4) We'd like to minimize

$$L = d^2 = x^2 + y^2 + z^2$$

with the constraint

$$g = xy + 2xz - 5\sqrt{5} = 0 .$$

To do so, we use the method of Lagrange multipliers, where for some λ we assume $\nabla L = \lambda \nabla g$

$$2x = \frac{z}{x}(5y) \rightarrow x^2 = \frac{5}{2}yz$$

$$\rightarrow \quad 2x = \lambda(y + 2z) \longrightarrow$$

$$2y = \lambda x \rightarrow \underline{2y \overset{*}{=} z}$$

$$2z = \lambda(2x) \rightarrow \lambda = z/x, \quad x \neq 0$$ [if $x = 0$, g is not satisfied]

continuing

$$\longrightarrow \quad x^2 = \frac{5}{\cancel{2}} y(\cancel{2}y) = 5y^2 \Big] \rightarrow \underline{x \overset{*}{=} \sqrt{5}y}$$

Plugging into g,

$$(\sqrt{5}y)y + 2(\sqrt{5}y)(2y) = 5\sqrt{5}$$

$$= \sqrt{5}y^2 + 4\sqrt{5}y^2 = 5\sqrt{5}$$

$$\rightarrow \quad 5y^2 = 5 \rightarrow \boxed{y = \pm 1}$$

From earlier

$x = \sqrt{5}y \rightarrow x = \pm\sqrt{5}$ and $z = 2y \rightarrow z = \pm 2$

Because of symmetry, any combination $\boxed{(\underset{x}{\pm\sqrt{5}}, \underset{y}{\pm 1}, \underset{z}{\pm 2})}$ will be a minimizer of d, and the minimum distance is

$$d = \sqrt{\sqrt{5}^2 + 1^2 + 2^2} = \sqrt{5+1+4} = \sqrt{10}$$

$$\rightarrow \boxed{d_{min} = \sqrt{10}}$$

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