AMATH 561 Autumn 2024
Problem Set 2

Due: Mon 10/14 at 10am

Note: Submit electronically to Canvas.

1. Suppose X and Y are random variables on $(\Omega, \mathcal{F}, P)$ and let $A \in \mathcal{F}$. Show that if we let $Z(\omega) = X(\omega)$ for $\omega \in A$ and $Z(\omega) = Y(\omega)$ for $\omega \in A^c$, then Z is a random variable.

2. Suppose X is a continuous random variable with distribution function F_X. Let g be a strictly increasing continuous function. Define $Y = g(X)$. a) What is F_Y, the distribution function of Y? b) What is f_Y, the density function of Y?

3. Suppose X is a continuous random variable with distribution function F_X. Find F_Y where Y is given by a) X^2 b) $\sqrt{|X|}$ c) $\sin X$ d) $F_X(X)$.

4. Let $X : [0,1] \to \mathbf{R}$ be a function that maps every rational number in the interval $[0,1]$ to 0, and every irrational number to 1. We assume that the probability space where X is defined is $([0,1], \mathcal{B}[0,1], P)$, where $\mathcal{B}[0,1]$ is the Borel σ-algebra on [0,1], and P is the Lebesgue measure.

(a) Is the set of rational numbers in [0,1] a Borel set? Show using definition of the Borel σ-algebra on $[0,1]$.

(b) Is X a random variable (and why)? If it is, what are its distribution function and expectation? Does X have a density function? Is X discrete?