

Use the constants  $a$ ,  $b$  and  $c$  of your ID

I. (12 Points) Answer the following questions:

(a) Find using Taylor Series the following limit

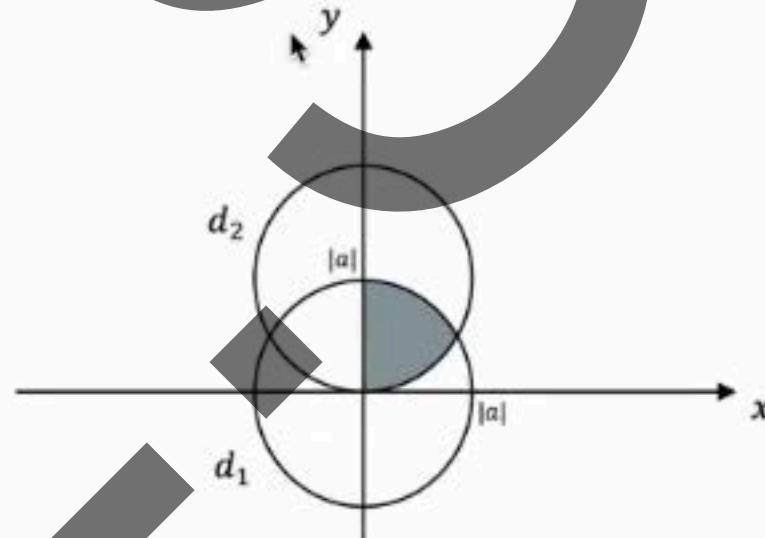
$$\lim_{x \rightarrow 0} \frac{\sin(ax) - ax \cos(bx)}{bx e^{ax} - bx - abx^2}.$$

IV. (15 Points) Let  $d_1$  be the disk of center  $(0,0)$  and of radius  $|a|$  and  $d_2$  the disk of center  $(0, |a|)$  and of radius  $|a|$ .

Let  $D$  be the part of the region of intersection of  $d_1$  and  $d_2$  situated in the first quadrant (see the figure below).

- (a) Define the region  $D$  using cartesian and polar coordinates.
- (b) Find the double integral

$$\iint_D \sqrt{x^2 + y^2} dx dy.$$

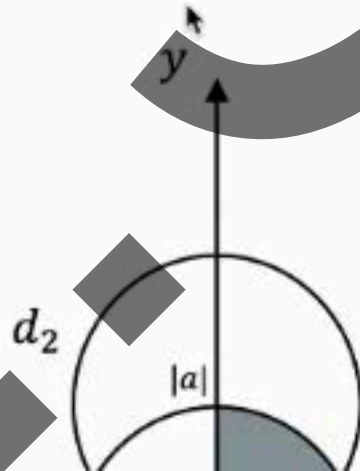


IV. (15 Points) Let  $d_1$  be the disk of center  $(0, 0)$  and of radius  $|a|$  and  $d_2$  the disk of center  $(0, |a|)$  and of radius  $|a|$ .

Let  $D$  be the part of the region of intersection of  $d_1$  and  $d_2$  situated in the first quadrant (see the figure below).

- (a) Define the region  $D$  using cartesian and polar coordinates.
- (b) Find the double integral

$$\iint_D \sqrt{x^2 + y^2} \, dx dy.$$



Use the constants  $a$ ,  $b$  and  $c$  of your ID

III. (12 Points) Consider the polynomial  $p(n) = an^2 + bn + c$  and the power series

$$S = \sum_{n=0}^{\infty} \frac{p(n)}{n!} x^n.$$

- (a) Study the convergence of  $S$ .
- (b) Show that  $p(n)$  can be written as  $p(n) = a'n(n-1) + b'n + c'$ , where  $a'$ ,  $b'$  and  $c'$  are three real numbers to be found.
- (c) Deduce the value of  $S$  in terms of  $x$ .

- (b) Study the convergence of the following power series and find its sum in terms of  $x$ :

$$\sum_{n=0}^{\infty} \cosh(an) x^n.$$

(b) Study the convergence of the following power series and find its sum in terms of  $x$ :

(a) 
$$\sum_{n=0}^{\infty} \cosh(an)x^n.$$

Use the constants  $a$ ,  $b$  and  $c$  of your ID

II. (12 Points) Answer the following questions:

(a) Find the first 3 terms of the Maclaurin series of the function:

$$(a) f(x) = e^x \cos^2(ax).$$

Use the constants  $a$ ,  $b$  and  $c$  of your ID

II. (12 Points) Answer the following questions:

(a) Find the first 3 terms of the Maclaurin series of the function:

$$(a) f(x) = e^x \cos^2(ax).$$



(b) Find the double integral

$$\iint_D \frac{|b|}{(|b|x + |c|y + 1)^2} dx dy$$

where  $D = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1, 0 \leq y \leq 1\}$ .