3.3.5 Ejercicios MM1

2) a) $R^i_j = \begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix}$ $\quad g^{ij} = g_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$\Rightarrow S^i_j = \frac{1}{2}\left(R^i_j + R^j_i\right) = \frac{1}{2}\left[\begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} + \begin{pmatrix} 1/2 & 2 & 7/2 \\ 1 & 5/2 & 4 \\ 3/2 & 3 & 9/2 \end{pmatrix}\right]$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 3 & 5 \\ 3 & 5 & 7 \\ 5 & 7 & 9 \end{pmatrix}$$

$$\Rightarrow A^i_j = \frac{1}{2}\left(R^i_j - R^j_i\right) = \frac{1}{2}\left[\begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} - \begin{pmatrix} 1/2 & 2 & 7/2 \\ 1 & 5/2 & 4 \\ 3/2 & 3 & 9/2 \end{pmatrix}\right]$$

$$= \frac{1}{2}\begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}$$

b) $g_{ik} R^i_j = A_{kj} = g_{1k} R^1_j + g_{2k} R^2_j + g_{3k} R^3_j$

$$\Rightarrow A_{1j} = \begin{cases} A_{11} = g_{11} R^1_1 + g_{21} R^2_1 + g_{31} R^3_1 = 1/2 \\ A_{12} = g_{11} R^1_2 + g_{21} R^2_2 + g_{31} R^3_2 = 1 \\ A_{13} = g_{11} R^1_3 + g_{21} R^2_3 + g_{31} R^3_3 = 3/2 \end{cases}$$

$$A_{2j} = \begin{cases} A_{21} = g_{12}R^1_1 + g_{22}R^2_1 + g_{32}R^3_1 = -2 \\ A_{22} = g_{12}R^1_2 + g_{22}R^2_2 + g_{32}R^3_2 = -5/2 \\ A_{23} = g_{12}R^1_3 + g_{22}R^2_3 + g_{32}R^3_3 = -3 \end{cases}$$

$$A_{3j} = \begin{cases} A_{31} = g_{13}R^1_1 + g_{23}R^2_1 + g_{33}R^3_1 = 7/2 \\ A_{32} = g_{13}R^1_2 + g_{23}R^2_2 + g_{33}R^3_2 = 4 \\ A_{33} = g_{13}R^1_3 + g_{23}R^2_3 + g_{33}R^3_3 = 9/2 \end{cases}$$

$R_{kj} = g_{ik}R^i_j \rightarrow$ se podría decir que al hacer la contracción, R hereda las propiedades de g_{ij} (cambio de signo en la segunda fila)

$$B^{ki} = g^{1k}R^i_1 + g^{2k}R^i_2 + g^{3k}R^i_3$$

$$B^{1i} = \begin{cases} B^{11} = g^{11}R^1_1 + g^{21}R^1_2 + g^{31}R^1_3 = 1/2 \\ B^{12} = g^{11}R^2_1 + g^{21}R^2_2 + g^{31}R^2_3 = 2 \\ B^{13} = g^{11}R^3_1 + g^{21}R^3_2 + g^{31}R^3_3 = 7/2 \end{cases}$$

$$B^{2i} = \begin{cases} B^{21} = g^{22} R^1_2 = -1 \\ B^{22} = g^{22} R^2_2 = -5/2 \\ B^{23} = g^{22} R^3_2 = -4 \end{cases}$$

$$B^{3i} = \begin{cases} B^{31} = g^{33} R^1_3 = 3/2 \\ B^{32} = g^{33} R^2_3 = 3 \\ B^{33} = g^{33} R^3_3 = 9/2 \end{cases}$$

R^{ki} es la traspuesta de R^i_j y se le aplica tambien el negativo, pero esta vez en vez de la segunda fila, la segunda columna

$$g_{ij} \tau^i = c_j = g_{1j} \tau^1 + g_{2j} \tau^2 + g_{3j} \tau^3$$

$$\Rightarrow c_1 = g_{11} \tau^1 + g_{21} \tau^2 + g_{31} \tau^3 = 1/3$$

$$c_2 = g_{12} \tau^1 + g_{22} \tau^2 + g_{32} \tau^3 = -2/3$$

$$c_3 = g_{13} \tau^1 + g_{23} \tau^2 + g_{33} \tau^3 = 1$$

$(1/3, -2/3, 1)$ la volbea y le pone negativo al segundo término

c) $R^i_j \tau_i = D_j = R^1_j \tau_1 + R^2_j \tau_2 + R^3_j \tau_3$

$\Rightarrow D_1 = R^1_1 \tau_1 + R^2_1 \tau_2 + R^3_1 \tau_3 = 5$

$D_2 = R^1_2 \tau_1 + R^2_2 \tau_2 + R^3_2 \tau_3 = 6$

$D_3 = R^1_3 \tau_1 + R^2_3 \tau_2 + R^3_3 \tau_3 = 7$

$\Rightarrow D_j = (5, 6, 7)$

$R^i_j \tau^j = E^i = R^i_1 \tau^1 + R^i_2 \tau^2 + R^i_3 \tau^3$

$$\left.\begin{aligned} E^1 &= R^1_1 \tau^1 + R^1_2 \tau^2 + R^1_3 \tau^3 = \tfrac{7}{3} \\ E^2 &= R^2_1 \tau^1 + R^2_2 \tau^2 + R^2_3 \tau^3 = \tfrac{16}{3} \\ E^3 &= R^3_1 \tau^1 + R^3_2 \tau^2 + R^3_3 \tau^3 = \tfrac{25}{3} \end{aligned}\right\} E^i = \frac{1}{3}\begin{pmatrix} 7 \\ 16 \\ 25 \end{pmatrix}$$

