Stat 3202 Lab 8

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Problem 1: Part a: $(1-\alpha)\%$ conf interval for μ (when σ is unknown) is:

$$\bar{x} - t_{1-\alpha/2,n-1}\frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{1-\alpha/2,n-1}\frac{s}{\sqrt{n}}$$

Part b:

```
set.seed(1)

mu = 10
sigma = 3
n = 5
alpha = 0.01

coverage = c()

for(i in 1:10000) {
samp = rnorm(n, mu, sigma)

xbar = mean(samp)
s = sd(samp)
t = qt(1-alpha/2, n -1)

lower = xbar - t*s/sqrt(n)
upper = xbar + t*s/sqrt(n)

coverage[i] = lower <= mu & mu <= upper
}
mean(coverage)
```

```
## [1] 0.9902
```

```
mean(lower)
```

```
## [1] 4.710318
```

```
mean(upper)
```

```
## [1] 10.99509
```

The coverage rate for these confidence intervals is about 99%.

Part c:

```
set.seed(1)

mu = 10
sigma = 3
n = 50
alpha = 0.01

coverage = c()

for (i in 1:10000) {
samp = rnorm(n, mu, sigma)

output = t.test(samp, conf.level = 1-alpha)

lower = output$conf.int[1]
upper = output$conf.int[2]

coverage[i] = lower <= mu & mu <= upper
}

mean(coverage)
```

```
## [1] 0.9903
```

Sample size does not impact the coverage rate since the coverage rate remained 99% at n = 5 and 50.

Part d: μ changed affects where the coverage is, but does not affect the distance between the lower and upper bound of the confidence interval. Changing σ affects how large the coverage is. A larger σ means a larger coverage. Smaller σ means smaller coverage. As n gets larger, the coverage gets smaller. As alpha gets larger, the coverage gets smaller.

Problem 2: Part a:

```
set.seed(1)
library(DescTools)

n = 40
p = 0.7

alpha = 0.05

lower = c()
upper = c()
coverage = c()

for(i in 1:10000) {
x = rbinom(1, n, p)
output = BinomCI(x, n, conf.level = 1-alpha, method = "wald")

lower[i] = output[2]
upper[i] = output[3]
```

```
coverage[i] = lower[i] <= p & p <= upper[i]
}

mean(coverage)
```

```
## [1] 0.9275
```

The confidence interval should have been 95%, but was 92.75% so no, coverage does not reach $(1 - \alpha)$.

Part b:

wilson: 0.9412 waldcc: 0.9597 agresti-coull: 0.9412 jeffreys: 0.9412

Waldcc is the closest. Strangely, wilson, agresti-coull, and jeffreys all came out the same.