

# Economics 631 IO - Fall 2019

## Problem Set 2

Nathan Mather and Tyler Radler

October 23, 2019

### 1 BLP - Random Coefficient

#### Preliminaries

Each firm chooses price to solve the problem

$$\max_{p_j} (p_j - mc_j) M s_j(\mathbf{p}, \mathbf{x}, \sigma)$$

The FOC is

$$0 = (p_j - mc_j) M \frac{\partial s_j}{\partial p_j} + M s_j$$

and so the price will be determined by the following condition:

$$p_j = mc_j - s_j \left( \frac{\partial s_j}{\partial p_j} \right)^{-1}$$

. The market share for product  $j$  is given by

$$s_j(\mathbf{p}, \mathbf{x}, \theta) = \int \frac{\exp(\beta_i x_j - \alpha p_j)}{1 + \sum_{j'} \exp(\beta_i x_{j'} - \alpha p_{j'})} dF(\beta_i)$$

Given our functional form assumptions we can rewrite  $\beta_i = \beta + \sigma v_i$  where  $\beta$  is the mean of the distribution and  $\sigma$  is the standard deviation,  $v_i \sim \mathcal{N}(0, 1)$ . Additionally we can define the mean utility of purchasing product  $j$  as  $\delta_j = \beta x_j - \alpha p_j$  and rewrite the market share expression in terms of  $\delta_j$  and  $\sigma$ .

$$s_j(\mathbf{p}, \mathbf{x}, \boldsymbol{\delta}, \sigma) = \int \frac{\exp(\delta_j + \sigma x_j v_i)}{1 + \sum_{j'} \exp(\delta_{j'} + \sigma x_{j'} v_i)} dF(v)$$

Note that if we rewrite the above expression as

$$s_j(\mathbf{p}, \mathbf{x}, \boldsymbol{\delta}, \sigma) = \int \tilde{s}_j(\mathbf{p}, \mathbf{x}, \boldsymbol{\delta}, \sigma) dF(v_i)$$

we can get a fairly-nice expression for the own-price derivative with respect to the price:

$$\frac{\partial s_j}{\partial p_j} = \int (-\alpha) \frac{\partial \tilde{s}_j}{\partial \delta_j} dF(v) = -\alpha \int \tilde{s}_j (1 - \tilde{s}_j) dF(v)$$

where the last equality is due to properties of the logit error. Thus our final price condition is

$$p_j = mc_j - \int \tilde{s}_j dF(v) [-\alpha \int \tilde{s}_j (1 - \tilde{s}_j) dF(v)]^{-1}$$

## 2 Q1

We are given that  $\alpha = 1, \beta = 1, \sigma = 1, x_1 = 1, x_2 = 2, x_3 = 3$ , and  $mc_j = x_j$ .

The price vector is:

write write write

## 3 Q2

Now  $\alpha = .5, \beta = .5, \sigma = .5, x_1 = 1, x_2 = 2, x_3 = 3$ , and  $mc_j = x_j$ . The price vector is:

write write write. It is different because blah.

## 4 Q3

After the merger the profit maximization problem for the new firm is

$$\max_{p_1, p_2} (p_1 - mc_1)s_1(\mathbf{p}, \mathbf{x}, \sigma) + (p_2 - mc_2)s_2(\mathbf{p}, \mathbf{x}, \sigma)$$

The FOC for  $p_1$  is

$$0 = s_1 + (p_1 - mc_1)\frac{\partial s_1}{\partial p_1} + (p_2 - mc_2)\frac{\partial s_2}{\partial p_1}$$

and so the price will be determined by the following condition:

$$p_1 = mc_1 - (s_1 + (p_2 - mc_2)\frac{\partial s_2}{\partial p_1})(\frac{\partial s_1}{\partial p_1})^{-1}$$

Note that using our previous notation,

$$\frac{\partial s_2}{\partial p_1} = -\alpha \int \tilde{s}_1 \tilde{s}_2 dF(v)$$

The FOC for  $p_2$  is symmetric to that of  $p_1$ , and thus the optimal  $p_1$  and  $p_2$  are given by

$$p_1 = mc_1 - [\int \tilde{s}_1 dF(v) + (p_2 - mc_2)(-\alpha \int \tilde{s}_1 \tilde{s}_2 dF(v))][-\alpha \int \tilde{s}_1(1 - \tilde{s}_1) dF(v)]^{-1}$$

$$p_2 = mc_2 - [\int \tilde{s}_2 dF(v) + (p_1 - mc_1)(-\alpha \int \tilde{s}_2 \tilde{s}_1 dF(v))][-\alpha \int \tilde{s}_2(1 - \tilde{s}_2) dF(v)]^{-1}$$

As firm 3 has not merged it's optimal price condition is the same:

$$p_3 = mc_3 - \int \tilde{s}_3 dF(v)[-\alpha \int \tilde{s}_3(1 - \tilde{s}_3) dF(v)]^{-1}$$