

Welfare or Unfair? Estimating Marginal Utility of Income From Willingness to Pay Conditional on Ability to Pay

Nathan Mather

September 29, 2022

[Click here for latest version](#)

Abstract

1 Introduction

What exactly is utility and is it comparable across people? Is it measurable? Despite playing a ubiquitous role in economic theory, the definition of utility and its use within that theory varies. Economic work generally models individual behavior as the result of utility maximization. If utility is defined as ordinal numbers corresponding to the preferences ranking of an individual then utility maximization is tautologically true so long as that person's preferences are consistent and rational. Rather than being a shortcoming, this tautology is a great strength of ordinal utility as a flexible and adaptable modeling tool. It doesn't matter *why* people choose something, if they do in fact choose it we can assign it more ordinal utility. However, normative economics research and tools frequently require that the utility describing individual actions actually correspond to a cardinal measure of individual's welfare or well-being. This need not be happiness or pleasure per se, but in these cases utility must be something that a policymaker would conceivably want to increase. This paper presents and implements a method of measuring the marginal utility of income under the stricter cardinal welfare interpretation of utility. The results indicate that if we want to treat decision utility as corresponding to well-being, then the marginal utility of income is flat or increases quite slowly. Put another way, a cardinal, welfare relevant decision utility is not consistent with diminishing marginal utility of income.

To better understand the motivation for this paper, consider the following hypothetical scenario. A city has enough money set aside in their budget to either expand public transit or widen the roads to allow more cars through at a time. Which policy should they implement? Of course, what they *should* do is partially a normative question. Some residents are going to prefer one outcome and others will prefer the alternative. Making a collective decision inevitably involves weighing the trade-offs between winners and losers along with other ethical concerns like racial disparities, inequality, or changes to individual rights. These are inherently normative ethical considerations. Good policy analysis, however, can objectively aid in decision making using instrumental rationality. Economists can lay out the outcomes from each policy in a way that allows policymakers to discern the option that best coincides with their ethical normative preferences. This goal, matching policymakers with the options that fit their preferences, is an objective and scientific approach to policy analysis.

Due to the wide range of ethical beliefs¹, Economic work often focuses on the subset of welfarist policymaker preferences. In this framework, policymakers have some weight they place on different people's utility and summing those weighted values up will lead us to the outcome that coincides

¹Virtue ethics, deontology, and consequentialism will lead to different sets of concerns and focus for analysis

with their values. Returning to the decision to widen roads or expand transit, a welfarist would, in their perfect world, take the utility gain for each person under each potential outcome, multiply their utility gain by that person’s corresponding welfare weight, and sum that product for each person under both policies. Whichever sum is higher, is the policy they prefer.

Doing this is, of course, easier said than done. There are many potential outcomes to consider and weigh against each other. How would widening the roads decrease commute times for drivers, what is the impact on carbon emissions, or how many people would use public transit if it was expanded? Economic analysis often tries to collapse these concerns to a single dimension using measures based on willingness to pay (WTP) ². Willingness to pay has a clear theoretical connection to an individual’s decision utility since trading money for a good implies one prefers the money less than or equal to the good traded for it. While the connection to an individual’s decision utility is clear, how to handle interpersonal comparisons is not. Marshall (1890) proposed simply summing up consumer surplus, but this would run counter to the idea of diminishing marginal utility³ first laid out by Menger, Jevons, and Walras (Moscati, 2018, pg 26). It also assumes that policymakers value only the sum total of utility and do not have distributional preferences as well.

Rather than tackling that question head on, economist often resort to efficiency arguments that rely only on an ordinal conception of utility. The sum of willingness to pay (WTP) for a policy is often referred to as “welfare” in economics. This practice stems from the idea put forth by Hicks (1940) and Kaldor (1939) that if we choose the option with the highest total WTP, the winners can compensate the losers and make everyone better off. Unfortunately, this logic relies on lump sum transfers which are not possible. Moreover, transfers of any kind are often politically or practically infeasible. Beyond that, Samuelson (1950) shows that the logic does not hold up generally, even with lump sum transfers, once we consider heterogeneous preferences. While measures based on the sum of WTP⁴ have their place, the allure of an ethics free conclusion is, in many cases, a mirage.

Since efficiency measures do not provide an easy fix, is there some way to rehabilitate willingness to pay measures to be compatible with diminishing marginal utility? Potentially, but it requires two things. First it requires assuming that individual behavior is the result of maximizing cardinal utility where utility corresponds to a meaningful measure of welfare or well-being. A simplest example of this *not* holding is people making behavioral mistakes, but the idea receives a more detailed discussion in section 8. The second requirement is we need welfare weights to apply to

²compensated and equivalent variation or consumer surplus

³interestingly, Marshall originally argued that the differences in marginal utility would average out and can be ignored (Moscati, 2018)

⁴Surplus measures as well as compensated and equivalent variation

these individual utility values.

We can better understand both of these requirements if we measure the marginal utility of income. If we assume utility is measurable with preferences, what is the marginal utility of income? What would a utilitarian, for example, prefer? Can the re-distributive preferences people express be explained with utilitarian ethics under this framework? Measuring utility has been recognized as an important step in policy assessment as early as 1871 when Jevons proposed that “We cannot really tell the effect of any change in trade or manufacture until we can with some approach to truth express the laws of the variation of utility numerically” (Jevons, [1871] 1911). While Jevons was optimistic about the future, stating it will eventually be a “mathematical problem of no great difficulty how to disentangle the functions expressing the degrees of utility of various commodities” (Jevons, [1871] 1911), a somewhat common view among late 19th and early 20th century scholars, it has turned out to be much more difficult (Fisher, 1927; Moscati, 2018).

So, how can marginal utility of income be measured? Rather than fulfilling Jevons’ vision of relying on modern computing power to disentangle utility from observed behavior and big data, a task I think he vastly underestimated, I instead rely on finding a particular type of good that meets assumptions necessary for identification. Suppose there is a good called a widget, and we know that the welfare benefit of receiving a widget is not affected by changes in income. That is, utility from widgets and income are independent. Now if we see that median Americans are willing to pay \$X, and billionaires are willing to pay \$Y for a widget, we know that they value X and Y dollars the same as well. This means the ratio of the average utility of money between billionaires and median Americans is Y/X.

While this may seem surprisingly simple there are a few important complexities to point out. The first is that we can only measure the average marginal utility of income conditional on observable traits like income or financial health. Focusing on the marginal utility of income across a particular subset of dimensions, like income, is somewhat standard in economic models that use welfare weights. While this narrowed focus is often accomplished in theoretical models with homogeneity assumptions, I show in section A.2 how the conditional expectation can be useful without assumptions of homogeneity. Next this is the marginal utility of a nominal dollar and so is conditional on a particular price vector. Last, consistent with the idea of the independence of common scale, marginal utility of income is identified up to a normalization. We can say things about the relative marginal utility across groups, but do not define an absolute value of a util.

Contrary to Jevons’ longing for advanced computational power, I show in section 5, that, with these assumptions, marginal utility is identified up to a normalization with a simple average. In

fact the main analysis in this paper was likely achievable in 1871. I do take some advantage of modern statistics to add controls and estimate a maximum likelihood model in section 7.2 and to analyze binary response style question in section 7.3; however, the core idea is the same as the simple example using the average price of a widget.

While the baseline computations are simple, the difficulty lies in finding real goods like the widget described. The marginal utility of any given good almost always depends on the quantity already consumed and/or the consumption of another set of goods. As an extreme example, suppose the taste of chocolate is equally enjoyable for everyone. This seems like the perfect fit for a widget, but it is unlikely to satisfy the necessary assumption. The marginal utility of a chocolate bar still depends on how much chocolate I already have as well as how many other desserts or food of any kind I consume. Since relaxing the budget constraint, changing income, will change the amount and quality of chocolate and other foods I eat, the marginal utility of a chocolate bar is unlikely to stay the same across income even in this extreme example. We don't actually need the taste to be identical for everyone, but even in that exceptional case, marginal utility is unlikely to be independent from income because the marginal utility is not independent from the consumption of other goods.

I field a survey that elicits the willingness to pay for relief from common minor pains. Using these questions, the identifying assumption is that pain relief from common minor pains is a good with marginal utility that is actually independent from income. Consider the following question. Imagine you bump your shin badly on a hard edge, for example, on the edge of a glass coffee table ⁵. What is the most you would pay in U.S. dollars to completely and immediately eliminate any pain caused by the situation described, as if the event never happened? What other consumption goods make bumping one's shin more or less desirable? There is nothing available to buy to immediately relieve the pain. Any medications will not be accessible or take affect before the pain has subsided naturally. The only real option to relieve the pain is basic things everyone has access to for free, such as sitting down and rubbing it. This also means that no one, regardless of income, is already consuming some quantity of the good because it doesn't actually exist. Everyone is considering a change from zero relief to total relief. I fielded a survey asking 4 open ended questions like the one above as well as three binary yes or no questions with a price randomly selected. The full survey can be seen in appendix E.

Surprisingly, the results indicate that the willingness to pay for relief from these situations, and by extension the marginal utility of income, does not change much across income. This implies that a cardinal measure of preference based welfare is not consistent with diminishing marginal

⁵Question pulled from (Ruscheweyh, Marziniak, Stumpfenhorst, Reinholz, & Knecht, 2009)

utility of income. Both of these assumption are quite common in economics, the latter being so widely accepted it is referred to as a law, and yet one must be relaxed. I discuss the justifications for and interpretations of both possibilities in section 8.

The rest of the paper precedes as follows. Before diving into the analysis, section 2 outlines why cardinal preferences and interpersonal comparisons would be useful or desirable. Section 3 outlines the main assumptions of the model. This covers some of the more nuanced theoretical points that are more implicit in the empirical model. Section 4 covers the previous literature on measuring utility. Section 5 outlines the empirical model used for my painful experience survey. Section 6 outlines the data from the survey, the survey method, and summary statistics about the population. Section 7 details the analysis and results from my survey while section 8 discusses the findings. Section A covers some theory extensions that support the discussion and results but are not central to the main analysis.

2 Why Cardinal Utility is Desirable

While much of economic theory works just fine with ordinal concepts of utility, where utility simply ranks a consumer’s preferences, interpersonal comparisons and discussions about welfare require more. Arrow’s impossibility theorem shows how ordinal preferences are insufficient for making collective decisions (Arrow, 1950; Pearce, 2021), and this motivated even him to look for an alternative to ordinal utility (Arrow, 1978).

Moreover, the assumption of ordinality doesn’t match up with people’s actual views on welfare. The intensity or magnitude of a preferences often conveys useful information. Imagine someone is having five people over for dinner and he knows four out of five prefer clam chowder to potato soup. Suppose, now, he additionally learns that the person who prefers potato soup is allergic to shellfish. Is this not useful information? Do we expect learning this to alter his collective soup choice? Many people actively care about the magnitude of welfare differences, not just the ordering, and so trying to understand those magnitudes is a worthy goal.

A common counter argument is that ordinality is enough to determine “efficient” outcomes. Kaldor (1939) and Hicks (1940) proposed measuring economic efficiency as the sum of aggregate real income. The core idea of their argument is that, of course policy creates winners and losers, but if the winners can compensate the losers, the size of the economic pie has increased and everyone can be made better off. This idea has become widely accepted in economic analysis and modern

practitioners often use such arguments to justify the using the sum of consumer and producer surplus as a measure of economic efficiency. The practice is ubiquitous, but some examples include industrial organization’s analysis of consumer surplus in merger litigation (Glick, 2018; Wilson, 2019), cost benefit analysis, or theoretical policy arguments for things like price gauging (Zwolinski, 2008).

Introductory economics textbooks frequently accept this definition of efficiency at face value and present policy analysis as a balance between equity and efficiency. Betsey Stevenson and Justin Wolfers’ book lays out the idea clearly

One argument for focusing on efficiency is that whenever economic surplus rises it’s possible for those who benefit to compensate those who were harmed, and to do so in a way that ensures everyone’s better off... In reality, it’s rare for new policies to compensate the people they harm. Thus, the argument that it’s possible to make everyone better off is just that, a possibility...consequently, real-world policy debates are rarely just about efficiency. They also focus on equity (Stevenson & Wolfers, 2019, Section 7.1).

While this logic is helpful, it is also an attempt to have our cake and eat it too. If we are not actually achieving a Pareto gain then we are not avoiding interpersonal comparisons or sticking to ordinal preferences. Weighing a gain in consumer surplus against a change in economic inequality, for example, also requires an assumption that preference satisfaction correspond to an inter-personally comparable and measurable metric of well-being and that the collective well-being can be compared to distributional concerns.

Hendren and Sprung-Keyser (2020) provide a helpful framework for thinking about this trade-off. They propose measuring the marginal value of public funds for individual policies using the following logic “Given two policies, A and B, suppose $MVPFA = 2$ and $MVPFB = 1$. Then one prefers more spending on policy A financed by less spending on policy B if and only if one prefers giving \$2 to policy A beneficiaries over giving \$1 to policy B beneficiaries.” As I show in section A.2, this logic works well if policies impact homogeneous groups but places a high informational burden on policymakers for heterogeneous populations. Most importantly, it very much assumes utility is cardinal and corresponds to individual well-being.

The premise of the equity efficiency trade-off actually merits even further challenging. Once we consider heterogeneity of preferences, increasing surplus does not necessarily mean everyone could be made better off (Samuelson, 1950). Suppose a policy actually leads to an outcome where

everyone is made better off, this does not necessarily mean that policy would be preferred by even by a utilitarian policymaker if the utility possibility frontier shifts as result (Samuelson, 1950). Samuelson (1950, pg 10) shows that without comparing an “Infinite number of points, no acceptable definition of an increase in potential real income can be devised at the non-ethical level of the new welfare economics.” The problem Samuelson outlines is simplified by abandoning the goal of a universal index number to measure “efficiency” and instead building on a given set of welfare weights or normative ethics supplied by our policymaker. Rather than trying to measure and compare changes in the entire utility possibility frontier, we can use instrumental rationality to point to the best of the feasible outcomes.

Samuelson’s above critique applies even when allowing for theoretically cost-less lump sum transfers, but as he continues to explain, movement along a utility possibility frontier would require “an ideally perfect and unattainable system of absolutely lump-sum taxes or subsidies (Samuelson, 1950, pg. 18)”. Lump sum transfers to increase equity simply don’t exist. Hendren (2020) takes this concern seriously and estimates the cost of redistribution through the income tax code. This provides an estimate of the utility *feasibility* frontier under the current policy regime.

García and Heckman (2022) challenges the Hendren (2020) approach by pointing out some issues with the MVPF framework, mainly that it is best suited for revenue neutral policy changes. However, Garcia and Heckman fall into the flawed logic of Kaldor and Hicks, stating that an increase in the net social benefit results in an expansion of the utility possibility frontier. This is only true if the utility possibility frontiers are parallel, which requires unrealistic restrictions on preference heterogeneity (Gorman, 1955).

While all of these efficiency measures can undoubtedly be useful, the above discussion shows it is rare for policy decisions of any consequence to be devoid of normative ethical considerations. If many hidden assumptions are not met, such as cost-less transfers or parallel utility possibility frontiers, efficiency measures have opaque normative implications and often require cardinality assumptions anyway.

3 Identifying Assumptions

The empirical model is incredibly simple. So simple, in fact, it glosses over some important assumptions that are implicitly being made. Like, for example, what is utility? The following assumptions lay out the theoretical requirements necessary to identify the marginal utility of

income.

The first assumption needed for identification is the following

Assumption 1. *The Marginal utility of the pain relief described in the survey questions is independent of income.*

So if we look across income groups, the average utility from pain relief would always be equal. We can see this in the diagram in figure 1 represented as equality 1 (corresponding with assumption 1). The estimation strategy will work for any conception of utility that satisfies this equality, but what exactly does it mean for utility to be equal across people?

Utility is a concept that is frequently used in economics and philosophy but it's definition is not consistent and it is often used in differing and conflicting ways. Assumption 1 requires a cardinal definition where the experiences of two different people, or the average of two groups to be more exact, can be considered equal. However, even among cardinal definitions of utility there is a wide variety of views about what exactly utility is.

Hausman and McPherson (2006) provide a clarifying framework for grouping theories of well being. Substantive theories say *what* things are inherently good. For example happiness, or pleasure could be considered the actual meaning of utility. The substantive approach fits with the work of utilitarian philosophers Jeremy Bentham, John Stuart Mill, and Henry Sidgwick (Driver, 2014). These theories fit well with assumption 1. It is reasonable to think, for example, that two people hitting their shins are experiencing the same pain or lack of pleasure.

Formal theories instead specify *how* to find out what is good, but not what those good things actually are (Hausman & McPherson, 2006). Welfare is considered to be the satisfaction of preferences and utility is the extent to which those preferences are satisfied. Irving Fisher actually preferred terms like “wantability” to avoid conflation of what economics is considering utility with the substantive theories of “Benthom and his school” (Fisher, 1927). However, some moral philosophers have accepted this as their preferred understanding of utility (Hare, 1981). If utility is the satisfaction of preferences how can we compare that across people?

Harsanyi (1955, 1986) proposed a way to think about how to compare the intensity of preferences across people using “extended preferences”. This idea was even considered by Arrow (1978) as a way past the independence of irrelevant alternatives assumption in his impossibility theorem (Arrow, 1950). The idea proposes a thought experiment for interpersonal comparisons of utility. To compare the utility cost of two people hitting their shins on a coffee table we must imagine

being each person and hitting our shin and compare those experiences. Harsanyi (1986) explains further that you have to imagine yourself with that person's preferences. MacKay (1986) calls this the mental shoehorn trick (Hausman & McPherson, 2006).

While this seems to be the dominant theory for rationalizing interpersonal comparison of preferences, it is controversial. Arrow (2012, pg 115) says of it, "The principle of extended sympathy as a basis for interpersonal comparisons seems basic to many of the welfare judgments made in ordinary practice. But it is not easy to see how to construct a theory of social choice from this principle." Moreover, theoretical work has considered it unsatisfactory (Hausman, 1995; MacKay, 1986). The advantage of the particular assumption in this paper, however, is that we need not be capable of comparing every possible situation, only the ones in my survey which have been specifically chosen for their ease of utility comparisons. Imagining oneself as a rich person hitting their shin and a poor person hitting their shin seems like a reasonable task. My conclusion is that I expect the experience to be much the same, and feel any of the above definitions of utility would satisfy assumption 1. The medical literature does provide some helpful references for substantiating that belief.

The International Association for the Study of Pain (IASP) defines pain as "An unpleasant sensory and emotional experience associated with actual or potential tissue damage, or described in terms of such damage" (Aydede, 2019). Under this definition, people have "epistemic authority with respect to their pain: they seem to be incorrigible, or even infallible, about their pains and pain reports" (Aydede, 2019). This means pain cannot be measured with a medical instrument, it must be elicited from the person experiencing it. The pain sensitivity questionnaire (PSQ) is a validated measure of pain sensitivity. First, respondents were asked to fill out the survey questions. Then, their responses are compared with questions asked during actual painful experiences (heat, cold, pressure, and pinprick) (Ruscheweyh et al., 2009). While this shows that the survey is consistent with more costly experimental outcomes, it is important to keep in mind that the PSQ scale is still ordinal and it may not be the case that a 5 is a similar experience across people.

While the very nature of pain makes it difficult to measure, the medical literature seems to suggest that, if anything, lower income people are less tolerant to pain. Miljković et al. (2014) shows that participants with a lower household possession index, suggesting lower income, are more sensitive to pain. Research has also shown that chronic pain increases pain sensitivity (Ruscheweyh et al., 2012). These results align with the idea that richer folks may have access to goods that decrease pain sensitivity. While nothing can immediately relieve the pain from bumping my shin, perhaps the knowledge that I can reward myself for enduring that pain with an expensive treat makes the experience less difficult. Moreover, the research on chronic pain suggests that perhaps

rich and poor folks are not coming from the same starting quantity of daily pain. If there are diminishing returns to pain relief, this would impact that benefit of relief from a marginal painful event.

If these relationships are true, my estimates will be biased since low income folks will be getting more of a benefit from pain relief. Despite the findings described, I do not find the medical literature to be conclusive. The chronic pain research, for example, suffers from selection bias. If 10 people actually have a chronic pain condition, the people that are least tolerant to pain are the ones who will actually go and get a diagnosis. This is not really addressed in the medical studies I have read probably because it is not important for their purposes. They are describing attributes of patients with chronic pain diagnosis and this is true even if it is because of selection. Additionally, studies assessing pain tolerance, like how long you can endure holding your hand in ice water may be testing a commitment to the study or looking tough as much as the epistemic experience Stephens and Robertson (2020). To assuage concerns that pain tolerance may be correlated with income, my survey includes questions from the pain sensitivity questionnaire so pain sensitivity can be controlled for.

The second assumption needed for identification is about connecting our concept of welfare from assumption 1 to people's preferences and action. Specifically,

Assumption 2. *Individual behavior is the result of maximizing utility where utility holds the same meaning as in assumption 1. This implies if a person is willing to trade two goods, they provide the same cardinal utility.*

This assumption gives us the equalities labeled with a 2 in figure 1. Since people are willing to trade \$X or \$Y dollars for pain relief on average, they must provide the same utility.

For this to hold, people need to do what gives them the most utility under the same theory of utility used in assumption 1. If the theory of utility used in assumption one is a substantive one, people must always prefer the things that increase their own substantive utility. Another way of putting this is that we are requiring the formal preference utility theory to hold, people prefer what gives them the most utility, but this does not preclude a substantive theory from also holding. The two can simultaneously hold if, for example "happiness is the ultimate object of preference, then it could be true both that well-being is the satisfaction of preference and that well-being is happiness" (Hausman & McPherson, 2006, Pg 119). Now if for someone who prefers the preference utility and extended sympathy arguments in the first place, assumption 2 is not adding much to assumption 1.

This assumption is a utilitarian theory of action, and it is important to clearly distinguish this from a utilitarian theory of ethics. Utilitarian ethics says the ethical action is the one that maximizes the most good for the most people. When I refer to utilitarian policymaker, this is the social preference I am referring to. A utilitarian theory of action requires people to act more like they ascribe to ethical egoism, since an individual's utility can often come into conflict with what maximizes collective utility. The theory of action is what is required by assumption 2

This is also distinctly different than saying that people's actions *can* be described *as if* they are maximizing *some* utility. If we define utility as an ordinal ranking of preferences then it is tautologically true that people with rational preferences maximize it. Jevons reflected insightfully on this distinction, saying "Call any motive which attracts to a certain action pleasure, and that which deters pain, and it becomes impossible to deny that all actions are prompted by pleasure and pain" (Jevons, 1879; Moscati, 2018). Here he is reflecting on the Benthamite definition of utility but the same is true of others.-

This is not an uncommon assumption. The MVPF framework in Finkelstein and Hendren (2020); Hendren (2020); Hendren and Sprung-Keyser (2020) is a prominent example in public finance, but the consumer welfare standard in industrial organization or really any cost benefit analysis using a willingness to pay measure is implicitly making this assumption.

