

Internship report

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Abstract

It's possible to distribute the Internet to users via drones. However, this raises the question of how to place the drones around the users, and how to distribute the bandwidth between the different users. A reinforcement AI has already been designed to address this problem. However, in this article, we will see how learning and optimization can be combined to further improve performance.

1 Problem presentation

We have m users, classified into 3 categories, each class having its own bandwidth demand for a drone. We then want to place n drones, and for each drone, decide how much of its bandwidth it gives to each class of drone, so that as many drones as possible are satisfied, i.e. the bandwidth available to them is greater than or equal to their demand.

2 Problem solving by constrained optimization

2.1 In-depth explanation

This problem can be solved by constrained optimization.

First, we have to define the equation that computes how well a user will receive a base station's connection depending on where it is.

We have the following Equations for the drones's SINRs, which is a factor that represents how much data is effectively received by the user, in relation to the data sent by the base station to the user:

$$\text{SINR}_{i,j}^t = \frac{pc\mu(y_j, u_i^t) \left((\|y_j - u_i^t\|)^2 + (h)^2 \right)^{-\alpha/2}}{\sum_{k \in \mathcal{U} \setminus i} pc\mu(y_j, u_k^t) \left((\|y_j - u_k^t\|)^2 + (h)^2 \right)^{-\alpha/2} + \sigma^2}$$

To evaluate a configuration we will thus proceed as follows:

You first need to associate each user with its nearest base station.

You then share the bandwidth of a base station dedicated to a given class between all users of this class associated to this base station.

Then, you compute each user SINR and decide for the user is satisfied or not.

The percentage of satisfied user is thus what you want to maximize.

2.2 Optimization problem

This problem can be resolved by as an optimization problem, I will show you how to resolve it for 2 base stations.

Let's first define u a matrix of dimensions $N * N$, with N being the number of positions possible for a base station. $u_{i,j}$ is a binary variable that is equal to one if and only if the first base station is in position i and the second is position j .

bw is a $3 * 2$ matrix that represents the bandwidth each base station allocates to each user class.

δ is a vector that represents for every whether it is satisfied or not.

The function to optimize is then: $\max_{u, bw} \sum_{g \in \mathcal{G}} \delta_g$

Under the following constraints:

$$1. \sum_{b \in |\mathcal{U}|} \sum_{i,j \in \mathcal{U}} u_{i,j} \times bw_{\text{slice}(g),b} \times \text{bps}_g \frac{100}{G_{\text{conn}}(\text{slice}(g),b)} \geq th_g \times \delta_g \quad \forall g \in \mathcal{G}$$

$$2. bw_{em,b} + bw_{ur,b} + bw_{mm,b} = 1 \quad \forall b \in |\mathcal{U}|$$

$$3. bw_{em,b} \geq 0; bw_{ur,b} \geq 0; bw_{mm,b} \geq 0 \quad \forall b \in |\mathcal{U}|$$

$$4. \sum_{i,j \in \mathcal{U}} u_{i,j} = 1$$

$$5. \delta_g \in \{0, 1\} \quad \forall g \in \mathcal{G}$$

$$6. u_{i,j} \in \{0, 1\} \quad \forall i, j \in \{0, 1, \dots, |\mathcal{U}| - 1\}$$

The resolution of this optimization problem can be implemented in Python with the help of a module such as pulp.

