Unit XXVI Assignment I

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Contents

1 Data

The following data is used in PII, PIII and PIV:

$$M = \begin{pmatrix} 3 & -1 \\ 4 & 2 \end{pmatrix} \tag{1}$$

$$N = \begin{pmatrix} 4 & 3 \\ -3 & -1 \end{pmatrix} \tag{2}$$

$$P = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \\ -3 & 4 & 3 \end{pmatrix} \tag{3}$$

$$Q = \begin{pmatrix} 2 & 3 & 3 \\ 4 & 4 & -2 \\ 3 & -4 & 8 \end{pmatrix} \tag{4}$$

$$R = \begin{pmatrix} 9 & 2 & 6 \\ 12 & -4 & 7 \end{pmatrix} \tag{5}$$

$$S = \begin{pmatrix} -6 & 3 \\ -3 & -2 \\ -6 & 6 \end{pmatrix} \tag{6}$$

2 Understanding Matrices and How Matrices Can Be Used To Represent Ordered Data

2.1 Overview

A Matrix is a way of displaying data in an ordered format. Matrices are in a rectanguar format with cells comprised of rows and columns. Matrices can be used with one another to add, subtract and multiply. When writing out a matrix calculation, regular mathematical symbols are used, except for the full stop symbol (.), which is used for multiplcation of matrices. When multipling matrices, the order of which matrix comes first is key. A . B is not the same as B . A.

2.2 Order

The order of a matrix is very important. A matrix with the numbers

$$\begin{pmatrix} 3 & 6 \\ 9 & 5 \end{pmatrix} \tag{7}$$

will have a different outcome if manipulated with another number than a matrix with the numbers

$$\begin{pmatrix} 6 & 3 \\ 5 & 9 \end{pmatrix} \tag{8}$$

This means that if the order of any individual number is changed, the whole calculation could be invalidated.

2.3 Indecies

Indexes of matrices are selected subsections of a matrix. For instance, a 3x3 matrix may be like this;

$$\begin{pmatrix} 9 & 2 & 8 \\ 3 & 1 & 4 \\ 7 & 6 & 5 \end{pmatrix} \tag{9}$$

But an index of the matrix would be only a small group, such as this 2x2 subsection.

$$\begin{pmatrix} 3 & 1 \\ 7 & 6 \end{pmatrix} \tag{10}$$

2.4 Real World Applications

Matrices can be used in the real world in many different applications. One use of matrixes in the real world is the traits of a population of people, webpage rankings and cryptography. Without matrices, many real world applications would be hindered.

3 Adding and Subtracting Matrices

The following are the questions that need to be answered:

1. M + N
2. P + Q
3. M - N
4. 3P
5. 3P - 2Q

The following are my answers to the question, with working out added to them as an intermediate step.

3.1 M + N

$$M + N = \begin{pmatrix} 3 & -1 \\ 4 & 2 \end{pmatrix} + \begin{pmatrix} 4 & 3 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} 3+4 & -1+3 \\ 4+-3 & 2+-1 \end{pmatrix} = \begin{pmatrix} 7 & 2 \\ 1 & 1 \end{pmatrix} \quad (11)$$

3.2 P + Q

$$P+Q = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \\ -1 & 4 & 3 \end{pmatrix} + \begin{pmatrix} 2 & 3 & 3 \\ 4 & 4 & -2 \\ 3 & -4 & 8 \end{pmatrix} = \begin{pmatrix} 1+2 & 3+3 & 5+3 \\ -1+4 & 2+4 & 4+-2 \\ -3+3 & 4+-4 & 3+8 \end{pmatrix} = \begin{pmatrix} 3 & 6 & 8 \\ 3 & 6 & 2 \\ 0 & 0 & 11 \end{pmatrix} \quad (12)$$

