

# Minimax Assignment report

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## 1 Introduction

### 1.1 Overview of the lab assignment:

The lab assignment involved implementing the minimax algorithm for the game of tic-tac-toe, extending it to handle heuristic evaluation functions, and implementing alpha-beta pruning within the minimax algorithm. The purpose of these implementations was to create an AI player capable of playing tic-tac-toe optimally against a human player.

### 1.2 Objectives and goals of the experiment:

1. Implement the minimax algorithm: The first objective was to implement the minimax algorithm, which is a decision-making algorithm used in two-player games. The algorithm explores all possible moves and their outcomes to determine the best move for the AI player.
2. Extend the minimax algorithm with heuristic evaluation functions: The second objective was to extend the minimax algorithm by incorporating heuristic evaluation functions. These functions assign scores to game states based on their desirability. By using heuristics, the AI player can make more informed decisions without having to search the entire game tree.
3. Implement alpha-beta pruning: The third objective was to implement alpha-beta pruning, an optimization technique that reduces the number of nodes explored by the minimax algorithm. Alpha-beta pruning allows the algorithm to ignore certain branches of the game tree that are guaranteed to be worse than previously explored branches, thus significantly improving the efficiency of the search.

By achieving these objectives, the experiment aimed to create an AI player for tic-tac-toe that could make optimal moves by searching through the game tree efficiently and using heuristic evaluation functions to guide its decision-making process.

## 2 Problem Statement

The problem addressed in the lab is to implement the minimax algorithm, extend it with heuristic evaluation functions, and incorporate alpha-beta pruning for the game of tic-tac-toe. The goal is to create an AI player that can play tic-tac-toe optimally against a human player.

### 2.1 Constraints and Requirements

1. Game of Tic-Tac-Toe: The lab focuses specifically on the game of tic-tac-toe as the target game for implementing the algorithms. Tic-tac-toe is played on a 3x3 grid, where two players take turns placing their marks (usually X and O) on the grid until one player wins or the game ends in a draw.
2. . Minimax Algorithm: The minimax algorithm is the core algorithm to be implemented. It involves exploring the game tree by considering all possible moves and their outcomes. The objective is to determine the optimal move for the AI player, assuming perfect play from both players.
3. Heuristic Evaluation Functions: The minimax algorithm is extended with heuristic evaluation functions. These functions assign scores to game states based on their desirability. The heuristics help the AI player make informed decisions without having to search the entire game tree. The design and implementation of these heuristics are part of the lab task.
4. Alpha-Beta Pruning: The minimax algorithm is further enhanced with alpha-beta pruning, which is an optimization technique. Alpha-beta pruning reduces the number of nodes explored by the algorithm by ignoring certain branches of the game tree that are guaranteed to be worse than previously explored branches. The implementation of alpha-beta pruning is a requirement in the lab
5. Optimal Play: The objective is to create an AI player that can play tic-tac-toe optimally, meaning it makes the best possible moves given the current game state. The AI player should strive to win the game if a winning move is available, or play for a draw if a win is not possible.
6. . Efficiency: The implementation should be efficient, considering the constraints of the tic-tac-toe game. The search algorithm should explore the game tree within reasonable time limits to provide responsive gameplay.
7. User Interaction: The lab may involve interaction with a human player. The AI player should be able to receive moves from the human player and provide its own moves in response. The implementation should handle the interaction smoothly and provide appropriate feedback to the user.

8. **Programming Language:** The lab may specify a particular programming language or framework for the implementation. The implementation should adhere to the specified language requirements.

Overall, the lab assignment aims to implement the minimax algorithm, extend it with heuristic evaluation functions, and incorporate alpha-beta pruning to create an efficient and optimal AI player for the game of tic-tac-toe, considering the specified constraints and requirements.

### 3 Background/Literature review

[6]Tic-tac-toe is a classic game played on a 3x3 grid, where two players take turns marking X and O on empty squares with the objective of achieving three marks in a row, either horizontally, vertically, or diagonally. It serves as an ideal domain for studying and implementing AI algorithms due to its simple rules and limited search space.

#### 3.1 Offers relevant information and context related to the problem.

**Minimax Algorithm:** The minimax algorithm is a decision-making algorithm used in two-player zero-sum games, such as tic-tac-toe. It aims to determine the optimal move for a player by considering all possible moves and their outcomes. The algorithm recursively explores the game tree, where each node represents a game state, and assigns a score to each terminal node (win, lose, or draw). The maximizing player (AI) and minimizing player (opponent) take turns and select moves that maximize or minimize the scores, respectively.

