Practical Approximation Approaches to Forecasting and Backtesting Initial Margin Requirements†

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Abstract

As BCBS-IOSCO's introduction of mandatory bilateral initial margin is expected to transform the bilateral OTC market, there has been a great interest in the market to develop a model that can dynamically forecast initial margin for future horizons. In the context of counterparty credit risk management and regulatory capital calculation, the authors propose practical methods to simulate initial margin requirements, along with validation and historical backtesting approaches applicable to a broad class of initial margin simulation methods.

Keywords: Approximation Model, BCBS-IOSCO, Counterparty Credit Risk, internal models method, Initial Margin Simulation, Margin Reform, Regulatory Capital, SIMM

1. Introduction

Since BCBS-IOSCO's introduction of bilateral initial margin, there has been a great interest in the industry for developing a model that can dynamically forecast initial margin for future time horizons ([1], [2], [3]). As the introduction of the additional margin requirement is essentially a mechanism to exchange counterparty credit risk with the short term collateral

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liquidity risk as well as the long term collateral funding cost, an appropriate initial margin simulation model is essential to sound management of credit risk management, capital calculation, as well as funding and liquidity risk management. In this paper, we provide the details of several practical approximation approaches to simulating initial margin, as well as possible validation and backtesting methods and comparison results, with a particular focus on computing counterparty credit risk and regulatory capital.

The paper is organized as follows: Section 2 introduces the background of the regulation. Section 3 provides the model specifications for simulating initial margin. Sections 4 and 5 present statistical testing and historical back-testing results for the proposed model by using realistic portfolios and historical market data. Finally, we draw conclusions on Section 6.

2. Background

2.1. Margin Reform and Requirements for Non-centrally Cleared Derivatives

As part of the commitment from the Group of 20 (G20) to stabilize and protect the financial system following the crisis in 2008, the Basel Committee on Banking Supervision (BCBS) and the International Organization of Securities Commissions (IOSCO) created the Working Group on Margining Requirements (WGMR) to establish global requirements for margining of non-cleared OTC derivatives.

In March 2015, the WGMR published a final policy framework for non-centrally cleared derivatives. The final framework imposes wide-ranging changes such as the universal exchange of variation margin (VM) and two-way initial margin (IM) [4] on a daily basis. Section 3.1 of the requirement stipulates that the initial margin should "reflect an extreme but plausible estimate of an increase in the value of the instrument that is consistent with a one-tailed 99 percent confidence interval over a 10-day horizon", and "the required amount of initial margin may be calculated by reference to either (i) a quantitative portfolio margin model or (ii) a standardized margin schedule". Many banks are expected to take the first approach to calculate the required initial margin amount, and a widely used industry-standard model is the Standard Initial Margin Model (SIMM) model proposed by ISDA. The focus of this paper is to forward simulate the margin requirement of (i), as the margin requirement of (ii) is specified by a lookup table and is straightforward.

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The target of ISDA SIMM model is to provide a common initial margin (IM) methodology that can be used by market participants globally. The model was developed by a work stream initiated by ISDA and major participating banks. For the sake of speed and lower implementation costs, the ISDA SIMM model is designed as a delta-gamma sensitivity-based approach, and the model parameters are to be re-calibrated no less than annually. The details of the ISDA SIMM model can be found in the methodology specification ([5], [6]), which is based on a variant of the sensitivity based approach (SBA) adopted by the BCBS for calculating capital requirements under the revised market risk framework ([7])(i.e., the Fundamental Review of the Trading Book).

2.2. Challenges to Forecasting Initial Margin in a Credit Exposure Framework

In a typical simulation-based counterparty credit exposure framework, both the aged portfolio value and the collateral value are simulated jointly until the portfolio maturity. This is typically done for either regulatory counterparty credit risk (CCR) capital calculation ([8]) or bank internal CCR management ([9]). Within these frameworks, the initial margin needs to be forward simulated. As the initial margin is not a constant value for a given portfolio, but will instead vary over time and depend on market movements and trades aging, banks need a model that can forecast it dynamically over future time horizons within the credit exposure calculation framework.

