

$$\begin{aligned}
 1) \quad m(a+bX) &= \frac{1}{N} \sum_{i=1}^N (a+bX_i) \\
 &= \frac{1}{N} \left( \sum_{i=1}^N a + b \sum_{i=1}^N X_i \right) \\
 &= \frac{1}{N} (Na) + \frac{b}{N} \sum_{i=1}^N X_i \\
 &= \boxed{a + b m(X)}
 \end{aligned}$$

$$\begin{aligned}
 2) \quad \text{cov}(X, X) &= \frac{1}{N} \sum_{i=1}^N (X_i - m(X))(X_i - m(X)) \\
 &= \frac{1}{N} \sum_{i=1}^N (X_i - m(X))^2 = \boxed{s^2}
 \end{aligned}$$

$$3) \quad \text{cov}(X, a+bY) = b \text{cov}(X, Y)$$

$$m(a+bY) = a + b m(Y)$$

MEAN:

$$\begin{aligned}
 (a+bY_i) - m(a+bY) &\rightarrow \text{cov}(X, a+bY) \\
 = (a+bY_i) - (a+b m(Y)) &= \frac{1}{N} \sum (X_i - m(X)) \cdot b(Y_i - m(Y)) \\
 = b(Y_i - m(Y)) &= \frac{b}{N} \sum (X_i - m(X)) (Y_i - m(Y)) \xrightarrow{\text{cov}(X, Y)} \\
 &= \boxed{b \text{cov}(X, Y)}
 \end{aligned}$$

$$4) \quad \text{cov}(a+bX, a+bY) = b^2 \text{cov}(X, Y)$$

$$\begin{aligned}
 (a+bX_i) - m(a+bX) &= b(X_i - m(X)) \\
 \text{AND} \\
 (a+bY_i) - m(a+bY) &= b(Y_i - m(Y)) \rightarrow \text{cov}(a+bX, a+bY) \\
 &= \frac{1}{N} \sum b(X_i - m(X)) \cdot b(Y_i - m(Y)) \\
 &= \boxed{b^2 \text{cov}(X, Y)}
 \end{aligned}$$

5) If  $b > 0$ ,  $\text{med}(X)$

- a) - since  $b > 0$ , transformation  $a+bX$  is strictly increasing, so it preserves order
- middle value shifts/scales the same way:
- Yes  $\text{med}(A+bX) = a + b \text{med}(X)$

b) for  $b > 0$ : Yes  
transformation of Quantiles:  
-  $Q_p(a + bX) = a + bQ_p(X)$

so:

$$IQR(a + bX) = bIQR(X)$$

6)  $X = \{1, 4\} \rightarrow Y = X^2 = \{1, 16\}$   
 $m(X) = \frac{1+4}{2} = 2.5$   
 $m(X^2) = \frac{1+16}{2} = 17/2$   
 $(m(X))^2 = \frac{25}{4}$   
 $\left. \begin{array}{l} m(X^2) = 17/2 \\ (m(X))^2 = 25/4 \end{array} \right\} \underline{m(X^2) \neq (m(X))^2}$

↓  
 $Z = \sqrt{X} = \{1, 2\}$

$$\left. \begin{array}{l} m(\sqrt{X}) = \frac{1+2}{2} = 3/2 \\ \sqrt{m(X)} = \sqrt{5/2} \end{array} \right\} \underline{m(\sqrt{X}) \neq \sqrt{m(X)}}$$