Asymmetric Learning Model With Wage Rigidity and Costly Firings

Nathan Mather

University of Michigan

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Introduction

- Analyze an asymmetric learning model as it relates to wages and tenure
- Incorporate sticky wages
- Incorporate fixed firing costs
- Basic model set up is similar to Acemoglu and Pischke
- Scaled back to focus on wage evolution
- Simplifying in certain ways allows adding other unexplored complexity

Outline

- Outline of basic model structure
- Equilibrium under flexible wages
- Equilibrium under sticky wages
- Equilibrium under sticky wages and various firing costs
- Implications
- Thoughts moving forward

Model Environment

My Model

- Workers have ability $\theta \in [0, 1]$
  - Produce θ per period
- Workers supply labor inelastically
- exogenously separate at rate δ
- Employers receive a good signal g w.p. θ and a bad signal b w.p. $1 - \theta$
- Employers offer wages each period (no long term contracts).
  - All firms can condition on employment status
  - Current employer can also condition wage on signal
- Outside firms see employment status and history (resume)
- There are many identical firms
  - Zero profit condition
- Downward wage rigidity
- Fixed cost to fire employee

Variable Definitions

Variable	Meaning
θ	Ability
g	Good Signal
b	Bad Signal
e	Employed Signal
f	Fired Signal
F_C	Fixed Cost to Firing
δ	Exogenous separation rate
w_1	Period 1 Wage
w_g	Wage After Good Signal
w_b	Wage After Bad Signal
w_u	Wage for unemployed worker
π	Profits

Time Line

- period 1 wage offers
- Workers produce output
- workers exogenously separate
- Workers send signal of ability
- Employers decide who to fire
- Employers offer period 2 wages conditional on signals
- Outside Firms offer wages conditional on resume
- Workers take the best offer and work for one more period
- Workers retire

Flexible Wages

- Let Q denote the fraction of workers that don't separate but then quit.

$$w_u(Q) = (\delta \mathrm{E}[\theta] + Q\mathrm{E}[\theta|\text{worker quit}])/(\delta + Q)$$

- Employers will not pay more than the expected output of a worker, giving

$$w_b = \mathrm{E}[\theta|b]$$

- This implies that all bad signal employees quit since if $\delta > 0$

$$\mathrm{E}[\theta|b] < \frac{\delta \mathrm{E}[\theta] + (1-\delta)p(b)(\mathrm{E}[\theta|b])}{\delta + (1-\delta)p(b)} = \min w_u(Q)$$

- Where the Right hand side is the lowest possible w_u since here $Q = (1-\delta)p(b)$

Flexible Wages

- If all bad signal employees quit than anyone still employed is identified as high ability. Giving

$$w_g = \mathrm{E}[\theta|g]$$

- This gives

$$w_u = \frac{\delta \mathrm{E}[\theta] + (1-\delta)p(b)(\mathrm{E}[\theta|b])}{\delta + (1-\delta)p(b)}$$

- And a first period wage

$$w_1 = \mathrm{E}[\theta]$$

Sticky Wages

- Now employers can't cut their low signal worker's wages.
- We know $w_1 > \mathrm{E}[\theta|b]$ because if it weren't profits would be positive
- This implies employers fire bad signal workers
- Wage outcomes are identical to flexible model but with workers fired rather than quitting

Sticky Wages and fixed firing costs

Low firing costs

- A low firing cost will not alter firms decision to fire bad signal employees
- Now that hiring an employee means I may have to pay a cost to fire them, their marginal benefit is decreased and so their wage offer is lower

$$w_1 = \mathrm{E}[\theta] + (1-\delta)p(b)F_C$$

- Firms continue to do this as long as

$$w_1 - \mathrm{E}[\theta|b] > F_C$$

$$\implies \mathrm{E}[\theta] + (1-\delta)p(b)F_C - \mathrm{E}[\theta|b] > F_C$$

$$\implies f_C < \frac{\mathrm{E}[\theta] - \mathrm{E}[\theta|b]}{1 + p(b)(1-\delta)}$$

Clarifying Equilibrium wages

An Aside

To clarify the wage offers in the next section, first consider this numerical example:

- $w_u = 1$

$\mathrm{E}[\theta]$	EQ Wage offer
3	2.5
2	2

- If the "raiding" firm offers $w = 2 + \epsilon$ to all employed workers, they get only those with $E[\theta] = 2$ for a loss
- If they offer $w = 2.5 + \epsilon$ they get workers with average $\mathrm{E}[\theta] = 2.5$ for a loss
- These are the lowest wages the firm can offer and still retain their employees
- In general, it means employers pay a worker the expected ability of all workers less than and equal to their own ability

