# Dynamic Modeling of a 6 Degree of Freedom Robotic Arm

Nate Dixon
*School of Engineering*
*Worcester Polytechnic Institute*
Kennebunk, ME
nddixon@wpi.edu

*Abstract* – In the fast-paced and demanding environment of industrial manufacturing, efficient and precise material handling is crucial for maintaining productivity and quality. This paper presents the development and evaluation of a pickup and place robotic arm designed to significantly enhance payload lifting capabilities in such settings. The study details the development of a simulated robotic arm that performs the task of picking up an object and placing it at a designated target location. The simulation will be made using MATLAB and provides a comprehensive testing of scenarios demonstrating the system's effectiveness in managing a multiple different payload.

## I. INTRODUCTION

In a continually advancing industrial manufacturing industry, the efficiency and precision of material handling processes are pivotal to operational success. As industries work to optimize productivity while maintaining high standards of safety and quality, the integration of advanced robotics has emerged as a promising aid. The introduction of robotics in manufacturing has revolutionized the way tasks are performed, offering increased accuracy, speed, and reliability (Groover, 2024). The introduction of robotics into the manufacturing industry has also bridged the gap for sustainability and showing potential for major roles in future application (Kangru et. al., 2019). However, the increasing complexity of manufacturing processes and the demand for higher payload capacities has called for continuous innovation in robotic technologies. This paper presents a comprehensive study on the dynamic modeling of a 6 degree of freedom robotic arm as it carries a payload along a preplanned trajectory.

The core objective of this research is to design, develop, and evaluate a robotic arm system capable of efficiently managing various payloads, thereby improving overall operational efficiency, and reducing manual labor. This study encompasses a detailed exploration of the robotic arm's mechanical design, control system architecture, and software integration. Through rigorous testing and validation in simulated industrial settings using MATLAB, the results of this research are expected to contribute significantly to the field of industrial robotics, offering practical insights and solutions for improving material handling processes in manufacturing.

## II. OVERVIEW OF MODEL

### *A. Forward Kinematics*

Forward kinematics is a fundamental concept in robotics, essential for determining the position and orientation of a robot's end effector based on the given joint angles. This process is pivotal for controlling and simulating robotic systems, as it allows the determination of the

end-effector's pose in space relative to the robot's base frame. Accurate forward kinematics calculations are crucial for tasks such as trajectory planning, motion control, and spatial reasoning in robotic systems.
In this project, forward kinematics were computed using the Denavit-Hartenberg (DH) parameters, a widely used convention for representing the relative transformations between adjacent links of a robotic manipulator. The DH parameters provide a systematic way to model the geometry of the robot by defining the relationship between consecutive coordinate frames attached to each link. Specifically, the DH convention uses four parameters for each joint, theta, d, a and alpha.

The DH parameters were used to calculate six transformation matrices from the base to the end effector of the robot. The matrices were then multiplied sequentially to obtain the final transformation matrix.

*B. Trajectory Planning*

Quintic trajectory generates trajectories for joint motion using quintic polynomials. Quintic polynomials are fifth-order polynomials that can provide smooth transitions in position, velocity, and acceleration, making them suitable for trajectory planning in robotic systems. For each joint of the robotic arm, the trajectory is described by a quintic polynomial of the form:

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

where $q(t)$ represents the joint position at time t. The coefficients $a_0$, $a_1$, $a_2$, $a_3$, $a_4$, and $a_5$ are determined based on the boundary conditions of the trajectory, which include initial and final positions, velocities, and accelerations.

To calculate the coefficients, a system of linear equations is set up based on the boundary conditions. The coefficients are obtained by the following equation.

$$M * a = b$$

Where M is a six-by-six matrix constructed using the time boundaries $t_0$ and $t_f$.

*C. Dynamic Model*

In the dynamic modeling of a robotic manipulator, the primary goal is to determine the torques required at each joint to achieve desired movements, taking into account the robot's mass, inertia, and gravitational effects. This process begins with the definition of symbolic variables for the joint angles, velocities, and accelerations, which are essential for expressing the robot's dynamic behavior. The forward kinematics is first calculated using the methods described in the previous section. The Jacobian matrix is then formed to relate the joint velocities to the linear velocities of specific points on the robot. The Jacobian matrix for a specific point can be found using the following equation.

$$J_i = \frac{\delta p}{\delta q_i}$$

The kinetic energy for each link is then calculated using the equation.

$$K_i = \frac{1}{2} m_i v_i^T v_i$$

The total kinetic energy for the system is the sum of each links kinetic energy. Next, the potential energy is calculated. For each link, the following equation can be used.

$$P_i = m_i g h$$

Where g is the gravitational acceleration and h is the height of the center of mass.

