Econ 675 Assignment 3

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1 Question 1: Estimating Equations

1.1 Q1 Part 1

To show that these are valid moment conditions we just need to show that they are all equal to zero. We start with the IPW condition

$$\mathrm{E}\left[\psi_{IPW}(\boldsymbol{Z}_i;\theta_t(g))\right] = \mathrm{E}\left[\frac{D_i(t)\cdot g(Y_i(t))}{p_t(\boldsymbol{X}_i)} - \theta(g)\right] = \mathrm{E}\left[\mathrm{E}\left[\frac{D_i(t)\cdot g(Y_i(t))}{p_t(\boldsymbol{X}_i)}|\boldsymbol{X}_i\right]\right] - \theta(g)$$
$$= \mathrm{E}\left[\frac{1}{p_t(\boldsymbol{X}_i)}\mathrm{E}\left[D_i(t)\cdot g(Y_i(t))|\boldsymbol{X}_i\right]\right] - \theta(g)$$

Now notice that

$$\mathrm{E}[D_i(t)|\boldsymbol{X}_i] = Pr[D_i(t)=1|\boldsymbol{X}_i] = Pr[T_i = t|\boldsymbol{X}_i] = p_t(\boldsymbol{X}_i)$$

using this we get

$$\mathrm{E}\left[\psi_{IPW}(\boldsymbol{Z}_i;\theta_t(g))\right] = \mathrm{E}\left[\mathrm{E}\left[g(Y_i(t))|\boldsymbol{X}_i\right]\right] - \theta(g) = \mathrm{E}\left[g(Y_i(t))\right] - \theta(g) = 0$$

Next we check $\psi_{RI1,t}$

$$\mathrm{E}[\psi_{RI1,t}(\boldsymbol{Z}_i;\theta_t(g))] = \mathrm{E}[e_t(g;\boldsymbol{X}_i)] - \theta_t(g) = \mathrm{E}[\mathrm{E}[g(Y_i(t))|\boldsymbol{X}_i]] - \theta_t(g) = \mathrm{E}[g(Y_i(t))] - \theta_t(g) = 0$$

Next check $\psi_{RI2,t}$

$$\mathrm{E}[\psi_{RI2,t}(\boldsymbol{Z}_i;\theta_t(g))] = \mathrm{E}\left[\frac{D_i(t)\cdot e_t(g;\boldsymbol{X_i})}{p_t(\boldsymbol{X}_i)}\right] - \theta(g) = \mathrm{E}\left[\mathrm{E}\left[\frac{D_i(t)\cdot e_t(g;\boldsymbol{X_i})}{p_t(\boldsymbol{X}_i)}|\boldsymbol{X}_i\right]\right] - \theta(g) = \mathrm{E}[e_t(g;\boldsymbol{X}_i)] - \theta_t(g) = 0$$

[*]Shouts out to Ani for the help with this

Finally we check $\psi_{DR,t}$

$$\mathrm{E}[\psi_{DR,t}(\boldsymbol{Z}_i;\theta_t(g))] = \mathrm{E}\left[\frac{D_i(t)\cdot g(Y_i(t))}{p_t(\boldsymbol{X}_i)} - \theta(g)\right] - \mathrm{E}\left[\frac{e_t(g;\boldsymbol{X}_i)}{p_t(\boldsymbol{X}_i)}(D_i(t) - p_t(\boldsymbol{X}_i))\right]$$

This first terms are identical to the IPW condition so we need only check the following.

$$\mathrm{E}\left[\frac{e_t(g;\boldsymbol{X}_i)}{p_t(\boldsymbol{X}_i)}(D_i(t) - p_t(\boldsymbol{X}_i))\right] = \mathrm{E}\left[\frac{e_t(g;\boldsymbol{X}_i)D_i(t)}{p_t(\boldsymbol{X}_i)} - pe_t(g;\boldsymbol{X}_i)\right] = \theta_t(g) - \theta_t(g) = 0$$

So all functions are valid moment conditions

1.2 Q1 Part 2

The plug-in IPW estimator is

$$\hat{\theta}_{\mathtt{IPW},t}(g) = \frac{1}{n}\sum_{i=1}^{n}\frac{D_i(t)g(Y_i)}{\hat{p}_t(\boldsymbol{X}_i)}$$

$\hat{p}_t(\boldsymbol{X}_i)$ is the estimated propensity score. Because this has multiple treatment levels we can estimate the propensity score with any suitable discrete choice model. For example the multinomial logit model.

The RD1 estimator is

$$\begin{aligned}\hat{\theta}_{\mathtt{RI1},t}(g) = \hat{\mathrm{E}}[e_t(g;\boldsymbol{X}_i)] = \frac{1}{n}\sum_{i=1}^{n}\hat{\mathrm{E}}[g(Y_i(t))|\boldsymbol{X}_i] &= \frac{1}{n}\sum_{i=1}^{n}\hat{\mathrm{E}}[g(Y_i(t))|\boldsymbol{X}_i, D_i(t)=1] \\ &= \frac{1}{n}\sum_{i=1}^{n}\hat{\mathrm{E}}[g(Y_i)|\boldsymbol{X}_i, D_i(t)=1],\end{aligned}$$

where the second last equality uses the ignorability assumption. We just need to decide how to estimate this last term. We could probably use NLS or some nonparametric methid.
The plug-in 'hybrid' imputation estimator is

$$\hat{\theta}_{\mathtt{RI2},t}(g) = \frac{1}{n}\sum_{i=1}^{n}\frac{D_i(t)\widehat{\mu}_t(\boldsymbol{X}_i)}{\hat{p}_t(\boldsymbol{X}_i)}.$$

