The following problems, unless specifically noted, refer to the exercises in the book *Numerical Linear Algebra*, by Lloyd N. Trefethen and David Bau, III, SIAM 1997.

Homework 8

Reading: Lectures 28-31.

Problems: Exercise 28.1, 30.1, 30.3.

One additional problem:

A1. Consider the 10×10 matrix:

$$A = \begin{bmatrix} 2 & -1 & & \\ -1 & \ddots & \ddots & \\ & \ddots & \ddots & -1 \\ & & -1 & 2 \end{bmatrix}.$$

(a) What information does the Gerschgorin's theorem tell you about this matrix?
(b) Implement the power method to compute an approximation to the largest eigenvalue in magnitude and its corresponding eigenvector. Choose a random vector to start with. Various stopping criteria can be set. For example, you may check if $\|A\mathbf{v}^{(k)} - \lambda^{(k)}\mathbf{v}^{(k)}\|$ is small enough.
(c) Implement the pure QR algorithm (without shifts) to take A to a diagonal form. You may use the MATLAB `qr` to perform the QR decomposition. Again various stopping criteria can be set. For example, you may check if the off-diagonal entries are small enough.
(d) Pick the fifth eigenvalue computed in your QR algorithm and use it as the shift in the inverse iteration to find the corresponding eigenvector.