Despite being commonly used in economics, this is not an uncontroversial or obvious assumption either⁶ Sen (1977) makes this point more clear with a distinction between sympathy and commitment, "The former corresponds to the case in which the concern for others directly affects one's own welfare. If the knowledge of torture of others makes you sick, it is a case of sympathy; if it does not make you feel personally worse off, but you think it is wrong and you are ready to do something to stop it, it is a case of commitment" (Sen, 1977, pg 326). In either case, an ordinal utility defining choice would put a higher utility index on intervening to stop torture, but only in the former does the choice indicate a higher personal welfare from intervening. Assumption 2 assumes all actions are coming from sympathy. Sen goes on to point out this assumption turns people into a kind of egoist since they only engage in actions that are in their own self interest⁷.

Now, believing assumption 2 and denying Sen's criticism is not a requirement to learn something from the results. Quite the contrary, as with any model based on an if-then statement, the results may lead to updating beliefs about the "then" results or updated beliefs about the "if" assumption.

⁶Although my experience grading papers for an ethics in economics class tells me that many undergrads trained in economics take this assumption as an indisputable fact of the universe.

⁷They are a *kind* of egoist only, Sen points out, because we aren't dictating the cause of their actions, which may not be self interest, only that the actions happen to coincide with self interest.

Results in line with estimates of subjective well-being or survey’s on perceptions of well-being would suggest the assumptions hold and give more precise estimates of well-being from revealed preference (Kimball, Ohtake, Reck, Tsutsui, & Zhang, 2015; Stevenson & Wolfers, 2013). Results far outside that range would support Sen’s criticism.

The final equality in figure 1 just follows by the transitive property and gives us the ratio of the average marginal utility of income between the two groups. While this diagram is a simple example with two groups, the same logic applies to more groups or a continuous function across income. Before connecting the assumptions here to the empirical model, the following section discusses the existing literature on utility measurement.

4 Previous Efforts to Measure Utility

Attempts to measure utility have a long history in economics. One of the main problems to overcome in estimating utility is that the marginal utility for any particular good will change with changes in income in unpredictable ways because of cross derivatives in cardinal utility. As I discussed above, even if the desire for chocolate is constant, as people gain income, the value of a marginal chocolate bar might change quite a bit depending on how consumption of chocolate and other goods changes with income. The value may go down if I consume other desserts, with a negative cross partial, or it may go up if I consume goods like strawberries or truffles, which might have positive cross partials⁸. These cross partials represent compliments and substitutes in utility when utility is cardinal and is how the terms were originally conceptualized (Auspitz & Lieben, 1889; Moscati, 2018).

While it may seem possible to control for consumption of other goods that impact the marginal utility of chocolate, the curse of dimensionality will make considering all the relevant goods impossible. Not only that, but if there is heterogeneity in preferences about those goods along the income distribution it will be impossible to disentangle differences in preferences about compliments and substitutes in utility from differences in the marginal utilities of income.

Irving Fisher recognized that these cross partials are the key complication in identifying utility and proposed overcoming this problem by looking at large consumption classes like food and housing. He proposes that “the utility derived from the consumption of each commodity group depends only upon the quantity of that commodity group that is consumed.” (Fisher, 1927). The

⁸Covering these thing in chocolate is good.

marginal utility of food, for example, must not depend on the type of housing or entertainment a person has. This independence assumption serves the same purpose as my assumption that pain relief is independent of other goods. It bypasses the need to look at compliments and substitutes because there are none. However, in Fisher's case, rich and poor folks are not going to consume the same amount of quality adjusted food or housing on average. He proposes using a different location with an alternative price vector where people consume the same amount of food as the low income group, but the same amount of housing has the high income group⁹. Using this method requires solving a difficult if not impossible index number problem.

Morgan (1945) actually attempts to implement fisher's method and points out several difficulties. However, both Morgan and Fisher treat the required index number problem as a problem of mathematical and statistical precision. In fact, The correct index number is inextricably linked with the question of marginal utility once we allow for heterogeneous preferences across regions. Fisher's method needs the index number to show that two families "enjoy the same or equivalent food rations (Fisher, 1927)". What is the equivalent consumption of quality adjusted food for an American in dollars and an English person in pounds? The relative enjoyment of the average English person for Shepherd's Pie compared to the relative enjoyment of the average American of a cheeseburger will differ and so the ability to purchase a given consumption basket in either country does not make the utility value of a given dollar expenditure on food equivalent to the average person in the two countries. This is a general problem with index numbers or measures of inflation under heterogeneous preferences (Samuelson, 1950). We can't equate the utility consumption of food with index numbers alone and instead need a measure of the utility of food consumption to say the two people get equivalent utility from their food. Unfortunately, this is as difficult to apprehend as the original question of marginal utility of income!

Rather than rely on aggregation to achieve a good without compliments and substitutes in utility, I use a good with it as an inherent quality. This avoids the concern about different preferences for a given food price vector. Since the pain relief I describe is also not something anyone can purchase, and so the rich and poor are considering the same marginal change, I also avoid the need for comparisons across different price indices, bypassing the index number problem entirely. The core idea however, getting a good with no cross partials, is the same as in my strategy.

For an interesting and more thorough overview of this early research see (Dimand, 2019) and for a more thorough account of the history of thought on measuring utility see (Moscati, 2018).

⁹Or vice versa. The consumer with alternative prices needs to act as a link between any two broad class of goods

5 Empirical Model

With these assumptions in mind we can now consider a more explicit empirical model. This model starts with an explicit functional form for cardinal utility and so takes assumption 2 as given.

$$U(m_i, X_i, \epsilon_i) = \phi(m_i) + r(X_i, \epsilon_i) \quad (1)$$

Where m is money and r is relief from one of the painful experiences in my survey. I have assumed that the marginal utility of income is constant across people, but is a function of income. There are different types of people, some more or less tolerant of pain. These types are described by the characteristics in the vector X_i . Additionally, each individual's utility from pain relief has an idiosyncratic error ϵ_i . It is crucial that $m_i \notin X_i$. That is, pain relief cannot also be a function of income. It is also crucial that the utility of pain relief and income are additive separable. This is a key part of satisfying assumption 1 since this ensures there are no non-zero cross partials. With these assumptions, we can see that the reservation price for person i for pain relief is the following

$$P_i^r = \frac{r'(X_i, \epsilon_i)}{\phi'(m_i)} \quad (2)$$

The price of money is normalized to 1 and $\phi'(m_i)$ is the marginal utility of income. $r'(X_i, \epsilon_i)$ is the marginal utility from taking the pain relief in my survey questions. While this model should be familiar to anyone who has taken an intermediate economics course, the key point here is that the cardinality and separability assumptions are meant to literally hold in this particular setting rather than just act as a simplification.

Building on equation 2, the following theorem shows when marginal utility of income can be identified with a conditional average.

Theorem 1. *If $r'_i \perp\!\!\!\perp m_i$,*

then the following holds up to a normalization α

$$\mathbb{E}[P_i^r(m)|m] = \frac{\alpha}{\mathbb{E}[\phi(m)|m]} \quad (3)$$

Not that the if statement here is equivalent to assumption 1. To show this start by taking the conditional expectation of both sides of equation 2. The expectations here are expectations across

people for a given income level m . Note that we could take an unconditional expectation, but that is not what we are interested in. The unconditional expectation would identify the average marginal utility of income within this sample¹⁰, I want to identify the marginal utility of income as a function of m . This gives

$$\begin{aligned}
\mathbb{E}[P^r(m_i)|m] &= \mathbb{E}\left[\frac{r'_i}{\phi'(m_i)}|m\right] \\
&= \frac{\mathbb{E}[r'_i|m]}{\mathbb{E}[\phi'(m_i)|m]} \\
&= \frac{\mathbb{E}[r'_i]}{\mathbb{E}[\phi'(m_i)|m]} \\
&= \frac{\alpha}{\mathbb{E}[\phi'(m_i)|m]}
\end{aligned}$$

After conditioning on m , $\phi'(m_i)$ is a constant and so independent of r'_i and so the expectation of the product is equal to the product of the expectation. Next, $r'_i \perp\!\!\!\perp m_i$ by assumption, and so we can remove the condition from the numerator. Finally, $\mathbb{E}[r'_i]$ is a constant that we can normalize to α

This normalization means we have not identified utility in the absolute sense, but in accordance with the independence of common scale, we have identified it up to a constant normalization. The ratio of $\phi'(m_1)$ to $\phi'(m_2)$ is the same for any normalization regardless of m_1 and m_2 . So we can say things like the marginal utility at m_1 is twice the marginal utility at m_2 . But we cannot say a specific number of utils. A useful normalization would be to choose particular income of interest. Let the average income be \bar{m} and set $\alpha = \mathbb{E}[P^r(m)|m = \bar{m}]$ so $\mathbb{E}[\phi'(\bar{m})] = 1$. Now setting welfare weights equal to the inverse of marginal utility will scale WTP to what it would be if marginal utility was unchanged, but everyone had the average income $m_i = \bar{m}$.

An important point that is implicit in this model is that the price vector is fixed. In this simplistic model where money is a single good, it is hard to see, but in appendix A.1, I present the same model using indirect utility, heterogeneity, and many goods. With that approach, it is clear that the model is conditional on a price vector. This implication of this is that a change in the relative price of goods, in particular the ratio of the average price of goods consumed by the

¹⁰This would only hold up to a normalization so on it's own it's also meaningless. You could compare it to a different sample though I suppose.

rich and the poor, will shift the marginal utility of income function. I discuss the implications of this in more detail in section 8

If income is categorical, we can simply take the average reservation price across income bins. Alternatively, the relationship between income and the reservation price can be estimated with any parametric or non-parametric estimation technique for a conditional average. For example, a quadratic polynomial. Before showing the results or building on this model, the next section reviews the data collection, population, and summary statistics.

6 Survey Data

The data for my main analysis comes from a new survey fielded to 1747 respondents through the survey panel company Centiment¹¹. Respondents are recruited through Facebook, LinkedIn, and partner networks to fill out surveys for money. While not a truly random sample, My respondents are matched to the census on age, race, gender, and region and these demographics are provided to me by Centiment.

The final full set of survey questions can be seen in appendix E. The first two sets of questions are variations of validated question on the pain sensitivity questionnaire (PSQ) Ruscheweyh et al. (2009). The first set asked an open ended question asking respondents to self report their willingness to pay. The second set asks a simple yes no question of the style, would you pay \$X to relieve that pain, where X is randomly selected. The next set of questions is a subset of the PSQ asking respondents to say how painful scenario's would be on a 0-10 scale. Many of these overlap with the WTP questions to get a sense of individual aversion to particular events

I chose the PSQ questions that were reported as the most severe, but also that resolve in a relatively short amount of time. The pain resolving in a short amount of time helps satisfy the assumption that the marginal utility of relief from these experiences is uncorrelated with income because other goods do not impact the experience. To better see why these are a good fit, consider an excluded question that asks about pain from a sunburn. Sunburn is much less likely to satisfy the independence assumption because the respondents job or access to soothing medication might make a difference on the actual pain sunburn causes.

One question in the first section and one in the third ask about scenarios that typically would not be painful at all. I refer to these questions as “catch questions”. These “catch questions”

¹¹website: <https://www.centiment.co/>

are used to determine who is engaging in good faith in the questions and paying attention. How exactly these were used to exclude responses is outlined in section 6.1. Additionally, in the third section of the survey there is a question asking respondents to enter 9 to ensure their full attention. Respondents who did not enter 9 for this are, at that point, removed from the survey.

The next section asks a few basic demographics. Marital status, number of children, education, employment. The final sections ask about income and financial health. The financial health questions come from the Fin-health survey¹².

6.1 Protest Answers and Outliers

It is common in open ended willingness to pay questions to receive protest answers to questions, yet there is not an agreed upon way in the literature of handling them (Boyle, 2017, pg.110-111). My first step to handling protest answers is removing entire respondents from the survey based on their response to the “catch” questions I mentioned above. An economist thinking about rational behavior may want to exclude anyone who did not answer 0 to both catch questions. The situations are not painful and so rational utility maximizing respondents should not pay anything to relieve the pain and should enter 0 when asked. I do not use this strict of a cutoff for my base specification. It is possible some people perceive some pain from these situations or simply did not consider 0 to be a viable option.

Instead of removing respondents with non-zero answers, I use the following conditions. First, I look at the response to the open ended catch question asking what is the most you would pay to relieve the pain of shaking hands with someone who has a normal grip. I exclude anyone who answered higher on this than any of the other open ended questions. The thought here is that, while they may not think to enter zero, it certainly should be less than or equal to questions that are clearly painful. Second, in order to check for attention, I remove anyone who answered the same thing for every question. This includes 263 respondents who entered zero for every question. I expect this in particular might be throwing away some useful information. Some people who actually have low WTP, perhaps below a dollar, may just default to zero. Especially since the question is not clear on how the payment is processed. I drop them for the main analysis since properly accounting for this truncation behavior will make things more complicated. Finally, I drop anyone who said more than \$5. This is, admittedly, a bit of an arbitrary cutoff but the goal is to allow people who defaulted to low, non-zero answers without including people who were not thinking about the question.

¹²Website: <https://finhealthnetwork.org/>

The last condition is based on the response to the pain sensitivity questionnaire catch question. Here, I ask again about shaking hands with someone who has a normal grip, but ask them to rank the pain from 0-10. I drop anyone who's answer to this question is greater than 3. I tried a few other more complicated conditions, like being less than the mean or less than all other PSQ Questions, but the vast majority of the cases overlapped with the three or less condition anyway. Table 1 shows how many responses were dropped for meeting each condition. 1021 responses remain after all of the drop conditions.

For the open response questions, I also top coded responses. The maximum answer, for example, was one trillion dollars. Even if this were in fact true, it will throw off any analysis using means. I top-code them rather than dropping them since top-coding uses some of the information. Whoever entered such a high amount likely has a true amount that is fairly high as well. The analysis in this paper is looking at the relationship between WTP and income. One potential impact of top coding based on the overall sample is to bias any measurable increase towards zero. To lessen this impact, I top-code within four income groups. The groups and counts per group are shown in table 5. A standard rule of thumb is to add 1.5 times the inter quartile range to the 75th percentile and treat anything above that as an outlier. As my data is fairly skewed, I use 4.5 times the IQR for each income group. The number of top codes can be seen in table 2. The distributions for each open response question after removing protest answers and top coding outliers can be seen in figure 2.

6.2 Summary Statistics

Before jumping in to the analysis of willingness to pay and marginal utility, it is important to understand the population. While Centiment matched my survey sample to the census on age, race, gender, and census region, some respondents were dropped from the sample as described above. Table 3 shows rates for the matched demographic variables in my sample compared to the census. Income was not matched explicitly, but figure 3 compares the results of my survey to the CPS family income. My survey is underrepresented in the \$200,000 and above categories, but is otherwise fairly representative in terms of income. Having a representative sample makes the results of the survey externally valid. The mean WTP conditional on income should represent the actual mean in the United States. However, to gauge internal validity we also want to consider how pain sensitivity differs across income.

How does pain sensitivity, measured by the mean PSQ score from 0-10, change with income? Figure 4 shows there is a slight negative relationship, as the medical literature seems to expect.

Table 4 shows the results of the simple regression from the plot. A one hundred thousand dollar increase in income is associated with a .12 decrease in average PSQ response (on a scale from 0-10). This is not statistically significant and Figure 4 shows that a .12 change is small relative to the within income variation. Nevertheless, I do control for the average responses to these questions in section 7.2.2.

Gender and age may also affect pain tolerance and by extension the utility from pain relief (Bartley & Fillingim, 2013; Lautenbacher, Peters, Heesen, Scheel, & Kunz, 2017). In order to get a clearer picture of overall trends in the following figures, I break up income into four groups. The counts for each income group are reported in table 5. These income groups are also used for some of the analysis. Figure 5 shows that men are more concentrated in the high income categories. Figure 6 and table 6 show that age increases some with income, the average goes from 42 in the lowest income group to 50 in the highest. This indicates a possibility for an omitted variable bias in the simple mean estimates.

While income is a big factor in someone's ability to pay, things like wealth, savings, family size, or expected future earnings may mean two people with the same yearly income have very different financial means and consumption habits. There are two reasons to be interested in how these characteristics vary with reported income. The first reason is the possibility of selection bias in my survey. For example, suppose people with higher ability to pay are over-represented in the survey. In this case, people with low reported income in my sample may be better off, on average, than typical U.S. respondents with the same reported income. In this case the estimated marginal utility of income for a particular income group, while internally valid, will not match a representative sample of the United States. If my sample is truly representative, then the marginal utility estimates will be valid for any policy or data set that is also representative of the United States. If, for example, higher income folks are older, as I observe, and older folks have more savings and so consume more than their income on average, this isn't "biasing" my estimate since that is a characteristic of people in that income group that is consistent in both samples. It is the same logic that makes a simple statistical average valid so long as the population is consistent.

While the above logic applies to the U.S. as a whole, what if we want to know the marginal utility for a sub-population with different characteristics, like single parents? This would be important for any targeted policy. Measuring WTP across these characteristics as well as income will allow for better estimates by matching to the sub-population. The differences may also be interesting in their own right as policymakers may have a normative preference for certain people, like those with kids. We can see in figures 7 and 8 that higher income people both have more kids and are more likely to be married. This will likely temper the relationship one might expect from

a change in income in a model where increasing income increases personal ability to consume one for one rather than one where some may go to a spouse or kids. One explanation is that kids cost money and so the marginal consumption of a parent might be equal to someone with less income but without kids. Another model that would predict this is if child or spousal utility has warm glow affects. Kids, who generally have no income, might have high marginal utility and spending on those kids might have large warm glow affects which in turn increases the marginal utility of a dollar to the parents despite a higher income. Basically, giving money to your kids might be very rewarding. Attempting to disentangle the specific impact is, however, beyond the scope of this paper.

Similarly, employment and race might be indicators for ability to consume. Someone who is temporarily unemployed or retired, for example, may have a low income despite being, on average, pretty well off. The extent to which their marginal utility of income is smoothed over time will impact how much their WTP for pain relief moves in response to a transitory income change. We can see in figures 9 that lower income folks are less likely to be employed, but I do not know if that is transitory or not.

Race is an indicator for facing racial oppression both past and present in the United States. Black Americans have far less wealth than white Americans with the same income Darity Jr et al. (2018). Similarly, facing more discrimination in the labor market will likely lead to lower expected lifetime income and job security which in turn will influence WTP for pain relief for the same present income. Figure 10 shows that, consistent with well documented Black income disparities, Black people make up a lower percentage of higher income groups. Asian Americans have increasing representation along income, while there are too few Native Respondents to discern a relationship. White representation is lowest in the lowest income bin, grows and then drops off a bit in the highest bin.

While the above variables are indirect indicators of consumption and financial well-being, I also asked five direct questions about financial well-being. These questions are drawn from the Financial Health Network’s financial health score. We can see in figure 13 that there is some relationship with income. Higher income levels do tend to answer the “most financially healthy” answers more often, but there is quite a bit of variability at lower and middle income levels. To further investigate the relationship I run a regression of dummies for every response on the five question on income. The adjusted R-squared is .1 and 13 of the 20 dummies are significant. Figure 14 shows the fitted incomes compared to true incomes. While it certainly seems like these things are related, it does seem like financial health is not just a complex indicator for income.

7 Survey Analysis

7.1 Open Ended Mean Results

As I showed in theorem 1, if the marginal utility of pain relief and income are actually independent, the expectation of the reservation price conditional on income gives marginal utility¹³. That conditional mean can be calculated in a variety of ways. Figures 15 through 18 show two ways to estimate the conditional mean for each question. Each plot has a point for every observation. The size of the points indicate multiple observations with the same income and WTP. The reservation prices are normalized so that the mean WTP of people with incomes between \$0-25k is 1. This makes comparisons across questions of the implied ratio of marginal utility more straightforward.

The first method for estimating the conditional average WTP is to estimate the mean for four income groups. The average for each bin is marked with the large diamond. Bootstrap standard errors are included for each mean¹⁴. The second method is the quadratic polynomial shown on each plot. In addition to each individual question, figure 19 shows all four questions grouped together. Since each question is normalized by dividing by the mean of the lowest income bin, the questions are all on the same scale of relative utility. The standard errors for this plot are bootstrapped with a cluster at the individual level. Surprisingly, these figures suggest very little relationship between WTP and income.

One concern with this simple approach is that the assumption that $r'_i \perp\!\!\!\perp m_i$, in theorem 1 might not hold. As I showed in the summary statistics above age and gender are correlated with income. While figure 3 suggests there is not an average relationship between pain tolerance and income, it is worth investigating specific factors. Surprisingly, neither age nor gender has a significant relationship with PSQ. That being said, The PSQ is not a perfect measure of pain sensitivity. In particular, “the most pain imaginable”, may not be constant across groups. One way to further investigate pain tolerance is to look at the unconditional relationship with WTP to see if there are differences across groups. There is not a significant relationship for gender and WTP on the open ended questions. There is, however, a significant relationships with age. Older respondents have higher incomes on average and are willing to pay less on average as we can see in figures 11 and 12. These observations suggest adding controls may make a difference, and doing so will make the results more robust.

¹³up to a normalization

¹⁴Bootstrapping is preferred here given the non-normal distribution

Unfortunately, we cannot simply add controls as a linear parameter into our model of conditional averages across income bins or with the quadratic polynomial. This is because our outcome is in terms of WTP. This means a control for gender, for example, would give a unique intercept in terms of WTP for men and women. But, the model implies that a gap in utility from pain relief between men and women should lead to a larger difference in WTP for higher incomes. So the difference in WTP should scale with income according to the marginal utility parameter that we need to simultaneously estimate. To more appropriately control for these variables, our model needs a bit more structure. The next sections outline a model with controls, prove it is identified, and provide the log-likelihood functions for MLE estimation.

7.2 Open Ended Response With Controls

To move beyond simple averages, we need to further parameterize our model. To start, I aggregate income groups into b income bins. I assume the the marginal utility of income function is

Assumption 3.

$$\phi'_i(m_i) = \sum_{k=1}^b \mathbb{1}_{ik}(m_i \in k) \phi'_k$$

This gives b marginal utility of income parameters, ϕ'_k , to identify. These will be the average marginal utility of income for each income bin. A continuous function, like a quadratic polynomial, could be used as well.

Now for the utility impact of r_i , I will assume X_i and ϵ_i enter the model with linear parameters in the following form

Assumption 4.

$$r(X_i, \epsilon_i) = \beta_1 + \beta X_i + \epsilon_i \tag{4}$$

Where

$$\epsilon_i \sim \mathcal{N}(0, \sigma^2) \tag{5}$$

and

$$\epsilon_i \perp\!\!\!\perp m_i \tag{6}$$

Together, these assumptions give the following equation for the reservation price of pain relief

Definition 1. *Given assumption 3 and 4 the reservation price for an individual is*

$$P_i^r = \frac{\beta_1 + \beta X_i + \epsilon_i}{\sum_{k=1}^B \mathbb{1}_{ik} \phi'_k} \quad (7)$$

In matrix form, the expected price vector \mathbb{P}^r for the full population can be written as

$$P^r = (\mathbb{X}_i \beta + \epsilon_i) \oslash \mathbb{M} \phi' \quad (8)$$

Where \mathbb{X} is the matrix of X_i traits influencing pain tolerance and a constant for the intercept of the numerator. In matrix form, let β be $[\beta_1 \quad \beta]$ from equation 7.¹⁵ Let \mathbb{M} be an n by b matrix indicating what income bin each person i is in. Let ϕ' be a b by 1 matrix of marginal utility parameters. \oslash is element wise matrix division or Hadamard division.