Finally, the plug-in doubly robust estimator is given by

$$\begin{aligned}\hat{\theta}_{\mathtt{DR},t}(g) &= \frac{1}{n}\sum_{i=1}^{n}\frac{D_i(t)g(Y_i)}{\hat{p}_t(\boldsymbol{X}_i)} - \frac{1}{n}\sum_{i=1}^{n}\frac{\widehat{\mu}_t(\boldsymbol{X}_i)}{\hat{p}_t(\boldsymbol{X}_i)}(D_i(t) - \hat{p}_t(\boldsymbol{X}_i)) \\ &= \frac{1}{n}\sum_{i=1}^{n}\left(\frac{D_i(t)(g(Y_i) - \widehat{\mu}_t(\boldsymbol{X}_i))}{\hat{p}_t(\boldsymbol{X}_i)} + \widehat{\mu}_t(\boldsymbol{X}_i)\right).\end{aligned}$$

As discussed in Abadie and Catteneo (2018), the relative performance of the above estimators depends on the features of the data generating process. In finite samples, IPW estimators become unstable when the propensity score approaches zero or one and regression imputation estimators may suffer from extrapolation biases. Doubly robust estimators include safeguards against bias caused by misspecification but impose additional specification choices that may affect the resulting estimate.

1.3 Q1 Part 3

Note that

$$\sigma_t^2 = \mathrm{V}[Y_i(t)] = \mathrm{E}\left[Y_i(t) - \mathrm{E}[Y_i(t)]\right]^2$$

Thus, we can estimate σ_t^2 using any of the Methods from 1.2, with $g(Y_i(t)) = \mathrm{E}\left[Y_i(t) - \mathrm{E}[Y_i(t)]\right]^2$. This would be a two-step estimator, since we would need to estimate $\mathrm{E}[Y_i(t)]$. To conduct the hypothesis test of $H_0 : \sigma_t^2 = \sigma^2\ \forall t \in \mathcal{T}$ we would need to use an appropriate joint hypothesis testing procedure. One way to proceed would be test $H_0 : \sigma_t^2 - \sigma^2 = 0\ \forall t \in \mathcal{T}$ and construct the vector $\widehat{\boldsymbol{\theta}} = (\hat{\sigma}_1^2 - \sigma^2, ..., \hat{\sigma}_T^2 - \sigma^2)'$, and then show $\sqrt{n}(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) \to \mathcal{N}(0, V)$. Then, the Delta method implies $\sqrt{n}(||\widehat{\boldsymbol{\theta}}||^2 - ||\boldsymbol{\theta}_0||^2) \to \mathcal{N}(0, 4\boldsymbol{\theta}_0' V \boldsymbol{\theta}_0)$. Note that under the null $\boldsymbol{\theta}_0 = 0$, so we can now conduct the hypothesis test $H_0 : \boldsymbol{\theta}_0 = 0$ in the usual way, using an estimator for the asymptotic variance.

1.4 Q1 Part 4

No Thanks

2 Question 2: Estimating Average Treatment Effects

A few things didn't run in R but it all went through in STATA. Results are below. I only did one table because making it is tedious but the code for both programs is in the appendix

ATE

statistic	specificaiton	estimate_exp	std.error_exp	CI_L	CI_U	estimate_PSID	std.error_PSID	CI_L	CI_U
Mean Diff		1794	670	479	3109	-15204	656	-16490	-13919
OLS	a	1582	659	291	2873	6302	1209	3932	8673
OLS	b	1507	657	219	2795	4699	1027	2686	6712
OLS	c	1501	663	202	2800	4284	1031	2263	6306
Reg. Impute	a	1462	630	228	2697	-11195	1741	-14608	-7782
Reg. Impute	b	1454	631	218	2690	-10398	3549	-17355	-3442
Reg. Impute	c	1428	642	170	2685	-11920	3498	-18776	-5065
IPW	a	1537	630	303	2772	-13507	2800	-18996	-8019
IPW	b	1470	631	234	2706	-7246	3550	-14204	-288
IPW	c	1468	642	210	2726	-7487	3499	-14344	-629
D. Robust	a	1473	630	239	2707	-13507	2800	-18996	-8019
D. Robust	b	1451	631	215	2687	-11419	3549	-18376	-4463
D. Robust	c	1423	642	166	2682	-12504	3498	-19360	-5649
N1 Match	a	1829	780	302	3358	-15619	1153	-17880	-13359
N1 Match	b	1876	735	435	3316	-9350	3975	-17140	-1559
N1 Match	c	1672	726	248	3095	-9560	4034	-17467	-1656
P Match	a	1542	646	275	2808	-15859	6750	-29089	-2629
P Match	b	1489	765	-12	2989	8646	15056	-20863	38156
P Match	c	1257	677	-70	2584	-9562	4034	-17468	1657

ATT

statistic	specificaiton	estimate_exp	std.error_exp	CI_L	CI_U	estimate_PSID	std.error_PSID	CI_L	CI_U
Mean Diff		1794	670	479	3109	-15204	656	-16490	-13919
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