$$E[s_p^2] = \frac{1}{n_x + n_y - 2}\left[E\left[\sum_{i=1}^{n_x}(x_i - \bar{x})^2\right] + E\left[\sum_{i=1}^{n_y}(y_i - \bar{y})^2\right]\right]$$

$$\sum_{i=1}^{n_x} E[x_i^2 - 2x_i\bar{x} + \bar{x}^2]$$

$$E[x_i^2] - 2E[x_i]E[\bar{x}] + E[\bar{x}^2]$$

$$= \mu_x^2 + \sigma^2 - 2\mu_x^2 + \mu_x^2 + \sigma^2$$

$$= 2\sum_{i=1}^{n_x} \sigma^2 = 2n_x\sigma^2$$

$$\frac{2n_x\sigma^2 + 2n_y\sigma^2}{n_x + n_y - 2}$$

$$= \frac{2\sigma^2(n_x + n_y)}{}$$

$$= \frac{4n\sigma^2}{2n-2} = \frac{2n\sigma^2}{n-1}$$

Part a

$$s_p^2 = \frac{\sum_{i=1}^{n_x}(x_i^2 - 2x_i\bar{x} + \bar{x}^2) + \sum_{i=1}^{n_y}(y_i^2 - 2y_i\bar{y} + \bar{y}^2)}{n_x + n_y - 2}$$

$$\frac{1}{n_x+n_y-2}$$

$$E[s_p^2] = E\left[\sum_{i=1}^{n_x}(x_i^2 - 2x_i\bar{x} + \bar{x}^2)\right] + E\left[\sum_{i=1}^{n_y}(y_i^2 - 2y_i\bar{y} + \bar{y}^2)\right]$$

$$\downarrow$$

$$\sum_{i=1}^{n_x}\left(E(x_i^2 - 2x_i\bar{x} + x^2)\right)$$

$$E[x_i^2] = \sigma^2 + \mu^2$$

$$E[x_i\bar{x}] = \mu^2$$

$$E[\bar{x}^2] = \sigma^2 + \mu^2$$

$$\frac{1}{n_x+n_y-2} \cdot \left(\sum_{i=1}^{n_x} 2\sigma^2 + \sum_{i=1}^{n_y} 2\sigma^2\right)$$

Part a answer

$$\frac{2n_x\sigma^2 + 2n_y\sigma^2}{n_x+n_y-2} = \frac{2\sigma^2(n_x+n_y)}{n_x+n_y-2}\ ?$$

-1

Part b:

$$s_p^2 = \frac{\sum_{i=1}^{n_x}(x_i - \bar{x})^2 + \sum_{i=1}^{n_y}(y_i - \bar{y})^2}{n_x - 1 + n_y - 1}$$

$$= \frac{\sum_{i=1}^{n_x} (x_i - \bar{x})^2}{n_x - 1 + n_y - 1} + \frac{\sum_{i=1}^{n_y} (y_i - \bar{y})^2}{n_x - 1 + n_y - 1}$$

$$= \frac{(n_x - 1) s_x^2}{n_x - 1 + n_y - 1} + \frac{(n_y - 1) s_y^2}{n_x - 1 + n_y - 1}$$

$n_x = n_y$

$$= \frac{(n-1) s_x^2 + (n-1) s_y^2}{2n - 2}$$

$2n-2 = 2(n-1)$

$$= \frac{s_x^2 + s_y^2}{2}$$

Part c:

Use Part b:

$$\frac{(n_x - 1) s_x^2}{n_x - 1 + n_y - 1} + \frac{(n_y - 1) s_y^2}{n_x - 1 + n_y - 1}$$

$$= \frac{(n_x - 1) s_x^2 + (n_y - 1) s_y^2}{n_x + n_y - 2}$$

$$= \frac{1}{n_x + n_y - 2} \left((n_x - 1) s_x^2 + (n_y - 1) s_y^2 \right)$$

Whichever has
the larger
n value will
have a higher value.
thus more weight.