3.3 M - N

$$M-N = \begin{pmatrix} 3 & -1 \\ 4 & 2 \end{pmatrix} - \begin{pmatrix} 4 & 3 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} 3-4 & -1-3 \\ 4--3 & 2--1 \end{pmatrix} = \begin{pmatrix} 3-4 & -1-3 \\ 4+3 & 2+1 \end{pmatrix} = \begin{pmatrix} -1 & -4 \\ 7 & 3 \end{pmatrix} \quad (13)$$

3.4 3P

$$3P = 3 \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \\ -3 & 4 & 3 \end{pmatrix} = \begin{pmatrix} 3(1) & 3(3) & 3(5) \\ 3(-1) & 3(2) & 3(4) \\ 3(-3) & 3(4) & 3(3) \end{pmatrix} = \begin{pmatrix} 3 & 9 & 15 \\ -3 & 6 & 12 \\ -9 & 12 & 9 \end{pmatrix} \quad (14)$$

3.5 3P - 2Q

Due to the fact that I have already calculated 3P, I shall now only calculate 2Q and then add them together at the end.

$$2Q = 2\begin{pmatrix} 2 & 3 & 3 \\ 4 & 4 & -2 \\ 3 & -4 & 8 \end{pmatrix} = \begin{pmatrix} 2(2) & 2(3) & 2(3) \\ 2(4) & 2(4) & 2(-2) \\ 2(3) & 2(-4) & 2(8) \end{pmatrix} = \begin{pmatrix} 4 & 6+6 & \\ 8 & 8 & -4 \\ 6 & -8 & 16 \end{pmatrix} \tag{15}$$

Now I will perform 3P - 2Q now that I have calculated 2Q.

$$\begin{pmatrix} 3 & 9 & 15 \\ -3 & 6 & 12 \\ -9 & 12 & 9 \end{pmatrix} - \begin{pmatrix} 4 & 6 & 6 \\ 8 & 8 & -4 \\ 6 & -8 & 16 \end{pmatrix} = \begin{pmatrix} 3-4 & 9-6 & 15-6 \\ -3-8 & 6-8 & 12--4 \\ -9-6 & 12--8 & 9-16 \end{pmatrix} = \begin{pmatrix} -1 & 3 & 9 \\ -11 & -2 & 16 \\ -15 & 20 & -7 \end{pmatrix} \tag{16}$$

4 Multiplying Matrices

The following are the questions that need to be answered:

1. M × N
2. P × Q
3. R × S
4. S × R

The following are my answers to the questions, along with the working out added to then as an intermediate step.

4.1 M × N

$$\begin{pmatrix} 3 & -1 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} (3 \times 4) + (-1 \times -3) & (3 \times 3) + (-1 \times -1) \\ (4 \times 4) + (2 \times -3) & (4 \times 3) + (2 \times -1) \end{pmatrix} \tag{17}$$

$$= \begin{pmatrix} 12 + 3 & 9 + 1 \\ 16 + -6 & 12 + -2 \end{pmatrix} = \begin{pmatrix} 15 & 10 \\ 10 & 10 \end{pmatrix} \tag{18}$$

4.2 P × Q

$$\begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \\ -3 & 4 & 3 \end{pmatrix} \begin{pmatrix} 2 & 3 & 3 \\ 4 & 4 & -2 \\ 3 & -4 & 8 \end{pmatrix} = \tag{19}$$

$$\begin{pmatrix} (1 \times 2) + (3 \times 4) + (5 \times 3) & (1 \times 3) + (3 \times 4) + (5 \times -4) & (1 \times 3) + (3 \times -2) + (5 \times 8) \\ (-1 \times 2) + (2 \times 4) + (4 \times 3) & (-1 \times 3) + (2 \times 4) + (4 \times -4) & (-1 \times 3) + (2 \times -2) + (4 \times 8) \\ (-3 \times 2) + (4 \times 4) + (3 \times 3) & (-3 \times 3) + (4 \times 4) + (3 \times -4) & (-3 \times 3) + (4 \times -2) + (3 \times 8) \end{pmatrix} \tag{20}$$

$$= \begin{pmatrix} 2 + 12 + 15 & 3 + 12 + -20 & 3 + -6 + 40 \\ -2 + 8 + 12 & -3 + 8 + -16 & -3 + -4 + 32 \\ -6 + 16 + 9 & -9 + 16 + -12 & -9 + -8 + 24 \end{pmatrix} = \begin{pmatrix} 29 & -5 & 37 \\ 18 & -11 & 25 \\ 19 & -6 & 7 \end{pmatrix} \tag{21}$$