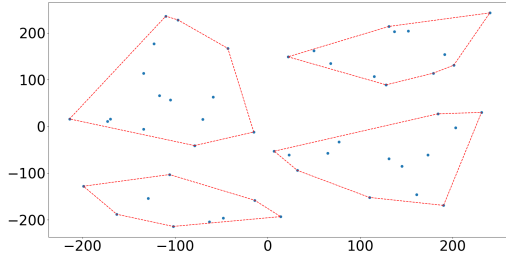
2.3 Time Optimizations

To speed up the resolution of this problem, I implemented a few optimizations:

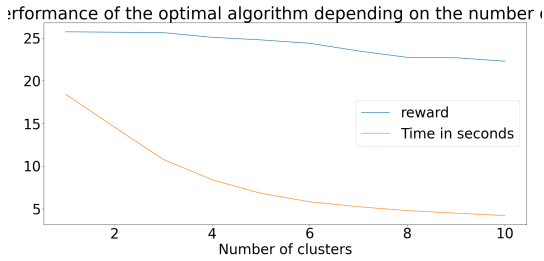
First, only positions in the convex envelope of the users can actually be candidates for optimal placements of the base stations. We can thus reduce by a lot the size of the matrix u .

Moreover, it is possible to go further but not without losing precision. Indeed, you can divide the users into k clusters and take the union of this convex envelope of this clusters.

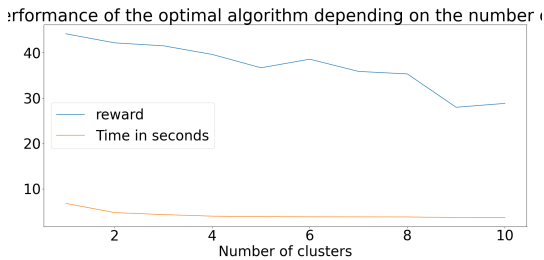
You can visualize the transformation done as follows:



The graph of performance and time in function of the number of clusters can be found below.



Moreover, I am using *Quote article here* to generate instances, thus as the instances generated have as many clusters as they have base stations, I am going to generate instances using clusters too. The performances are then as follows:



However, since the computation times are pretty reasonable for 2 base stations, I decided to go with only 1 cluster.

Finally, multithreading can be used to further accelerate the computations.

3 Machine learning to solve the problem

However, a major issue is that this method of solving the problem by optimization is still pretty long. This can be problematic, especially if the users are moving, and it thus becomes necessary to recalculate the optimal position often.

Thus, a solution using machine learning was considered. Indeed, solving an instance of the problem would only require a forward pass of the neural network, which is negligible in time.

3.1 Reinforcement learning

This is why a reinforcement learning solution was imagined and implemented by Lorenzo Bellone. *Quote article*

He tried to solve the same problem as the one I presented and, to put it shortly, managed to get an average reward of 72-73% of users satisfied.

However, there are multiple differences between how he approached this problem and how I did:

1. First, he moves the base stations setp by step and computes the mean reward over 100 movements. However, since the base stations spend most of their times idling (because they have already reached their best possible position), I took the bias of putting the base stations directly at the computed best position, and then computing the reward only once (which also allows to test the different agents over more instances in the same amount of time).
2. Secondly, what is plotted in his article is the average reward during the reinforcement process. What it means is that the neural network is only tested on the instances it just learnt from. What I did instead is I have a separate test set, which I use to evaluate the different agents.

3.2 Supervised learning from the optimal solution

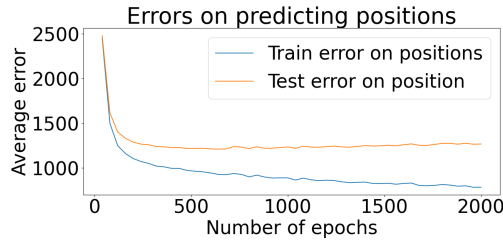
But, an other idea is to use supervised learning. By using an optimal agent, the neural network can learn to replicate the optimal solutions.

The neural network is then divided in 2 neural networks:

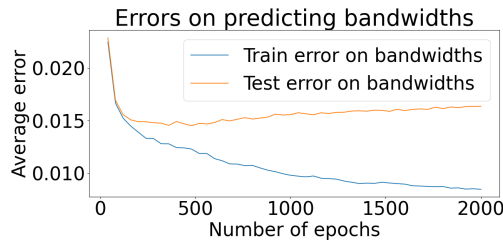
1. A neural network that takes as inputs the positions of the users and learns to output the best positions for the 2 base stations.
2. A neural network that takes as inputs the positions of the users and the positions of the base stations and learns to output the best slicing of bandwidths between the 3 classes of users.