Appendix D proves that the model is identified, but the basic intuition is that it is identified so long as demographic characteristics do not show up in both the numerator and the denominator. For example, differences in pain relief across age cannot be separately identified from differences in marginal utility of income across age. This is why the independence of income and the utility from pain relief is crucial.

7.2.1 Likelihood Functions

We can estimate the model with MLE. Definition 1 gives the following probability density function

Definition 2.

$$P(P_i^r | \mathbb{X}_i, m_i \in k) = \pi \frac{1}{\sqrt{2\pi \frac{\sigma^2}{\phi'_{k_i}}}} e^{-\frac{(P_i^r - \hat{P}_i^r)^2}{2(\frac{\sigma^2}{\phi'_{k_i}})^2}} \quad (9)$$

and the following log likelihood for a population of size n

¹⁵I realize the notation is confusing because I am using beta for two things. I'm not sure how to indicate it in a less confusing way.

Definition 3.

$$L(\mathbb{X}_i, m_i) = -\frac{n}{2} \log(2\pi) + \sum_i^n -\log\left(\frac{\sigma}{\phi'_{k_i}}\right) - \frac{(P_i^r - \frac{1+\beta\mathbb{X}_i}{\phi'_{k_i}})}{2(\frac{\sigma}{\phi'_{k_i}})^2} \quad (10)$$

Where ϕ'_{k_i} indicates the marginal utility parameter for the corresponding bin for m_i . Recall that in assumption 4 the variance of the error term, and by extensions the variance in the utility from pain relief, is constant across income. This assumption allows the ϕ'_k terms to be identified off of both mean differences in reservation prices and differences in the variance. We can see this since the marginal utility of income enters the same terms as σ . The intuition here is that the underlying variance in utility from pain relief is constant. So, observed differences in the variance of reservation prices across incomes must be because of a larger marginal utility of income. Marginal utility also identified in relation to the controls since the model assumes that the WTP gap between, for example, men and woman grows according to the inverse of the marginal utility of income.

7.2.2 MLE Results

I run this model with controls for age, gender, and mean PSQ. The results of the model for each open ended question are show in table 7. Keep in mind the estimates are of marginal utility ϕ_k and not WTP directly like in the unconditional means. Here, a higher value indicates a higher marginal utility. The “lemon juice in a cut” and “burn tongue on a hot drink” are both basically constant, while “touching a hot pot” and “bumping shin on a sharp edge” both actually show an increase in the marginal utility of income after \$25K. The “Hot pot” question increases to 1.74 in the 50-100K group but drops back down to 1.18 in the “more than 100K” group. A nice sanity check is that the coefficients on PSQ are positive and significant for every question. This indicates that people who are more sensitive to pain will pay more to avoid it.

I also run the model grouping the responses for all four open ended questions. This shows a slight increase in marginal utility of income in the \$25-100k range and then a slight drop above \$100k, but not as dramatic as typical assumptions would make it. Note the standard errors are in Red. I have not yet accounted for the within respondent and between question correlation (clustering at the individual level). This will almost certainly make them larger.

The contingent valuation literature suggests that open ended questions are more difficult to respond accurately to and have high rates of protest answers (Boyle, 2017, pg 110-111). With this in mind, I also ask a series of binary choice, yes or no, questions. The price proposed is randomized across respondents, but each respondent only answers each question one time, for one price. This

is the opposite extreme to open ended questions in that it carries less information, but places very less burden on the respondents.

7.3 Binary Choice Model

While a binary choice question may be more familiar for respondents, leading to more accurate answers, assessing the results is more difficult and requires more assumptions. The underlying economic model is the same as in theorem 1, but now we have to model and estimate the reservation price rather than just observing it in the data. I follow the general strategy from Hanemann (1984), but the technique outlined in that paper is to calculate an overall average or median WTP. I update the technique to analyze the change in WTP with respect to income rather than a single collective estimate. The first step is to run a random utility logit regression of the following form.

$$V_i = \sum_{j=1}^4 \delta_j \mathbb{1}(M_i = j) + \gamma X_i + \sum_{j=1}^4 \beta_j \mathbb{1}(M_i = j) * P_i + \epsilon_i \quad (11)$$

Where δ_j is an intercept coefficient for income level j , γ is a vector of coefficients for controls X_i and β_j is the price coefficient for income group j . It might seem incorrect, at first pass, to include unique intercepts for income in the utility model. This seems to imply different utility levels across incomes (the opposite of the identifying assumption). However, it is important to remember that V_i is ordinal utility that should not be compared across individuals and is, in my opinion, better thought of as just modeling choice probability. The differing utility intercepts allow for different choice probability levels across incomes while the differing price coefficient allows the choice probability to decrease differentially with price across incomes. The intercepts allow the model to more flexibly estimate WTP with less strict functional form assumptions. A nice clarifying example is to imagine a good where people of different incomes buy it with the same probability at a price near zero (indicating a similar choice probability intercept). This observation would not in anyway indicate that the good provides the same marginal benefit to both people. Similarly the converse, having different purchase probability intercepts, does not imply different marginal benefits.

Putting our above equation in terms of Hanemann (1984), let α_1 and α_0 be the utility from pain relief and no pain relief respectively. We can write the difference in utility from paying for pain relief as

$$\Delta V = (\alpha_1 - \alpha_0) - \beta_j P \quad (12)$$

Now $Prob_1 = F_n(\Delta V) = (1 + e^{-\Delta V})^{-1}$ is the probability of purchasing relief. This gives a distribution of the reservation price of $1 - F_n(\Delta V) = 1 - G_{P_r}(P)$. Now the mean reservation price, \bar{P}_r^j , for an income group j is represented by

$$\bar{P}_r^j = \int_0^\infty [1 - G_{P_r}(P)] dP \quad (13)$$

Connecting this back to the actual logit model we can write this as

$$\bar{P}_r^j = \int_0^\infty [1 - \frac{1}{1 + e^{\delta_j + \bar{X}\gamma + \beta_j P}}] dP \quad (14)$$

Where \bar{X} is the average value of the controls, age and gender and mean PSQ in this case, for the entire population. This assigns each group the ordinal utility for the average age and gender, but varies the ordinal utility associated with the income group and then scales the ordinal utility by the income specific ordinal marginal utility of income. The difference across groups shows the difference in WTP attributable to changes in income.

A slight variation I also show in the results, which is used in Bishop and Heberlein (1979) and mentioned in Hanemann (1984), is to cap the integral at the maximum price. I report both values, but prefer this truncated version. The main reason is that, with no money on the line, I suspect some subset of respondents are prone to exaggeration and will say yes to wildly high values. Capping the integral limits how inflated those value get. If the exaggerators are evenly distributed across income groups, it won't matter as much for the analysis since what ultimately matters is the ratio across income groups.

Similar to the means for the open ended questions, this binned approach gives more flexibility but throws away some information by grouping different incomes. Instead we can parameterize the marginal utility of income as quadratic function.

7.3.1 Binary choice Results

Figures 20 to 22 show the percentage of respondents that accepted the price at each income level and at the various prices. We can see that, generally, the probability of accepting the offer decreases with the price, but a relationship with income is not clear. Table 9 shows the results of the binary choice model described above with controls for age, gender, and the mean response to the 0-10 PSQ questions. Confidence intervals are calculated by taking the corresponding percentile in a

bootstrapped sample of estimates. The mean and truncated mean are what are described above, but I also report the median estimate described in Hanemann (1984). The first two questions seem to support the same story as the open ended responses while the last suggest an increased WTP, and so decrease in marginal utility. Between the lowest and highest income groups the WTP doubles if we look at the means and increases five times for the median. The confidence intervals for all of these estimates, however, are quite large. This is the biggest limitation of the binary choice approach.

8 Discussion of Results

The results indicate that assumption 2, preferences correspond to a cardinal utility representing well-being, is not consistent with diminishing marginal utility of income (or at least not sharply diminishing). Given the popularity of both of these assumptions this is an unexpected, but interesting result. While the set up of the paper suggests the later should be discarded, the surprising result also casts a new light on what is being revealed with this analysis. The idea that preferences are welfare is, as I discussed above, contested (Hausman & McPherson, 2006; Sen, 1977). Furthermore, the idea that utility is diminishing in income is often an important part of utilitarian ideology (Crimmins, 2021). If we instead treat diminishing marginal utility of income as a known assumption, than since assumption 2 implies there is *not* diminishing marginal utility, than assumption 2 must not hold ¹⁶. Either has interesting implications and while they may seem like drastically different conclusions, the implications are not necessarily as different as one might expect.

First, what it would mean if marginal utility of income was not diminishing? The following welfare function will help to distinguish between two possible interpretations of a constant marginal utility of income.

$$\sum_i W(m_i) = \sum_i \gamma(U(m_i)) \quad (15)$$

In this equation, $W(m_i)$ is the total weight the social planner places on a person with income m_i . $U(m_i)$ is the utility at income level m_i and γ is a function that expresses the policymakers preferences over the distribution of utility. So, A pure utilitarian would have $\gamma(U(m_i)) = U(m_i)$

¹⁶This is the contrapositive of the hypothesis test as laid out in the paper

and value only the sum total of utility. Suppose instead, I have a preference for equity. In this case, even though I agree a rich and poor person lose the same amount of utility from hitting their shin on a table, I would rather it happen to the rich person since they are at a higher level of utility. This means $\gamma(U)$ will be concave in utility. In this case I am still a welfarist, but am not a pure utilitarian since I do not *only* value the sum total of utility.

The results in this paper inform us about the shape of the $U(m_i)$ function. Since marginal utility is flat, utility will be linear. We can treat $\gamma(U)$ as fixed and update our assumptions about $W(m_i)$. This would make sense if we know a lot about policy-maker’s preferences over the distribution of utility, but not much about their preferences over the end distribution of money. Or, we can treat $W(m_i)$ as fixed and update our assumptions about $\gamma(U)$. This makes sense if we know a lot about policy-maker’s preferences for the total redistribution of money, but not much about their utilitarian vs egalitarian preferences.

In this case where I have clear well defined sense of equity preferences for a given amount of utility, learning that the utility of money is linear would not change $\gamma(U)$. A pure utilitarian with $\gamma(U) = u$, would also revert to a linear social welfare function. Interestingly, however, for an egalitarian the impact on the concavity of W is not obvious. Consider a welfarist with a strong equity preference. Similarly to the utilitarian, a linear utility function makes the egalitarian realize that money to the wealthy creates more utility than they thought. However, this also means the *distribution* of utility is much more unequal than they thought. Under diminishing marginal utility someone with twice the income would have less than twice the utility, but with constant marginal utility they have exactly twice the utility. An egalitarian would be more unhappy with the latter scenario. Depending on the shape of the $\gamma(U)$ functions it’s possible that this shifts our egalitarian welfarist towards more redistribution policy rather than less.

While I expect this to be a common first reaction, on further inspection, I actually find it to be the an odd choice. We typically don’t illicit views about equity or ask if people really subscribe to a welfarist or utilitarian ideology, we just ask how much redistribution they would like or infer it from existing policy (Hendren, 2020; Kimball et al., 2015). I don’t have strong priors about people’s equity vs sum total utility views, but I know that most people think giving \$1 to a poor person is better than giving it to a doctor. People do use terms like “it would make a bigger difference to the poor person”, but it’s not actually obvious what they mean by that. Do they mean the poor person desires it more in the utilitarian sense, or do they mean it in some other way? Perhaps they see the things its being spent on as more worthy or more important beyond simple desire. In this case, it makes sense to think of their preference for redistribution, W , as fixed and update our beliefs about $\gamma(U)$. This would suggest the welfarist, but non-utilitarian,

portion of people's re-distributive preferences is larger than we thought and that very few people subscribe to a pure utilitarian view when utility is restricted to reflect preferences.

Is a constant marginal utility of income actually plausible? Is this a reasonable assumption to drop? The idea bumps up against some common intuition but may not cause as much conflict as one would expect. Constant marginal utility of income doesn't mean that there is not still diminishing marginal utility for particular goods. The tenth apple is likely less enjoyable than the first. This is the intuition often provided for diminishing marginal utility. But if I were to double any person's income they almost certainly would not just double the same goods they already purchase. People don't have homothetic preferences and so the quality and type of goods might change significantly.

Non-homothetic preferences play another key role as well. The average marginal utility of income depends on the price vector in the economy. Suppose necessities become more expensive and the consumer goods available at low incomes shift to a lower quality. Suppose at the same time efficiency gains are coming to expensive luxury items and services that the low income workers performing those services could never afford themselves. In this case a dollar might just buy a lot more value in the hands of a wealthy person. This idea is also exacerbated by the related idea that it is expensive to be poor. Berkouwer and Dean, for example, show that households in Nairobi are only willing to pay \$12 for a stove that would save \$237 over two years, and that a low interest loan increases willingness to pay to the actual savings over the life of the loan (Berkouwer & Dean, 2021). Lower income folks may end up buying lower quality goods that don't last as long and so receive less consumption per dollar. These examples together show that our intuition about the tenth apple being less valuable than the first, is not enough to understand the marginal utility income. This is a much more complex problem with many more moving parts.

Another important consideration for the plausibility is how to reconcile a linear utility function with risk aversion? A constant marginal utility of income would imply risk neutrality. Here I think behavioral economics can easily square the circle. For example, reference dependent utility can make a big difference. People might assume that if they won \$100,000,000 they would suddenly be much more flippant with money because it no longer matters. This expectation would influence their risk taking behavior even if, in the event they win, they update their reference point and still desire \$100 about the same.

A final argument is that our understanding and intuition of what utility is under assumption 2 just is not very good. Under this assumption, the marginal utility of income is really about people's attitudes towards spending. How willing are you to part with money for a given gain?

How much do you desire money relative to particular goods? This desire just might not stack up well to our notions of well-being or material benefit. This argument, however, makes the line dividing this reasoning from the following section relaxing assumption 2 instead very,very fine. Maybe desire doesn't scale with income the way we expected, but so long as desire is still an internally consistent measure we can, as discussed above, adjust the $\gamma(U)$ in our welfare functions to match our perceptions of fairness to the implications of assumption 2. But what if desire or a persons actions are not even internally consistent with what is good or welfare maximizing? This brings us to the second possibility. Relaxing the assumption that preferences reveal cardinal well-being.

Sen (1977) presents the issue with assuming preferences reveal cardinal well-being quite clearly

A person is given one preference ordering, and as and when the need arises this is supposed to reflect his interests, represent his welfare, summarize his idea of what should be done, and describe his actual choices and behavior (Sen, 1977).

A single utility function to describe what is, ultimately, multiple different things may just be too simple. So if we actually need multiple functions, what would they all be? The first, which behavioral economics considers, is a difference between a well informed decision without bounded rationality compared to the decisions people actually make. In this case, for example, perhaps people are not good at determining precise reservation wages and end up rounding to whole bill numbers 1,5,10,20,100. This will mask differences in WTP along income for goods with low WTP since the differences are small in dollar terms. The question with the highest average WTP, slamming your finger in a drawer, is the only question we see with an increasing WTP across income, consistent with this theory. While this is still quite comfortable territory for a neoclassical economist, the reality might be even more complicated.

A point that was briefly considered earlier is the idea that people can behave through sympathy or commitment (Sen, 1977). This isn't an intuitive idea to many economists and I find the following example helpful. Suppose we see a person, call them Bob, sacrifice their life to save a stranger. If we are using ordinal utility we put a higher utility value for Bob on saving the stranger than not and that accurately describes Bob's behavior. But, how do we rationalize assigning a higher well-being to the scenario where Bob dies? We can say Bob would experience extreme guilt had he not intervened, so much so that intervening and dying was better for him. This allows us to explain this behavior using sympathy, but this just as easily could not hold. It is physically possible for Bob to not be racked with guilt in the event he stands aside and yet he may still intervene

through an act of commitment. While I find this particular example compelling, a more difficult question is to what extent this is relevant for economists? How much does this dictate behavior in normal economic exchanges? Prior to writing this paper I would have argued that it is probably an uncommon occurrence, but the results make the counter argument more convincing.

Consider for example, a commitment to fair an ethical pricing. It could be the case that people will only buy a good if they desire it more than the money it would cost *and* if they perceive a price as fair. Perhaps for the questions in my survey, the perceived “fair” price is below the preference based reservation price for most people across incomes. So, for example, a rich person declines pain relief at \$10 because it seems unfair, in this hypothetical situation, to ask for such a high amount even though accepting would make them materially better off. This would lead to an observed WTP that does not change across income despite the value based reservation price changing. The questions in Kahneman, Knetsch, and Thaler (1991) indicate that people view firms increasing profits at the direct expense of consumers as unfair. Since there are no clear costs to produce relief in my examples, a higher price may be perceived as the imagined firm simply taking advantage of their situation. If this is how people are behaving, it can not simply be written off as a quirk of my questions. Deriving value from the maximum WTP will be hindered whenever fairness binds faster than the inherent value and that could be quite often. This would support not just having a well informed and miss-informed behavior function, but an entirely separate well-being function that ignores the fairness concerns.

Finally, another important distinction is when actions are dictated by sympathy. Should we include sympathy in well-being? Suppose, for example, people really do donate money because they get a warm glow feeling of satisfaction from donating. Should we include that warm glow in our sum total of utility? Should we weigh the benefit of solving a problem against the cost of eliminating the opportunity for donors to get that warm glow from donating to solve the problem, for example?

What would accepting the idea of many different utility functions mean for economic analysis? While it might seem like serious deviation, it leaves a lot of work completely untouched. Of course any ordinal positive observations are unchanged. Optimal tax models would need to be re-framed, but not really reworked. The social welfare welfare functions used would need to reflect the true underlying value and not necessarily revealed preferences. Policy analysis that uses a welfare framework, but not revealed preferences, would also be okay. For example, in Eastmond, Mather, Ricks, and Betts (2022) we use a welfare framework weighting changes to test scores according to their impact on student well-being. The students don’t need to desire higher test scores and so nothing is changed. What would need changing are things like Finkelstein and Hendren (2020),

where willingness to pay is summed to measure the welfare impact of a given policy since people may be making decisions based on commitment or behavioral mistakes, for example.

A third possibility that I have not yet touched on is that the results are just biased. I certainly do not see this as the most likely outcome, but I do want to touch on the weakness so that future work can improve upon the methodology.

The biggest area of concern is selection bias into the survey. What kind of rich person does a survey for a few dollars? One who is willing to do things for little money. This is a characteristic that will lead them to systematically give lower answers than their average peer. This motivated me to originally go with the company pollfish. They collect responses from phone app users and pay the users with “in app” benefits related to the apps purpose like a free yoga lesson or news article. My expectation is that a wealthy person is more likely to do a survey to get free lives while playing a game like candy crush, viewed more as an ad experience, than to sign up to do surveys for money, viewed more as a job. This is speculation on my part, and the sample in the pilot I did with Pollfish was not representative on observables of the United States. The non-Representative sample is what convinced me to switch to Centiment. However, the results from the pollfish survey show an increase marginal utility of income with income. Appendix F shows the mean figures and MLE results for the questions in my pilot. For the pain questions, the marginal utility for the 0-25 group was high, which I expected was due to non-working spouses or temporary unemployment making up a disproportionate share of that group. However, after this initial income group, we see an increase in WTP, implying a decrease in marginal utility of income. The sample is smaller and does not reflect national demographics, but the difference across these two panels suggests more work is warranted.

It is also possible pain tolerance changes significantly across income. Beyond showing that the PSQ does not change across income, there is not much that is possible to say on the matter. The best way to address this concern would be with follow up studies using other goods that are conceivably uncorrelated with income. In the Pollfish pilot I also asked question about disgusting scenarios. The variance in these questions was much higher. My intuition is that there is much more variance in what people find disgusting and how much of it they can handle. This convinced me to focus on the pain questions in my final survey in order to make the most of my limited budget, but perhaps another look at things like disgust is warranted in future work.

9 Conclusion

Measuring utility is a difficult problem with a long history. Without any structure, measuring the marginal utility of income is not even a well defined problem , but with the right structure and example it is empirically possible. I present a model for a feasible identification strategy for the average marginal utility of income and implement that strategy with a new survey. The results suggest that treating utility as a cardinal measure of well-being is inconsistent with diminishing marginal utility of income. While both of these assumptions are pillars of many economic analyses, I discussed the implications of relaxing both. While the implications of relaxing one or the other seem far apart, a common through line is that our understanding of individual preferences, utility, well-being, and distributional ethics do not fit together as seamlessly as it once seemed.

Appendix A Theory Extensions

A.1 Generalizing the empirical model

The simple empirical model laid out in the body of the paper includes money as a numeraire good. However, the same estimation strategy can be supported using heterogeneous preferences and indirect utility. I think this model is more accurate to the real world where there are many goods and a whole set of prices, but the estimation is ultimately identical and it is more confusing. Thus, it is here in the appendix.

First consider an indirect utility function V that is a function of prices P and income y , but also other characteristics θ like gender, age, pain sensitivity, or anything else that might influence preferences. This allows V to be heterogeneous across different people. This gives

$$V(P, Y, \theta) \tag{16}$$

Now for the pain relief described in our questions needs to be re-characterized as a price change. Let P be the price vector in our current world, and let P' be a price vector where immediate pain relief is free. Now we can define the change in utility from pain relief as follows.

$$V(P', Y, \theta) - V(P, Y, \theta) = \Delta V^j \tag{17}$$

Since these are relatively small changes we can, as in the empirical model in the body, treat this as a marginal change. That means the following equality holds

$$EV(P', P, Y, \theta) U'_y(P, Y) = \Delta V^j \tag{18}$$

Where U'_y is the marginal utility of income and EV is equivalent variation. To shorten the notation, let $EV(P', P, Y, \theta) = EV^j$.

An important note regarding the previous step is that the marginal utility of income is not a function θ . So, while the preference for pain relief may vary by characteristics like age, the marginal utility of income cannot.

Now to identify the marginal utility of income we need the same assumption as in the simpler model. That is, we need the utility from pain relief to be independent of income. With that we get the following

Theorem 2. *if $\Delta V^j \perp\!\!\!\perp Y$ then*

$$\mathbb{E}[U'_y(P, Y)|Y, P] = \frac{\alpha}{\mathbb{E}[EV(P', P, Y, \theta)|Y, P]} \quad (19)$$

Proof. Given $\Delta V^j \perp\!\!\!\perp Y$ we get that

$$\mathbb{E}[\Delta V^j|P, Y] = \mathbb{E}[\Delta V^j|P] = \alpha \quad (20)$$

□

The first equation follows from the independence of the change in utility, ΔV^j and income Y . The second equality is a bit of an abuse of notation, but here the price vector is actually set to our current price vector P , and so this is a constant we can normalize to any level. Now, given this equality we get

$$\alpha \quad (21)$$

$$= \mathbb{E}[\Delta V^j|Y, P] \quad (22)$$

$$= \mathbb{E}[EV(P', Y, P, \theta)U'_y(Y, P)|Y, P] \quad (23)$$

$$= \mathbb{E}[EV(P', Y, P, \theta)|Y, P]E[U'_y(Y, P)|Y, P] \quad (24)$$

The second line is the normalization explained above, the third line is the identify also explained above, the last line comes from the fact that $U'_y(Y, P)$ is only a function of Y and P and so conditioning on those makes it a constant. Rearranging the equation gives us the theorem.