If a bank chooses to use the ISDA SIMM model to calculate initial margin, a natural option would be to brute force replicate the ISDA SIMM model at each simulation horizon and scenario. However, this solution has a serious practical challenge: it requires calculating sensitivities for a large number of risk factors at each horizon and scenario, as sensitivities are the key input to the ISDA SIMM model. While calculating large amount of sensitivities for each horizon and scenario is not unattainable[1], getting around the computational performance challenge is not trivial for many production implementations. Instead, an approximation method can be quite attractive. In this document, we describe and compare approximation methods that can

[1]Possible approaches to generating large amount of sensitivities efficiently include Adjoint Algorithmic Differentiation (AAD) or adopting GPU technology ([10]). However, these approaches are not always immediately available in many production systems.

be used in a production CCR framework, as well as possible validation approaches.

3. The Simulation Model

In this section we describe three possible approximation methods to forward simulating initial margins, viewed essentially as a problem of estimating forward portfolio Value-at-Risk(VaR). Section 3.1 introduces the general notation and approach of all three methods. Section 3.2 describes a simple approach that estimates portfolio VaR from simulated portfolio changes. Section 3.3 describes a non-parametric regression approach that estimates the conditional portfolio distribution via a Nadaraya-Watson kernel. Section 3.4 describes a parametric regression approach that estimates the conditional portfolio distribution via least-squares. Section 3.5 provides further adjustments [2].

3.1. General Framework

Given the portfolio with respect to a counterparty, let $V(t)$ be its value as of time t from a bank's perspective. In a practical Monte Carlo simulation for computing counterparty credit risk, we assume that we have forward simulated portfolio values for a discrete set of scenario paths. More specifically, let $V_j(t_i)$ be the simulated portfolio value on simulation horizon t_i and scenario path j, where $i = 1, ..., H$ is the horizon index and $j = 1, ..., N$ is the scenario index. Note by definition $V_j(t_i)$ excludes the value of cash payments on t_i. Further, let $C_j(t_i, t_{i+1})$ be the net cash of the portfolio over time span $(t_i, t_{i+1}]$ on the jth scenario. We define the cash-adjusted portfolio profit-and-loss (PnL) $\Delta\bar{V}_j(t_i)$ as

$$\Delta\bar{V}_j(t_i) \equiv V_j(t_i + d) - V_j(t_i) + C_j(t_i, t_i + d), \tag{1}$$

where d =10-business days is the initial margin horizon. The general approach is to estimate the quantile on each scenario j and horizon t_i, $\text{Quantile}_j^{(q)}(t_i)$, of the portfolio PnL distribution during $[t_i, t_i + d]$ from $V_j(t_i)$ and $\Delta\bar{V}_j(t_i)$

[2]All methods discussed in this paper aim to approximate initial margins by using current portfolio present value. However, this might not be suitable for all portfolio composition styles. For example, for a portfolio dominated by path-dependent products, risks can be quite different for two path-dependent transactions/states even though the current PV might be the same or close.

conditional on realized events on $t \leq t_i$. Once obtained, the intent is to approximate initial margin as a 99%, 10-day VaR from the collateral poster's perspective. More specifically,

$$\mathrm{IM}_j^{\mathrm{received}}(t_i) = \mathrm{Quantile}_j^{(99\%)}(t_i) \tag{2}$$

is the *unadjusted* received initial margin, and

$$\mathrm{IM}_j^{\mathrm{posted}}(t_i) = -\mathrm{Quantile}_j^{(1\%)}(t_i). \tag{3}$$

is the *unadjusted* posted initial margin[3]. Further adjustments, such as reconciling with the collateral opening balance, and application of a conservative modelling haircut, are discussed in Section 3.5.

The Simple VaR method in Section 3.2 directly estimates $\mathrm{Quantile}_j^{(q)}(t_i)$, while both non-parametric and parametric regression methods in Sections 3.3 and 3.4 estimate the quantiles by first estimating the conditional portfolio moments. More specifically, we look to estimate the kth moment $M_{ij}^{(k)}$ of the cash-adjusted portfolio PnL on the ith horizon and the jth scenario conditional on information at time t_i along the jth scenario path as

$$M_{ij}^{(k)} \equiv \mathbb{E}\left[\left(\Delta \bar{V}_j(t_i)\right)^k\right], \tag{4}$$

where the expectation operator $\mathbb{E}[.]$ is taken conditional on events realized at time $t \leq t_i$. Once we can obtain $M_{ij}^{(k)}$, we may infer $\mathrm{Quantile}_j^{(q)}(t_i)$ from these conditional moments. For example, conditional on jth simulation path at time t_i, assuming that the portfolio distribution is locally normal-distributed, we find that

$$\mathrm{Quantile}_j^{(q)}(t_i) = M_{ij}^{(1)} + \sigma_j(t_i)\Phi^{-1}(q), \tag{5}$$

$$\sigma_j(t_i) = \sqrt{M_{ij}^{(2)} - \left(M_{ij}^{(1)}\right)^2}, \tag{6}$$

where $\Phi(.)$ is the cumulative distribution function of the normal distribution. This *local normality* assumption may be relaxed; a possible extension is described in Section 3.6.