$R^i_j \tau_i \tau^j = R^1_j \tau_1 \tau^j + R^2_j \tau_2 \tau^j + R^3_j \tau_3 \tau^j = F$

$\Rightarrow F = R^1_1 \tau_1 \tau^1 + R^1_2 \tau_1 \tau^2 + R^1_3 \tau_1 \tau^3 + R^2_1 \tau_2 \tau^1 + \ldots$

$\ldots + R^2_2 \tau_2 \tau^2 + R^2_3 \tau_2 \tau^3 + R^3_1 \tau_3 \tau^1 + R^3_2 \tau_3 \tau^2 + R^3_3 \tau_3 \tau^3$

$= \frac{38}{3}$

$\frac{273}{2}$ $\frac{273}{4}$

d) $R^i_j S^j_i = G = R^1_j S^j_1 + R^2_j S^j_2 + R^3_j S^j_3$

$= R^1_1 S^1_1 + R^1_2 S^2_1 + R^1_3 S^3_1 + R^2_1 S^1_2 + R^2_2 S^2_2 + R^2_3 S^3_2 + \cdots$

$\cdots + R^3_1 S^1_3 + R^3_2 S^2_3 + R^3_3 S^3_3 = \frac{273}{4}$

$$R^i_j A^j_i = \begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} \cdot \begin{pmatrix} 0 & -\frac{1}{2} & -1 \\ 1/2 & 0 & -1/2 \\ 1 & 1/2 & 0 \end{pmatrix} = \frac{-1}{2} - \frac{3}{2} + 1 - \frac{3}{2} + \frac{7}{2} + 2 = 3$$

$A^j_i \tau^i = H^j = A^j_1 \tau^1 + A^j_2 \tau^2 + A^j_3 \tau^3$

$$\Rightarrow \left.\begin{aligned} H^1 &= A^1_1 \tau^1 + A^1_2 \tau^2 + A^1_3 \tau^3 = -\frac{4}{3} \\ H^2 &= A^2_1 \tau^1 + A^2_2 \tau^2 + A^2_3 \tau^3 = -\frac{1}{3} \\ H^3 &= A^3_1 \tau^1 + A^3_2 \tau^2 + A^3_3 \tau^3 = \frac{2}{3} \end{aligned}\right\} \frac{1}{3}(-4,-1,2) = H^i$$

$\tau^i A^j_i \tau_j = 0$

e) $R^i_j - 2\delta^i_j R^l_l \Rightarrow$ Como $\delta^i_j = \begin{cases} 1 & \text{si } i = j \\ 0 & \text{si } i \neq j \end{cases}$

$\delta^i_j R^l_l = \delta^1_j R^l_l + \delta^2_j R^l_l + \delta^3_j R^l_l = C^i_l$

$$\Rightarrow \delta^1_1 R^l_l = \begin{bmatrix} \delta^1_1 R^1_1 = 1/2 \\ \delta^1_1 R^2_2 = 5/2 \\ \delta^1_1 R^3_3 = 9/2 \end{bmatrix} \rightarrow C^1_l$$

$$C^2_l = \delta^2_2 R^l_l = \begin{cases} \delta^2_2 R^1_1 = 1/2 \\ \delta^2_2 R^2_2 = 5/2 \\ \delta^2_2 R^3_3 = 9/2 \end{cases}$$

$$C^3_l = \delta^3_3 R^l_l = \begin{cases} \delta^3_3 R^1_1 = 1/2 \\ \delta^3_3 R^2_2 = 5/2 \\ \delta^3_3 R^3_3 = 9/2 \end{cases}$$

$$\Rightarrow C^j_l = \begin{pmatrix} 1/2 & 5/2 & 9/2 \\ 1/2 & 5/2 & 9/2 \\ 1/2 & 5/2 & 9/2 \end{pmatrix} \Rightarrow R^i_j - 2C^i_l$$

$$\Rightarrow \begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} - \begin{pmatrix} 1 & 5 & 9 \\ 1 & 5 & 9 \\ 1 & 5 & 9 \end{pmatrix} = \begin{pmatrix} \frac{-1}{2} & -4 & \frac{-15}{2} \\ 1 & -5/2 & -6 \\ 5/2 & -1 & \frac{-9}{2} \end{pmatrix}$$

$$\left(R^i_j - 2\delta^i_j R^l_l\right) t_i = \begin{pmatrix} \frac{-1}{2} & -4 & \frac{-15}{2} \\ 1 & \frac{-5}{2} & -6 \\ 5/2 & -1 & -9/2 \end{pmatrix} \begin{pmatrix} 1/3 \\ 2/3 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{-31}{3} \\ \frac{-22}{3} \\ \frac{-13}{3} \end{pmatrix}$$

$$\left(R^i_j - 2\delta^i_j R^l_l\right) t_i t^j = \begin{pmatrix} \frac{-31}{3} \\ \frac{-22}{3} \\ \frac{-13}{3} \end{pmatrix} (1/3, 2/3, 1)$$

$$\Big(\frac{-31}{} \quad \frac{-62}{} \quad -31$$

$$\frac{}{3}$$
$$\frac{-13}{3}$$

$$= \frac{1}{3}\begin{pmatrix} \frac{-31}{3} & \frac{-62}{3} & -31 \\ \frac{-22}{3} & \frac{-44}{9} & \frac{-22}{3} \\ \frac{-13}{3} & \frac{-26}{3} & -13 \end{pmatrix}$$