

$$\begin{aligned}
 1) \quad m(a+bX) &= \frac{1}{N} \sum_{i=1}^N (a+bx_i) \\
 &= \frac{1}{N} \left(\sum_{i=1}^N a + b \sum_{i=1}^N x_i \right) \\
 &= \frac{1}{N} (Na) + \frac{b}{N} \sum_{i=1}^N x_i \\
 &= \boxed{a + b m(X)}
 \end{aligned}$$

$$\begin{aligned}
 2) \quad \text{cov}(X, X) &= \frac{1}{N} \sum_{i=1}^N (x_i - m(X))(x_i - m(X)) \\
 &= \frac{1}{N} \sum_{i=1}^N (x_i - m(X))^2 = \boxed{S^2}
 \end{aligned}$$

$$3) \quad \text{cov}(X, a+bY) = b \text{cov}(X, Y)$$

$$m(a+bY) = a + b m(Y)$$

MEAN:

$$\begin{aligned}
 (a+bx_i) - m(a+bY) &\rightarrow \text{cov}(X, a+bY) \\
 = (a+bx_i) - (a + b m(Y)) &= \frac{1}{N} \sum_{i=1}^N (x_i - m(X)) \cdot b(y_i - m(Y)) \\
 = b(x_i - m(Y)) &= \frac{b}{N} \sum_{i=1}^N (x_i - m(X))(y_i - m(Y)) \xrightarrow{\text{cov}(X, Y)} \\
 &= \boxed{b \text{cov}(X, Y)}
 \end{aligned}$$

$$4) \quad \text{cov}(a+bX, a+bY) = b^2 \text{cov}(X, Y)$$

$$\begin{aligned}
 (a+bx_i) - m(a+bX) &= b(x_i - m(X)) \quad \rightarrow \text{cov}(a+bX, a+bY) \\
 \text{AND} \\
 (a+bY_i) - m(a+bY) &= b(y_i - m(Y)) \\
 &= \frac{1}{N} \sum_{i=1}^N b(x_i - m(X)) \cdot b(y_i - m(Y)) \\
 &= \boxed{b^2 \text{cov}(X, Y)}
 \end{aligned}$$

$$5) \quad \text{If } b > 0, \text{med}(X) \rightarrow$$

- a) - since $b > 0$, transformation $a+bX$ is strictly increasing, so it preserves order
 - middle value shifts/scales the same way:
 - Yes $\text{med}(a+bX) = a + b \text{med}(X)$

b) for $b > 0$: Yes

transformation of Quantiles:

$$\cdot Q_p(a+bX) = a + bQ_p(X)$$

so:

$$IQR(a+bX) = bIQR(X)$$

(6)

$$X = \{1, 4\}$$

$$m(X) = \frac{1+4}{2} = 2.5$$

$$Y = X^2;$$

$$Y = \{1, 16\}$$

$$m(X^2) = \frac{1+16}{2} = 17/2 \quad \left. \right\} m(X^2) \neq (m(X))^2$$

$$(m(X))^2 = \frac{25}{4} \quad \left. \right\} \text{red}$$



$$Z = \sqrt{X} = \{1, 2\}$$

$$m(\sqrt{X}) = \frac{1+2}{2} = 3/2 \quad \left. \right\} m(\sqrt{X}) \neq \sqrt{m(X)}$$

$$\sqrt{m(X)} = \sqrt{5/2}$$