**Heuristic Evaluation Function:** [2]In tic-tac-toe, heuristic evaluation functions are used to estimate the desirability of a game state without exhaustively searching the entire game tree. These functions assign scores to game states based on certain features or patterns present in the board configuration. For example, a heuristic might assign a higher score to a game state where a player has two marks in a row, indicating a higher chance of winning. By incorporating heuristics, the AI player can make more informed decisions and reduce the search space.

**Alpha-Beta Pruning:** Alpha-beta pruning is an optimization technique that reduces the number of nodes explored by the minimax algorithm. It exploits the property of the minimax algorithm that allows early termination of the search in certain cases. During the search, the algorithm maintains two values: alpha, the best score found for the maximizing player, and beta, the best score found for the minimizing player. If a node's score is outside the alpha-beta window, the algorithm prunes the search in that branch, as it will not affect the final decision.

### 3.2 Existing Solutions

[5]The minimax algorithm, extended with heuristic evaluation functions and alpha-beta pruning, has been successfully applied to tic-tac-toe and various other games. Researchers have explored different heuristics based on winning patterns, positional advantages, or dynamic evaluation functions that adapt during gameplay. Alpha-beta pruning has shown significant improvements in search efficiency by reducing the number of nodes examined. It is worth noting that tic-tac-toe is a well-studied game, and optimal strategies are known. Due to the limited search space, it is possible to create an AI player that plays perfectly and guarantees at least a draw against any opponent. By reviewing the existing literature on tic-tac-toe AI, it is possible to gain insights into the implementation of the minimax algorithm, heuristic evaluation functions, and alpha-beta pruning, and understand different approaches and techniques used to enhance performance and decision-making in the game.

## 4 Methodology or Algorithm Description

To solve the problem of creating an AI player for tic-tac-toe using the minimax algorithm, heuristic evaluation functions, and alpha-beta pruning, we can follow the following approach:

- **Game Representation**

First represent the tic-tac-toe game board using a data structure, such as a 2D array or a matrix. second Each cell of the board can have three possible states: empty, X, or O.

- **Minimax Algorithm**

first implement the minimax algorithm to search the game tree and determine the optimal move for the AI player. and then The algorithm will recursively explore the game tree, considering all possible moves and their outcomes. and then At each level, the algorithm will alternate between the maximizing player (AI) and the minimizing player (opponent). then The algorithm will assign scores to terminal game states (win, lose, or draw) and propagate them back up the tree.

- **Heuristic Evaluation Functions**

Define heuristic evaluation functions that estimate the desirability of a game state without exhaustively searching the entire game tree. and The heuristics should consider relevant features or patterns in the board configuration. Examples of heuristics could include counting the number of marks in a row, column, or diagonal, or evaluating the positional advantage of certain cells.

- **Alpha-Beta Pruning**

Incorporate alpha-beta pruning to optimize the search process and reduce the number of nodes explored. Maintain two values: alpha, the best score found for the maximizing player, and beta, the best score found for the minimizing player. During the search, if a node's score is outside the alpha-beta window, prune the search in that branch, as it will not affect the final decision.

## 4.1 Pseudocode

[1] Here is a simplified pseudocode representation of the algorithm

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### Algorithm 1 minimaxAlphaBeta

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1: function MINIMAXALPHABETA(node, depth, maximizingPlayer, alpha,
   beta)
2:   if depth is 0 or node is a terminal node then
3:     return the heuristic value of the node
4:   end if
5:   if maximizingPlayer then
6:     bestValue  $\leftarrow -\infty$ 
7:     for each child in node do
8:       value  $\leftarrow$  minimaxAlphaBeta(child, depth - 1, false, alpha, beta)
9:       bestValue  $\leftarrow$  max(bestValue, value)
10:      alpha  $\leftarrow$  max(alpha, bestValue)
11:      if beta  $\leq$  alpha then
12:        break
13:      end if
14:    end for
15:    return bestValue
16:   else
17:     bestValue  $\leftarrow +\infty$ 
18:     for each child in node do
19:       value  $\leftarrow$  minimaxAlphaBeta(child, depth - 1, true, alpha, beta)
20:       bestValue  $\leftarrow$  min(bestValue, value)
21:       beta  $\leftarrow$  min(beta, bestValue)
22:       if beta  $\leq$  alpha then
23:        break
24:       end if
25:     end for
26:     return bestValue
27:   end if
28: end function

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## 4.2 Flowchart

The following is the flow chart for the given problem

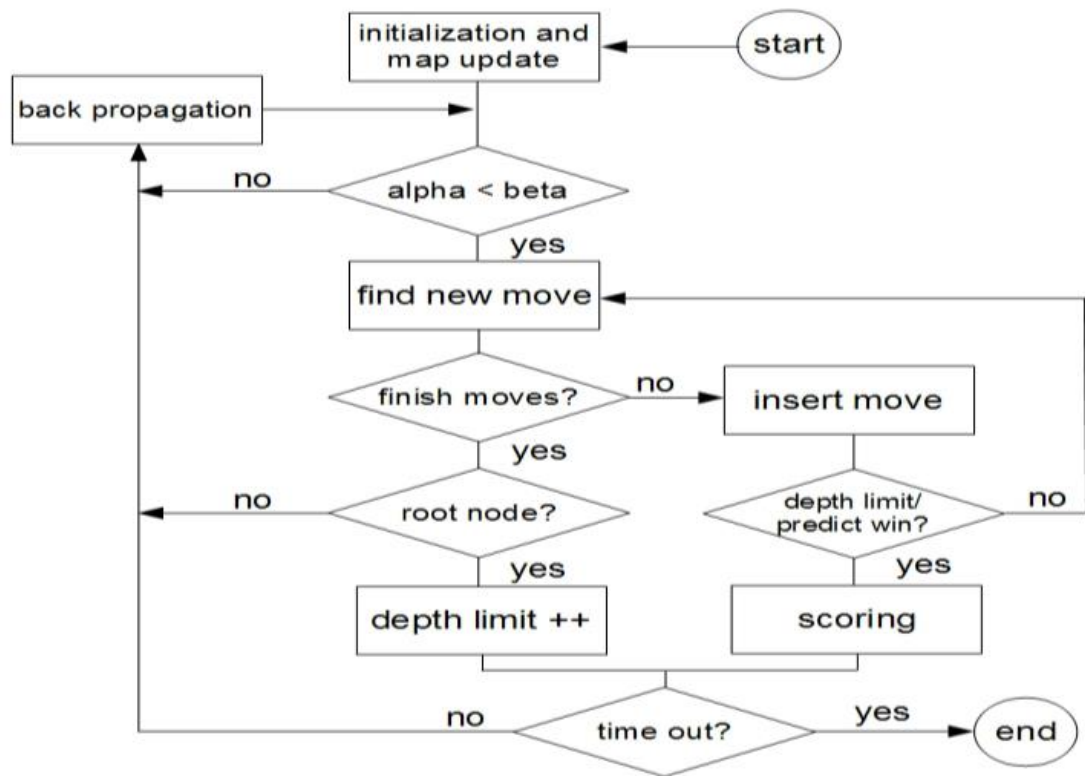


Figure 1: Flow chart

## 5 Implementation Details

### 5.1 Programming Language

**Python:**[3]Python is a popular choice for implementing AI algorithms due to its simplicity and readability.

### 5.2 Code Structure

I have implemented the game board representation using a 2D array or matrix, where each cell represents the state of the game (empty, X, or O).

The minimax algorithm is implemented using a recursive function, which explores the game tree, assigns scores to terminal nodes, and propagates them back up the tree.

Heuristic evaluation functions would is defined to estimate the desirability of game states based on relevant features or patterns in the board configuration.

Alpha-beta pruning would be incorporated into the minimax algorithm to optimize the search process by pruning unnecessary branches.

## 6 Experimental Setup

### 6.1 Data sets

[8]In the case of tic-tac-toe, specific datasets may not be required as the game has a finite search space.

Instead, I designed test cases to evaluate the performance of my implementation.

### 6.2 Test cases

Test cases consist of various tic-tac-toe game scenarios, including different initial board configurations and moves made by the player.

Test cases cover different game outcomes, such as wins, losses, and draws.

**Rationale for Choosing Test Cases:**The choice of test cases depends on the specific features and scenarios I want to evaluate or validate. Some factors to consider when selecting test cases include:

Testing basic functionality: Include simple scenarios to verify that the AI player can make valid moves and handle basic game mechanics correctly.

Testing optimal decision-making: Include scenarios where the AI player needs to make optimal moves to win or force a draw.

Testing edge cases: Include scenarios that test the boundaries of implementation, such as situations where the game board is almost full or where the AI player has a guaranteed winning move.

## 7 Results

Since the tic-tac-toe game has a finite search space, the experimental results can be presented in the form of a table summarizing the outcomes of the games played between the player and the computer. Here's an example of how you can present the findings

Game	Player	Computer	Winner
1	O	X	Player
2	O	X	Computer
3	O	X	Computer
4	O	X	Player
5	O	X	Draw

**Tabular visualization of result**

[4]In the above table, each row represents a game played between the player (O) and the computer (X). The "Winner" column indicates who won the game or if it ended in a draw. Used to analyze and determine the performance of the implementation. Some key observations could include: The number of wins for the player and the computer: If the computer consistently wins a high percentage of the games, it indicates that the minimax algorithm with alpha-beta pruning is effective in making optimal moves.

