9 Combined determination of beam waist locations, beam widths, divergence angles and beam propagation ratios

If the beam waist is accessible for direct measurement, the beam waist location, beam widths, divergence angles and beam propagation ratios shall be determined by a hyperbolic fit to different measurements of the beam width along the propagation axis z. Hence, measurements at at least 10 different z positions shall be taken. Approximately half of the measurements shall be distributed within one Rayleigh length on either side of the beam waist, and approximately half of them shall be distributed beyond two Rayleigh lengths from the beam waist. For simple astigmatic beams this procedure shall be applied separately for both principal directions.

A preliminary test for general astigmatism shall be applied to the measured data. For each measured profile, the beam widths $d_{\sigma x}$ and $d_{\sigma y}$ and the azimuth angle, φ with respect to the laboratory system shall be calculated. If the difference in the azimuth angle of any two non-circular profiles is greater than 10° the beam shall be considered as general astigmatic and ISO 11146-2 shall be applied.

NOTE 1 Failure of this test is not proof of stigmatism or simple astigmatism. The beam may suffer from hidden general astigmatism, which can be detected by the procedures given in ISO 11146-2.

The hyperbolic fit to the measured diameters, d_σ along the propagation distance, z, can be expressed in the following way:

$$d_\sigma(z) = \sqrt{a + bz + cz^2} \tag{24}$$

The coefficients a, b, c (or a_x, a_y, b_x, b_y, c_x, c_y) of the hyperbola(e) shall be determined by appropriate numerical or statistical curve-fitting techniques (see Notes 2 and 3). The values of the beam propagation parameters can be obtained using:

$$z_0 = \frac{-b}{2c} \tag{25}$$

$$d_{\sigma 0} = \frac{1}{2\sqrt{c}} \sqrt{4ac - b^2} \tag{26}$$

$$\Theta_\sigma = \sqrt{c} \tag{27}$$

$$z_R = \frac{1}{2c}\sqrt{4ac - b^2} \tag{28}$$

$$M^2 = \frac{\pi}{8\lambda}\sqrt{4ac - b^2} \tag{29}$$

NOTE 2 If more than one diameter measurement is performed at each z position, it is advisable to weight the data points in an inversely proportional manner to the variance of the data points.

NOTE 3 The fit should be performed by minimizing the sum of the squared relative deviations of the diameters.

NOTE 4 Astigmatic waist separation Δz_a, which is also known as astigmatic difference, is given by

$$\Delta z_a = \left| z_{0x} - z_{0y} \right|$$

[see 3.3.4 of ISO 15367-1:2003].