Stat 3202 Lab 3

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```
tidy.opts=list(width.cutoff=60, tidy=TRUE)
```

Problem 1:

CLT: Normalize a particular distribution $\bar{X}$ has approximately mean μ and $\frac{\sigma^2}{n}$

That is: $\frac{\bar{X}-\mu}{\frac{\sigma}{n}} \overset{approx}{\sim} N(0,1)$

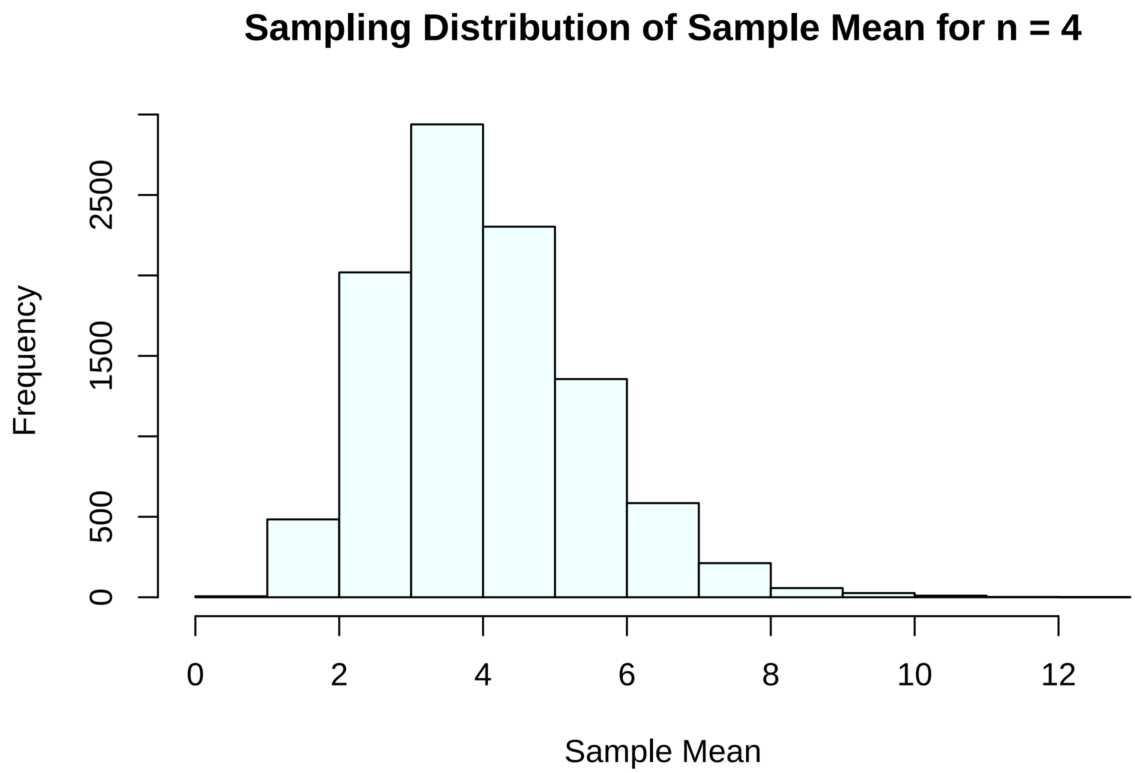
Part A:

```
set.seed(1)

mc = 10000
n = 4
xbar = c()