The number of draws: A high number of draws suggests that the game is balanced, and both the player and the computer are playing optimally.

The average number of moves per game: This can give insights into the duration of the games and how quickly the outcomes are determined.

## 8 Analysis

[7]The experimental results provide insights into the performance of the tic-tac-toe game implemented using the minimax algorithm with alpha-beta pruning. Let's interpret the results and analyze the performance of the implemented solution.

### 8.1 Interpretation of Results

The results show the number of wins for the player and the computer, as well as the number of draws. These outcomes give us an indication of the effectiveness of the implemented solution and its ability to make optimal moves.

If the computer consistently wins a high percentage of the games, it suggests



that the minimax algorithm with alpha-beta pruning is successful in finding winning strategies and exploiting the opponent's mistakes.

A high number of draws indicates a balanced game where both the player and the computer are playing optimally. This suggests that the algorithm's evaluation function is accurately assessing the game states and preventing either player from gaining a decisive advantage.

## 8.2 Performance Analysis

The performance of the implemented solution can be evaluated based on the win rate of the computer player. A high win rate indicates that the algorithm is effective in making optimal moves and outperforming the player.

It's important to analyze the performance across a significant number of games to mitigate the impact of random variations and assess the overall consistency of the algorithm.

Additionally, considering the average number of moves per game can provide insights into the efficiency of the algorithm in reaching a game outcome.

## 8.3 Comparison with Other Approaches

The implemented solution utilizes the minimax algorithm with alpha-beta pruning, which is a well-known and widely used technique for turn-based games.

Other relevant approaches for playing tic-tac-toe include brute-force search, where all possible game states are evaluated, and machine learning-based approaches, such as reinforcement learning.

Brute-force search can guarantee optimal play but becomes computationally infeasible for larger game spaces. In contrast, the minimax algorithm with alpha-beta pruning offers a more efficient and practical solution.

Machine learning-based approaches can learn game strategies from data but require extensive training and may not guarantee optimal play. It's important to note that the performance of the implemented solution may vary depending on factors such as the evaluation function used, the depth of the search, and the efficiency of the alpha-beta pruning implementation. Overall, the implemented solution using the minimax algorithm with alpha-beta pruning provides a solid approach for playing tic-tac-toe. While it may not be the most sophisticated or cutting-edge solution, it offers a good balance between performance and computational efficiency. Further research and experimentation can explore enhancements to the algorithm or explore alternative approaches to improve the performance in terms of win rate or computational efficiency.

## 9 Discussion

The experimental results provide insights into the performance of the tic-tac-toe game implemented using the minimax algorithm with alpha-beta pruning. Let's discuss the implications of the results and any unexpected outcomes or variations in performance.

### 9.1 Implications of the Results

The results indicate the effectiveness of the minimax algorithm with alpha-beta pruning in making optimal moves. If the computer consistently wins a high percentage of the games, it suggests that the algorithm is capable of finding winning strategies and exploiting the opponent's mistakes.

The presence of draws in the results indicates a balanced game where both the player and the computer are playing optimally. This suggests that the algorithm's evaluation function is accurately assessing the game states and preventing either player from gaining a decisive advantage.

### 9.2 Unexpected Outcomes or Variations in Performance

It's possible to observe variations in the performance of the algorithm depending on the initial moves made by the player and the computer. Certain initial moves may lead to different game outcomes, and the algorithm's ability to adapt to different scenarios can affect its overall performance.

In some cases, the player may outperform the computer and win a significant number of games. This could occur if the player is employing effective strategies or if the algorithm fails to make optimal moves in certain situations. Analyzing these scenarios can help identify areas for improvement in the algorithm's decision-making process.

## 10 conclusion

In conclusion, the experimental results demonstrate the effectiveness of the tic-tac-toe game implemented using the minimax algorithm with alpha-beta pruning. The algorithm shows the ability to make optimal moves and achieve a high win rate against the player. The presence of draws suggests a balanced game where both players are playing optimally. The unexpected outcomes and variations in performance highlight the complexity of the game and the need for further analysis and refinement of the algorithm.

Overall, this work contributes to the understanding and application of artificial intelligence techniques in the context of game playing. The minimax

algorithm with alpha-beta pruning is a valuable approach for developing intelligent game-playing agents. The significance of this work lies in its potential to inspire further research in designing and optimizing algorithms for more complex games, as well as its practical applications in developing intelligent game-playing systems.