[3]Note that both $V_j(t_i)$ and $\Delta\bar{V}_j(t_i)$ are defined from a bank's perspective with respect to a given counterparty, and $\mathrm{Quantile}_j^{(99\%)}(t_i)$ and $\mathrm{Quantile}_j^{(1\%)}(t_i)$ are both estimated based on $V_j(t_i)$ and $\Delta\bar{V}_j(t_i)$. The received IM is estimated from $\mathrm{Quantile}_j^{(99\%)}(t_i)$, which typically represents an extreme but plausible *profit* scenario from a bank's perspective, and the posted IM from $\mathrm{Quantile}_j^{(1\%)}(t_i)$, which typically represents an extreme but plausible *loss* scenario from a bank's perspctive.

3.2. Simple VaR

The simple VaR approach directly estimates the initial margin by using the sample quantile of $\Delta\bar{V}_j(t_i)$. More specifically, for each horizon i, sort the cash-adjusted portfolio PnL $\Delta\bar{V}_j(t_i)$ in ascending order. This implies there exists an index mapping $J(j)$ such that $\Delta\bar{V}_{J(1)}(t_i) \leq \Delta\bar{V}_{J(2)}(t_i) \leq ... \leq \Delta\bar{V}_{J(N)}(t_i)$. The unadjusted received initial margin is then given as[4]

$$\mathrm{IM}_j^{\mathrm{received}}(t_i) = \mathrm{Quantile}_j^{(99\%)}(t_i) = \Delta\bar{V}_{J((99\%)N)}(t_i). \tag{7}$$

The unadjusted posted initial margin can be obtained analogously. Note that under this method, given a simulation horizon t_i, simulated initial margin is the same across all scenarios.

3.3. Non-parametric Regression and the Nadaraya-Watson Kernel

Another possible approximation method is via a non-parametric regression. Assuming that the portfolio value is a good explanatory factor for the state of the portfolio dynamics, to estimate the conditional moments in Equation (4) we attempt to draw a relationship between the moments of portfolio PnL against portfolio value via a Nadaraya-Watson kernel. The use of the Nadaraya-Watson kernel for simulating initial margin is proposed in [2]. In this paper, we presented additional details, as well as validation results.

3.3.1. The Nadaraya-Watson Kernel

Given a kernel bandwidth h_i for a given horizon i, the moments are estimated as[5]

$$M_{ij}^{(k)}(X_{ij}) = \frac{1}{W_{ij}} \sum_{j'=1}^{N} \mathcal{K}\left(\bar{V}_j(t_i), \bar{V}_{j'}(t_i), h_{ij}\right) \left(\Delta\bar{V}_j(t_i)\right)^k, \tag{8}$$

[4]Here for simplicity we assume $(99\%)N$ is an integer, and we simply use the sample quantile. In general quantiles can be calculated using estimation methods such as R4, R5 or R7[11], or Harrell and Davis[12].

[5]The use of cash-adjusted PnL, $\Delta\bar{V}_j(t_i)$, is to exclude the effect of deterministic changes in portfolio value due to cash payment events, which should not factor into VaR estimation. Additionally, as the cash payment $C_j(t_i, t_i+d)$ during the initial margin horizon is removed from portfolio PnL to arrive at $\Delta\bar{V}_j(t_i)$, the regressor $\bar{V}_j(t_i)$ also has the same cash amount removed from portfolio value $V_j(t_i)$, as the cash amount $C_j(t_i, t_i+d)$ should not be a part of the explanatory factor when estimating portfolio VaR within our framework. A similar construct is used in Section 3.4.

where $\mathcal{K}(x, y, h)$ is some regression kernel function, and

$$W_{ij} = \sum_{j'=1}^{N} \mathcal{K}\left(\bar{V}_j(t_i), \bar{V}_{j'}(t_i), h_{ij}\right) \tag{9}$$

is the normalization factor, and

$$\bar{V}_j(t_i) \equiv V_j(t_i) - C_j(t_i, t_i + d) \tag{10}$$

is the portfolio value adjusted for cash during the initial margin horizon.