The Lagrangian is then formed by subtracting the potential energy from the kinetic energy. The Euler-Lagrange equations can then be employed to find the torque applied to each of the joints in the system.

$$A_i = \frac{d}{dt}\frac{\delta L}{\delta q_i}$$

$$B_i = \frac{\delta L}{\delta q_i}$$

Finally, the torque on the system can be represented by,

$$\tau_i = A_i - B_i$$

## III. SIMULATION

To visualize and evaluate the torques applied to the joints of the robotic arm manipulator, matlab was used. This simulation environment was crucial to test and refine the calculations done.

Initially, symbolic variables are defined for the joint angles (q1, q2, q3, q4, q5, q6), their velocities, and accelerations. The simulation begins with the robot in a defined initial position (q0) and progresses to a final position (qf), with joint angles transitioning from 0 to specified values. The quinticTrajectory function is employed to generate smooth trajectories for each joint, ensuring continuous position, velocity, and acceleration profiles over the time span (0 to 3 seconds) sampled at 50 intervals. These trajectories are stored in a cell array (paths) for each joint.

Subsequently, a dynamic model of the robot is created using the dynamicModel function, incorporating the physical parameters and specifications of the robot. The calculateTorque function then computes the torques required for each joint to follow the generated trajectories. This involves substituting the generated joint positions, velocities, and accelerations into the dynamic model. The resulting torques are then plotted to provide insights into the forces needed for the robot's actuators to achieve the desired movements, allowing for further analysis and optimization of the robotic system's performance.

## IV. Results

The first simulation, using massless links and a 5 kg payload, revealed the highest torques in joints 2 and 4. This outcome is expected, given that these joints have the longest moment arms.

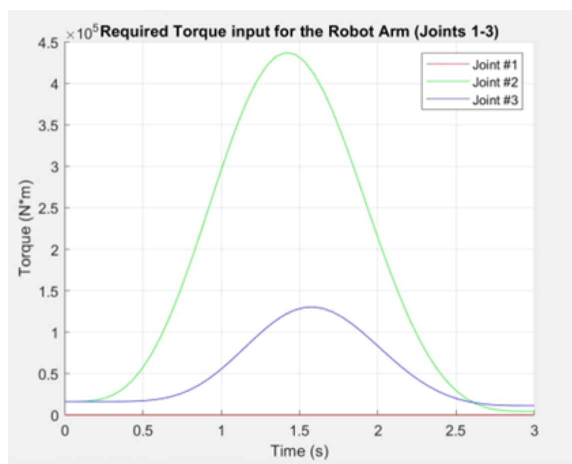


*Figure 1: required torque input for joints 1-3 sim 1*

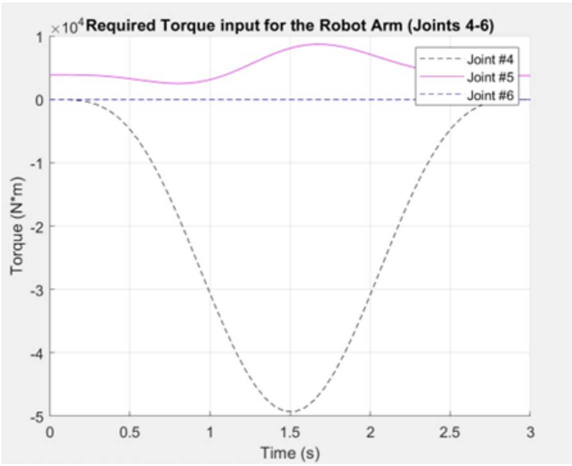


*Figure 2: Required Torque Input for Joints 4-6*

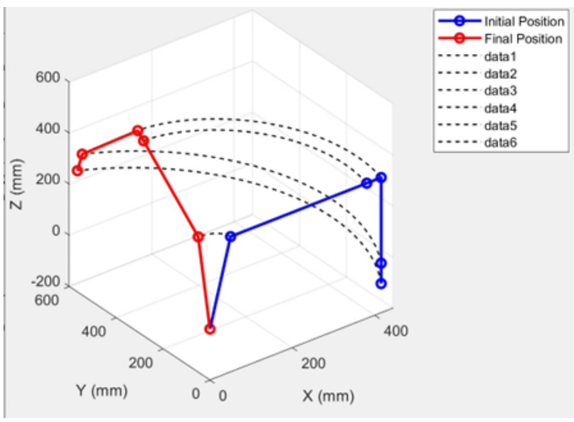


*Figure 3: Stick Plot of Simulation 1*

The second test was done with the same trajectory and the same payload, but now with the link mass incorporated. The results are very similar to the simulation 1 results with higher peak torques on joints 2 and 4.

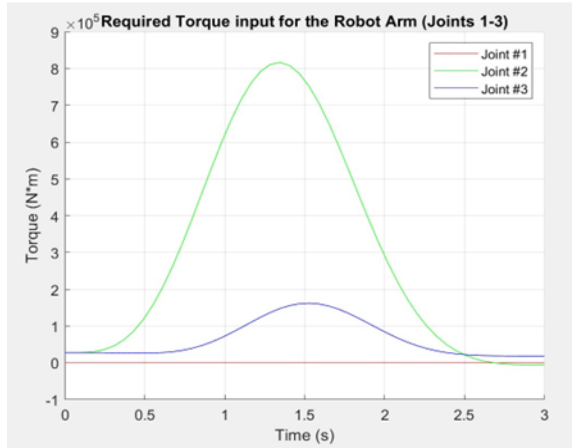


*Figure 4: Required Torque Input for Joints 1-3 in Simulation 2*

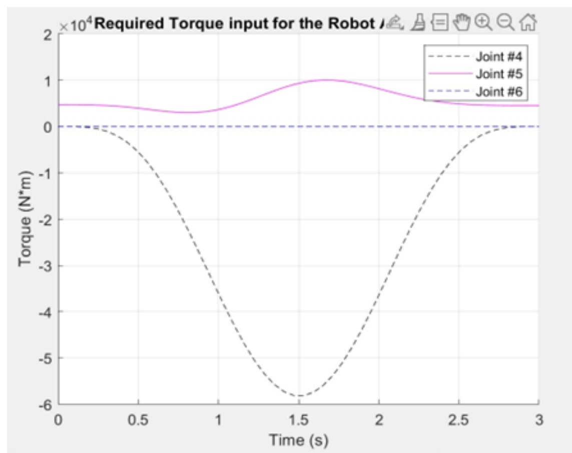


*Figure 5: Required Torque Input for Joints 4-6 in Simulation 2*

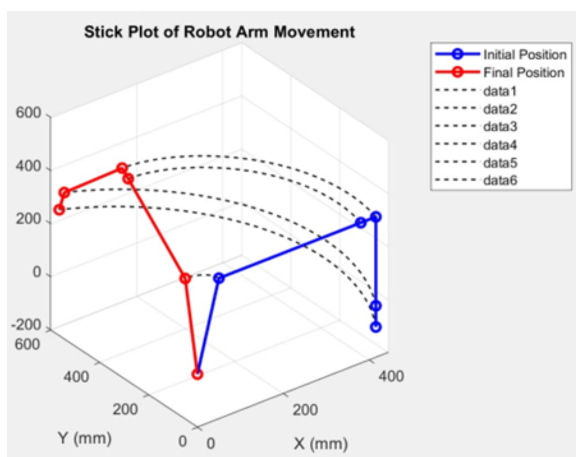


*Figure 6: Stick Plot of Simulation 2*

The third and final simulation was done on a new trajectory, link masses incorporated and the payload increased to 10 kg. This simulation had the highest peak torque, which was expected due to it having the largest payload. The highest torques were on joints 3 and 5, which is again expected as those can be seen with the largest moment arm about their axis of rotation.

