THE LEBANESE AMERICAN UNIVERSITY
Summer I 2015
MTH305 EXAM 2

July 6, 2015

Name: ID:

This exam is comprised of 8 problems. Answer the questions in the space provided for each problem. If more space is needed, use the back of the page. Make sure to justify all your answers.

Duration: 75 minutes

1. (10 pts) One of the largest universities in the US is looking for 15 students to join its swimming varsity team. 245 students showed interest in joining the team. To select the best 15 from among the interested students, the university's athletic department arranges a contest in order to select the fastest 15 students swimming 50 meters freestyle. The time it took students to swim 50 meters follows the normal distribution with mean 38 seconds and standard deviation of 6 seconds. Can you tell whether a student who made it in 29.3 seconds is expected to be selected to the swimming varsity team? Justify your answer.

2. (20 pts.) The test scores on MTH315 exam1 follow the normal distribution with mean 65 and standard deviation 8. Let X denote the test score of a randomly selected student.

   a. find $P(65<X<85)$

   b. find $P(X>76)$

   c. if $P(X<k)=0.04$ find k.

   d. If a random sample of 12 MTH315 students was selected, calculate the probability that the sample mean score will be less than 62

3. (10 pts.) The number of deaths as a result of a dreadful disease per month has a Poisson distribution with a mean of 5. Let RV T denote **the time** in days that elapses before 2 deaths are reported. (Assume that 1 month = 30 days).
   a. What is the probability distribution of T and what are its parameters?

   b. What is the probability that more than 10 days elapse before 2 deaths are reported?

   c. Solve part (b) using the Poisson Probability.

4. (10 pts) Solve the following:
   a. Find $\chi^2_{0.05}$ when $\nu = 19$

   b. $Find\ k\ if\ P(37.652 < \chi^2 < k) = 0.045\ with\ \nu = 25$

5. (15 pts) A Lebanese student who never studied Chinese is taking a multiple-choice-exam in the Chinese language. The exam is made of 75 multiple choice questions each with 3 possible answers of which only one is correct. If the student is purely relying on guessing, let X be the number of correct answers the student gets.
    a. Show that the random variable X follows the Binomial distribution and find its parameters

    b. Calculate $P(22 \le X \le 23)$

    c. Can we use the Poisson distribution to approximate the above probability? Explain.

    d. Show that the Normal distribution can be used to approximate the above probability.

    e. Approximate $P(22 \le X \le 23)$ using the Normal distribution.

6. (15 pts.) The random variable X, representing the number of car accidents occurring at Yohati intersection per day, has the following probability distribution:

X	0	1	2	3
$P(X=x)$	0.2	0.4	0.3	0.1

   a. Calculate the probability of having at least one accident at Yohati Intersection on a randomly selected day.

   b. For random samples of 49 days, find $\mu_{\bar{X}}$ and $\sigma_{\bar{X}}$

   c. Find the probability that the average number of car accidents at Yohati Intersection in 49 randomly selected days will be at least one accident.

7. (10 pts.) Solve the following:
   a. The random variable X follows the Binomial distribution with mean 7 and variance 21/4. Find n and p

   b. On which toss of a balance die will you expect to get the first "4"?

8. (10 pts.) A sample of 25 observations was randomly selected from a normal population with mean μ and standard deviation σ. If s^2 represents the sample variance and assuming that $\sigma = 3.5$,

   a. calculate $P(s^2 < 18.5865)$

   b. calculate $P(16.944 < s^2 < 20.092)$

The density function of the normal distribution

$$f(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-(1/2)[(x-\mu)/\sigma]^2}, \qquad -\infty<x<+\infty$$

The density function of the Gamma distribution

$$f(x)=\begin{cases}\dfrac{1}{\beta^{\alpha}\Gamma(\alpha)}x^{\alpha-1}e^{-x/\beta} & x>0\\ 0, & elsewhere\end{cases}$$

Critical Values of the Chi-Squared Distribution

ν \ α	0.995	0.990	0.980	0.975	0.950	0.900	0.100	0.050	0.025	0.020	0.010	0.005
10	2.156	2.558	3.059	3.247	3.940	4.865	15.987	18.307	20.483	21.161	23.209	25.188
11	2.603	3.053	3.609	3.816	4.575	5.578	17.275	19.675	21.920	22.618	24.725	26.757
12	3.074	3.571	4.178	4.404	5.226	6.304	18.549	21.026	23.337	24.054	26.217	28.300
13	3.565	4.107	4.765	5.009	5.892	7.042	19.812	22.362	24.736	25.472	27.688	29.819
14	4.075	4.660	5.368	5.629	6.571	7.790	21.064	23.685	26.119	26.873	29.141	31.319
15	4.601	5.229	5.985	6.262	7.261	8.547	22.307	24.996	27.488	28.259	30.578	32.801
16	5.142	5.812	6.614	6.908	7.962	9.312	23.542	26.296	28.845	29.633	32.000	34.267
17	5.697	6.408	7.255	7.564	8.672	10.085	24.769	27.587	30.191	30.995	33.409	35.718
18	6.265	7.015	7.906	8.231	9.390	10.865	25.989	28.869	31.526	32.346	34.805	37.156
19	6.844	7.633	8.567	8.907	10.117	11.651	27.204	30.144	32.852	33.687	36.191	38.582
20	7.434	8.260	9.237	9.591	10.851	12.443	28.412	31.410	34.170	35.020	37.566	39.997
21	8.034	8.897	9.915	10.283	11.591	13.240	29.615	32.671	35.479	36.343	38.932	41.401
22	8.643	9.542	10.600	10.982	12.338	14.041	30.813	33.924	36.781	37.659	40.289	42.796
23	9.260	10.196	11.293	11.689	13.091	14.848	32.007	35.172	38.076	38.968	41.638	44.181
24	9.886	10.856	11.992	12.401	13.848	15.659	33.196	36.415	39.364	40.270	42.980	45.559
25	10.520	11.524	12.697	13.120	14.611	16.473	34.382	37.652	40.646	41.566	44.314	46.928
26	11.160	12.198	13.409	13.844	15.379	17.292	35.563	38.885	41.923	42.856	45.642	48.290
27	11.808	12.879	14.125	14.573	16.151	18.114	36.741	40.113	43.195	44.140	46.963	49.645
28	12.461	13.565	14.847	15.308	16.928	18.939	37.916	41.337	44.461	45.419	48.278	50.993
29	13.121	14.256	15.574	16.047	17.708	19.768	39.087	42.557	45.722	46.693	49.588	52.336
30	13.787	14.953	16.306	16.791	18.493	20.599	40.256	43.773	46.979	47.962	50.892	53.672