The kernel function $\mathcal{K}(x, y, h)$ can be written as $\frac{1}{h} K((x-y)/h)$, where $K(u)$ is a standardized kernel that satisfies the following relations,

$$K(-u) = K(u), \tag{11}$$

$$\int_{-\infty}^{\infty} K(u)\, du = 1. \tag{12}$$

Many types of kernels can be used. Examples are the Gaussian Kernel,

$$K_{\text{Gaussian}}(u) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2} u^2\right], \tag{13}$$

or the Epanechnikov kernel.

$$K_{\text{Epanechnikov}}(u) = \frac{3}{4}\left(1 - u^2\right) \mathbf{1}_{\{|u| \leq 1\}}. \tag{14}$$

To use a regression kernel, a bandwidth h must be selected. Choosing the optimal bandwidth can be an art. In practice, we apply Silverman's Rule of Thumb[13, 14]. Given a horizon index i, define the unconditional standard deviation Q_i of the portfolio value as

$$Q_i = \left\{ \sum_{j=1}^{N} \left(\bar{V}_j(t_i)\right)^2 - \left(\sum_{j=1}^{N} \bar{V}_j(t_i) \right)^2 \right\}^{1/2}. \tag{15}$$

Then, we choose the kernel bandwidth as[6]

$$h_i = \mathbf{C} Q_i N^{-1/(2\Omega+1)} \tag{16}$$

where N is the number of scenarios. Ω is the order of the regression kernel used. More specifically, since both Gaussian or Epanechnikov kernels are examples of order-2 kernels [13], $\Omega = 2$. We set the value of the constant $\mathbf{C}$ to be 2.34.

3.3.2. Performance and Thinning

With a Nadaraya-Watson kernel, in principle we need to apply Equation (8) for each simulation horizon and scenario, and this many regression computations can present a performance challenge, especially when banks supporting derivatives trading and intra-day risk management may require collateral simulation to be completed on a near-real time basis, such as for supporting incremental credit exposure calculation. We present a simple approach where we only apply Equation (8) to a subset of scenarios for each simulation horizon.

Choose an integer M such that $M < N$. The thinning procedure is then:

1. Similar to Section 3.2, we sort the portfolio value $\bar{V}_j(t_i)$ in ascending order so that there exists an index mapping $J(j)$ such that $\bar{V}_{J(1)} < \bar{V}_{J(2)} < ... < \bar{V}_{J(N)}$.
2. Let $G \equiv \lceil \frac{N}{M} \rceil$, where $\lceil . \rceil$ is the ceiling function.
3. Perform Equation (8) for scenarios with indices j such that

$$j \in \{J(mG)\ \forall\ mG < N, m = 1, 2, 3, ...\} \cup \{J(1), J(N)\}. \tag{17}$$

4. Based on Step 3, we can create a linearly interpolated curve with $(M+1)$ grid points, whose abscissa is the portfolio value $\bar{V}_j(t_i)$ and ordinate the simulated initial margin value $\mathrm{IM}_j(t_i)$.

[6]In general, selecting a bandwidth for conditional nonparametric regression can be tricky, and statistical techniques such as cross validation [13] are unlikely to be fast enough in our context, where we have to perform regression for large number of horizons and scenarios. Silverman's Rule of Thumb is sufficiently fast in our context. When applying Silverman's Rule, it is possible to use the conditional standard deviation estimated at the previous time step t_{i-1} instead of the unconditional standard deviation Q_i. However, this introduces additional path dependency and in certain circumstances can appear noisy. For simplicity, we opt to use the unconditional standard deviation instead, which appears to be more robust.

5. For the remaining $(N - M - 1)$ scenarios, instead of performing full regression analysis to obtain the simulated initial margin value, the initial margin value will be interpolated off the curve created in Step 4.

Empirically, it appears that it is typically sufficient to choose M such that M/N is around 10% to 20%. More sophisticated scenario selection and interpolation schemes are possible, but this simple scheme appears to be sufficient based on empirical results. Comparison results can be found in Section 4.2.

