Asymmetric Learning Model of Resume Building With Wage Rigidity and Costly Firings

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1 Introduction

The purpose of this document is to clearly outline the two period version of my model. The main idea and focus of my model is to investigate how wages are related to tenure in an asymmetric learning environment where outside firms only observe a resume like signal. The two period version of my model has a lot in common with the two period Acemoglu and Picshke model of asymmetric learning and general training

(Acemoglu & Pischke, 1998). I, however, am not focusing on training, and differ from there set up in some key ways.

Employers in my model will only observe a noisy signal of worker ability. In the two period model this is essentially indistinguishable from learning the true type. A firm that receives a good signal will consider that worker to have a high *expected* ability rather than simply knowing they have high ability. I believe this departure will have more significant implications in a 3 period or more model. The second difference from the Acemoglu and Picshke model is that I assume outside firms can observe an employees resume. That is, they can tell after period one if a worker was fired or separated from their firm, or if the firm decided to keep them. This has implications for the two period model, and will allow for a more realistic framework for labor force signals in 3 or more periods. Finally, I consider the impact of sticky wages and costly firings.

The results of the model so far suggest that in an otherwise frictionless market, asymmetric information of this type will tend to lower wages and increase wage dispersion with tenure. A market with asymmetric information and sticky wages, but with high fixed costs to firing, will see relatively constant and even wages over tenure. Finally, a market with wage rigidity but moderate costs to firing will see wage growth with tenure and wage cuts for fired workers.

2 Model

2.1 outline

I will start by discussing a model without wage rigidity. I then consider what happens to this model if we introduce wage rigidity, but let the cost to firing by extremely high. Finally, I consider what happens as the cost to firing is lowered. The Final model is the most interesting. However, considering the simpler versions in turn is crucial for solving the final model and makes the implications of each additional assumption more apparent.

2.2 Environment

This model lasts for two periods. In the first period no firm has any information abut individual workers. The following information about the distribution of workers is common knowledge to all firms. Workers have ability $\theta \in [0, 1]$ with distribution $f(\theta)$. They produce θ per period and supply labor inelastically. They take the highest wage offer they receive, and in the event of a tie stay with their current employer or pick randomly if unemployed.

In the second period firms receive a noisy signal about employee's ability. Workers with a ability θ send a good signal with probability θ and a bad signal with probability $1-\theta$. Employers offer wages for period 2 employees conditional on this signal. Firms can also fire employees for a fixed cost F_C. Outside firms observe employee's resume. That is, if a worker is employed or has been fired after period one. Outside firms offer wages conditional on these signals. employees decide where to work and produce for one more period before retiring. There are many competitive firms so this implies a zero profit condition on all firms.

The variables I will be using are summarized in the table below.

Variable	Meaning
θ	Ability
g	Good Signal
b	Bad Signal
e	Employed Signal
f	Fired Signal
F_C	Fixed Cost to Firing
w_1	Period 1 Wage
w_g	Wage After Good Signal
w_b	Wage After Bad Signal
w_e	Wage Offered to Worker With a Year of Employment
w_f	Wage for Fired Worker
π	Profits

The time-line is also laid out below.

- period 1 wage offers
- Workers produce output
- Workers send signal of ability
- Employers decide who to fire
- Employers offer period 2 wages conditional on signals
- Outside Firms offer wages conditional on resume
- Workers take the best offer and work for one more period

- Workers retire

2.3 Flexible Wage Model

First we will consider a model with totally flexible wages. In this model there is no reason to ever fire workers. Employers can simply lower wages to whatever level is profitable or induce a worker to quit. Note that in period 2 the expected output of employees who sent a good or bad signal is $\mathrm{E}[\theta|g]$ and $\mathrm{E}[\theta|b]$ respectively. To determine wages we begin in the second period.

Proposition 1: Second period wages in the flexible model will be $w_g = \mathrm{E}[\theta]$ and $w_b = \mathrm{E}[\theta|b]$. No workers will switch firms.

