AMATH 561 Autumn 2024
Problem Set 7

Due: Fri 11/22 at 10am

Note: Submit electronically to Canvas. Exercises 1-4 are from Matt Lorig's notes (link on course website).

1. Exercise 4.1.

2. Exercise 4.2.

3. Exercise 4.3.

4. Exercise 4.4.

5. Stationary distribution of Ehrenfest chain. (a) Let X_n be the number of balls in the left urn at time n (total number of balls in both urns is r). At each time step, one of the r balls is picked at random and moved to the other urn.

(a) Let $G_n(s)$ be the generating function of X_n. Derive a formula for G_{n+1} as a function of G_n.

(b) Let $G(s) = \lim_{n\to\infty} G_n(s)$. Use the relation in part a) to derive an equation for G. Solve it and find G.

(c) Find the stationary distribution π of Ehrenfest chain. What is the connection between G and π?