Economics 631 IO - Fall 2019
Problem Set 2

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1 BLP - Random Coefficient

Preliminaries

Each firm chooses price to solve the problem

$$\max_{p_j}(p_j - mc_j)Ms_j(\boldsymbol{p}, \boldsymbol{x}, \sigma)$$

The FOC is

$$0 = (p_j - mc_j)M\frac{\partial s_j}{\partial p_j} + Ms_j$$

and so the price will be determined by the following condition:

$$p_j = mc_j - s_j(\frac{\partial s_j}{\partial p_j})^{-1}$$

. The market share for product j is given by

$$s_j(\boldsymbol{p}, \boldsymbol{x}, \theta) = \int \frac{\exp(\beta_i x_j - \alpha p_j)}{1 + \sum_{j'} \exp(\beta_i x_{j'} - \alpha p_{j'})} dF(\beta_i)$$

Given our functional form assumptions we can rewrite $\beta_i = \beta + \sigma v_i$ where β is the mean of the distribution and σ is the standard deviation, $v_i \sim \mathcal{N}(0, 1)$. Additionally we can define the mean utility of purchasing product j as $\delta_j = \beta x_j - \alpha p_j$ and rewrite the market share expression in terms of δ_j and σ.

$$s_j(\boldsymbol{p}, \boldsymbol{x}, \boldsymbol{\delta}, \sigma) = \int \frac{\exp(\delta_j + \sigma x_j v_i)}{1 + \sum_{j'} \exp(\delta_{j'} + \sigma x_j' v_i)} dF(v)$$

Note that if we rewrite the above expression as

$$s_j(\boldsymbol{p}, \boldsymbol{x}, \boldsymbol{\delta}, \sigma) = \int \tilde{s}_j(\boldsymbol{p}, \boldsymbol{x}, \boldsymbol{\delta}, \sigma) dF(v_i)$$

we can get a fairly-nice expression for the own-price derivative with respect to the price:

$$\frac{\partial s_j}{p_j} = \int (-\alpha)\frac{\partial \tilde{s}_j}{\partial \delta_j} dF(v) = -\alpha \int \tilde{s}_j(1 - \tilde{s}_j) dF(v)$$

where the last equality is due to properties of the logit error. Thus our final price condition is

$$p_j = mc_j - \int \tilde{s}_j dF(v)[-\alpha \int \tilde{s}_j(1 - \tilde{s}_j) dF(v)]^{-1}$$

2 Q1

We are given that $\alpha = 1, \beta = 1, \sigma = 1, x_1 = 1, x_2 = 2, x_3 = 3$, and $mc_j = x_j$.

The price vector is:

write write write

3 Q2

Now $\alpha = .5, \beta = .5, \sigma = .5, x_1 = 1, x_2 = 2, x_3 = 3$, and $mc_j = x_j$. The price vector is:

write write write. It is different because blah.

4 Q3

After the merger the profit maximization problem for the new firm is

$$\max_{p_1,p_2}(p_1 - mc_1)s_1(\boldsymbol{p}, \boldsymbol{x}, \sigma) + (p_2 - mc_2)s_2(\boldsymbol{p}, \boldsymbol{x}, \sigma)$$

The FOC for p_1 is

$$0 = s_1 + (p_1 - mc_1)\frac{\partial s_1}{\partial p_1} + (p_2 - mc_2)\frac{\partial s_2}{\partial p_1}$$

and so the price will be determined by the following condition:

$$p_1 = mc_1 - (s_1 + (p_2 - mc_2)\frac{\partial s_2}{\partial p_1})(\frac{\partial s_1}{\partial p_1})^{-1}$$

Note that using our previous notation,

$$\frac{\partial s_2}{\partial p_1} = -\alpha \int \tilde{s}_1 \tilde{s}_2 dF(v)$$

The FOC for p_2 is symmetric to that of p_1, and thus the optimal p_1 and p_2 are given by

$$p_1 = mc_1 - [\int \tilde{s}_1 dF(v) + (p_2 - mc_2)(-\alpha \int \tilde{s}_1 \tilde{s}_2 dF(v))][-\alpha \int \tilde{s}_1(1 - \tilde{s}_1)dF(v)]^{-1}$$

$$p_2 = mc_2 - [\int \tilde{s}_2 dF(v) + (p_1 - mc_1)(-\alpha \int \tilde{s}_2 \tilde{s}_1 dF(v))][-\alpha \int \tilde{s}_2(1 - \tilde{s}_2)dF(v)]^{-1}$$

As firm 3 has not merged it's optimal price condition is the same:

$$p_3 = mc_3 - \int \tilde{s}_3 dF(v)[-\alpha \int \tilde{s}_3(1 - \tilde{s}_3)dF(v)]^{-1}$$

5 Q4

The change in consumer surplus is given by the compensating variation, which we can calculate as follows:

$$CV_i = \frac{\log(\sum_j \exp(V_{ij}^{old}) - (\sum_j \exp(V_{ij}^{new})}{\alpha}$$

where $V_{ij} = \beta_i x_j - \alpha p_j$. The total consumer surplus is found by

$$CV = M * \int CV_i dF(v)$$

where M is the number of people in the market. The change in producer surplus is just the change in profits, and is given by

$$\Delta\pi = \sum_j (p_j^{new} - mc_j^{new})Ms_j^{new} - (p_j^{olds} - mc_j^{old})Ms_j^{old}$$

and the total change in welfare is

$$\Delta\text{Surplus} = \Delta\pi - CV = M[(\sum_j (p_j^{new} - mc_j^{new})s_j^{new} - (p_j^{olds} - mc_j^{old})s_j^{old}) - \int CV_i dF(v)]$$

The change in surplus when we normalize $M = 1$ is
insert ish here

6 Q5

If we allow the merging firm's marginal costs to decrease from x_j to $\frac{x_j}{2}$ we get the following pricing equilibrium and change in consumer, producer and total surplus:
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