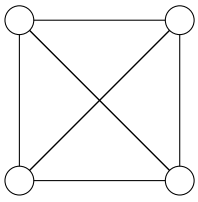
MAT 230: Sets and Proofs
In Class Assignment: Graph Theory

Names: _____

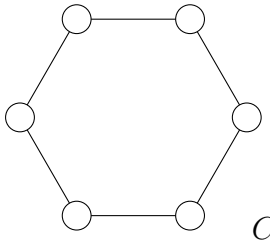
1. VERTEX DEGREES and EULER

   (a) The degree of a vertex v is the number of edges with v being one of the endpoints. For each graph below, write the degree of each vertex next to that vertex. Now for each graph, add the degrees for the graph. Can you relate these sums to another quantity, such as the number of vertices or the number of edges?

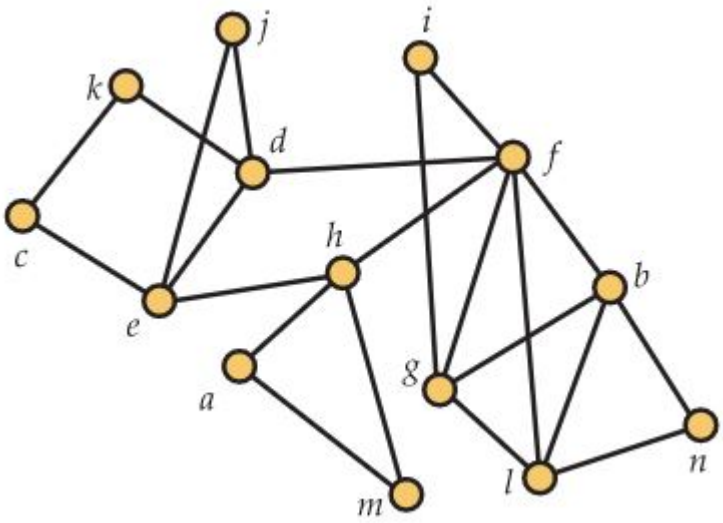


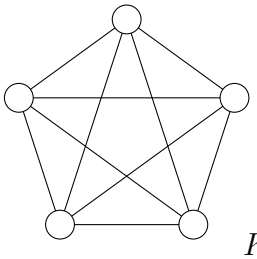
K_4

P_4



C_6





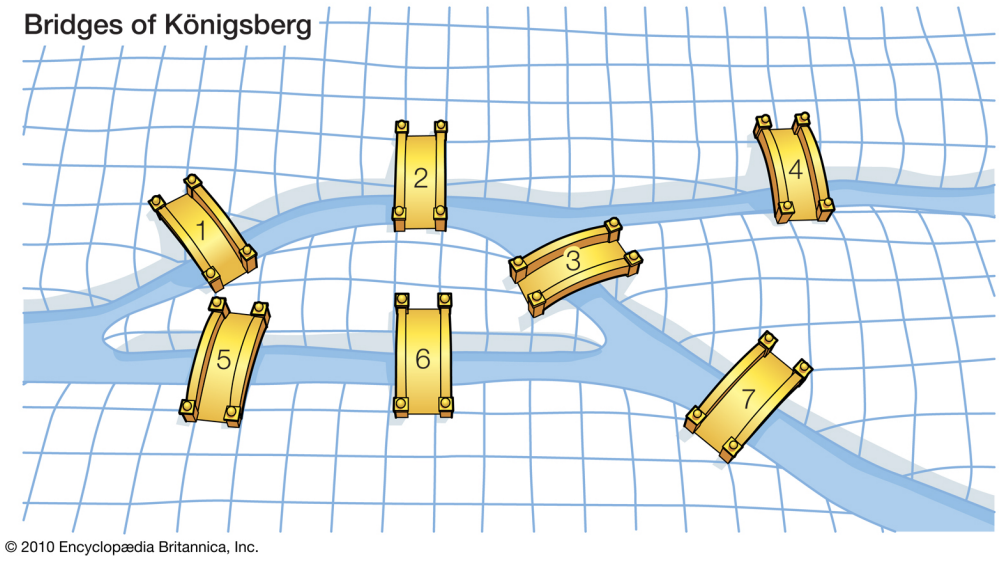
K_5

   (b) Draw a graph with 6 vertices where every vertex has degree 3. If this is impossible, tell why.

   (c) Draw a graph with 7 vertices where every vertex has degree 3. If this is impossible, tell why.

   (d) A graph is called *Eulerian* if you can start at a vertex, move along the edges of the graph, touching each edge exactly once, and return to start (all without lifting your pencil). Which of the graphs above are Eulerian? Find a rule for when a graph is Eulerian.

(e) Euler was told the following puzzle: The town Königberg has 7 bridges as shown in this picture. Is there a way for someone to start at their house on one of the land masses and cross each bridge exactly once before returning home? What do you think?



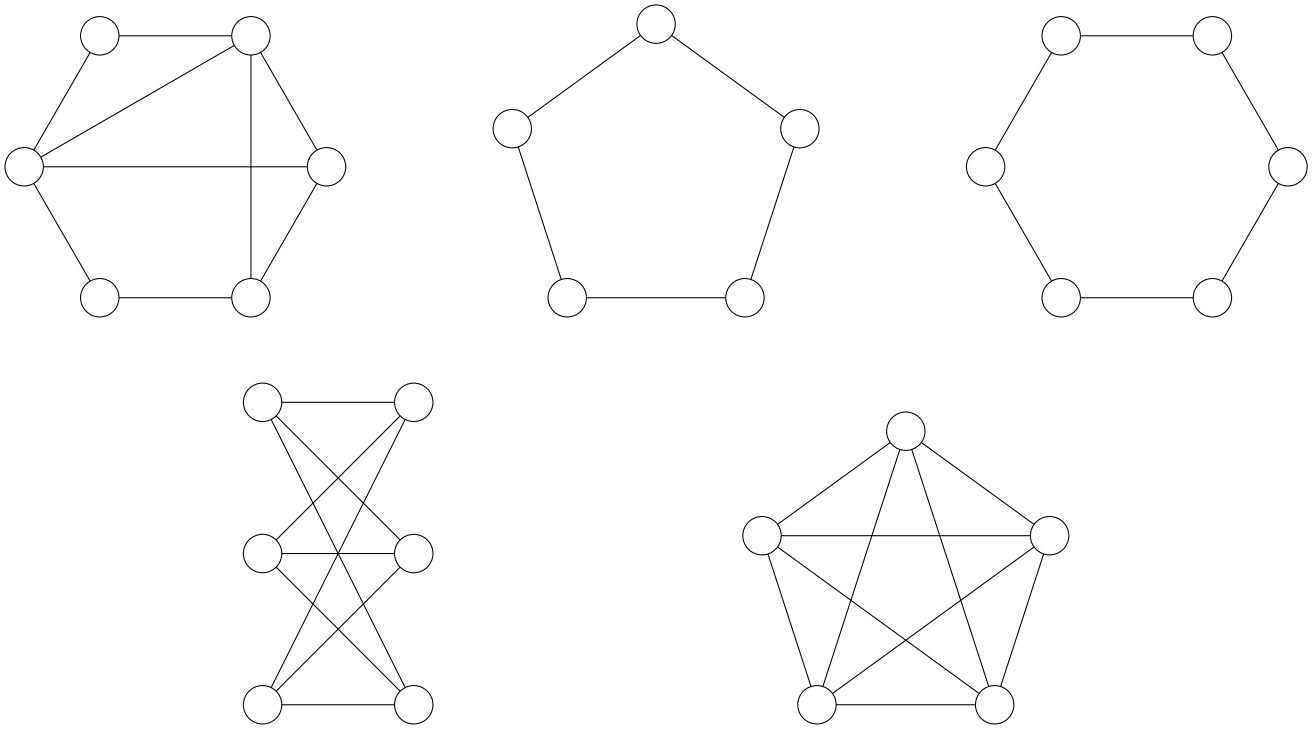
2. SCHEDULING and COLORING

(a) In the transition to online courses, the class times are being re-evaluated to compact the schedule. Below we list 6 students and the courses they are in.

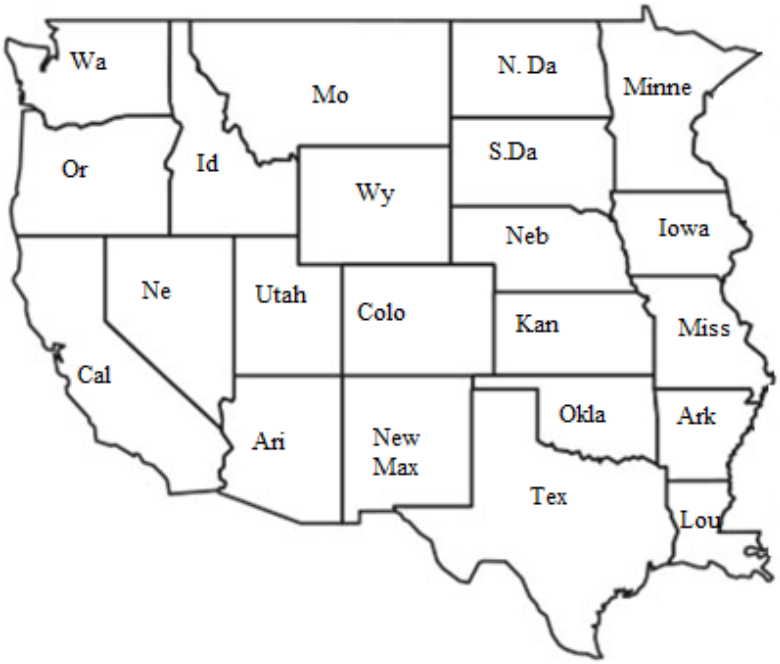
- First draw a graph with one vertex for each class. Then draw an edge between two vertices if the classes need to be at different times.
- Find a way to schedule these courses using the fewest number of class periods.
- Explain why fewer class periods cannot be used.

Student	Courses
Alice	Chemistry, Physics, Economics
Bob	English, German, Statistics
Cynthia	Statistics, Calculus, German
David	Chemistry, Physics
Evan	English, Chemistry
Florrie	Chemistry, Economics

(b) Color the vertices of these graphs so that if two vertices have an edge between them, they get different colors. Do this with the fewest number of colors possible.



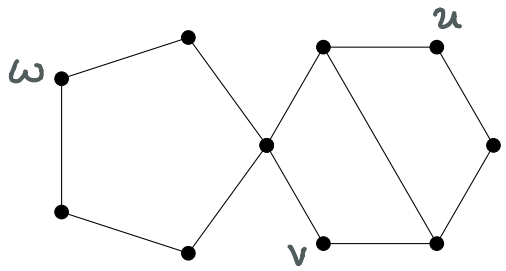
(c) Color the states in this map of the western USA so that two states which share a border receive different colors. What is the fewest number of colors you need? How is this related to coloring vertices in graphs like we did in the last question?

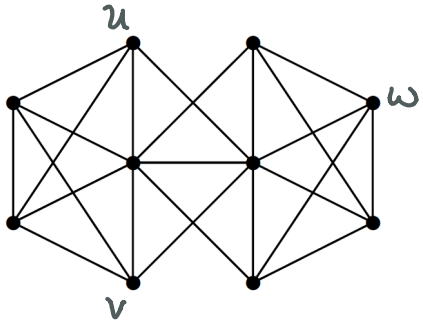


3. CONNECTIVITY

The following graphs represent networks of people. Each vertex represents a person and there is an edge between two people if they regularly come within 6 feet of one another.

Right now, if anyone gets the coronavirus, it is likely to spread to everyone in the graph. In order to slow the spread of the coronavirus, we will put a few people in quarantine.





(a) For each graph above, what is the fewest number of people we can quarantine so that if u gets the virus, it will not spread to v? [Assume that u and v are essential workers, so they cannot be put into quarantine.]

(b) For each graph above, what is the fewest number of people we can quarantine so that if u gets the virus, it will not spread to w? [Now assume that u and w are essential workers, so they cannot be put into quarantine.]