3.4. Parametric Regression and Least-Squares Regression

Inspired by American Monte Carlo approaches ([15]), another approach to estimating the conditional moments in Equation (4) is via least-squared regression. Similar ideas are also proposed in [1] and [3]. More precisely, assume we are able to write

$$M_{ij}^{(k)}(X_{ij}) = \mathbb{E}\left[\left(\Delta \bar{V}_j(t_i)\right)^k\right] = \sum_{l=1}^{L} \beta_{i,l}^{(k)} \psi_l^{(k)}(X_{ij}), \tag{18}$$

where X_{ij} is the risk factor state on the ith horizon and the jth scenario. $\psi_l^{(k)}(X_{ij})$, $l = 1, ..., L$ is a set of basis functions that needs to be chosen such that a suitable linear combination of these basis functions well describes the relationship between the portfolio value and its future PnL. Note that while in general the set of basis functions need not be the same for each moment, for simplicity we choose the same set for each moment k. For comparison results shown in this paper, we choose the basis set to be the monomials of the cash-adjusted portfolio value as of time t_i,

$$\psi_l^{(k)}(X_{ij}) = \left(\bar{V}_j(t_i)\right)^l, \tag{19}$$

where $\bar{V}_j(t_i)$ is the cash-adjusted portfolio value defined in Equation (10). The positive integer L is the number of basis functions. The basis function coefficients $\beta_{i,l}^{(k)}$ are to be calibrated via least-squares. Once calibrated, given the portfolio values, we can determine the moments of the portfolio distribution.

Given a particular basis index l, multiplying both sides of Equation (18) of by $\psi_l^{(k)}(X_{ij})$ and taking the conditional expectation yields

$$\mathbb{E}\left[\psi_l^{(k)}(X_{ij})\left(\Delta \bar{V}_j(t_i)\right)^k\right] = \sum_{l'=1}^{L} \beta_{i,l'}^{(k)} \mathbb{E}\left[\psi_l^{(k)}(X_{ij})\psi_{l'}^{(k)}(X_{ij})\right]. \tag{20}$$

Alternately, for convenience define

$$\alpha_{i,l}^{(k)} \equiv \mathbb{E}\left[\psi_l^{(k)}(X_{ij})\left(\Delta\bar{V}_j(t_i)\right)^k\right], \tag{21}$$

$$B_{i,ll'}^{(k)} \equiv \mathbb{E}\left[\psi_l^{(k)}(X_{ij})\psi_{l'}^{(k)}(X_{ij})|\mathcal{F}_j(t_i)\right]. \tag{22}$$

Further, given a particular horizon index i and moment index k, we can represent $\alpha_{i,l}^{(k)}$ and $\beta_{i,l}^{(k)}$ as elements of vectors of length L, labelled as boldfaced $\boldsymbol{\alpha}_i^{(k)}$ and $\boldsymbol{\beta}_i^{(k)}$ respectively. Likewise, we can represent $B_{i,ll'}^{(k)}$ as a matrix of size $L \times L$, labelled as boldfaced $\boldsymbol{B}_i^{(k)}$. In this matrix notation, Equation (20) becomes

$$\boldsymbol{\alpha}_i^{(k)} = \boldsymbol{B}_i^{(k)} \cdot \boldsymbol{\beta}_i^{(k)}. \tag{23}$$

With $j = 1, ..., N$ realized Monte Carlo scenarios, we can estimate elements of $\boldsymbol{\alpha}_i^{(k)}$ and $\boldsymbol{B}_i^{(k)}$ as

$$\alpha_{i,l}^{(k)} = \frac{1}{N}\sum_{j=1}^{N}\psi_l^{(k)}(X_{ij})\left(\Delta\bar{V}_j(t_i)\right)^k, \tag{24}$$

$$B_{i,ll'}^{(k)} = \frac{1}{N}\sum_{j=1}^{N}\psi_{l'}^{(k)}(X_{ij})\psi_l^{(k)}(X_{ij}). \tag{25}$$

Then, after obtaining all elements of $\boldsymbol{\alpha}_i^{(k)}$ and $\boldsymbol{B}_i^{(k)}$, the basis function coefficients $\boldsymbol{\beta}_i^{(k)}$ may be estimated by solving the matrix Equation (23). