The following problems, unless specifically noted, refer to the exercises in the book *Numerical Linear Algebra*, by Lloyd N. Trefethen and David Bau, III, SIAM 1997.

Homework 7

Reading: Lectures 25-27.

Problems: Exercise 26.1, 27.1.

Three additional problems:

A1. Prove the *Bauer-Fike theorem*: suppose $A \in \mathbb{C}^m \times \mathbb{C}^m$ is diagonalizable with $A = V\Lambda V^{-1}$, and let $\delta A \in \mathbb{C}^m \times \mathbb{C}^m$ be arbitrary. If $\tilde{\lambda}_j$ is an eigenvalue of $A + \delta A$, show that there exists an eigenvalue λ_j of A such that

$$|\lambda_j - \tilde{\lambda}_j| \leq \kappa_2(V)\|\delta A\|_2,$$

where $\kappa_2(V) = \|V\|_2\|V^{-1}\|_2$ is the condition number of matrix V. In particular, when A is normal, show that

$$|\lambda_j - \tilde{\lambda}_j| \leq \|\delta A\|_2.$$

This theorem implies that the problem of computing eigenvalues of a normal matrix is well-conditioned.

A2. Consider the two matrices

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad A_\varepsilon = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \varepsilon & 0 & 0 & 0 \end{bmatrix},$$

where ε is a small real number. What are the eigenvalues of A and A_ε? Would you say that the problem of computing the eigenvalues of the matrix A is well-conditioned or ill-conditioned?

A3. Find the Householder reflector Q and an upper Hessenberg matrix H by hand such that $Q^*AQ = H$, where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}.$$