4.3 R × S

$$\begin{pmatrix} 9 & 2 & 6 \\ 12 & -4 & 7 \end{pmatrix} \begin{pmatrix} -6 & 3 \\ -3 & -2 \\ -6 & 6 \end{pmatrix} \tag{22}$$

$$= \begin{pmatrix} (9 \times -6) + (2 \times -3) + (6 \times -6) & (9 \times 3) + (2 \times -2) + (6 \times 6) \\ (12 \times -6) + (-4 \times -3) + (7 \times -6) & (12 \times 3) + (-4 \times -2) + (7 \times 6) \end{pmatrix} \tag{23}$$

$$= \begin{pmatrix} 54 + -6 + -36 & 27 + -4 + 36 \\ -72 + 12 + -42 & -36 + 8 + 42 \end{pmatrix} = \begin{pmatrix} -96 & 59 \\ -102 & -86 \end{pmatrix} \tag{24}$$

4.4 S × R

$$\begin{pmatrix} -6 & 3 \\ -3 & -2 \\ -6 & 6 \end{pmatrix} \begin{pmatrix} 9 & 2 & 6 \\ 12 & -4 & 7 \end{pmatrix} \tag{25}$$

$$= \begin{pmatrix} (-6 \times 9) + (3 \times 12) & (-6 \times 2) + (3 \times -4) & (-6 \times 6) + (3 \times 7) \\ (-3 \times 9) + (-2 \times 12) & (-3 \times 2) + (-2 \times -4) & (-3 \times 6) + (-2 \times 7) \\ (-6 \times 9) + (6 \times 12) & (-6 \times 2) + (6 \times -4) & (-6 \times 6) + (6 \times 7) \end{pmatrix} \tag{26}$$

$$= \begin{pmatrix} -54 + 48 & -12 + -12 & -36 + 21 \\ -27 + -24 & -6 + 8 & -18 + -14 \\ -54 + 72 & -12 + -24 & -36 + 42 \end{pmatrix} = \begin{pmatrix} -18 & -24 & -15 \\ -51 & 2 & -32 \\ 18 & -36 & 6 \end{pmatrix} \tag{27}$$

5 Inverse and Transpose

5.1 Inverse

When generating an Inverse Matrix, there are many steps to follow that can make the process seem difficult as they all need to be performed in the right in order to ensure that the calculations are done correctly. Fortunately, I shall show how to perform an Inverse function by breaking it down into simple steps. The first method I shall show is a general rule of thumb for generating matrices.

I shall start off this explaination by naming some terms and definining them. An inverse square is one where the original and an inverse of that are multiplied with one another to get the answer which will give you another matrix. This tertiary matrix is called an indentity matrix due to the fact that it contains nothing but zeros, apart from a diagonal line going from the top left corner down to the bottom right corner of the matrix. The following is an example of an identity matrix:

$$\begin{pmatrix} 1 & 0 & 0 & & 0 \\ 0 & 1 & 0 & & 0 \\ 0 & 0 & 1 & & 0 \\ & & & & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{28}$$

for the 2x2 matrix.

The following are the questions that need to be answered:

1. M^-1
2. N^-1
3. P^-1
4. Q^-1
5. M^T
6. P^T
7. R^T

The following are my answers to the questions, along with the working out added to them as an intermediate step.

For reference, when calculating the inverse of a matrix you need to calculate the inverse of each individual element.

5.2 $\mathbf{M^{-}1}$

$$A = \begin{pmatrix} 3 & -1 \\ 4 & 2 \end{pmatrix} \tag{29}$$