for (i in 1:mc) {
  sample = rgamma(n, 2, 0.5)
  xbar[i] = mean(sample)
}

hist(xbar, main = "Sampling Distribution of Sample Mean for n = 4", xlab = "Sample Mean", col = "azure"
```



This sampling distribution is slightly right skewed, so it is not normally distributed.

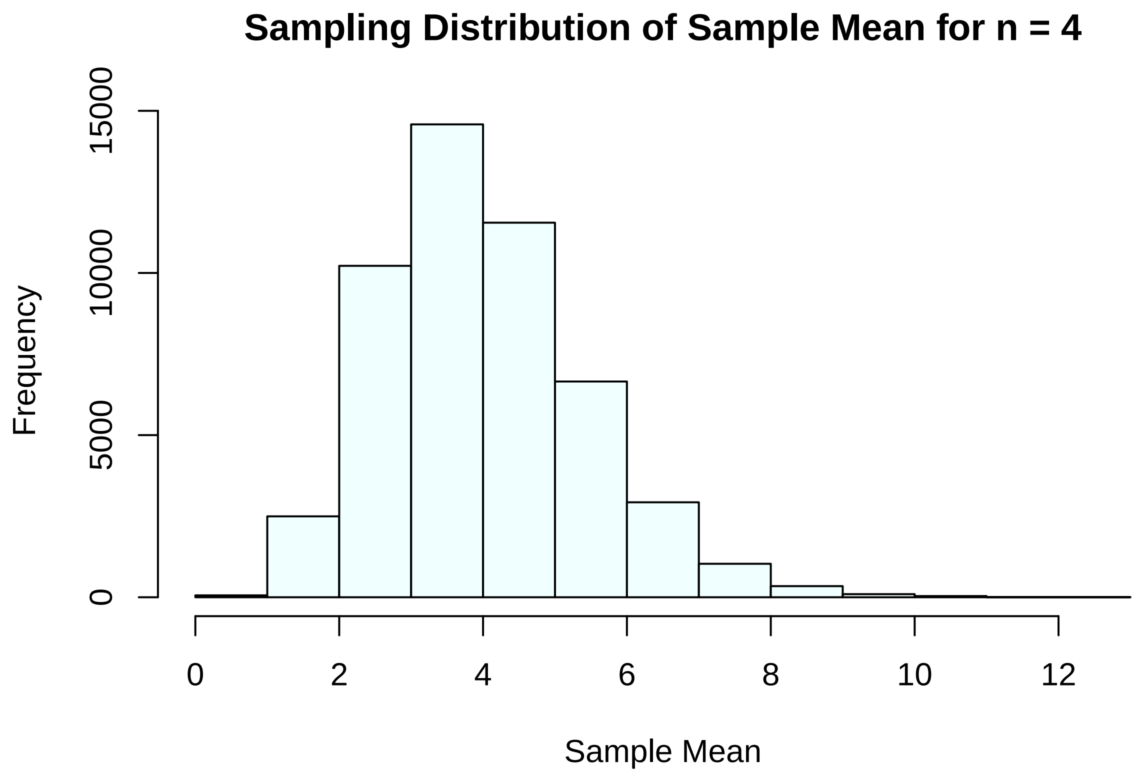
Part B:

```
set.seed(1)

mc = 50000
n = 4
xbar = c()

for (i in 1:mc) {
  sample = rgamma(n, 2, 0.5)
  xbar[i] = mean(sample)
}

hist(xbar, main = "Sampling Distribution of Sample Mean for n = 4", xlab = "Sample Mean", col = "azure")
```



No, it doesn't change because all was changed was the simulations, the number of samples was not changed.

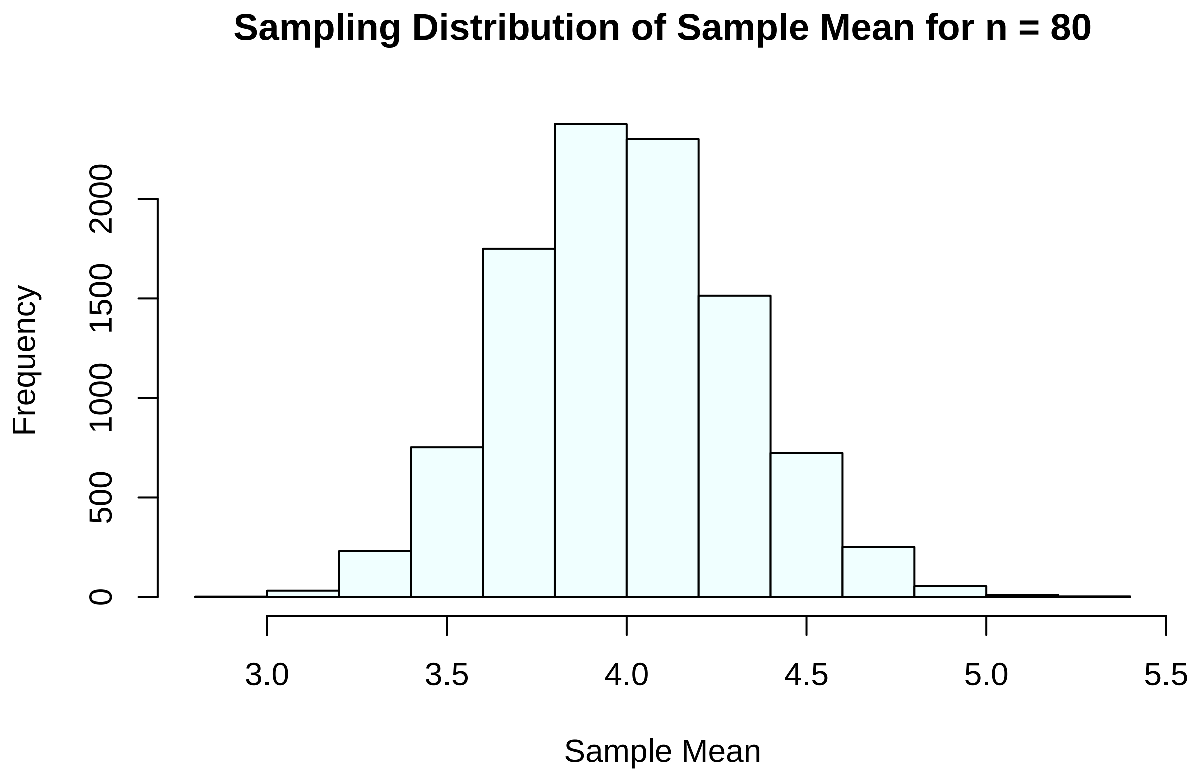
Part C:

```
set.seed(1)

mc = 10000
n = 80
xbar = c()

for (i in 1:mc) {
  sample = rgamma(n, 2, 0.5)
  xbar[i] = mean(sample)
}

hist(xbar, main = "Sampling Distribution of Sample Mean for n = 80", xlab = "Sample Mean", col = "azure
```



Yes, the sample was changed.

Part D:

The true mean of $\bar{X}$ is 4 and the variance is 8 as per the gamma distribution.

```
mean(xbar)
```

```
## [1] 3.996224
```

```
var(xbar)*80
```

```
## [1] 7.976128
```

The mean is slightly under 4 (3.996224) and the variance is also slightly under 8 (7.976128).

Due to Central Limit Theorem, $\bar{X}$ approximately has the distribution $N(4, 8/80)$.

Part E:

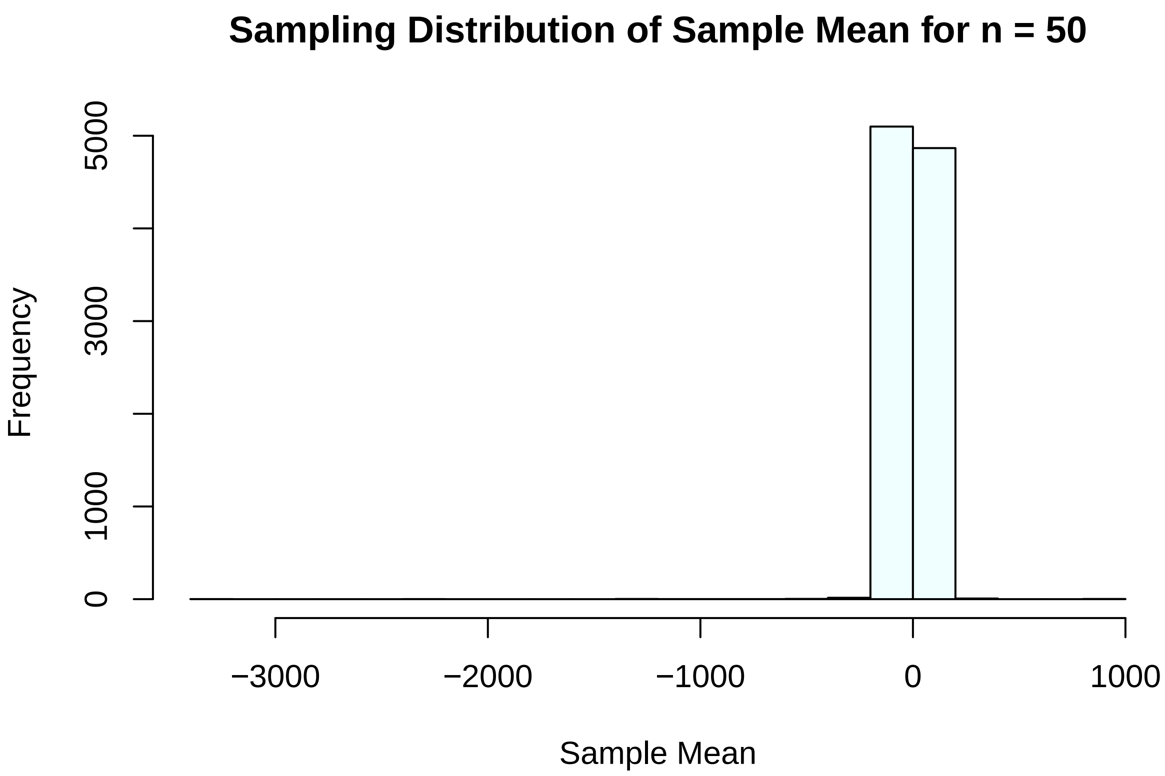
```
set.seed(1)

