EJERCICIO 5.

Suponga el operador $\mathbb{L} = \mathbb{L}_- \mathbb{L}_+$ con $[\mathbb{L}_-, \mathbb{L}_+] = \mathbb{I}$

$\rightarrow$ Hipótesis: $\mathbb{L}|x\rangle = \lambda|x\rangle \wedge |y\rangle = \mathbb{L}_+|x\rangle$

Tesis: $\mathbb{L}|y\rangle = (\lambda+1)|y\rangle = \lambda|y\rangle + |y\rangle$

Demostración: de $\mathbb{L}|x\rangle$ se tiene que $\mathbb{L}$ trasforma multiplicando por un λ el elemento

Luego $\mathbb{L}|y\rangle = \mathbb{L}_-\mathbb{L}_+|y\rangle = (\mathbb{L}_-\mathbb{L}_+ - \mathbb{L}_+\mathbb{L}_- + \mathbb{L}_+\mathbb{L}_-)|y\rangle$
$= ([\mathbb{L}_-, \mathbb{L}_+] + \mathbb{L}_+\mathbb{L}_-)|y\rangle = (\mathbb{I} + \mathbb{L}_+\mathbb{L}_-)|y\rangle$
$= \mathbb{I}|y\rangle + \mathbb{L}_+\mathbb{L}_-|y\rangle = |y\rangle + \mathbb{L}|y\rangle = |y\rangle + \lambda|y\rangle$
$= (\lambda+1)|y\rangle$

$\rightarrow$ Hipótesis: $\mathbb{L}|x\rangle = \lambda|x\rangle \wedge |z\rangle = \mathbb{L}_-|x\rangle$

Tesis: $\mathbb{L}|z\rangle = (\lambda-1)|z\rangle$

Demostración: $\mathbb{L}_-|x\rangle = |z\rangle$

$$\mathbb{L}_+\mathbb{L}_-|x\rangle = \mathbb{L}_+|z\rangle$$
$$(\mathbb{L}_-\mathbb{L}_+ - \mathbb{L}_-\mathbb{L}_+ + \mathbb{L}_+\mathbb{L}_-)|x\rangle = \mathbb{L}_+|z\rangle$$
$$((\mathbb{L}_+\mathbb{L}_- - \mathbb{L}_-\mathbb{L}_+) + \mathbb{L}_-\mathbb{L}_+)|x\rangle = \mathbb{L}_+|z\rangle$$
$$([\mathbb{L}_+, \mathbb{L}_-] + \mathbb{L}_-\mathbb{L}_+)|x\rangle = \mathbb{L}_+|z\rangle$$
$$(-[\mathbb{L}_-, \mathbb{L}_+] + \mathbb{L})|x\rangle = \mathbb{L}_+|z\rangle$$
$$(-\mathbb{I} + \mathbb{L})|x\rangle = \mathbb{L}_+|z\rangle$$
$$-\mathbb{I}|x\rangle + \mathbb{L}|x\rangle = \mathbb{L}_+|z\rangle$$
$$-|x\rangle + \lambda|x\rangle = \mathbb{L}_+|z\rangle$$
$$(\lambda-1)|x\rangle = \mathbb{L}_+|z\rangle$$
$$\mathbb{L}_-(\lambda-1)|x\rangle = \mathbb{L}_-\mathbb{L}_+|z\rangle$$
$$(\lambda-1)\mathbb{L}_-|x\rangle = \mathbb{L}|z\rangle$$

$\rightarrow \mathbb{L}|z\rangle = (\lambda-1)|z\rangle$