Sticky Wages and fixed firing costs

High firing costs

- If the fixed costs become sufficiently large, it is clear that firms will not fire any employees
- Firms must pay their employees enough to keep them from getting "raided"

$$w_g = E[\theta|e] = \mathrm{E}[\theta]$$

$$w_b = w_1$$

- Unemployed workers get their expected output

$$w_u = E[\theta]$$

Sticky Wages and fixed firing costs

High firing costs

- This gives a first period wage through the zero profit condition

$$w_1 = \mathrm{E}[\theta] + (1-\delta)(p(g)(\mathrm{E}[\theta|g] - \mathrm{E}[\theta]) + p(b)(\mathrm{E}[\theta|b] - w_1))$$

- using the following fact: $p(g)\mathrm{E}[\theta|g] + p(b)\mathrm{E}[\theta|b] = \mathrm{E}[\theta]$

$$\implies w_1 = \mathrm{E}[\theta] + (1-\delta)(\mathrm{E}[\theta] - p(g)\mathrm{E}[\theta] - p(b)w_1)$$

- Next, using

$$\mathrm{E}[\theta] - p(g)\mathrm{E}[\theta] = p(b)\mathrm{E}[\theta]$$

$$\implies w_1 = \mathrm{E}[\theta] + (1-\delta)p(b)(\mathrm{E}[\theta] - w_1)$$

$$\implies w_1 + (1-\delta)p(b)w_1 = \mathrm{E}[\theta] + (1-\delta)p(b)\mathrm{E}[\theta]$$

$$\implies w_1 = \mathrm{E}[\theta]$$

- all workers get paid $\mathrm{E}[\theta]$

Sticky Wages and fixed firing costs

High firing costs: stability

This equilibrium will be stable whenever it is more costly to fire an employee then to keep them at this wage.

$$F_C > w_1 - \mathrm{E}[\theta|b] = \mathrm{E}[\theta] - \mathrm{E}[\theta|b]$$

Sticky Wages and fixed firing costs

Mixed Equilibrium

- Now consider the case between the two extremes.

$$F_C \in \left[\frac{\mathrm{E}[\theta] - \mathrm{E}[\theta|b]}{1 + p(b)(1-\delta)}, \mathrm{E}[\theta] - \mathrm{E}[\theta|b] \right]$$

- In this case employers will fire a fraction of their bad signal workers δ_F until they are indifferent between firing and keeping them

$$F_C = w_1 - E[\theta|b]$$

- Good signal employees need to be paid the expected output of an average worker employee in period 2

$$w_g(\delta_F) = \frac{p(g)\mathrm{E}[\theta|g] + p(b)(1-\delta_F)\mathrm{E}[\theta|b]}{p(g) + p(b)(1-\delta_F)}$$

Sticky Wages and fixed firing costs

Mixed Equilibrium

- bad signal employees are paid the lowest wage possible $w_b = w_1$ and unemployed get their expected output

$$w_u = \frac{\delta \mathrm{E}[\theta] + (1-\delta)\delta_F \mathrm{E}[\theta|b]}{\delta + (1-\delta)\delta_F}$$

- by the zero profit condition we get the first period wages

$$w_1 = \mathrm{E}[\theta] + (1-\delta)\Big(p(g)\Big(\mathrm{E}[\theta|g] - w_g(\delta_F)\Big) - p(b)F_C\Big)$$

Sticky Wages and fixed firing costs

Mixed Equilibrium

- We have two equations for w_1 with the only unknown being δ_F
- I have solved for it, but it not very intuitive or generally helpful. Can use it to solve numeric examples.

$$\delta_F = \frac{p(g)\left[\frac{\mathrm{E}[\theta]-F_C-(1-\delta)p(b)F_C-\mathrm{E}[\theta|b]}{(1-\delta)p(g)}\right]}{p(b)\left[\mathrm{E}[\theta|b]-\frac{\mathrm{E}[\theta]-F_C-(1-\delta)p(b)F_C-\mathrm{E}[\theta|b]}{(1-\delta)p(g)}+\mathrm{E}[\theta|g]\right]}$$

What Next

- - vThree period version is not intuitive.
    - I have made progress solving general numeric examples, but unsure how useful this is

- More than two signal levels seems promising
  - I expect I will also need to resort to numeric examples

- A continuous signal or just employers fully learning workers continuous ability measure
  - I started this and do not think there is a general solution. (Depends on θ distribution)

- Promotions

- An Acemoglu and Pischke type utility shock

Questions and Concerns

- How to apply model to data?
- Are numerical examples and graphs useful?
- Trouble fitting this into literature
- Are the assumptions reasonable?

Thank You