mc = 10000
n = 50
xbar = c()

for (i in 1:mc) {
```

```
  sample = rcauchy(n)
  xbar[i] = mean(sample)
}

hist(xbar, main = "Sampling Distribution of Sample Mean for n = 50", xlab = "Sample Mean", col = "azure
```

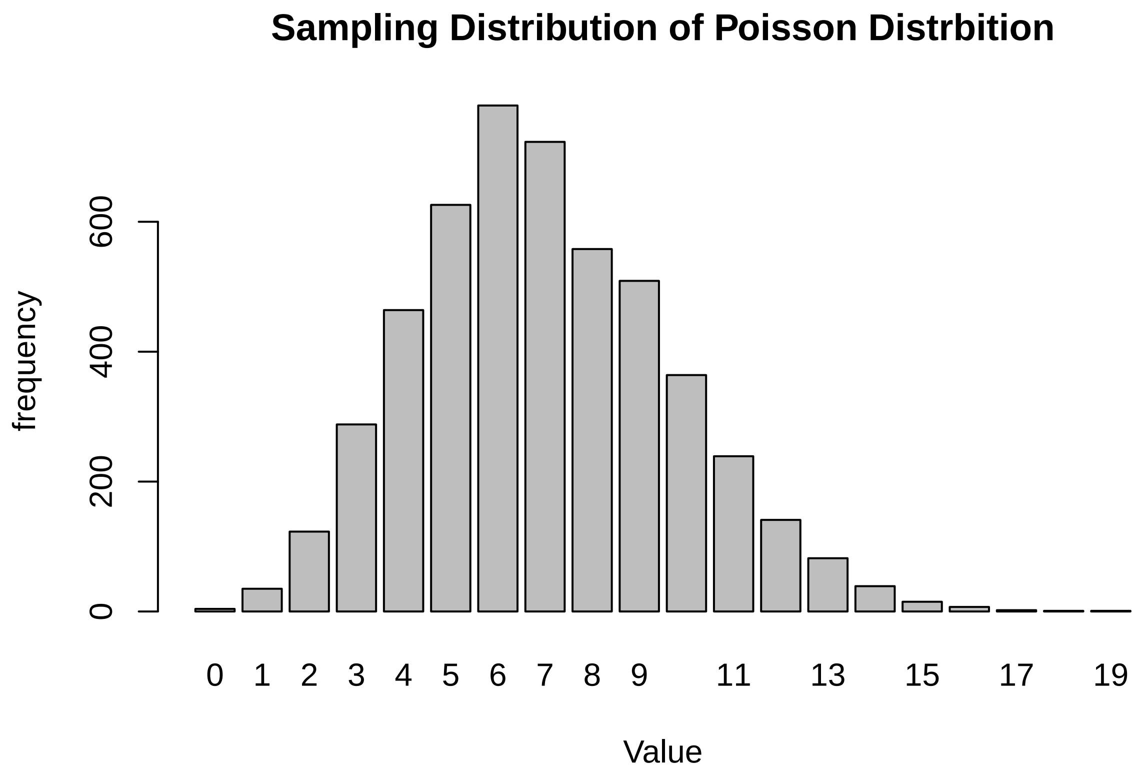


Problem 2:

Part A:

```
set.seed(1)

pois = rpois(5000, 7)
barplot(table(pois), main = "Sampling Distribution of Poisson Distrbition", xlab = "Value", ylab = "fre
```



Part B:

```
set.seed(1)

