P(y) \rightarrow P(x \div y))$**
      iii. "For every value of x and y, if x is positive and y is positive, the ratio of x to y is positive."
   e. The reciprocal of every positive number less than one is greater than one.
      i. P(x): x is a positive number
      ii. R(x): the reciprocal of x is greater than one
      iii. **$\forall x (P(x) \land (x < 1) \rightarrow R(x))$**
      iv. "For every value of x, if x is positive and x is less than one, than the reciprocal of x is greater than one."
   f. There is no smallest number.
      i. N (x, y): y is greater than x
      ii. **$\neg\exists x \forall y N(x, y)$**
      iii. "For every value of y, there does not exist a value x that is smaller than y."
   g. Every number other than 0 has a multiplicative inverse.
      i. I(x): has a multiplicative inverse
      ii. **$\forall x ((x \neq 0) \rightarrow I(x))$**
      iii. "For every value of x, if x is not 0 then x has a multiplicative inverse."

2. Solve Exercise 1.10.7, Sections C, D, E, F
   The domain is a group working on a project at a company. One of the members of the group is named Sam. Define the following predicates.
   - P (x, y): x knows y's phone number. (A person may or may not know their own phone number
   - D(x): x missed the deadline.
   - N(x): x is a new employee.

   c. There is at least one new employee who missed the deadline.
      i. **$\exists x (N(x) \land D(x))$**
   d. Sam knows the phone number of everyone who missed the deadline.
      i. **$\forall x (D(x) \rightarrow P(\text{"Sam"}, x))$**
   e. There is a new employee who knows everyone's phone number.
      i. **$\exists x (N(x) \land \forall y P(x, y))$**
   f. Exactly one new employee missed the deadline.
      i. **$\exists x (N(x) \land D(x)) \land \forall y (N(y) \land D(y) \rightarrow y = x)$**

3. Solve exercise 1.10.10, Sections C, D, E, F

   The domain for the first input variable to predicate T is a set of students at a university. The domain for the second input variable to predicate T is the set of Math classes offered at that university. The predicate T (x, y) indicates that student x has taken class y. Sam is a student at the university and Math 101 is one of the courses offered at the university. Give a logical expression for each sentence.

   c. Every student has taken at least one class other than Math 101.
      i. **$\forall x\, \exists y\, (T(x, y) \land \neg T(x, \text{"Math 101"}))$**
   d. There is a student who has taken every math class other than Math 101.
      i. **$\exists x\, \forall y\, (y \neq \text{"Math 101"} \rightarrow T(x, y))$**
   e. Everyone other than Sam has taken at least two different math classes.
      i. **$\forall x\, (x \neq \text{"Sam"} \rightarrow \exists y1\, \exists y2\, (y1 \neq y2 \land T(x, y1) \land T(x, y2)))$**
   a. Sam has taken exactly two math classes.
      i. **$\exists y1\, \exists y2\, (y1 \neq y2 \land T(\text{"Sam"}, y1) \land T(\text{"Sam"}, y2) \land \forall y\, (T(\text{"Sam"}, y) \rightarrow (y = y1 \lor y = y2)))$**

Question 12:

1. Solve Exercise 1.8.2, Sections B, C, D, E

In the following question, the domain is a set of male patients in a clinical study. Define the following predicates:

- P(x): x was given the placebo
- D(x): x was given the medication
- M(x): x had migraines

Translate each statement into a logical expression. Then negate the expression by adding a negation operation to the beginning of the expression. Apply De Morgan's law until each negation operation applies directly to a predicate and then translate the logical expression back into English.

b. Every patient was given the medication or the placebo or both.
   i. Translation: $\forall x\ (D(x) \vee P(x))$
   ii. Negation: $\neg\ (\forall x\ (D(x) \vee P(x)))$
   iii. De Morgan's: $\exists x\ (\neg(D(x) \vee P(x)))$
   iv. Distribute: $\exists x\ (\neg D(x) \wedge \neg P(x))$
   v. **Translation: "There exists a patient who was not given the medication and was not given the placebo."**

c. There is a patient who took the medication and had migraines.
   i. Translation: $\exists x\ (D(x) \wedge M(x))$
   ii. Negation: $\neg\ (\exists x\ (D(x) \wedge M(x)))$
   iii. De Morgan's: $\forall x\ (\neg D(x) \vee \neg M(x))$
   iv. **Translation: "All patients either did not take the medication or did not have migraines."**

d. Every patient who took the placebo had migraines.
   i. Translation: $\forall x\ (P(x) \rightarrow M(x))$
   ii. Negation: $\neg\ (\forall x\ (P(x) \rightarrow M(x)))$
   iii. De Morgan's: $\exists x\ (\neg(P(x) \rightarrow M(x)))$
   iv. Conditional Identity: $\exists x\ (P(x) \wedge \neg M(x))$
   v. **Translation: "There is a patient who took the placebo and did not have migraines."**

e. There is a patient who had migraines and was given the placebo.
   i. Translation: $\exists x\ (M(x) \wedge P(x))$
   ii. Negation: $\neg\ (\exists x\ (M(x) \wedge P(x)))$
   iii. De Morgan's: $\forall x\ (\neg M(x) \vee \neg P(x))$
   iv. **Translation: "Every patient either did not have migraines or was not given the placebo."**

3. Solve Exercise 1.9.4, Sections C, D, E

   Write the negation of each of the following logical expressions so that all negations immediately precede predicates. In some cases, it may be necessary to apply one or more laws of propositional logic.

c. $\exists x\ \forall y\ (P(x, y) \rightarrow Q(x, y))$
   i. $\neg\ (\exists x\ \forall y\ (P(x, y) \rightarrow Q(x, y)))$
   ii. $\boldsymbol{\forall x\ \exists y\ (P(x, y) \wedge \neg\ Q(x, y))}$

d. $\exists x\ \forall y\ (P(x, y) \leftrightarrow P(y, x))$
   i. $\neg\ (\exists x\ \forall y\ (P(x, y) \leftrightarrow P(y, x)))$

  ii. $\exists x\ \forall y\ \neg\ (P\ (x, y) \leftrightarrow P\ (y, x))$
  iii. $\exists x\ \forall y\ (\neg\ (P\ (x, y) \land P\ (y, x)) \lor \neg\ (\neg P\ (x, y) \land \neg P\ (y, x)))$
  iv. **$\underline{\exists x\ \forall y\ ((\neg\ P\ (x, y) \lor \neg\ P\ (y, x)) \lor (P\ (x, y) \lor P\ (y, x)))}$**

e. $\exists x\ \exists y\ P\ (x, y) \land \forall x\ \forall y\ Q\ (x, y)$
  i. $\neg\ (\exists x\ \exists y\ P\ (x, y) \land \forall x\ \forall y\ Q\ (x, y))$
  ii. $(\neg \exists x\ \exists y\ P\ (x, y)) \lor (\neg \forall x\ \forall y\ Q\ (x, y))$
  iii. **$\underline{(\forall x\ \forall y\ \neg P\ (x, y)) \lor (\exists x\ \exists y\ \neg Q\ (x, y))}$**