As a side note, the type of each users is represented by 3 one-hots, one for each of the class.

To conclude with my results, here is the average error for the neural network that learns to output the positions:

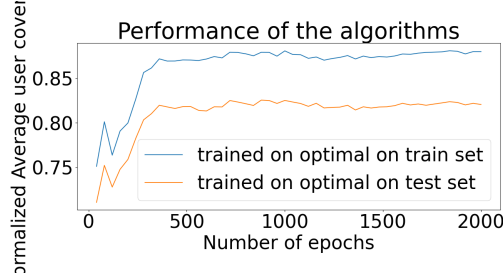


And here is the average error for the neural network that learns to output the bandwidths:



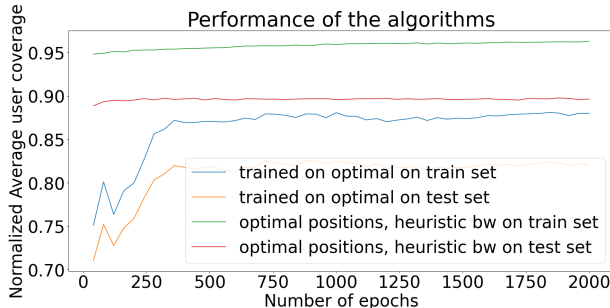
With these neural networks, we can test it on instances and evaluate the performances it gives.

(We plot here the normalized average user coverage, i.e. the average user coverage divided by the average user coverage of the optimal solution)



3.3 Learn only what's necessary

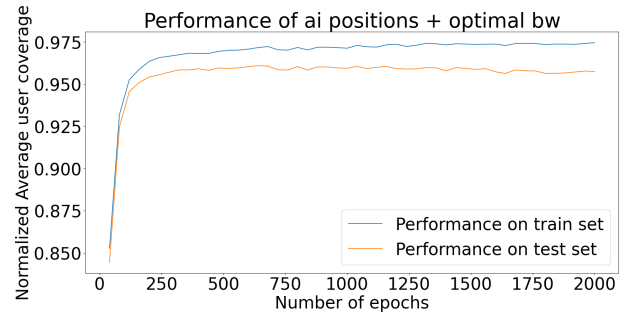
However, you can see the test error on bandwidths stops to decrease rapidly. That's why I decided to plot the performances when the positions are decided optimally, but the bandwidths are decided with the AI.



As we can see, the bandwidth part seems to be the issue.

Thus, I implemented an agent which decides positions with a neural network, but decides the bandwidths optimally, which is pretty fast once the positions are fixed.

We then have the following results:



To do a complete comparison of the different methods, I computed the mean rewards and time to execute of the 3 methods quoted until now:

Type of agent	Normalized performance	E
AI positions and bandwidths	cell5	
AI positions and optimal bandwidths	cell8	
Optimal positions and bandwidths	cell5	

3.4 Generalizing to a variable number of users

However, until now, I have only resolved instances with a fix number of users.

To fix this, the first solution we imagined was to use graph neural networks. *Detail how gnn work*

The graph given to the neural network was constructed using a k-neighbour algorithm. I have made a graphic plotting the final test error in function of the k in the k-neighbour algorithm.

However, as you can see, whatever the chosen k is, the results are always lacking.

I thus opted for a simpler but less beautiful approach: I just add a one-hot to each user stating if the drone exists or not. Thus, the neural network can be used to resolve instances with a variable number of users, as long as the number in question is inferior to a certain maximum.

The precision is very similar to what it used to be with a fix number of users as you can see in the following graphics:

Put some graphics here

4 Ouverture: Utiliser la distance à l'optimum comme erreur pour l'apprentissage

5 Conclusion