AMATH 584 HW 4

Nate Whybra

10.1) a) Put $u = \frac{v}{\sqrt{v^* v}}$ so we can write our Householder reflector as

$$F = I - 2uu^*$$

We want to find eigenvectors q_i such that

$$Fq_i = \lambda q_i$$

for some $\lambda \neq 0$. Let $q_1 = u$, then

$$Fq_1 = Fu = u - 2u(u^* u)$$

$$= u - 2u$$

$$= -1 \cdot u$$

So u is an eigenvector of F with corresponding eigenvalue $\lambda_1 = -1$. Now consider a vector $q_2 = w$ where w is orthogonal to u. Since F is Hermitian all the eigenvectors of F should be orthogonal to each other. Then

$$Fq_2 = Fw = w - 2w(\overset{0}{\cancel{w^* w}}) \text{ b/c orthogonal}$$

$$= 1 \cdot w$$

So w is an eigenvector with corresponding eigenvalue $\lambda_2 = 1$. If F is $m \times m$, we found an eigenvector q_1 with $\lambda_1 = -1$. Since any vector orthogonal to q_1 gives an eigenvalue $\lambda_2 = 1$, since F is Hermitian it is determined that there are $m-1$ eigenvalues of value 1 (multiplicity m-1) and 1 eigenvalue with value -1, in summary

$\lambda = 1$ w/ multiplicity $m-1$

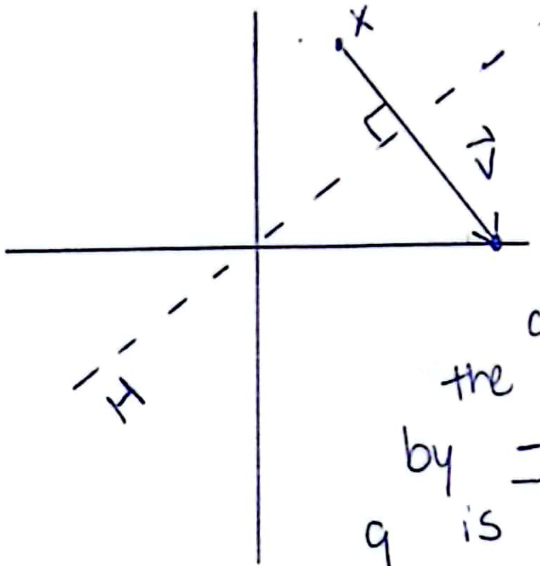
$\lambda = -1$ w/ multiplicity 1

b) For any square matrix, the determinant is the product of the eigenvalues, so

$$\det(F) = 1 \cdot (-1) = \boxed{-1}$$

c) Since F is Hermitian, the singular values are the absolute values of the eigenvalues, so in this case F has m singular values equal to 1.

a) continued:



As F maps vectors q by reflecting q across the hyperplane H orthogonal to v, if q is parallel to v, q will simply have its direction reversed, wich is the same as being multiplied by -1 (one of our eigenvalues). If q is orthogonal to v, it lies in the hyperplane H and won't get moved anywhere by F (corresponding to $\lambda = 1$).