One point this more complex model makes clearer is that the model estimates the marginal utility of nominal dollars, conditional on a price vector. A nominal dollar might not provide the same purchasing power to everyone in a world of non-linear pricing, quality variation, credit constraints, non-convex preferences, and other complications outside of basic economics models. In particular, a marginal dollar may have more, or less, purchasing power the more dollars someone

has. A simple example would be to consider geographic sorting. Suppose richer people live in more expensive areas. In this case richer people value dollars less because the marginal value of consumption is lower, but also because a dollar literally buys less at stores in their area. This example might theoretically be controlled for with geographic price indices, but other examples are more complicated.

The following story popularized by novelist Terry Pratchett illustrating why the rich are able to spend less. Suppose a quality pair of boots that will last ten years is \$50, but a cheap pair that will last only a year is \$10. A poor person, with a marginal dollar, may only be able to purchase the cheaper option despite it being more costly in the long run (Flood, n.d.). The poor person may appreciate the boots more, in line with diminishing marginal utility of consumption, but the richer person is able to purchase boot years at half the price, making their marginal consumption per dollar higher. This example supports the colloquial saying¹⁷, “it’s expensive to be poor”. Higher credit rates, an inability to buy in bulk or take advantage of off peak sales might mean the poor just can’t buy as much with an additional dollar. Berkouwer and Dean, for example, show that households in Nairobi are only willing to pay \$12 for a stove that would save \$237 over two years, and that a low interest loan increases willingness to pay to the actual savings over the life of the loan (Berkouwer & Dean, 2021).

Perhaps a more fundamental consideration is that consumption is not homothetic and so the types of goods people consume with their first \$10,000 a year look very different than after making \$100,000. The first priorities for consumption are essentials like food and shelter. If food staples and housing become more expensive relative to luxury or entertainment goods, than we will see the marginal utility of a dollar for the poor fall relative to the rich. So, in general, the marginal utility of a nominal dollar also captures the relative cost of the differing consumption baskets of each income group. Interestingly, looking at these utility estimates over time could capture to what extent inflation has been concentrated on essentials or low quality items compared to luxury goods.

A.2 Using Condition Expected Marginal Utility

One goal of the empirical methodology I develop in his paper is to better understand and use welfare weights for real world policy analysis. The model below shows the general case for a policy change with completely unrestricted heterogeneous individual preference. That is, preferences can be so heterogeneous that literally every person is different. In the case, which is almost surely the

¹⁷I’ve at least heard this a lot among family and friends. Not sure how common it actually is

case in real life, is there any hope to actually recovering the welfare impact of a policy change even if we know the conditional marginal utility of income? I expand on the approach from Hendren and Sprung-Keyser (2020) to show how estimating a conditional WTP lowers the informational burden on policymakers. This compliments the empirical exercises in this paper which is estimating the conditional marginal utility of income.

To start I want to show the limitations of using the sum of WTP to estimate welfare. Suppose cardinal indirect utility is represented by

$$V(P, Y, \theta) \tag{25}$$

where Y is a vector of “observed by the econometrician” terms that impact marginal utility of income (income, financial health, race, in my case), θ is a vector of “un-observed by the econometrician” terms that influence preferences (and may also influence the marginal utility of income). This can be things like color preference, but also can be as idiosyncratic as individual indicators for an entire individual specific ordinal preference function over all goods. The later example demonstrates that heterogeneity in individual preferences over goods is not restricted in any way with this general parameterization.

Now a social planner is considering a policy j that will change prices from P^0 to P' . The social planner wants to know the sum of the welfare weighted change in utility from their proposed policy j ,

$$\sum_i \gamma_i (V(P', Y_i, \theta_i) - V(P^0, Y_i, \theta_i)) = \sum_i \gamma_i \Delta V_i^j \tag{26}$$

where γ_i is the welfare weight they place on individual i 's utility.

Now let $MUy(P, Y, \theta)$ be marginal utility of income s.t.

$$EV(P^0, P', Y_i, \theta_i) MUy(P^0, Y_i, \theta_i) = \Delta V_i$$

where EV is the equivalent variation for policy j . Let EV_i^j be shorthand for $EV(P^0, P', Y_i, \theta_i)$. Now how does WTP or the conditional marginal utility of income fit into this? If we expect policy makers to make use of simple total WTP estimates, we are using the approach outlined in Hendren and Sprung-Keyser (2020) which gives the following equation.

$$\sum_i \gamma_i \Delta V_i^j = \sum_i \gamma_i MUy(P^0, Y_i, \theta_i) EV_i^j \quad (27)$$

$$= \frac{1}{\sum_i EV_i^j} \sum_i \gamma_i MUy(P^0, Y_i, \theta_i) EV_i^j \sum_i EV_i^j \quad (28)$$

$$= \bar{\omega}^j \sum_i EV_i^j \quad (29)$$

This reorganizing of terms makes it seem like $\sum_i EV_i^j$ will be an extremely useful metric for policymakers since all they need to know is $\bar{\omega}^j$ which is partially normative since it contains the welfare weights γ_i . Even if the policymaker knows γ_i and $MUy(P^0, Y_i, \theta_i)$, it's not clear that they would know $\bar{\omega}^j$ since it includes the interaction of each individuals marginal utility and their equivalent variation for the policy. So knowing $\bar{\omega}^j$ requires knowing about the joint distribution of the policy impact and the marginal utility of the people impacted. This is a high informational burden!

There is a special case where the informational burden is low. Consider the following

Theorem 3. *If $\gamma_i * MUy_i^j$ and EV_i^j are independent then*

$$\begin{aligned} \sum_i \mathbb{E}[\gamma_i \Delta V_i^j] &= \sum_i \mathbb{E}[\gamma_i MUy_i^j EV_i^j] \\ &= n * \mathbb{E}[\gamma_i MUy(P^0, Y_i, \theta_i)] \mathbb{E}[EV(P^0, P', Y_i, \theta_i)] \end{aligned}$$

Cases where this hold would be policies that impact people essentially at random. Another case is where a policy impacts a very homogeneous subgroup that all have similar $\gamma_i \Delta V_i^j$. A welfare policy targeted at single parents with a specific income level is a good example that approximates this situation. Since the welfare weighted marginal utility is essentially constant, than it is independent of the equivalent variation of the policy. In these cases we can multiply the sum of WTP by the average welfare weight for the group, which is the process Hendren and Sprung-Keyser (2020) suggest, and recover welfare.

While this sometimes works, knowing the conditional marginal utility of income allows us to relax this assumption and recover welfare in more cases. With conditional marginal utility, we get

the following

Theorem 4. *If $\gamma_i * MU y_i^j \perp\!\!\!\perp EV_i^j | P, Y$*

$$\begin{aligned} \sum_i \mathbb{E}[\gamma_i \Delta V_i^j] &= \sum_i \mathbb{E}[\gamma_i MU y_i^j EV_i^j | P, Y] \\ &= n * \mathbb{E}[\gamma_i MU y(P^0, Y_i, \theta_i) | P, Y] \mathbb{E}[EV(P^0, P', Y_i, \theta_i) | P, Y] \end{aligned}$$

Now we can still recover welfare from policies that have differential impacts across Y so long as there is no joint distribution between the marginal utility and the EV of the policy in the θ terms. For example, the policy can have differential impacts by income, observed and conditioned on in y , but not color preference, unobserved and unconditioned in term in θ . An example where this would not hold, and so separating the expectations biases the estimate, is if a policy disproportionately helps people who like the color red and we live in a world where red things are expensive and so people who like red have lower marginal utility of income. Color preferences is intentionally pedantic because it demonstrates the difficulty of incorporating heterogeneous preference into a welfare model. We want to allow people to have different color preferences. However, if we don't want to restrict preferences, this also means that if the price of red things increases, the marginal utility of income for people who like red decreases faster than their peers who do not like red. This makes recovering welfare in general seem impossible since so many factors influence the marginal utility of income. On the contrary, this is in fact a mathematical distraction. If red dye is more expensive than blue, should the government transfer money from people who like blue to those who like red? I've never heard of such a request. The vast majority of people simply do not see things like color preference as a valid reason for redistribution.

We can adapt the model to better reflect the actual normative preferences of most people by using a specific set of adaptive welfare weights. For simplicity, let's suppose our policymaker is a special kind of utilitarian. They want to maximize utility, but only by considering differences in marginal utility along dimensions in y , things like income wealth etc, but not along dimensions in θ , things like color preference. Then their welfare weights are the following

$$\lambda_i = \frac{\frac{1}{n} \sum_k MU y(P, Y, \theta_k)}{MU y(p, Y_i, \theta_i)} \quad (30)$$

Note that the numerator is an average of a function. What we are doing here is getting the mean marginal utility of income as a function of P and Y across all θ terms. This gives the following welfare weighted marginal utility of income for an individual

$$\lambda_i MUy(P, Y) = \frac{1}{n} \sum_k MUy(P, Y, \theta_k) \quad (31)$$

In words, this is just the average marginal utility of income across θ . Rather than try to discern the marginal utility difference between people who prefer red and blue, we just assign everyone the average. We can see how this is useful because it makes the welfare weighted marginal utility of income a function of P and Y only. Returning to analyzing the welfare of policy j , since $\lambda_i MUy$ is only a function of P and Y we get

$$EV(P, P', Y, \theta) \perp\!\!\!\perp \lambda_i MUy(P, Y) \mid P^0, Y$$

which gives

$$\begin{aligned} \sum_i \mathbb{E}[EV(P, P', Y, \theta) \mid P^0, Y] - \mathbb{E}[\lambda_i MUy(P, Y) \mid P^0, Y] = \\ \sum_i \mathbb{E}[\gamma_i \Delta V_{ij} \mid P^0, Y] \end{aligned}$$

For a policymaker with these presences, this relationship holds for all policies. Now Y may contain more things than we can observe in a given case. For example it may include income and wealth. but we only observe income in a particular data set. In this case, the same logic applies as before the welfare weight normalization. If the policy does not differentially impact wealthy or un-wealthy people, conditioning on income will give an unbiased estimate of the welfare impact of the policy. The same idea can actually be applied to any outcome, not just WTP, to understand how to recover welfare with heterogeneity. We show this using test scores in Mather, Eastmond, Ricks, and Betts (2022)

Appendix B Tables

Drop Conditions				
All Identical	Not the Minimum	WTP greater than 5	PSQ Greater Than 3	N
Pass	Pass	Pass	Pass	1021
Fail	Pass	Pass	Pass	258
Fail	Pass	Fail	Pass	6
Fail	Pass	Pass	Fail	22
Fail	Pass	Fail	Fail	20
Pass	Fail	Pass	Pass	19
Pass	Fail	Pass	Fail	11
Pass	Fail	Fail	Pass	47
Pass	Fail	Fail	Fail	112
Pass	Pass	Fail	Pass	84
Pass	Pass	Fail	Fail	63
Pass	Pass	Pass	Fail	84

Table 1: Each Row indicates what conditions are passed or failed. One failure leads to the response being dropped. The total for each combination of conditions are in the N column

Top Code Counts		
Question	Outliers Top Coded	Percent Top Coded
q1	62	6%
q2	58	6%
q4	56	5%
q5	74	7%

Table 2

Age Percent Comparison		
Age	Census Perc	Percent
18 to 24 years	12.0	15.8
25 to 29 years	9.2	7.7
30 to 34 years	8.8	9.0
35 to 39 years	8.4	8.8
40 to 44 years	7.9	7.4
45 to 49 years	8.1	8.8
50 to 54 years	8.3	7.3
55 to 59 years	8.6	7.1
60 to 64 years	8.0	5.1
65 to 69 years	6.8	7.1
70 to 74 years	5.2	7.3
75 to 79 years	3.6	4.5
80 to 84 years	2.4	3.1
85 years and over	2.6	0.8

Gender Percent Comparison		
Gender	Census Perc	Percent
female	51.3	49.8
male	48.7	50.2

Race Percent Comparison		
Race	Census Perc	Percent
White	75.1	74.8
Black	14.2	11.5
Native Or Pacific Islander	2.2	1.5
Asian	6.8	3.0
Other	7.4	7.1
No Response	NA	2.2

Region Percent Comparison		
Region	Census Perc	Percent
Midwest	24.0	21
Northeast	17.6	17
South	38.5	38
West	19.9	24

Table 3

Mean PSQ by Income		
Term	Estimate	Std Error
Intercept	5.5014	0.0759
Income Thousands	-0.0012	0.0008

Table 4: Independent variable of regression is individual's average response to all 0-10 pain questions, dependent variable is income. Sample is after removing protest answers

Aggregated Income Counts	
Income	Count
0-25	242
25-50	239
50-100	334
more than 100	206

Table 5

Mean Age by Income	
Income (Thousands)	Mean Age
0-25	42
25-50	46
50-100	48
more than 100	50

Table 6

	Lemon Juice		Hot Pot		Burn Tongue		Bump Shin	
label	est	se	est	se	est	se	est	se
intercept	0.610	0.2100	0.650000	0.2200	-0.0130	0.2200	-0.1100	0.2000
sigma	1.500	0.0610	1.500000	0.0650	1.5000	0.0660	1.4000	0.0600
Mean psq	0.130	0.0270	0.100000	0.0290	0.2100	0.0290	0.2600	0.0280
Age	-0.009	0.0025	0.000055	0.0026	-0.0039	0.0026	-0.0071	0.0025
Male	0.084	0.0940	-0.290000	0.0980	0.1300	0.1000	0.0380	0.0920
0-25	1.000	NA	1.000000	NA	1.0000	NA	1.0000	NA
25-50	1.100	0.0610	1.500000	0.0870	0.9900	0.0570	1.4000	0.0790
50-100	1.000	0.0540	1.700000	0.0920	0.9900	0.0530	1.3000	0.0670
more than 100	1.100	0.0640	1.200000	0.0710	0.9800	0.0600	1.2000	0.0730

Table 7

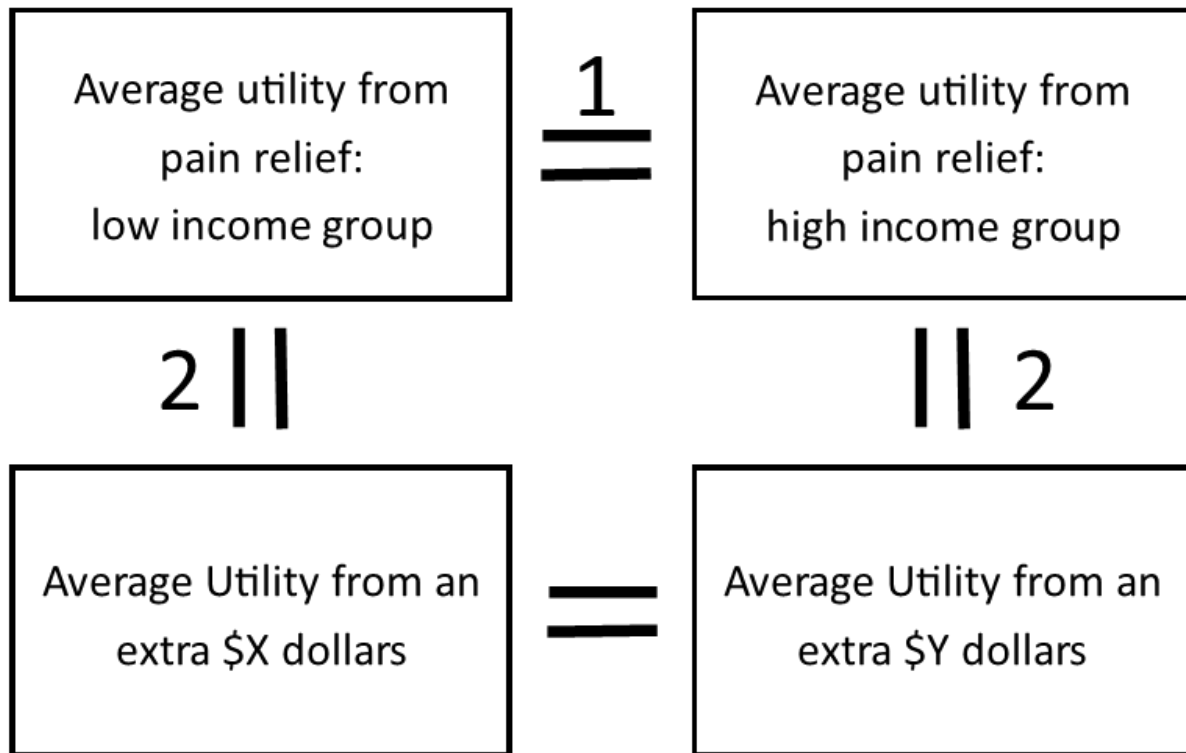
MLE Normed Pain Qs			
label	est	se	P_ratio
intercept	0.36	0.11	NA
sigma	1.50	0.03	NA
Mean psq	0.17	0.01	NA
Age	-0.01	0.00	NA
Male	0.00	0.05	NA
0-25	1.00	NA	1.00
25-50	1.20	0.03	0.83
50-100	1.20	0.03	0.83
more than 100	1.10	0.03	0.91
all questions normed to lowest mean = 1			
mean psq control			

Table 8

Binary Choice With Bootstrap CI									
	Bump Elbow			Bite Cheek			Slam Finger		
	Est	LB	UB	Est	LB	UB	Est	LB	UB
meanWTP									
0-25	25	18	31	36	28	44	69	45	107
25-50	37	19	68	41	19	66	115	66	138
50-100	25	15	34	33	16	47	131	91	183
more than 100	34	19	48	25	19	38	147	123	249
trunc.meanWTP									
0-25	24	18	29	35	28	43	68	45	103
25-50	31	18	41	41	19	59	111	66	129
50-100	23	15	30	33	16	45	120	88	145
more than 100	30	19	38	25	19	38	132	116	177
medianWTP									
0-25	10	3	15	16	5	25	39	16	59
25-50	8	1	21	19	10	30	76	50	94
50-100	3	-2	12	20	12	25	70	42	101
more than 100	14	4	24	16	10	25	83	69	143

Table 9

Appendix C Figures



- X and Y are the average reservation prices for each group respectively

Figure 1

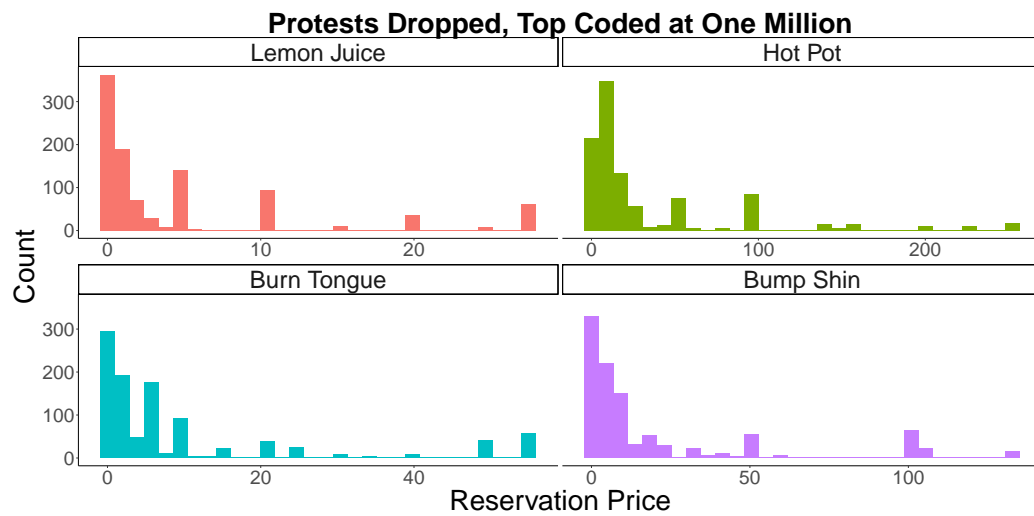


Figure 2

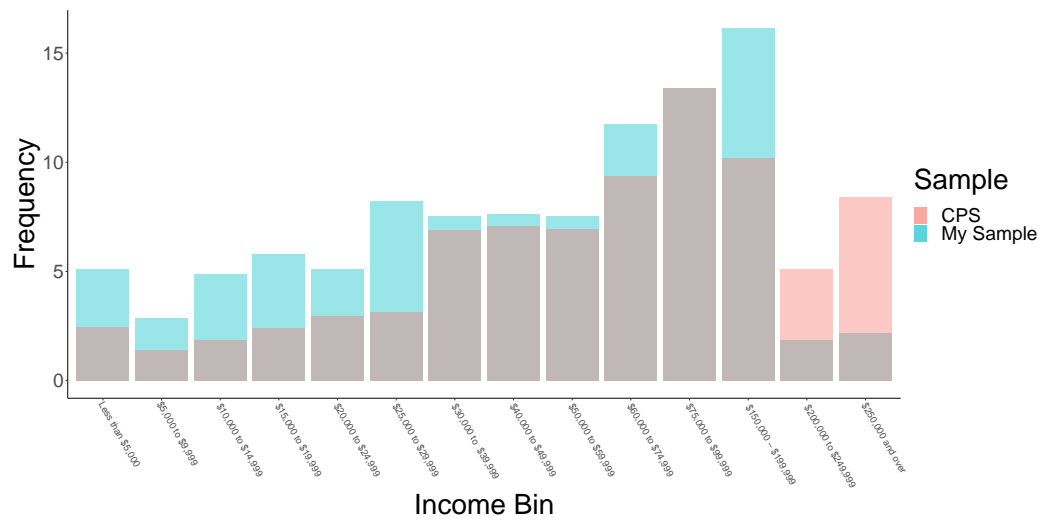


Figure 3

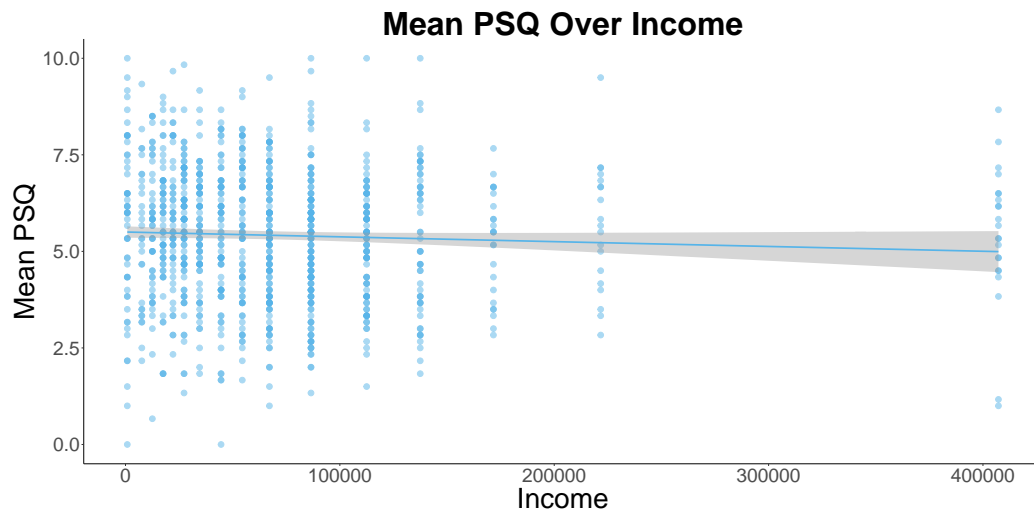


Figure 4: each individual's average response to all 0-10 pain questions after removing protest answers

