

$$E(s_p^2) = \frac{1}{n_x + n_y - 2} \left( E \left[ \sum_{i=1}^{n_x} (x_i - \bar{x})^2 \right] + E \left[ \sum_{i=1}^{n_y} (y_i - \bar{y})^2 \right] \right)$$

$$\sum_{i=1}^{n_x} E [x_i^2 - 2x_i \bar{x} + \bar{x}^2]$$

$$E[x_i^2] - 2E[x_i]E[\bar{x}] + E[\bar{x}^2]$$

$$= \mu_x^2 + \sigma^2 - 2\mu_x^2 + \mu_x^2 + \sigma^2$$

$$= 2 \sum_{i=1}^{n_x} \sigma^2 = 2n_x \sigma^2$$

$$\frac{2n_x \sigma^2 + 2n_y \sigma^2}{n_x + n_y - 2}$$

$$= \frac{2\sigma^2(n_x + n_y)}{n - 2}$$

$$= \frac{4n\sigma^2}{2n - 2} = \frac{2n\sigma^2}{n - 1}$$

Part a

$$s_p^2 = \frac{\sum_{i=1}^{n_x} (x_i^2 - 2x_i\bar{x} + \bar{x}^2) + \sum_{i=1}^{n_y} (y_i^2 - 2y_i\bar{y} + \bar{y}^2)}{n_x + n_y - 2}$$

$$E[s_p^2] = E\left[\frac{1}{n_x + n_y - 2} \left( \sum_{i=1}^{n_x} (x_i^2 - 2x_i\bar{x} + \bar{x}^2) + \sum_{i=1}^{n_y} (y_i^2 - 2y_i\bar{y} + \bar{y}^2) \right)\right]$$

$$\downarrow$$

$$\sum_{i=1}^{n_x} (E[x_i^2 - 2x_i\bar{x} + \bar{x}^2])$$

$$E[x_i^2] = \sigma^2 + \mu^2$$

$$E[x_i\bar{x}] = \mu^2$$

$$E[\bar{x}^2] = \sigma^2 + \mu^2$$

$$\frac{1}{n_x + n_y - 2} \cdot \left( \sum_{i=1}^{n_x} 2\sigma^2 + \sum_{i=1}^{n_y} 2\sigma^2 \right)$$

$$\frac{2n_x\sigma^2 + 2n_y\sigma^2}{n_x + n_y - 2}$$

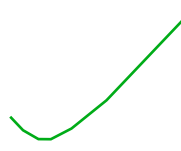
$$= \frac{2\sigma^2(n_x + n_y)}{n_x + n_y - 2}$$

Part a answer

(-1)

Part b:

$$s_p^2 = \frac{\sum_{i=1}^{n_x} (x_i - \bar{x})^2 + \sum_{i=1}^{n_y} (y_i - \bar{y})^2}{n_x - 1 + n_y - 1}$$

$$\begin{aligned}
&= \frac{\sum_{i=1}^{n_x} (x_i - \bar{x})^2}{n_x - 1 + n_y - 1} + \frac{\sum_{i=1}^{n_y} (y_i - \bar{y})^2}{n_x - 1 + n_y - 1} \\
&= \frac{(n_x - 1) s_x^2}{n_x - 1 + n_y - 1} + \frac{(n_y - 1) s_y^2}{n_x - 1 + n_y - 1} \\
&\quad n_x = n_y \\
&= \frac{(n - 1) s_x^2 + (n - 1) s_y^2}{2n - 2 = 2(n - 1)} \\
&= \frac{s_x^2 + s_y^2}{2}
\end{aligned}$$


Part c:

Use Part b:

$$\begin{aligned}
&\frac{(n_x - 1) s_x^2}{n_x - 1 + n_y - 1} + \frac{(n_y - 1) s_y^2}{n_x - 1 + n_y - 1} \\
&= \frac{(n_x - 1) s_x^2 + (n_y - 1) s_y^2}{n_x + n_y - 2} \\
&= \frac{1}{n_x + n_y - 2} \left( (n_x - 1) s_x^2 + (n_y - 1) s_y^2 \right)
\end{aligned}$$

✓

Whichever has  
the larger  
 $n$  value will  
have a higher value.  
thus more weight.