AMATH 483 / 583 (Roche) - Homework Set 2

Due Wednesday April 16, 5pm PT

April 10, 2025

Homework 2 (82 points)

1. (+10) Write a C ++ program that finds a practical measure of your machine's SP (32 bit) and DP (64 bit) floating point precision by taking the difference of 2 numbers and comparing these to zero in the same precision. What value do you obtain for $\epsilon_{\text{machine}}$ in both precisions? Hint: A loop (j) will be helpful here $1 - \left(1 + \frac{1}{2^j}\right)$ should do it.

2. (+12) What are the largest and smallest SP (32 bit) and DP (64 bit) numbers that can be represented in IEEE floating point representation? Show work in terms of sign, mantissa, and exponent.

3. (+5) Write a C++ program to multiply the integers $200 * 300 * 400 * 500$ on your computer? What is the result? Name the effect you observe.

4. (+5) Given C++ code segment below, what is the final value of *counter*?

```
unsigned int counter = 0;
for (int i = 0; i < 3; ++i) --counter;
```

5. (+10) Count and report how many IEEE SP (32 bit) normalized and denormalized floating point numbers there are. Please count and label infinities and NANs as well. Show work.

6. (+15) Consider a 6 bit floating point system with $s = 1$ (1 sign bit), $k = 3$ (3 bit exponent field), and $n = 2$ (2 bit mantissa).
   (a) Calculate by hand all the representable normalized numbers. Show work.
   (b) Calculate by hand all the representable denormalized numbers. Show work.
   (c) Plot both sets of numbers (ignoring NANs and infinities) as a number line to see the gaps of (un)representable numbers.

7. (+10) Conversions. Show work.
   (a) Write $(D3B701)_{16}$ as an integer in base-10.
   (b) Write $(1010000100111111)_2$ as an integer in base-16, i.e. as a hexadecimal number.

8. (+5) Are there $a, b, c \in \mathbb{Z}$ s.t. $6a + 9b + 15c = 107$? Show work.

9. (+10) Equivalence classes modulo n. $\forall a, b \in \mathbf{Z}$ then $a \equiv b \pmod{n}$ means $n \mid (a - b)$ or $a = b + k \cdot n$ and $k \in \mathbf{Z}$. $\mathbf{Z}_n$ is the set of equivalence classes $\{[0], [1], \ldots, [n-1]\}$. Is $(\mathbf{Z}_n, +, \cdot)$ a ring? Hint: If $s \in [i]$, then $n \mid (s - i)$. Show work (use ring properties).