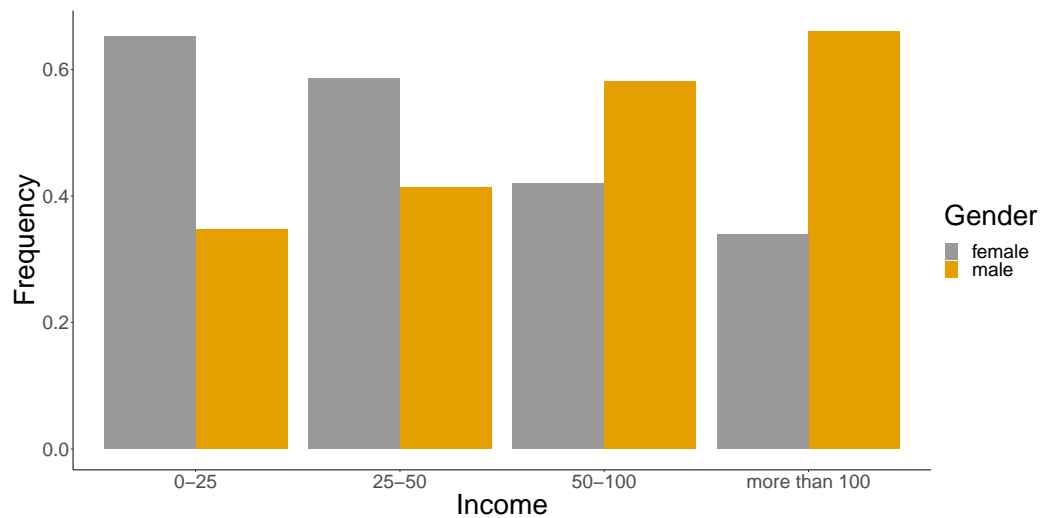


Figure 5: each individual's average response to all 0-10 pain questions after removing protest answers

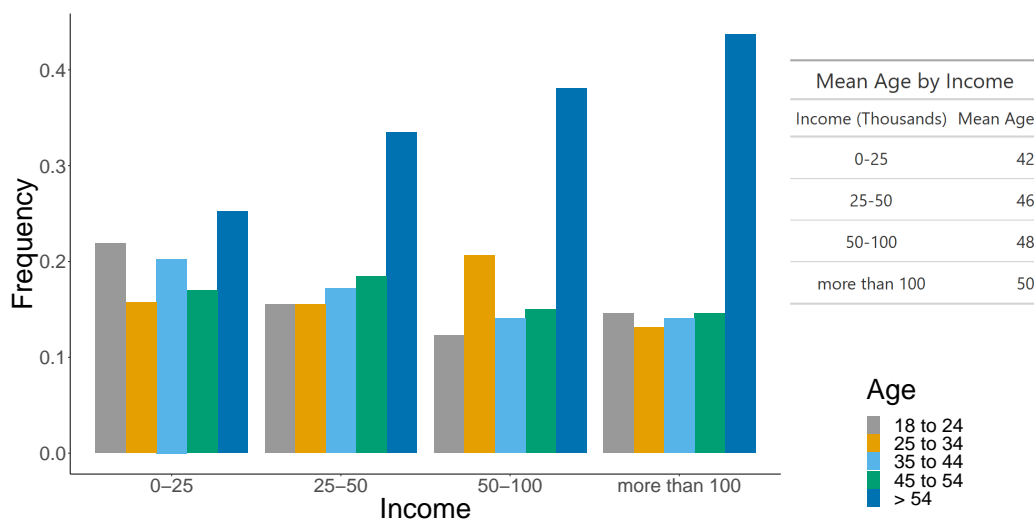


Figure 6

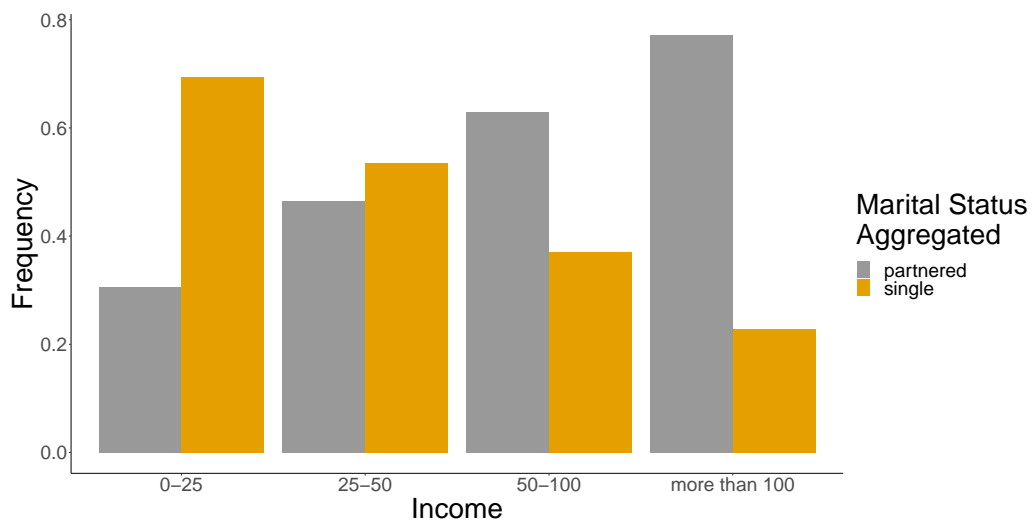


Figure 7: Partnered includes "Living with a partner in a long term relationship" and married, while single is "Single, never married", Widowed, Divorced, Separated.

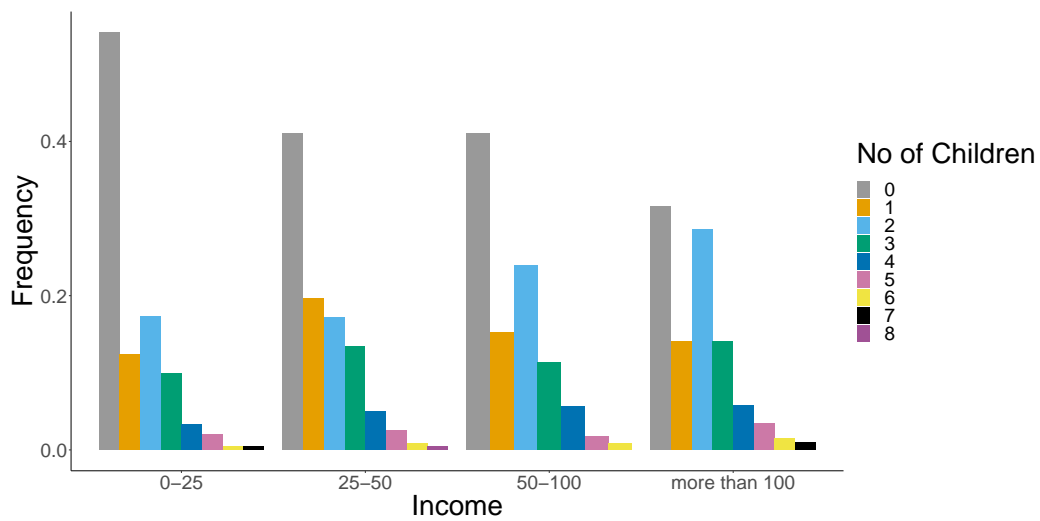


Figure 8: 8 was the max reported in my survey

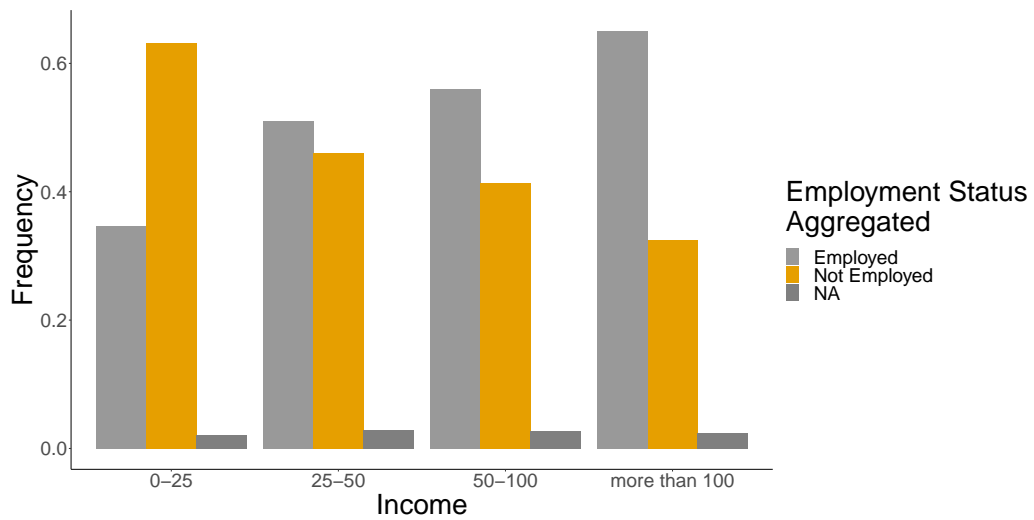


Figure 9

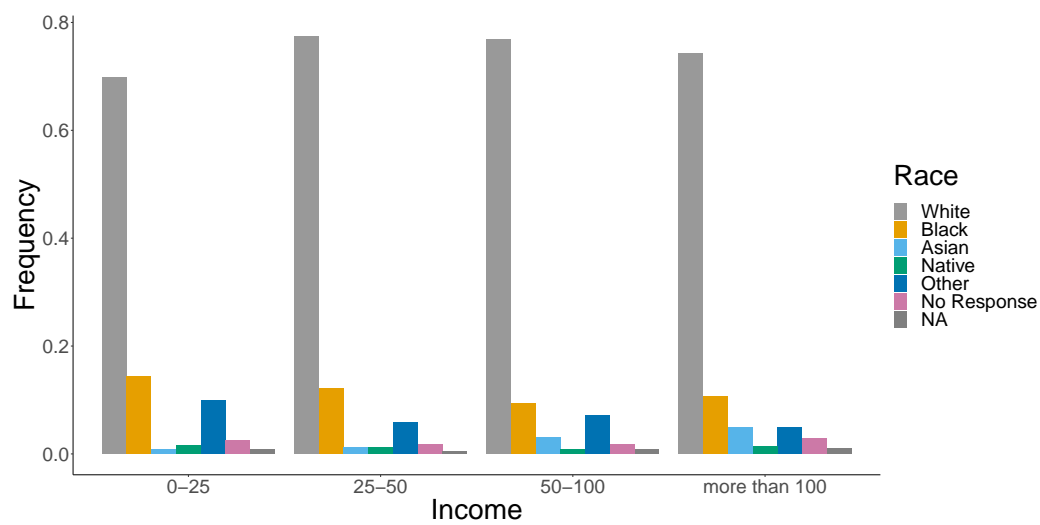


Figure 10



Figure 11

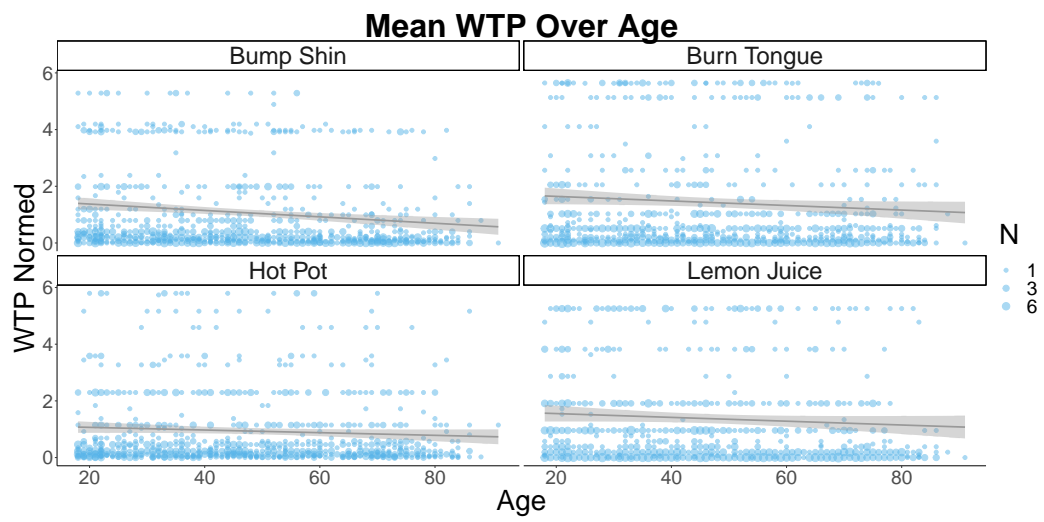


Figure 12

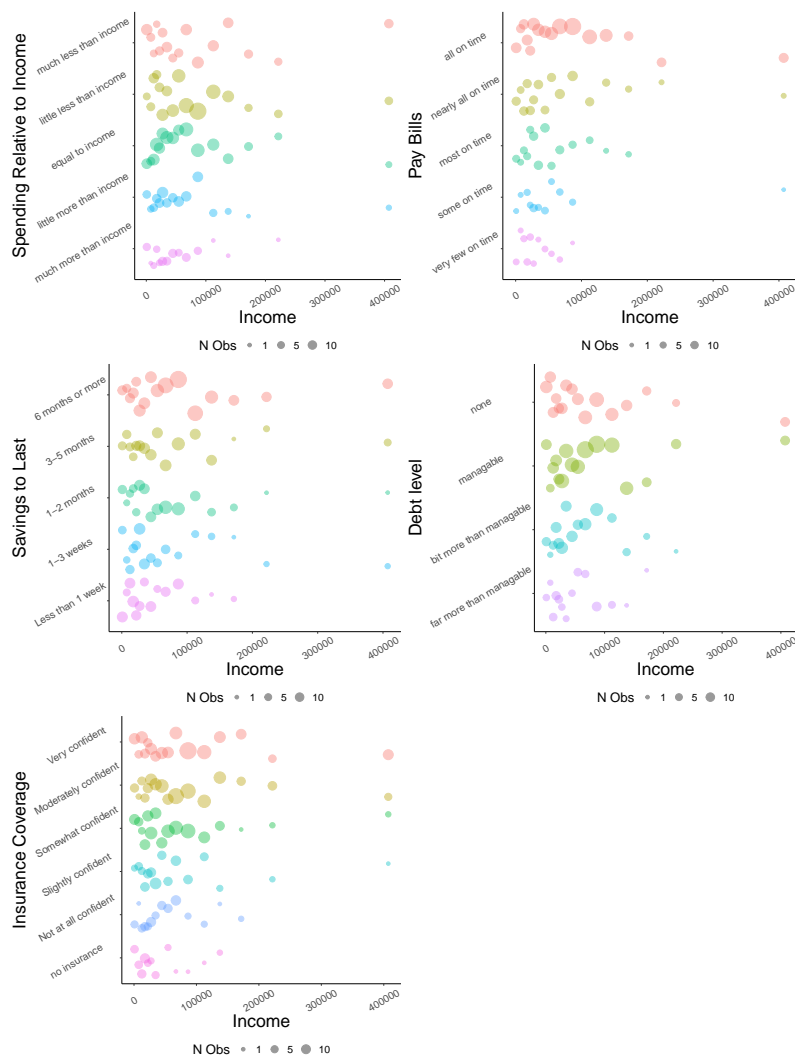


Figure 13: The answers correspond to questions in appendix

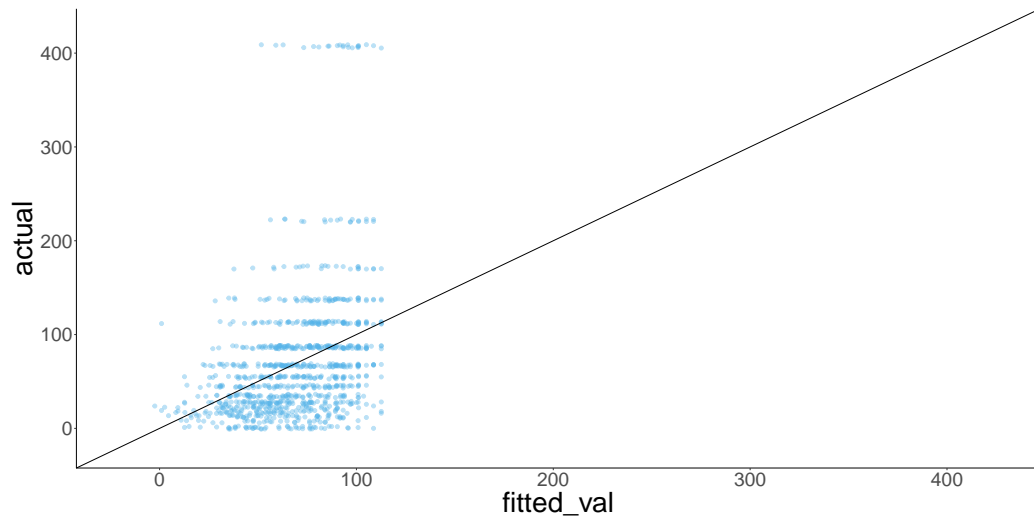
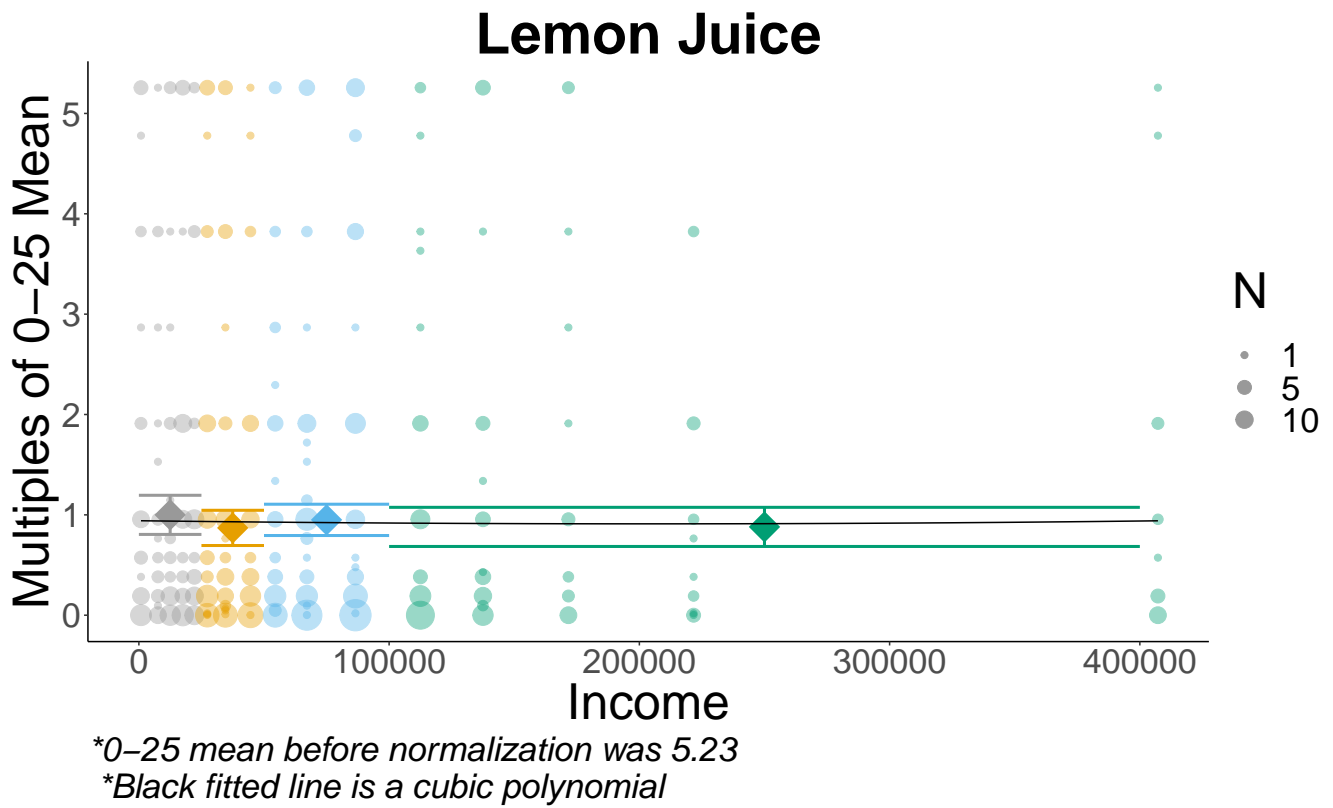


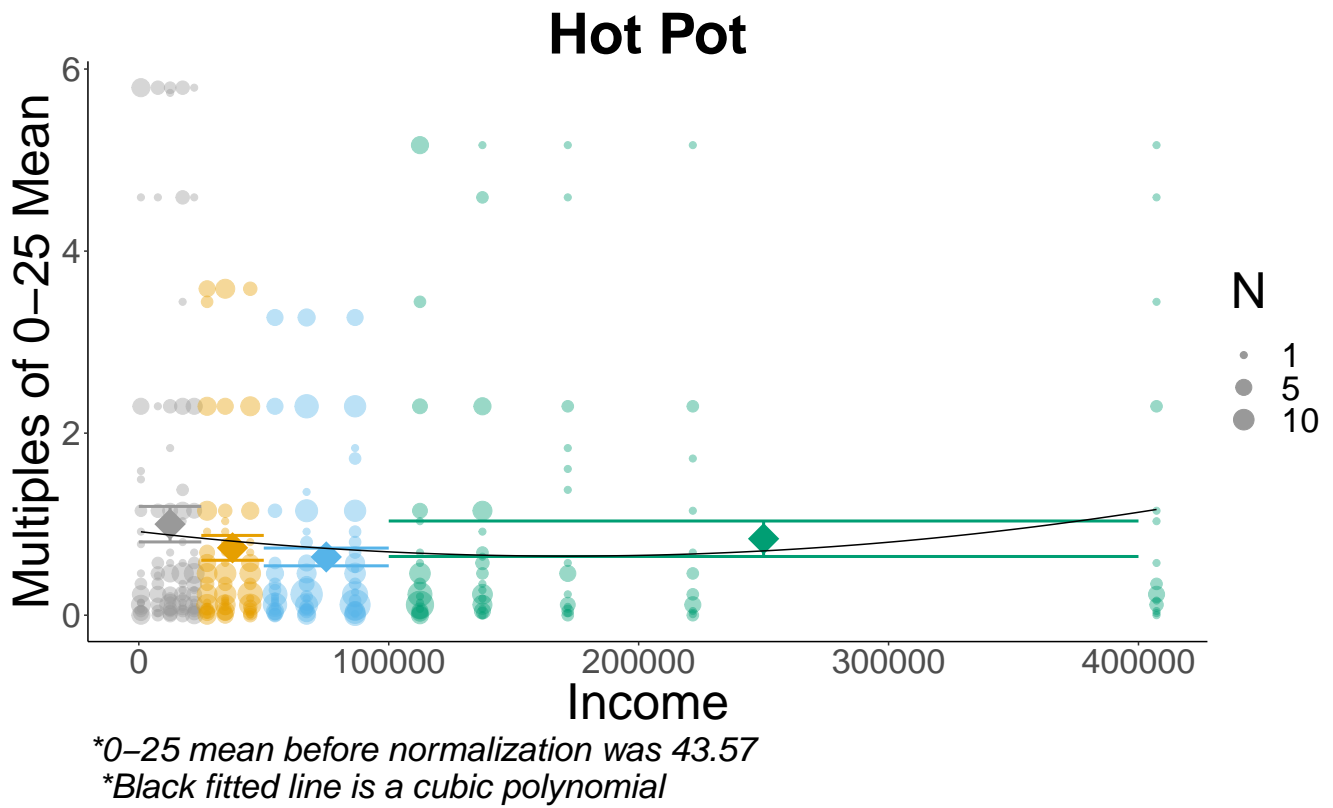
Figure 14



Quadratic Regression		
Term	Estiamte	Std Error
Intercept	0.94	0.088
Income (1000's)	-0.00030	0.0016
Income^2 (1000's)	0.00000074	0.0000045

Group Means		
Income	Mean	Std Error
0-25	1.00	0.10
25-50	0.87	0.09
50-100	0.95	0.08
more than 100	0.88	0.10

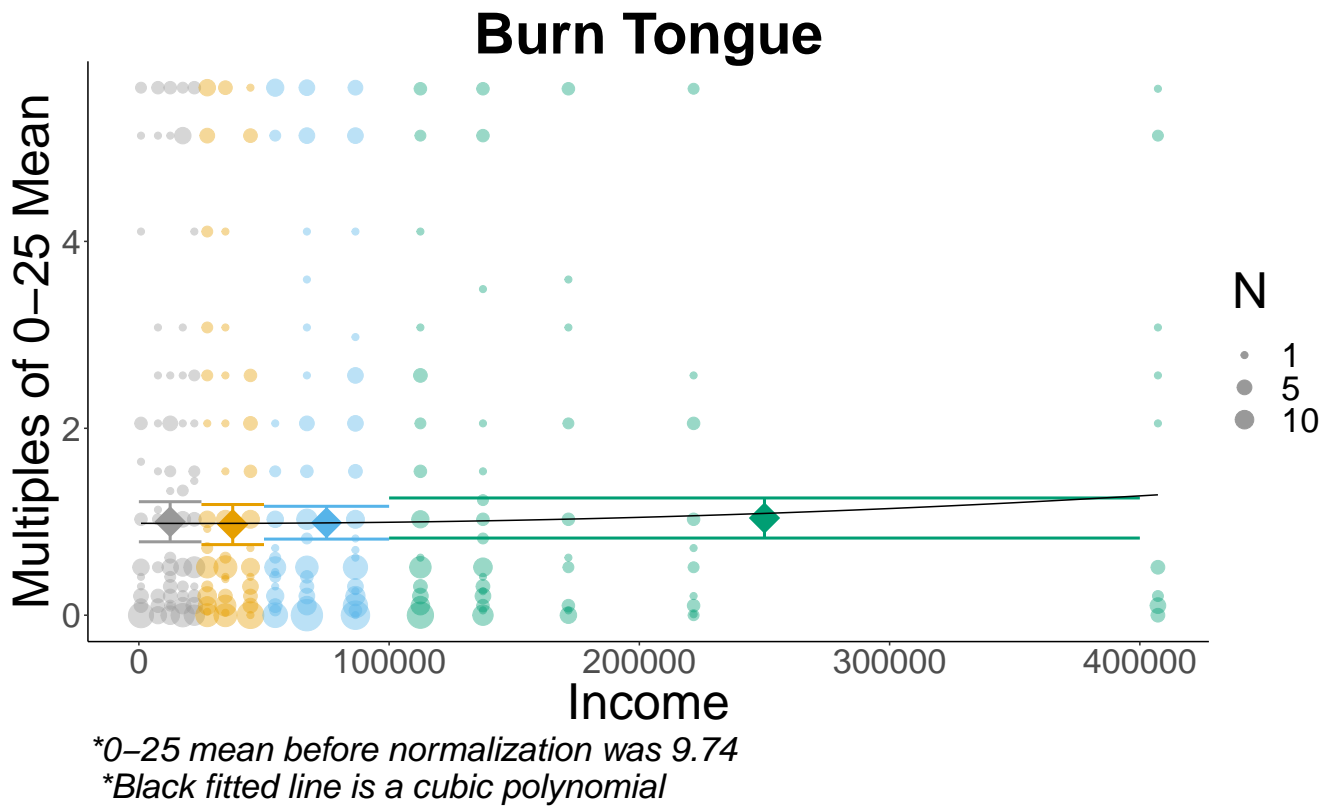
Figure 15



Quadratic Regression		
Term	Estiamte	Std Error
Intercept	0.92	0.073
Income (1000's)	-0.0031	0.0013
Income^2 (1000's)	0.0000092	0.0000037

Group Means		
Income	Mean	Std Error
0-25	1.00	0.10
25-50	0.74	0.07
50-100	0.64	0.05
more than 100	0.84	0.10

Figure 16

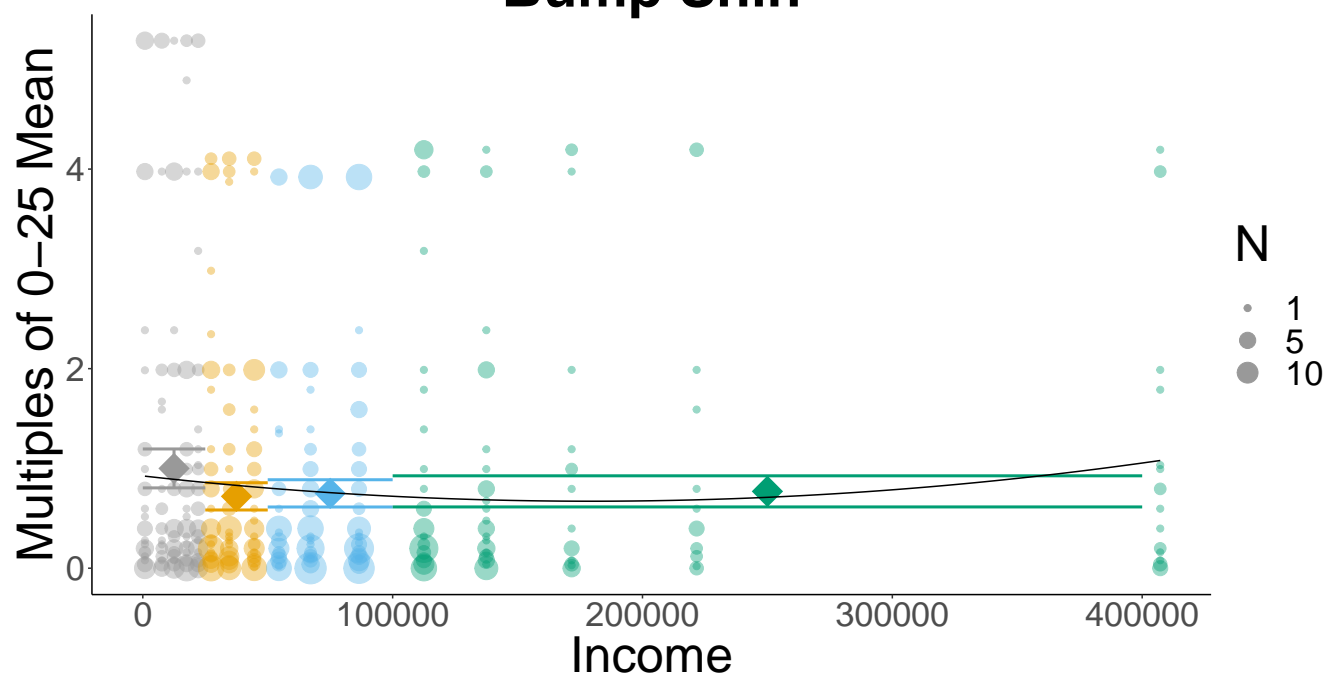


Quadratic Regression		
Term	Estiamte	Std Error
Intercept	0.98	0.097
Income (1000's)	-0.000095	0.0018
Income^2 (1000's)	0.0000021	0.0000049

Group Means		
Income	Mean	Std Error
0-25	1.00	0.11
25-50	0.97	0.11
50-100	0.99	0.09
more than 100	1.04	0.11

Figure 17

Bump Shin



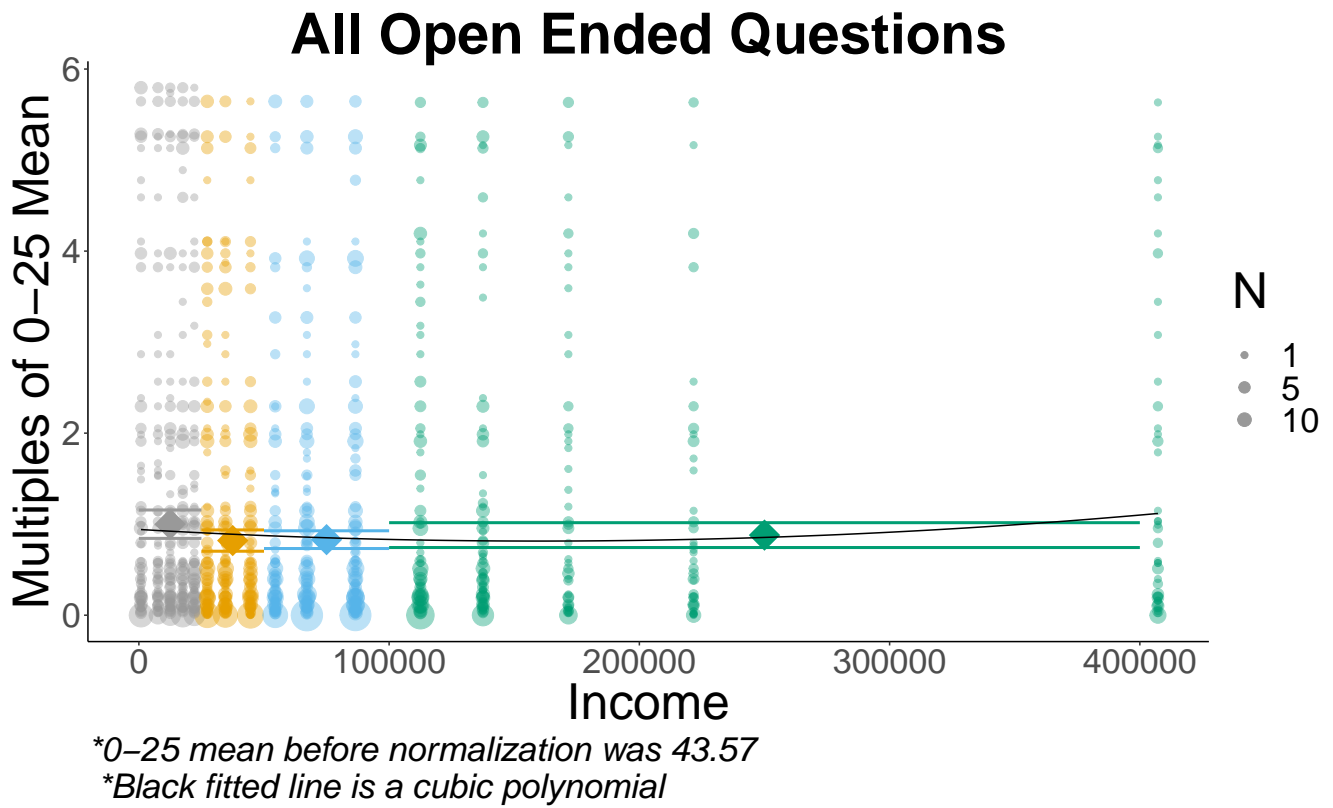
*0-25 mean before normalization was 25.16

*Black fitted line is a cubic polynomial

Quadratic Regression		
Term	Estiamte	Std Error
Intercept	0.92	0.077
Income (1000's)	-0.0028	0.0014
Income^2 (1000's)	0.0000079	0.0000039

Group Means		
Income	Mean	Std Error
0-25	1.00	0.10
25-50	0.72	0.07
50-100	0.75	0.07
more than 100	0.77	0.08

Figure 18



Quadratic Regression		
Term	Estiamte	Std Error
Intercept	0.94	0.042
Income (1000's)	-0.0016	0.00078
Income^2 (1000's)	0.0000050	0.0000021

Group Means		
Income	Mean	Std Error
0-25	1.00	0.08
25-50	0.82	0.06
50-100	0.83	0.05
more than 100	0.88	0.07

Figure 19

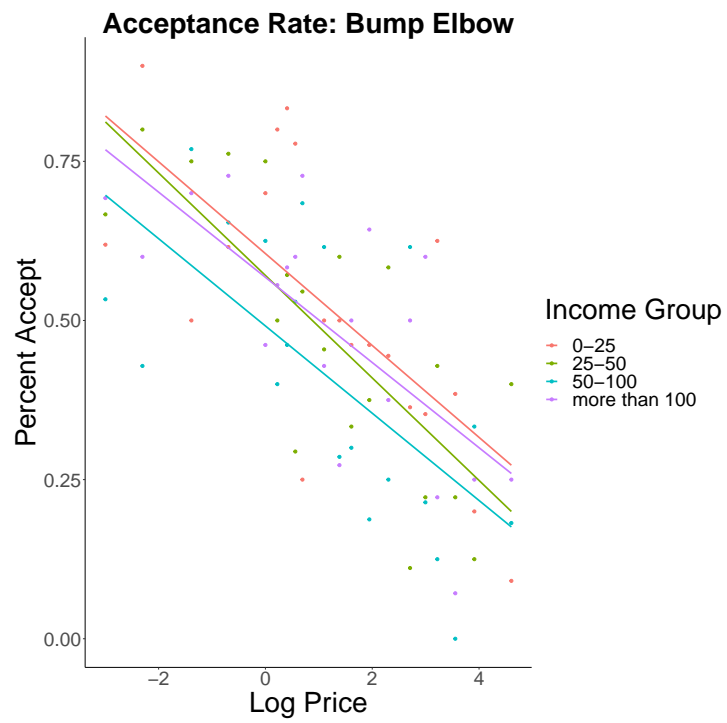


Figure 20



Figure 21

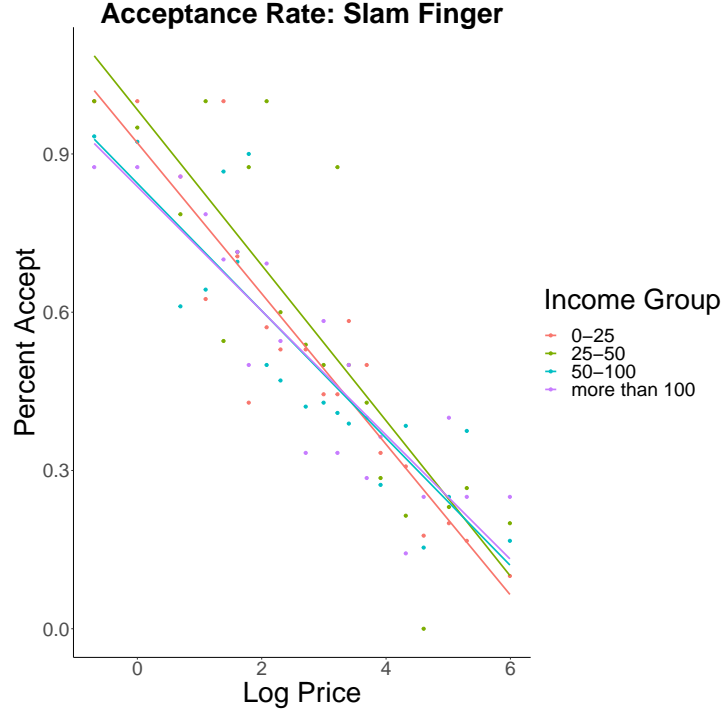


Figure 22

Appendix D Maximum Likelihood Identification

equations 7 and 1 lead to the following theorem specifying the identification of the parameters. Let $\theta = (\beta, \phi')$ be the full set of parameters in the model.

Theorem 5. *If the conditions in assumption 3 and 4 and definition 1 hold and we also have that the matrix $[\mathbb{X} \quad \mathbb{M}]$ is full rank, then the ratio of any two parameters in θ is identified. If we normalize the marginal utility of income for the lowest income group to one, that is $\phi'_1 = 1$, then the remaining parameters in θ are identified.*

To prove this Suppose there exists a $\theta^* \neq \theta$ s.t. $\mathbb{E}[\mathbb{P}^r | \theta^*] = \mathbb{E}[\mathbb{P}^r | \theta]$. This implies

$$\mathbb{X}_i \beta^* \odot \mathbb{M} \phi'^* = \mathbb{X}_i \beta \odot \mathbb{M} \phi' \quad (32)$$

or that

$$\frac{\beta_1^* + X_i \beta^*}{\sum_{k=1}^b \mathbb{1}_{ik} \phi_k'^*} = \frac{\beta_1 + X_i \beta}{\sum_{k=1}^b \mathbb{1}_{ik} \phi_k'} \quad \forall \quad i \quad (33)$$

Now it is true that $\theta = \alpha\theta^*$, where α is any constant, satisfies the condition since α cancels out in the numerator and denominator. Once we have normalized $\phi_1 = 1$, however, α no longer appears in the denominator for i's in income group 1 and so does not cancel. If $\phi'_2 = .5$, for example, it implies the marginal utility of a dollar for income group 2 is half that of group 1.

With the $\phi_1 = 1$ normalization, any change to a parameter in the numerator would alter the expected reservation price for those in group one and violate the equality in equation 33, assuming, as in a regression, that \mathbb{X} is full rank. Any change to the other marginal utility parameters ϕ'_k could be cancelled out for that group by appropriately scaling the numerator, but, since the marginal utility of group one is fixed and they share the numerator parameters, this would again change the expected reservation price for group 1 and violate 33. Thus, there does not exist a $\theta^* \neq \theta$ satisfying the condition.

Can this same identification strategy be used for any good? No, the marginal utility of income cannot be identified from just any reservation price. Recall the assumption that $\epsilon_i \perp\!\!\!\perp m_i$ and notice that income does not enter the utility function for pain relief and the factors impacting pain relief do not impact the marginal utility of income. If either of these appeared in both the numerator and denominator, we would not be able to uniquely identify the parameters.

Appendix E Full Centiment Survey

Centiment Survey

The instructions were first written alone on an initial page and required respondents to wait five seconds before continuing.

Instructions copy ***Instructions Repeated (if needed for reference):***

Imagine yourself in each of the following situations.

Consider if each situation would be painful for you and if yes, how painful it would be.

Then, enter the MOST you would pay in U.S. dollars to completely and immediately eliminate any pain caused by the situation, as if the event described never happened.

Q1 Imagine you have a minor cut on your finger and you accidentally get lemon juice in the wound.

☐ \$ _____

Q2 Imagine you pick up a hot pot by accidentally grabbing its equally hot handles.

☐ \$ _____

Q3 Imagine you shake hands with someone who has a normal grip.

☐ \$ _____

Q4 Imagine you burn your tongue on a very hot drink.

☐ \$ _____

Q5 Imagine you bump your shin badly on a hard edge, for example, on the edge of a glass coffee table.

☐ \$ _____

End of Block: WTP Pain

Start of Block: bump your elbow

Q6 Imagine you bump your elbow on the edge of a table ("funny bone"). Would you pay \$X to completely and immediately eliminate any pain caused by this situation, as if it never happened?

☐ Yes, I would pay \$X

☐ No, I would not pay \$X

NOTE: X is chosen Randomly from (0.05 0.10 0.25 0.50 1 1.25 1.50 1.75 2 3 4 5 7 10 15 20 25 35 50 100) and a single value is displayed

End of Block: bump your elbow

Start of Block: Bite cheek

Q7 Imagine you accidentally bite your tongue or cheek badly while eating. Would you pay \$X to completely and immediately eliminate any pain caused by this situation, as if it never happened?

☐ Yes, I would pay \$X

☐ No, I would not pay \$X

NOTE: X is chosen Randomly from (0.10 0.25 0.50 0.75 1 2 3 4 5 6 8 10 15 20 25 35 50 75 100 200) and a single value is displayed

End of Block: Bite cheek

Start of Block: Trap finger drawer

Q8 Imagine you slam your finger in a drawer. Would you pay \$X to completely and immediately eliminate any pain caused by this situation, as if it never happened?

☐ Yes, I would pay \$X

☐ No, I would not pay \$X

NOTE: X is chosen Randomly from 0.5 1 2 3 4 5 6 8 10 15 20 25 30 40 50 75 100 150 200 400) and a single value is displayed

End of Block: Trap finger drawer

Start of Block: PSQ Match

Q9 Imagine yourself in each of the following situations. Decide if each situation would be painful for you and if yes, how painful it would be. Let 0 stand for no pain; 1 is just noticeable pain and 10 is the most severe pain that you can imagine.

[illegible]

Imagine
you bump
your elbow
on the
edge of a
table
("funny
bone"). (6)

☐☐☐☐☐☐☐☐☐☐☐☐

Imagine
you bump
your shin
badly on a
hard edge,
for
example,
on the
edge of a
glass
coffee
table. (7)

☐☐☐☐☐☐☐☐☐☐☐☐

Skip To: End of Block If Imagine yourself in each of the following situations. Decide if each situation would be painful f... != To ensure your full attention, please select 9 [9]

End of Block: PSQ Match

Start of Block: Demographics

Q10 Which of the following best describes your status?

- ☐ Married
 - ☐ Divorced
 - ☐ Widowed
 - ☐ Separated
 - ☐ Single, never married
 - ☐ Living with a partner in a long term relationship
-



Q11 How many children do you have?

Q13 What is the highest level of school you have completed or the highest degree you have received?

- ☐ Less than high school
- ☐ High School Graduate- high school DIPLOMA or the equivalent (For example: GED)
- ☐ Some college but no degree
- ☐ Occupational/vocational program
- ☐ Associate degree in college
- ☐ Bachelor's degree (For example: BA, AB, BS)
- ☐ Master's degree (For example: MA, MS, MEng, MEd, MSW, MBA)
- ☐ Professional School Degree (For example: MD,DDS,DVM,LLB,JD)
- ☐ Doctorate degree (For example: PhD, EdD)

Q14 What is your current employment status? Mark all that apply.

- ☐ Employed for wages full time
- ☐ Employed for wages part time
- ☐ Self-employed
- ☐ Out of work and looking for work
- ☐ Out of work but not currently looking for work
- ☐ A home-maker
- ☐ A student
- ☐ Military
- ☐ Retired
- ☐ Unable to work

End of Block: Demographics

Start of Block: Financial Health FHN

Title For the following questions, think of a HOUSEHOLD as all people related by birth, marriage, or adoption and residing together. If you live alone, or do not consider anyone else to be a member of your HOUSEHOLD, please answer these questions as an individual.

Q15 Which category represents the total combined income of all members of your HOUSEHOLD during the past 12 months? This includes money from jobs, net income from business, farm or rent, pensions, dividends, interest, social security payments and any other money income received by members of your HOUSEHOLD who are 15 years of age or older.

- ☐ Less than \$5,000
 - ☐ \$5,000 to \$9,999
 - ☐ \$10,000 to \$14,999
 - ☐ \$15,000 to \$19,999
 - ☐ \$20,000 to \$24,999
 - ☐ \$25,000 to \$29,999
 - ☐ \$30,000 to \$39,999
 - ☐ \$40,000 to \$49,999
 - ☐ \$50,000 to \$59,999
 - ☐ \$60,000 to \$74,999
 - ☐ \$75,000 to \$99,999
 - ☐ \$100,000 to \$124,999
 - ☐ \$125,000- \$149,999
 - ☐ \$150,000 - \$199,999
 - ☐ \$200,000 to \$249,999
 - ☐ \$250,000 and over
-

Q16 Which of the following statements best describes how your household's total spending compared to total income over the last 12 months?

- ☐ Spending was much less than income
 - ☐ Spending was a little less than income
 - ☐ Spending was about equal to income
 - ☐ Spending was a little more than income
 - ☐ Spending was much more than income
-

Q17 Which of the following statements best describes how your household has paid its bills over the last 12 months? My household has been financially able to:

- ☐ Pay all our bills on time
 - ☐ Pay nearly all our bills on time
 - ☐ Pay most of our bills on time
 - ☐ Pay some of our bills on time
 - ☐ Pay very few of our bills on time
-

Q18 At your current level of spending, how long could you and your household afford to cover expenses, if you had to live on only the money you have readily available, without withdrawing money from retirement accounts or borrowing?

- ☐ 6 months or more
 - ☐ 3-5 months
 - ☐ 1-2 months
 - ☐ 1-3 weeks
 - ☐ Less than 1 week
-

Q19 Thinking about all of your household's current debts, including mortgages, bank loans, student loans, money owed to people, medical debt, past-due bills, and credit card balances that are carried over from prior months...

As of today, which of the following statements describes how manageable your household debt is?

- ☐ Do not have any debt
 - ☐ Have a manageable amount of debt
 - ☐ Have a bit more debt than is manageable
 - ☐ Have far more debt than is manageable
-

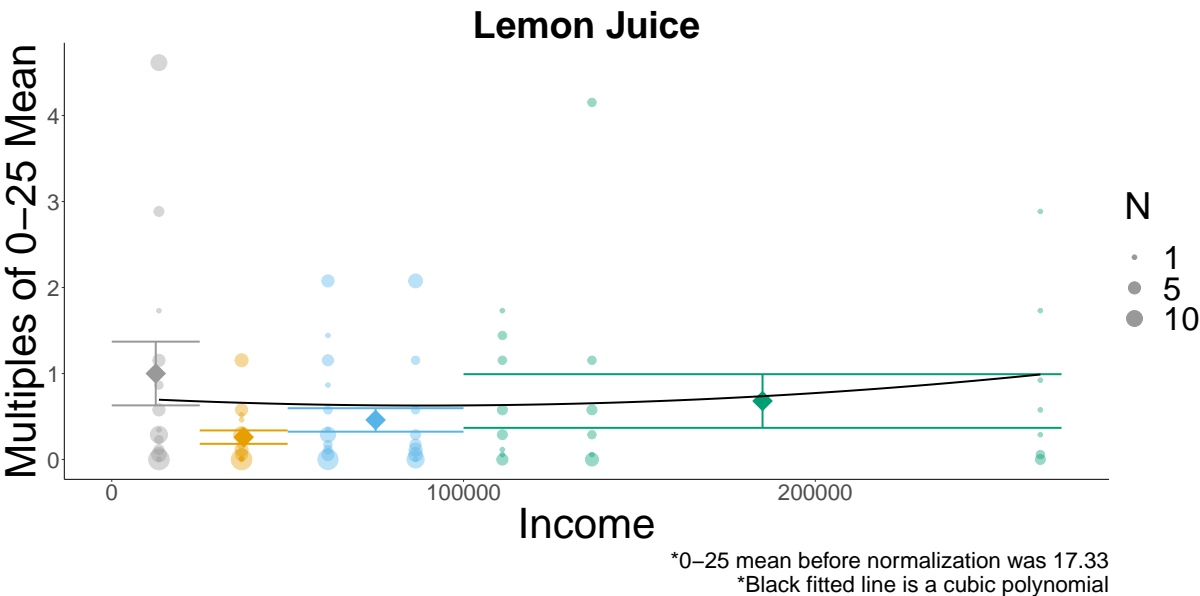
Q20 Thinking about all of the types of insurance you and others in your household currently might have, including health insurance, vehicle insurance, home or rental insurance, life insurance, and disability insurance...

How confident are you that those insurance policies will provide enough support in case of an emergency?

- ☐ Very confident
- ☐ Moderately confident
- ☐ Somewhat confident
- ☐ Slightly confident
- ☐ Not at all confident
- ☐ No one in my household has any insurance

End of Block: Financial Health FHN

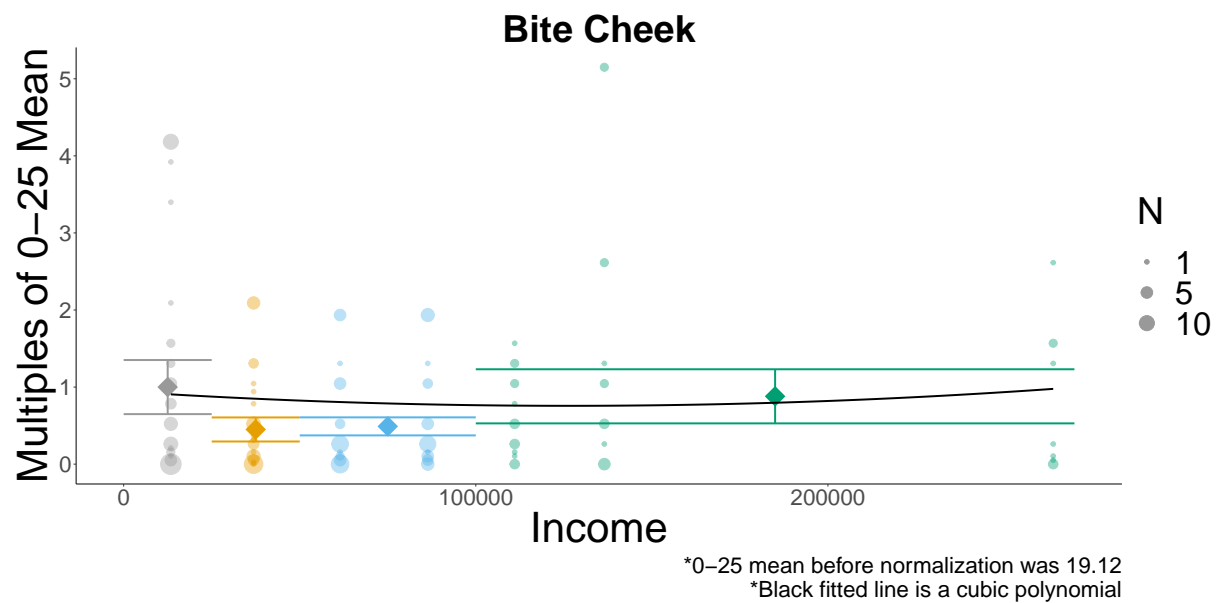
Appendix F Pollfish results



Lemon Juice					
Income	Mean	se	Std Dev	Std Dev Norm	Median Norm
0-25	1.00	0.18	1.59	1.00	1.0
25-50	0.26	0.05	0.35	0.22	0.4
50-100	0.46	0.07	0.70	0.44	0.4
more than 100	0.68	0.16	1.00	0.63	1.0

MLE Lemon Juice			
label	est	se	P_ratio
male	1.42	3.39	NA
psq	4.91	1.20	NA
intercept	-0.70	4.74	NA
sigma	26.11	2.00	NA
0-25	1.00	NA	1.0000000
25-50	4.51	0.49	0.2219545
50-100	2.21	0.22	0.4524891
more than 100	1.52	0.19	0.6570436
mean psq control			

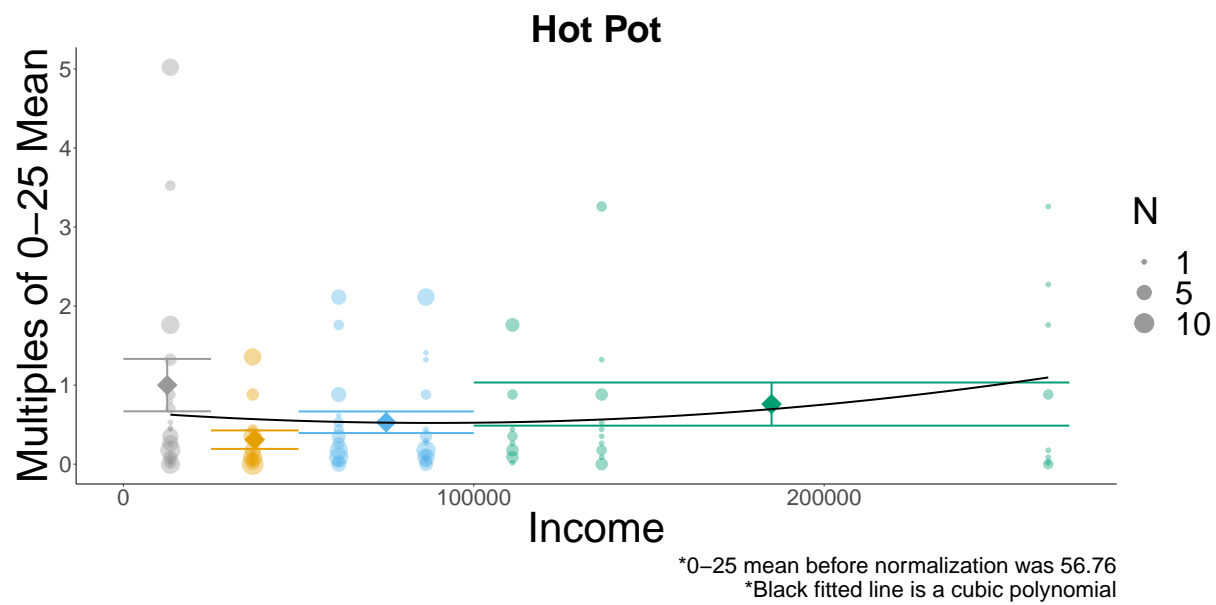
Figure 23: imagine you have a minor cut on your finger and you inadvertently get lemon juice in the wound



Bite Cheek					
Income	Mean	se	Std Dev	Std Dev Norm	Median Norm
0-25	1.00	0.17	1.47	1.00	1.0
25-50	0.45	0.08	0.65	0.45	0.4
50-100	0.49	0.07	0.64	0.44	1.0
more than 100	0.88	0.18	1.21	0.82	2.0

MLE Bite Cheek			
label	est	se	P_ratio
male	5.05	3.42	NA
psq	5.34	1.20	NA
intercept	-1.80	4.75	NA
sigma	26.22	1.98	NA
0-25	1.00	NA	1.0000000
25-50	2.29	0.24	0.4374243
50-100	2.19	0.21	0.4575944
more than 100	1.16	0.14	0.8607178
mean psq control			

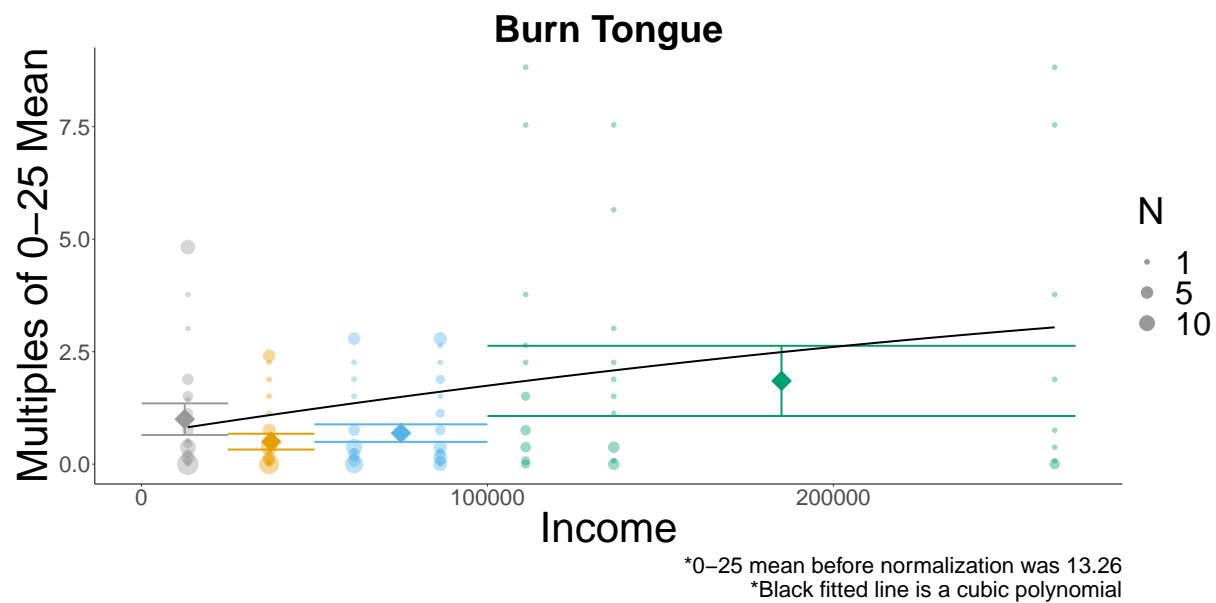
Figure 24: imagine you accidentally bite your tongue or cheek badly while eating



Hot Pot					
Income	Mean	se	Std Dev	Std Dev Norm	Median Norm
0-25	1.00	0.17	1.52	1.00	1.00
25-50	0.31	0.06	0.44	0.29	0.33
50-100	0.53	0.07	0.71	0.46	0.67
more than 100	0.76	0.15	0.93	0.61	1.33

MLE Hot Pot			
label	est	se	P_ratio
male	12.86	10.49	NA
psq	14.20	3.65	NA
intercept	3.24	14.62	NA
sigma	80.50	6.10	NA
0-25	1.00	NA	1.0000000
25-50	3.45	0.37	0.2899111
50-100	2.05	0.20	0.4885869
more than 100	1.51	0.18	0.6608440
mean psq control			

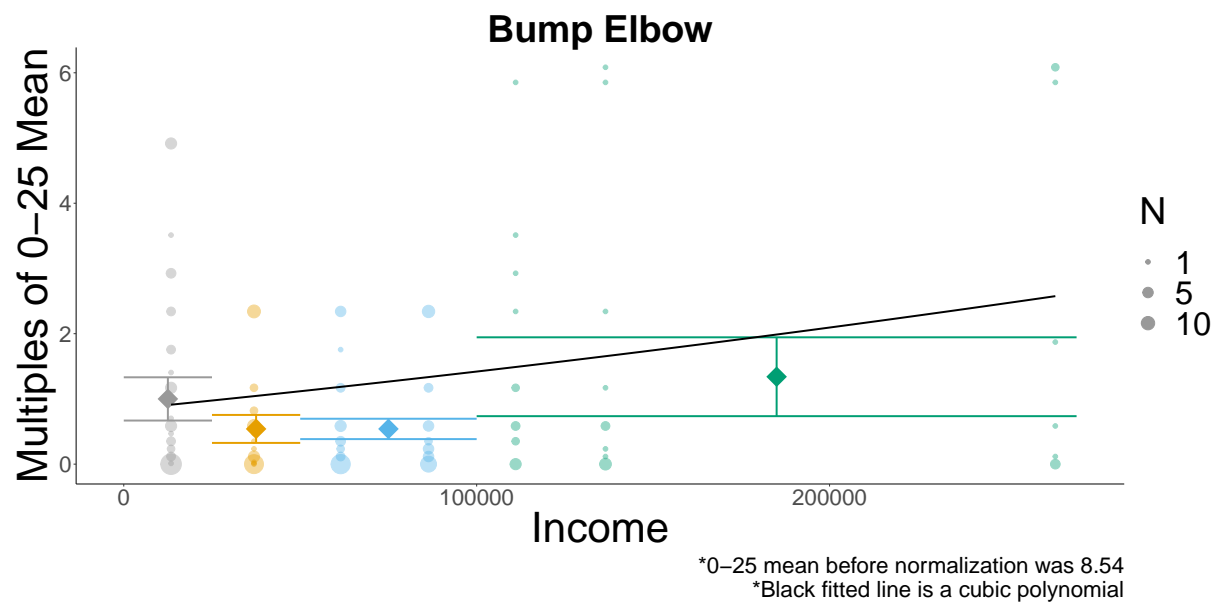
Figure 25: imagine you pick up a hot pot by inadvertently grabbing its equally hot handles



Burn Tongue					
Income	Mean	se	Std Dev	Std Dev Norm	Median Norm
0-25	1.00	0.18	1.54	1.00	1.0
25-50	0.50	0.10	0.74	0.48	0.4
50-100	0.69	0.10	0.95	0.61	0.7
more than 100	1.85	0.40	2.60	1.69	2.0

MLE Burn Tongue			
label	est	se	P_ratio
male	6.96	2.50	NA
psq	3.75	0.88	NA
intercept	-2.46	3.45	NA
sigma	19.01	1.44	NA
0-25	1.00	NA	1.0000000
25-50	2.12	0.23	0.4716562
50-100	1.56	0.15	0.6407617
more than 100	0.58	0.07	1.7384238
mean psq control			

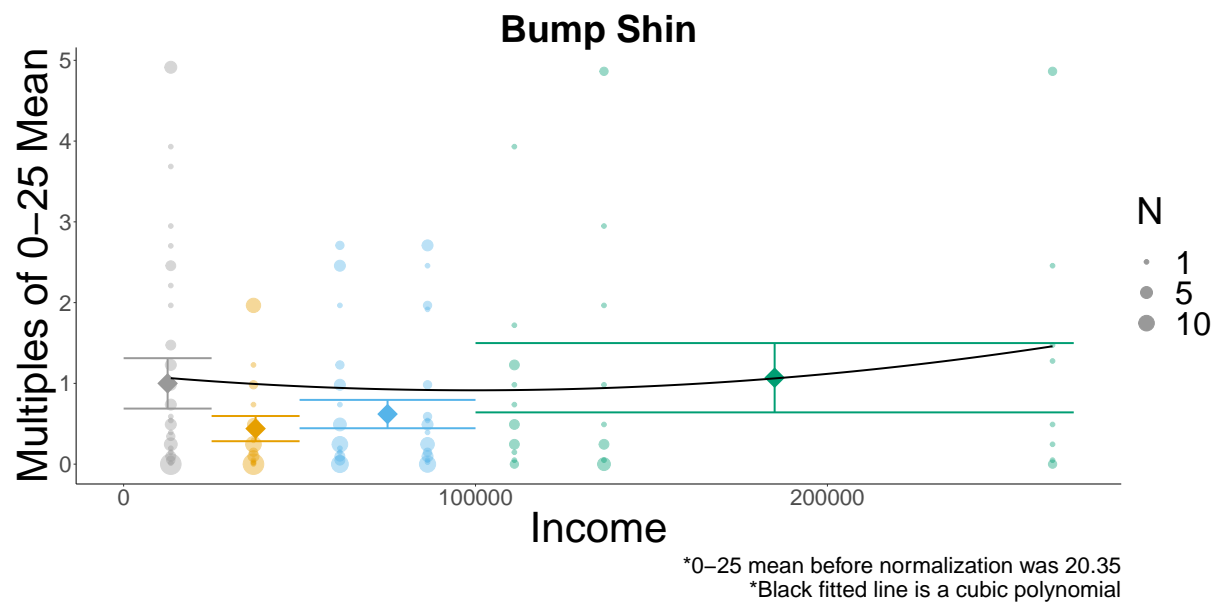
Figure 26: imagine you burn your tongue on a very hot drink.



Bump Elbow					
Income	Mean	se	Std Dev	Std Dev Norm	Median Norm
0-25	1.00	0.18	1.50	1.00	1.0
25-50	0.54	0.11	0.83	0.55	0.5
50-100	0.54	0.09	0.81	0.54	0.5
more than 100	1.34	0.30	2.06	1.38	1.5

MLE Bump Elbow			
label	est	se	P_ratio
male	1.71	1.58	NA
psq	2.50	0.56	NA
intercept	-1.52	2.19	NA
sigma	12.11	0.92	NA
0-25	1.00	NA	1.0000000
25-50	1.86	0.20	0.5375826
50-100	1.81	0.18	0.5511262
more than 100	0.71	0.09	1.4090528
mean psq control			

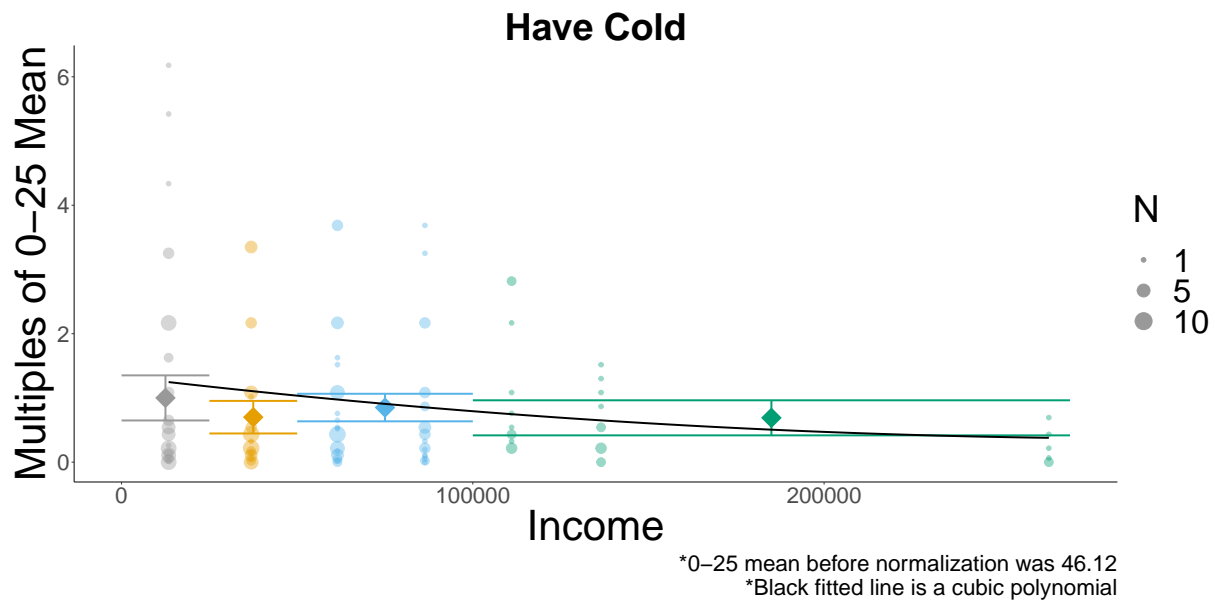
Figure 27: imagine you bump your elbow on the edge of a table ("funny bone").