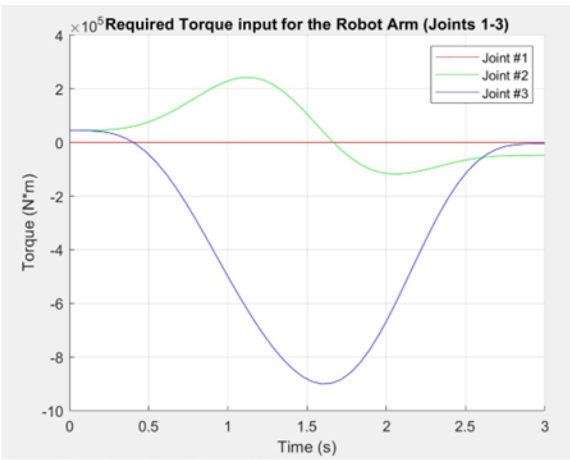


*Figure 7: Required Torque Input for Joints 1-3 in Simulation 3*

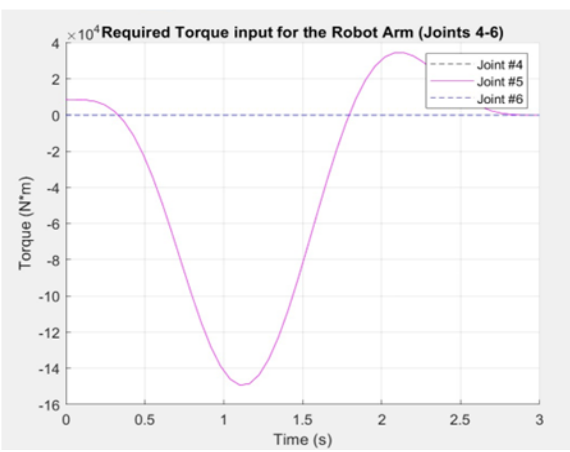


*Figure 8: Required Torque Input for Joints 4-6 in Simulation 3*

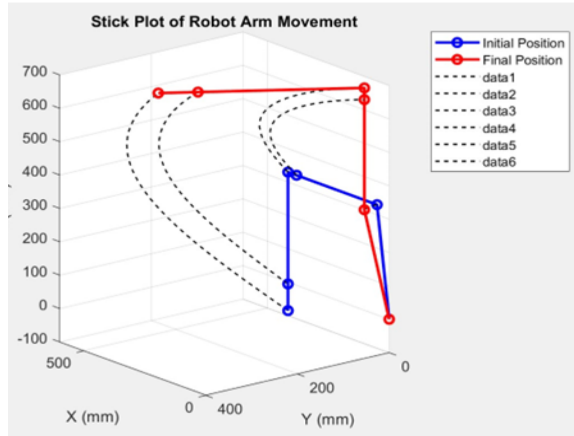


*Figure 9: Stick Plot of Simulation 3*

## V. CONCLUSION

This project focuses on the dynamic modeling of a 6 degree of freedom robotic arm manipulator with the purpose of studying the forces applied to the joints as it carries a payload through a planned trajectory. It uses key components including forward kinematics, quintic trajectory and dynamic modeling using MATLAB. Simulations were used to test the effectiveness and accuracy of the dynamic model and the robot arm performed various movements with different payloads. The findings offer potential insights into improving manufacturing efficiency and precision, contributing to further advancements in industrial robotics.

## REFERENCES


Bhangale, P., Saha, S., & Agrawal, V. P. (2004). A Dynamic Model Based Robot Arm Selection Criterion. *Multibody System Dynamics C*, *12*, 95–115. https://doi.org/10.1023/B:MUBO.0000044363.57485.39

Groover, M. (2024, July 19). *Automation—Robotics, Manufacturing, Automation*. Encyclopedia Britannica. https://www.britannica.com/technology/automation

Kangru, T., Riives, J., Mahmood, K., & Otto, T. (2019). Suitability analysis of using industrial robots in manufacturing. Proceedings of the Estonian Academy of Sciences, 68(4), 383. https://doi.org/10.3176/proc.2019.4.06

Mengyao, P., & Daoxiong, G. (2021). Dynamics Modeling and Control of Humanoid Robot Arm with 7DOF Actuated by Pneumatic Artificial Muscles. *2021 33rd Chinese Control and Decision Conference (CCDC)*, 3050–3055. https://doi.org/10.1109/CCDC52312.2021.9602846