pois = rpois(5000, 7)
mean = mean(pois)
mean
```

```
## [1] 6.9784
```

```
var = var(pois)
var
```

```
## [1] 7.335401
```

Mean is closer to the true value of λ.

Part C: I believe we would choose the sample mean. The sample variance can be changed more than the mean will be by outliers.

Problem 3:

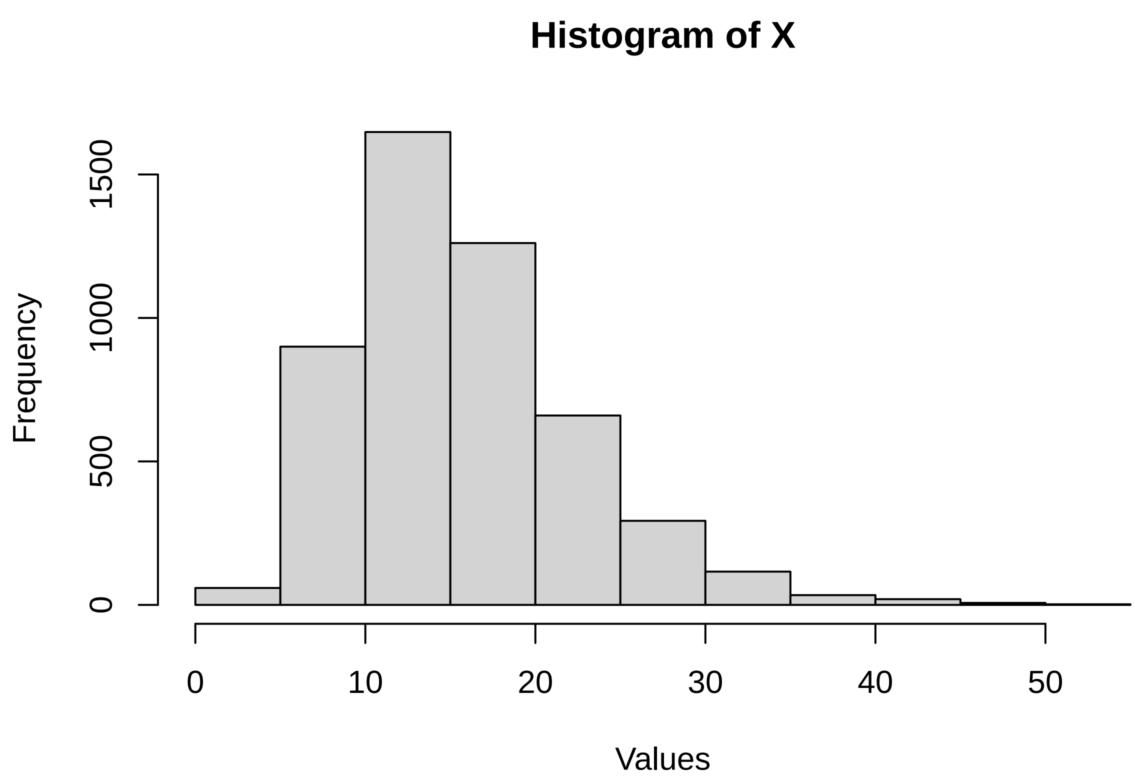
Part A: As per the random variables: $E(A) = 6, V(A) = 4, E(B) = 5, V(B) = 10, E(C) = \frac{1}{10}, V(C) = \frac{1}{100}$ To find the expectation of X, we have to do $E(X) = E(A+2B-3C) = E(A)+2E(B)-3E(C) = 6+2*5-3*\frac{1}{10} = 15.7$ To find the variance of X, we have to do $V(X) = V(A+2B-3C) = V(A)+4V(B)+9V(C) = 4+4*10+9*\frac{1}{100} = 44.09$

Part B:

```r
set.seed(1)
n=5000

A=rnorm(n,6,2)
B=rchisq(n,5)
C=rexp(n,10)
X=A+2*B-3*C
EX=mean(X)
VX=var(X)

hist(X,main="Histogram of X", xlab = "Values")
```




```r
EX
```

```
## [1] 15.75409
```

```r
VX
```

```
## [1] 44.63352
```

The expectation of X is 15.75409 which is 0.05409 above the true mean and the variance of X is 44.63352 which is 0.543532 above the variance.

Part C:

```
set.seed(1)
A=pnorm(3.5,6,2,lower.tail=TRUE)
A
```

```
## [1] 0.1056498
```

The probability that an observation from distribution A is less than 3.5 is 0.1056498.

Part D:

```
set.seed(1)
A=rnorm(5000,6,2)
lessThan=round(A<3.5)
lessThanMean=mean(lessThan)
lessThanMean
```

```
## [1] 0.1128
```

The true probability is 0.1056498 while the given probability is 0.1128 so they are pretty close. They aren't identical, but with greater and greater values of n, the distribution of A should get closer and closer to the true probability.

Part E:

```
set.seed(1)
greaterThan=round(X>10)
greaterThanMean=mean(greaterThan)
greaterThanMean
```

```
## [1] 0.8082
```

The approximate probability of observations of X greater than 10 is 0.8082.