Bump Shin					
Income	Mean	se	Std Dev	Std Dev Norm	Median Norm
0-25	1.00	0.17	1.41	1.00	1.00
25-50	0.44	0.08	0.66	0.47	0.40
50-100	0.62	0.09	0.85	0.60	0.67
more than 100	1.07	0.23	1.50	1.06	1.33

MLE Bump Shin			
label	est	se	P_ratio
male	10.52	3.61	NA
psq	4.48	1.23	NA
intercept	-0.20	4.92	NA
sigma	27.17	2.05	NA
0-25	1.00	NA	1.0000000
25-50	2.16	0.23	0.4628516
50-100	1.63	0.16	0.6149244
more than 100	0.92	0.11	1.0864257
mean psq control			

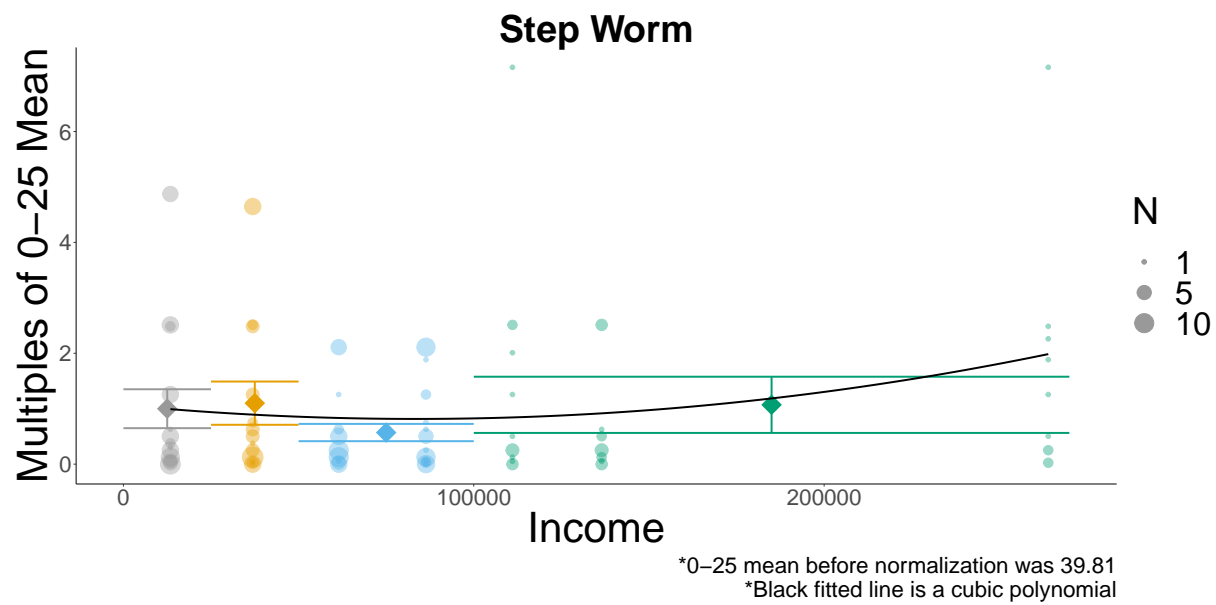
Figure 28: imagine you bump your shin badly on a hard edge, for example, on the edge of a glass coffee table.



Have Cold					
Income	Mean	se	Std Dev	Std Dev Norm	Median Norm
0-25	1.00	0.18	1.38	1.00	1.00
25-50	0.70	0.13	0.95	0.69	0.87
50-100	0.85	0.12	0.99	0.72	1.00
more than 100	0.69	0.14	0.78	0.56	1.00

MLE Have Cold			
label	est	se	P_ratio
male	-0.43	9.19	NA
psq	4.78	2.95	NA
intercept	31.65	12.26	NA
sigma	61.41	5.27	NA
0-25	1.00	NA	1.0000000
25-50	1.45	0.17	0.6903644
50-100	1.32	0.15	0.7562644
more than 100	1.68	0.24	0.5951526
mean psq control			

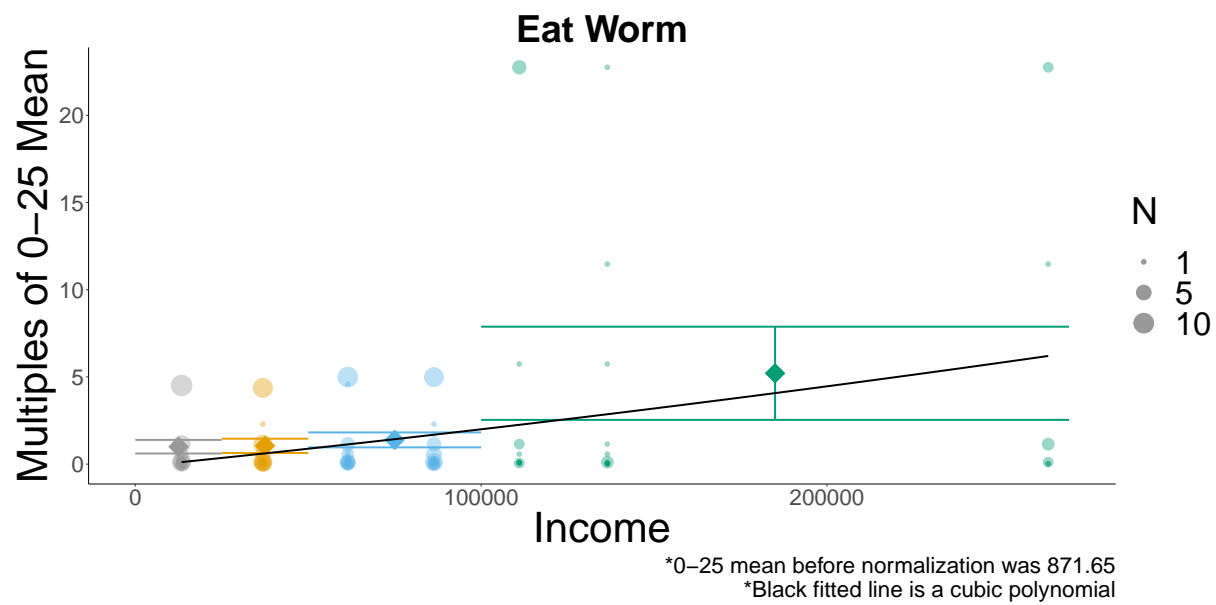
Figure 29: imagine you have a sore throat and cough which will last a week with no other complications. drugs will not relieve the symptoms.



Step Worm					
Income	Mean	se	Std Dev	Std Dev Norm	Median Norm
0-25	1.00	0.17	1.46	1.00	1.00
25-50	1.10	0.20	1.55	1.06	1.25
50-100	0.57	0.08	0.76	0.52	1.00
more than 100	1.07	0.26	1.67	1.15	1.00

MLE Step Worm			
label	est	se	P_ratio
male	-10.28	7.45	NA
psq	8.80	2.64	NA
intercept	11.15	10.47	NA
sigma	56.19	4.42	NA
0-25	1.00	NA	1.0000000
25-50	0.94	0.11	1.0640200
50-100	1.90	0.19	0.5271882
more than 100	0.87	0.11	1.1440865
mean psq control			

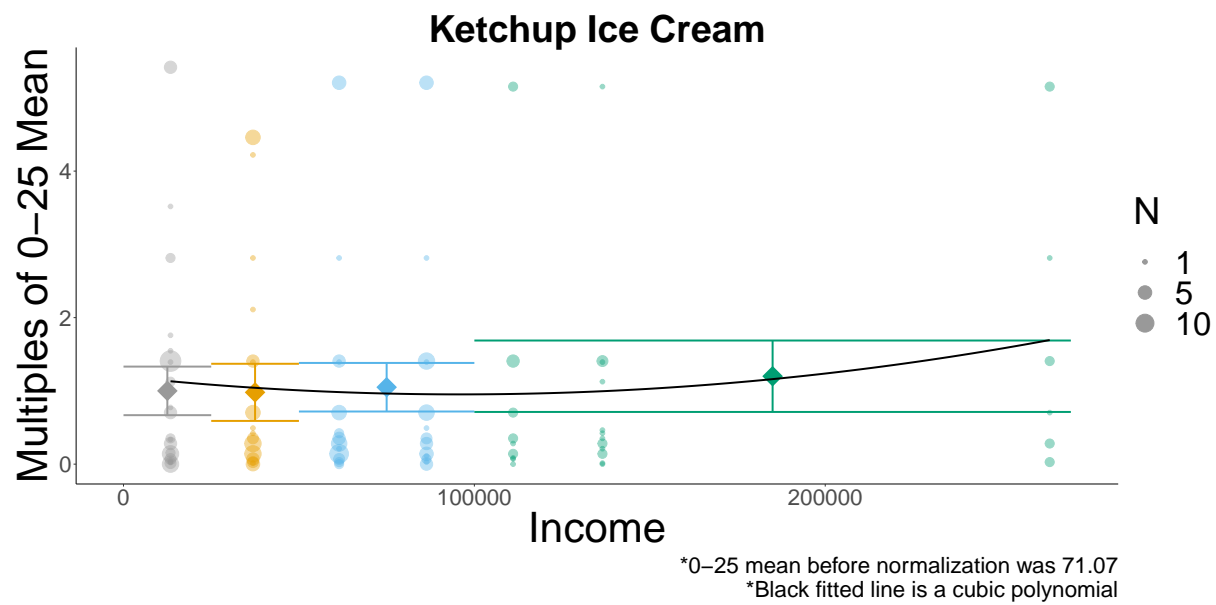
Figure 30: walk barefoot on concrete and step on a dead earthworm.



Eat Worm					
Income	Mean	se	Std Dev	Std Dev Norm	Median Norm
0-25	1.00	0.20	1.65	1.00	1
25-50	1.05	0.21	1.57	0.95	1
50-100	1.39	0.21	2.00	1.21	1
more than 100	5.21	1.34	8.74	5.30	1

MLE Eat Worm			
label	est	se	P_ratio
male	15.35	190.38	NA
psq	221.45	67.95	NA
intercept	54.94	270.47	NA
sigma	1,385.03	116.31	NA
0-25	1.00	NA	1.000000
25-50	1.04	0.12	0.962051
50-100	0.80	0.08	1.253803
more than 100	0.18	0.02	5.438409
mean psq control			

Figure 31: eat a dead earthworm, 6 inches long.



Ketchup Ice Cream					
Income	Mean	se	Std Dev	Std Dev Norm	Median Norm
0-25	1.00	0.18	1.38	1.00	1.00
25-50	0.98	0.19	1.46	1.05	0.80
50-100	1.05	0.16	1.56	1.13	1.00
more than 100	1.20	0.24	1.60	1.16	1.26

MLE Ketchup Ice Cream			
label	est	se	P_ratio
male	-20.43	13.08	NA
psq	13.85	4.61	NA
intercept	22.47	18.30	NA
sigma	97.00	7.83	NA
0-25	1.00	NA	1.000000
25-50	0.97	0.11	1.028962
50-100	0.90	0.09	1.107206
more than 100	0.85	0.11	1.178982
mean psq control			

Figure 32: put two tablespoons of ketchup on one cup of vanilla ice cream and eat it.

label	Lemon Juice		Bite Cheek		Hot Pot		Burn Tongue		Bump Elbow		Bump Shin	
	est	se	est	se	est	se	est	se	est	se	est	se
male	1.4248556	3.3938385	5.048771	3.4221608	12.857169	10.4872904	6.9620718	2.50459055	1.7069535	1.57755635	10.5154235	3.6075709
psq	4.9132981	1.2014617	5.337530	1.2025717	14.201043	3.6514827	3.7493992	0.87605874	2.5032895	0.55823489	4.4849073	1.2271831
intercept	-0.7043961	4.7383238	-1.797299	4.7490253	3.235489	14.6205175	-2.4556670	3.44633306	-1.5198520	2.19186640	-0.1995436	4.9161006
sigma	26.1061732	2.0019957	26.215477	1.9792671	80.501951	6.0978881	19.0063680	1.43743405	12.1052190	0.92273439	27.1674595	2.0540339
0-25	1.0000000	NA	1.000000	NA	1.000000	NA	1.0000000	NA	1.0000000	NA	1.0000000	NA
25-50	4.5054288	0.4899229	2.286110	0.2444016	3.449333	0.3709161	2.1201884	0.22743035	1.8601794	0.20151247	2.1605195	0.2329265
50-100	2.2099982	0.2182993	2.185342	0.2105613	2.046719	0.1987606	1.5606426	0.15116269	1.8144664	0.17828592	1.6262162	0.1572062
more than 100	1.5219689	0.1867252	1.161821	0.1390792	1.513216	0.1811029	0.5752337	0.06865261	0.7096966	0.08613581	0.9204495	0.1100030

label	Have Cold		Step Worm		Eat Worm		Ketchup Ice Cream	
	est	se	est	se	est	se	est	se
male	-0.4336552	9.1943315	-10.2760847	7.4511514	15.3547878	190.37874196	-20.4301353	13.07856097
psq	4.7768386	2.9515214	8.8045662	2.6367809	221.4533352	67.94913386	13.8538416	4.60867985
intercept	31.6474190	12.2647548	11.1454741	10.4689092	54.9444072	270.46596055	22.4739752	18.30406629
sigma	61.4140409	5.2724097	56.1932747	4.4198356	1385.0300168	116.30663366	96.9989201	7.83042050
0-25	1.0000000	NA	1.0000000	NA	1.0000000	NA	1.0000000	NA
25-50	1.4485104	0.1735345	0.9398320	0.1059102	1.0394459	0.12284045	0.9718535	0.11212166
50-100	1.3222888	0.1450972	1.8968559	0.1894838	0.7975737	0.08411142	0.9031741	0.09184296
more than 100	1.6802415	0.2357622	0.8740598	0.1092534	0.1838773	0.02418736	0.8481892	0.10600972

Average Pain Questions				
Income	Mean	Std Dev	Std Dev Norm	Median Norm
0-25	1.00	1.50	1.00	1.00
25-50	0.42	0.64	0.43	0.41
50-100	0.55	0.78	0.52	0.78
more than 100	1.10	1.70	1.13	1.31

MLE Normed Pain Qs			
label	est	se	P_ratio
male	0.25	0.07	NA
psq	0.26	0.03	NA
intercept	-0.06	0.10	NA
sigma	1.42	0.04	NA
0-25	1.00	NA	1.0000000
25-50	2.37	0.10	0.4223848
50-100	1.87	0.07	0.5356272
more than 100	0.87	0.04	1.1513940

all questions normed to lowest mean = 1

mean psq control

References

- Arrow, K. J. (1950). A difficulty in the concept of social welfare. *Journal of political economy*, 58(4), 328–346.
- Arrow, K. J. (1978). Extended sympathy and the possibility of social choice. *Philosophia*, 7(2), 223–237.
- Arrow, K. J. (2012). *Social choice and individual values* (Vol. 12). Yale university press.
- Auspitz, R., & Lieben, R. (1889). *Untersuchungen über die theorie des preises*. Duncker & Humblot.
- Aydede, M. (2019). Pain. In E. N. Zalta (Ed.), *The Stanford encyclopedia of philosophy* (Spring 2019 ed.). Metaphysics Research Lab, Stanford University. <https://plato.stanford.edu/archives/spr2019/entries/pain/>.
- Bartley, E. J., & Fillingim, R. B. (2013). Sex differences in pain: a brief review of clinical and experimental findings. *British journal of anaesthesia*, 111(1), 52–58.
- Berkouwer, S. B., & Dean, J. T. (2021). Credit, attention, and externalities in the adoption of energy efficient technologies by low-income households. *Unpublished manuscript, University of Pennsylvania, Philadelphia, PA*.
- Bishop, R. C., & Heberlein, T. A. (1979). Measuring values of extramarket goods: Are indirect measures biased? *American journal of agricultural economics*, 61(5), 926–930.
- Boyle, K. J. (2017). Contingent valuation in practice. In *A primer on nonmarket valuation* (pp. 83–131). Springer.
- Crimmins, J. (2021). Jeremy Bentham. In *The Stanford encyclopedia of philosophy* (Winter 2021 ed.). Metaphysics Research Lab, Stanford University. <https://plato.stanford.edu/entries/bentham/>.
- Darity Jr, W., Hamilton, D., Paul, M., Aja, A., Price, A., Moore, A., & Chiopris, C. (2018). What we get wrong about closing the racial wealth gap. *Samuel DuBois Cook Center on Social Equity and Insight Center for Community Economic Development*, 1(1), 1–67.
- Dimand, R. (2019). Irving fisher, ragnar frisch and the elusive quest for measurable utility. Retrieved from <https://ssrn.com/abstract=3495946>
- Driver, J. (2014, Sep). *The history of utilitarianism*. Stanford University. Retrieved from <https://plato.stanford.edu/entries/utilitarianism-history/>
- Eastmond, T., Mather, N., Ricks, M. D., & Betts, J. (2022). From value added to welfare added: A social planner approach to education policy and statistics.
- Finkelstein, A., & Hendren, N. (2020, November). Welfare analysis meets causal inference. *Journal of Economic Perspectives*, 34(4), 146–67. Retrieved from <https://www.aeaweb.org/articles?id=10.1257/jep.34.4.146> doi: 10.1257/jep.34.4.146

- Fisher, I. (1927). *A statistical method for measuring" marginal utility" and testing the justice of a progressive income tax.*
- Flood, A. (n.d.). Terry pratchett estate backs jack monroe's idea for 'vimes boots' poverty index. *The Guardian*. Retrieved 6-27-2022, from <https://www.theguardian.com/books/2022/jan/26/terry-pratchett-jack-monroe-vimes-boots-poverty-index>
- García, J. L., & Heckman, J. J. (2022). *On criteria for evaluating social programs* (Tech. Rep.). National Bureau of Economic Research.
- Glick, M. (2018). The unsound theory behind the consumer (and total) welfare goal in antitrust. *The Antitrust Bulletin*, 63(4), 455–493.
- Gorman, W. M. (1955). The intransitivity of certain criteria used in welfare economics. *Oxford Economic Papers*, 7(1), 25–34.
- Hanemann, W. M. (1984). Welfare evaluations in contingent valuation experiments with discrete responses. *American journal of agricultural economics*, 66(3), 332–341.
- Hare, R. M. (1981). *Moral thinking: Its levels, method, and point*. Oxford: Clarendon Press; New York: Oxford University Press.
- Harsanyi, J. C. (1955). Cardinal welfare, individualistic ethics, and interpersonal comparisons of utility. *Journal of political economy*, 63(4), 309–321.
- Harsanyi, J. C. (1986). *Rational behaviour and bargaining equilibrium in games and social situations*. CUP Archive.
- Hausman, D. M. (1995). The impossibility of interpersonal utility comparisons. *Mind*, 104(415), 473–490.
- Hausman, D. M., & McPherson, M. S. (2006). *Economic analysis, moral philosophy and public policy*. (Vol. 2nd ed). Cambridge University Press. Retrieved from <http://proxy.lib.umich.edu/login?url=https://search.ebscohost.com/login.aspx?direct=true&db=e000xna&AN=149844&site=ehost-live&scope=site>
- Hendren, N. (2020). Measuring economic efficiency using inverse-optimum weights. *Journal of Public Economics*, 187, 104198. Retrieved from <https://www.sciencedirect.com/science/article/pii/S0047272720300621> doi: <https://doi.org/10.1016/j.jpubeco.2020.104198>
- Hendren, N., & Sprung-Keyser, B. (2020). A unified welfare analysis of government policies. , 135(3), 1209-1318. Retrieved from <https://academic.oup.com/qje/artjournal={QuarterlyJournalofEconomics}> doi: 10.1257/jep.34.4.146
- Hicks, J. R. (1940). The valuation of the social income. *Economica*, 7(26), 105–124. Retrieved from <http://www.jstor.org/stable/2548691>
- Jevons, W. S. ([1871] 1911). *The theory of political economy*, 4th ed. london: Macmillan.
- Jevons, W. S. (1879). *The theory of political economy*. Macmillan and Company.
- Kahneman, D., Knetsch, J. L., & Thaler, R. H. (1991). Anomalies: The endowment effect, loss

- aversion, and status quo bias. *Journal of Economic perspectives*, 5(1), 193–206.
- Kaldor, N. (1939). Welfare propositions of economics and interpersonal comparisons of utility. *The Economic Journal*, 49(195), 549–552. Retrieved from <http://www.jstor.org/stable/2224835>
- Kimball, M. S., Ohtake, F., Reck, D., Tsutsui, Y., & Zhang, F. (2015). Diminishing marginal utility revisited. *Available at SSRN 2592935*.
- Lautenbacher, S., Peters, J. H., Heesen, M., Scheel, J., & Kunz, M. (2017). Age changes in pain perception: a systematic-review and meta-analysis of age effects on pain and tolerance thresholds. *Neuroscience & Biobehavioral Reviews*, 75, 104–113.
- MacKay, A. F. (1986). Extended sympathy and interpersonal utility comparisons. *The Journal of philosophy*, 83(6), 305–322.
- Marshall, A. (1890). *The principles of economics*. McMaster University Archive for the History of Economic Thought. Retrieved from <https://EconPapers.repec.org/RePEc:hay:hetboo:marshall1890>
- Mather, N., Eastmond, T., Ricks, M., & Betts, J. (2022). From value added to welfare added: A social planner approach to education policy and statistics.
- Miljković, A., Stipčić, A., Braš, M., orević, V., Brajković, L., Hayward, C., ... Polašek, O. (2014). Is experimentally induced pain associated with socioeconomic status? do poor people hurt more? *Medical science monitor: international medical journal of experimental and clinical research*, 20, 1232.
- Morgan, J. N. (1945). Can we measure the marginal utility of money? *Econometrica: Journal of the Econometric Society*, 129–152.
- Moscatti, I. (2018). *Measuring utility: From the marginal revolution to behavioral economics*. Oxford Studies in History of E.
- Pearce, D. (2021, Mar). Individual and social welfare: A bayesian perspectiveâ.
- Ruscheweyh, R., Marziniak, M., Stumpenhorst, F., Reinholz, J., & Knecht, S. (2009). Pain sensitivity can be assessed by self-rating: Development and validation of the pain sensitivity questionnaire. *Pain*, 146(1-2), 65–74.
- Ruscheweyh, R., Verneuer, B., Dany, K., Marziniak, M., Wolowski, A., Çolak-Ekici, R., ... others (2012). Validation of the pain sensitivity questionnaire in chronic pain patients. *Pain*, 153(6), 1210–1218.
- Samuelson, P. A. (1950). Evaluation of real national income. *Oxford Economic Papers*, 2(1), 1–29.
- Sen, A. K. (1977). Rational fools: A critique of the behavioral foundations of economic theory. *Philosophy & Public Affairs*, 317–344.
- Stephens, R., & Robertson, O. (2020). Swearing as a response to pain: Assessing hypoalgesic

- effects of novel “swear” words. *Frontiers in Psychology*, 11. Retrieved from <https://www.frontiersin.org/article/10.3389/fpsyg.2020.00723> doi: 10.3389/fpsyg.2020.00723
- Stevenson, B., & Wolfers, J. (2013). Subjective well-being and income: Is there any evidence of satiation? *American Economic Review*, 103(3), 598–604.
- Stevenson, B., & Wolfers, J. (2019). *Principles of macroeconomics*. Macmillan Higher Education.
- Wilson, C. S. (2019). Welfare standards underlying antitrust enforcement: What you measure is what you get. In *In: Luncheon keynote address at george mason law review 22nd antitrust symposium. arlington: Antitrust at the crossroads*.
- Zwolinski, M. (2008). The ethics of price gouging. *Business Ethics Quarterly*, 18(3), 347–378.