These wages are an equilibrium because they are the lowest possible wages an employer can make, and the firm maximize profits by offering the lowest possible wages. Consider if employers offered their good signal employees $w_g = \mathrm{E}[\theta] - \epsilon$. An outside firm could offer a wage of $w_e > w_g$ to my workers with a year of experience. All of them would leave, yielding the outside firm an expected profit of $\mathrm{E}[\theta] - w_e$. This will be profitable for outside firms for any $w_e < \mathrm{E}[\theta]$ and so employers must offer at least $w_g = \mathrm{E}[\theta]$ to retain their good signal workers.

Given that $w_g = \mathrm{E}[\theta]$ any outside offer $w_e < \mathrm{E}[\theta]$ will only attract bad signal employees. This implies outside firms will offer $w_e = \mathrm{E}[\theta|b]$. So, employers can offer their bad signal employees $w_b = \mathrm{E}[\theta|b]$. the intuition is that outside firms can either offer a high wage and get both good and bad signal workers, for which they would pay at most $\mathrm{E}[\theta]$, or they can offer a lower wage and attract only bad signal employees, for which they wold offer at most $\mathrm{E}[\theta|b]$. Employers only need to match these offers to retain their workers. The result of these wages is that workers cannot gain by leaving their employer and employers have no incentive to fire. So, there is no turn over.

proposition 2: First period wages in the flexible model will be $w_1 = \mathrm{E}[\theta] + p(g)(\mathrm{E}[\theta|g] - \mathrm{E}[\theta])$

Given the wages in period 2, in order to determine period one wages all we need to do is apply the zero profit condition. The period one wage will be equal to period one output plus expected profits in period two.

$$0 = \mathrm{E}[\theta] - w_1 + p(g)(\mathrm{E}[\theta|g] - w_g) + p(b)(\mathrm{E}[\theta|b] - w_b)$$

$$\implies w_1 = \mathrm{E}[\theta] + p(g)(\mathrm{E}[\theta|g] - w_g) + p(b)(\mathrm{E}[\theta|b] - w_b) = \mathrm{E}[\theta] + p(g)(\mathrm{E}[\theta|g] - \mathrm{E}[\theta])$$

Putting this together we get $w_1 > w_g > w_b$. The intuition here is that employers are able to earn expected profits on good signal workers in the second period and break even on bad signal workers. So, they bid up the wage of first period workers until their profits are back to zero. The wage tenure pattern here, wages falling with time, is inconsistent with what we see in reality. We can see that asymmetric information in an otherwise frictionless market works to lower wages with tenure.

2.4 Wage rigidity with High Fixed Cost to Firing

Now we will introduce downward wage rigidity. By "downward wage rigidity" I mean that wages cannot go down in period 2. We will also start by assuming that the fixed cost to firing employees, F_C, is so high that no firm fires anyone. This is partially to help solve the final model, but also is of some interest. Some firms may face extremely high fixed costs to firing through legal risk, difficulty building a case for firing workers, difficulty getting managers to fire employees, or strong unions. The implications for this on the impact of wages and tenure is of interest. The conclusion is stated below.

Proposition 3: With sticky wages and high F_C we get $w_1 = w_b = w_g = \text{E}[\theta]$

The logic here is that offering a wage lower than $\text{E}[\theta]$ in the first period would lead to positive profits, but offering a wage larger than $\text{E}[\theta]$ in the first period would lead to negative profits. To start the solution I make an observation about possible values for period one wages.

