The following problems, unless specifically noted, refer to the exercises in the book *Numerical Linear Algebra*, by Lloyd N. Trefethen and David Bau, III, SIAM 1997.

Homework 4

Reading: Lectures 10-11.

Problems: Exercises 10.1, 11.3 (ignore parts (b) and (c)).

One additional problem:

A1. Implement the Algorithm 10.1 (Householder QR Factorization) on page 73 and add a part that generates the unitary matrix Q (or its reduced version $\hat{Q}$). Together with the two algorithms (classical Gram-Schmidt and modified Gram-Schmidt) from last homework, you now have three algorithms to perform the QR decomposition. Apply these three algorithms to the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{bmatrix}$$

and compute $\|Q^*Q - I\|_2$, where Q is the numerically computed Q by each method. This is to see how far/close the numerical Q is from the unitary matrix. Report the results of each method for $\varepsilon = 1, 10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}$. Comment on your observation. (This is the same problem I demonstrated in class and serves as a good test to illustrate the stability of each algorithm.)