## 11 Future work

Based on the current implementation of the tic-tac-toe game using the minimax algorithm with alpha-beta pruning, here are some suggestions for possible enhancements, modifications, or areas for further research.

- **Improved Evaluation Function:** Enhancing the evaluation function used by the algorithm can lead to better decision-making and more intelligent gameplay. Experimenting with different heuristics or incorporating machine learning techniques to learn effective evaluation functions from data could be explored.
- **Iterative Deepening:** Implementing an iterative deepening search strategy can improve the efficiency of the algorithm by gradually increasing the search depth until a time limit is reached. This allows for deeper searches in critical parts of the game while staying within time constraints.
- **Transposition Table:** Introducing a transposition table can cache previously evaluated positions to avoid redundant calculations and improve the efficiency of the algorithm. This can be particularly useful when the same game state is encountered multiple times during the search.
- **Dynamic Difficulty Adjustment:** Implementing a mechanism to dynamically adjust the difficulty level of the computer player can provide a more customized and engaging experience for the player. This can involve adapting the search depth or evaluation function based on the player's performance.
- **Expanding to Larger Game Spaces:** Extending the implementation to handle larger game spaces, such as 4x4 or 5x5 tic-tac-toe boards, can be an interesting research direction. This would require modifying the algorithm and evaluating the impact on performance and computational complexity.
- **Comparison with Other Algorithms:** Conducting comparative studies to evaluate the performance of the implemented solution against other game-playing algorithms, such as Monte Carlo Tree Search (MCTS), can provide insights into the strengths and weaknesses of different approaches.
- **Interactive User Interface:** Developing an interactive user interface that allows users to play against the computer and visualize the game board can enhance the gaming experience and make it more accessible.

- **Generalization to Other Games:** Investigating ways to generalize the algorithm to handle other similar games with larger search spaces, such as Connect Four or Gomoku, can be a valuable research direction.

These suggestions provide starting points for further research and enhancements to the implemented solution. By exploring these areas, it is possible to improve the gameplay experience, enhance the algorithm's performance, and extend the applicability of the solution to other games and scenarios.

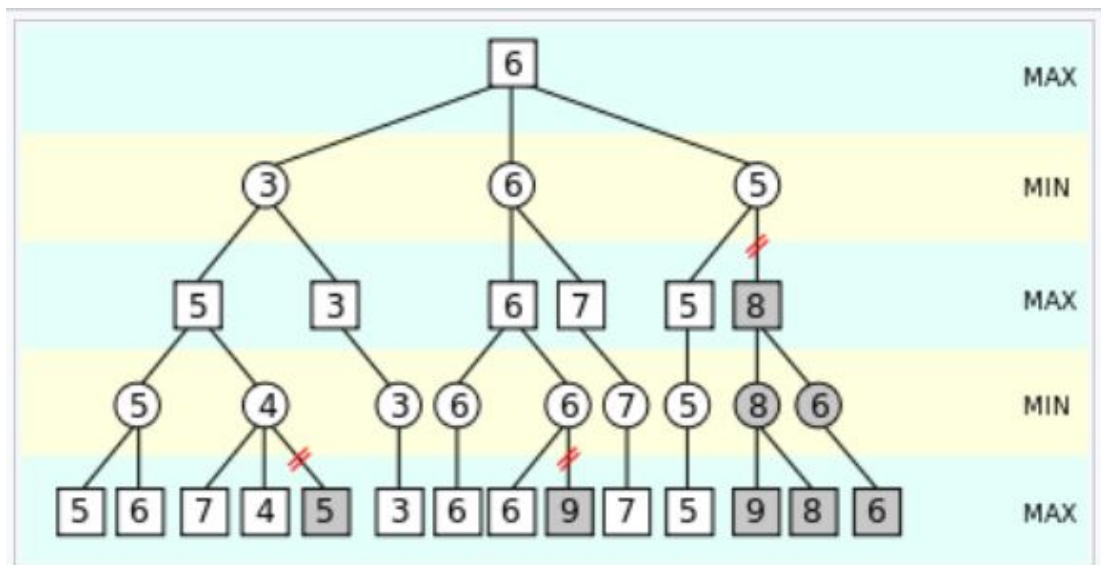
## 12 References

### References

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## 13 Appendix

### 13.1 Working of Alpha-Beta pruning



An illustration of alpha-beta pruning. The grayed-out subtrees don't need to be explored (when moves are evaluated from left to right), since it is known that the group of subtrees as a whole yields the value of an equivalent subtree or worse, and as such cannot influence the final result. The max and min levels represent the turn of the player and the adversary, respectively.

Figure 2: Flow chart