Lemma 1: Given the high firing costs, $w_1 \in [\text{E}[\theta|b], \text{E}[\theta]]$

We know that the profit maximizing wage offers, without wage rigidity, for firms in period two will be the wages offered in the flexible mode. So, if the firm offers period one wages $w_1 \leq \text{E}[\theta|b]$ they can offer the same wages as in the flexible model in period two and receive positive profits. So this will not be an equilibrium. If a firm offers first period wages $w_1 \geq \text{E}[\theta]$ they will not be able to lower wages in the second period. Thus the firm earns profits $\pi = \text{E}[\theta] - w_1 + p(g)(\text{E}[\theta|g] - w1) + p(b)(\text{E}[\theta|b] - w_1) = 2 \cdot (\text{E}[\theta] - w1) < 0$. Using this Lemma, we can determine optimal period two wages. First, we know

Corollary 1: $w_b = w_1$ because w_b will be "stuck"

Firms will want to lower their wages for their bad workers in period two to $\text{E}[\theta|b]$, but because of sticky wages they will not be able to lower it. Thus, their best course of action is to make it as low as possible. Which is $w_b = w_1$.

Now for their good signal workers, the same logic as the flexible model applies. The profit maximizing wage offer will be $\text{E}[\theta]$. Since by lemma 1 $w_1 \leq \text{E}[\theta]$ we know this will be possible. Thus $w_g = \text{E}[\theta]$.

Now to find period 1 wages we simply apply the zero profit condition with sticky wages. This is equivalent to setting w_1 equal to expected revenue since wages are the only cost. i.e.

$$w_1 = \mathrm{E}[\theta] + p(g)(\mathrm{E}[\theta|g] - \mathrm{E}[\theta]) + p(b)(\mathrm{E}[\theta|b] - w_1)$$

using the fact that $p(g)\mathrm{E}[\theta|g] + p(b)\mathrm{E}[\theta|b] = \mathrm{E}[\theta]$ we get

$$w_1 = \mathrm{E}[\theta] + \mathrm{E}[\theta] - p(g)\mathrm{E}[\theta] - p(b)w_1$$

using the fact that $\mathrm{E}[\theta] - p(g)\mathrm{E}[\theta] = p(b)\mathrm{E}[\theta]$

$$\implies w_1 + p(g)w_1 = \mathrm{E}[\theta] + p(b)\mathrm{E}[\theta]$$

$$\implies w_1 = \mathrm{E}[\theta]$$

Putting this all together we get proposition 3 $w_1 = w_b = w_g = \mathrm{E}[\theta]$. The implication of this is that sticky wages make it impossible for firms to capitalize on their inside information in period 2. Thus they earn no profits in period two and period one wages are not bid up past expected revenue in period 1. We will show in the next section that if we give firms an avenue to rid themselves of bad workers, through firing, they will do so.

But how high is “High fixed cost”? At what F_C is this no longer an equilibrium? In order for the above equilibrium to hold we need that no firm could earn a profit by deviating and firing their bad workers. If a firm unilaterally deviates and fires workers after a bad signal they receive profits

$$\pi = p(g)(\mathrm{E}[\theta|g] - \mathrm{E}[\theta]) - p(b)F_C$$

This will not be optimal when it is less than profits from not firing bad signal worker. i.e. when

$$p(g)(\mathrm{E}[\theta|g] - \mathrm{E}[\theta]) - p(b)F_C < p(g)(\mathrm{E}[\theta|g] - E[\theta]) + p(b)(E[\theta|b] - E[\theta]) = 0$$

Which occurs when

$$F_C > \mathrm{E}[\theta] - \mathrm{E}[\theta|b]$$

Thus when $F_C > \mathrm{E}[\theta] - \mathrm{E}[\theta|b]$ the equilibrium is to not to fire any workers and pay a constant wage of $\mathrm{E}[\theta]$.

2.5 Equilibrium with Firing

Next we consider when firing all bad workers is an equilibrium. First let's determine what the wages would be if all firms decided to fire their bad signal employees.

proposition 4: Wages in an equilibrium where bad signal employees are fired will be $w_1 = \mathrm{E}[\theta] - p(b)F_c$ and $w_g = \mathrm{E}[\theta|g]$ and $w_f = \mathrm{E}[\theta|b]$.

If bad signal employees are all fired than the outside employers will know that workers with a resume of e are all good signal workers. So $w_e = \mathrm{E}[\theta|e] = \mathrm{E}[\theta|g]$. Since any worker that hasn't been fired can receive this wage, firms will have to pay their good signal workers $w_g = \mathrm{E}[\theta|g]$. Bad signal workers will have been fired and identified as bad signal and so they will be rehired for a wage of $w_f = \mathrm{E}[\theta|b]$ by other firms.

Given these second period actions and wages the zero profit condition will give us optimal first period wages of

$$w_1 = \mathrm{E}[\theta] + p(g)(\mathrm{E}[\theta|g] - \mathrm{E}[\theta|g]) - p(b)F_C = \mathrm{E}[\theta] - p(b)F_C$$

These wages will be a stable equilibrium whenever firms cannot deviate and receive positive profits. If a firm deviates and keeps their bad signal employees they can pay them w_1. They will not leave for another firm because leaving would identify them as a low skilled worker and yield them $\mathrm{E}[\theta|b] < w_1$. Therefore keeping bad signal employees gives

$$\pi_{keep} = \mathrm{E}[\theta] - w1 + p(g)(\mathrm{E}[\theta|g] - w_g) + p(b)(\mathrm{E}[\theta|b] - w_1) = \mathrm{E}[\theta] - w_1 + p(b)(\mathrm{E}[\theta|b] - w_1)$$

compared to firing them and receiving

$$\pi_{fire} = \mathrm{E}[\theta] - w1 + p(g)(\mathrm{E}[\theta|g] - w_g) - p(b)F_C$$

and so firing workers is optimal for all firms when

$$F_C < w_1 - \mathrm{E}[\theta|b]$$

Given the w_1 we found above this implies firing all bad signal employees is optimal when

$$F_C < \mathrm{E}[\theta] - p(b)F_C - \mathrm{E}[\theta|b] \implies F_C < \frac{\mathrm{E}[\theta] - \mathrm{E}[\theta|b]}{1 + p(b)}$$

This equilibrium implies $w_f < w_1 < \mathrm{E}[\theta] < w_g$. The intuition here is that in period two firms either make 0 expected profits on good signal workers or have to pay to fire bad signal workers. Given this, workers in the first period are paid below their expected first period output.

2.6 Mixed Equilibrium

If the fixed cost is $F_C \in \left(\frac{\mathrm{E}[\theta]-\mathrm{E}[\theta|b]}{1+p(b)}, \mathrm{E}[\theta]-\mathrm{E}[\theta|b]\right)$ than we will have a mixed equilibrium where firms fire a fraction δ of their bad signal employees.

Note that in a mixed equilibrium firms would need to be indifferent between firing and keeping employees after a bad signal. This implies that the fixed cost to firing must be equivalent to the loss from holding on to a low signal worker. That is

$$F_C = w_1 - E[\theta|b]$$

Given that δ of the bad signal employees are fired, the wages required to keep a good signal employee are

$$w_g(\delta) = \frac{p(g)E[\theta|g] + p(g)(1-\delta)E[\theta|b]}{p(g) + p(b)(1-\delta)}$$

This is just the expected output of a worker who is not fired. That is the only information a worker can prove to an outside employer and so this is the highest offer they could receive. Given that they are being offered this by their current employers, no good signal workers would leave for any outside offer and so low wage workers receive their "stuck" wage of w_1.

Now given those period 2 wages the period 1 wages will satisfy the zero profit condition.

$$w_1 = E[\theta] + p(g)\left(E[\theta|g] - w_g(\delta)\right) - p(b)F_C$$

Using these equations we can get a closed form solution for the fraction of bad signal workers fired δ.

$$\delta = 1 - \frac{p(g)\left[E[\theta] - (1+p(b))F_C - E[\theta|b]\right]}{p(b)\left[-E[\theta] + (1+p(b))F_C + (1+p(g))E[\theta|b] - p(g)E[\theta|g]\right]}$$

References

Acemoglu, D., & Pischke, J.-S. (1998). Why do firms train? theory and evidence. *The Quarterly Journal of Economics*, *113*(1), 79–119. Retrieved from http://www.jstor.org/stable/2586986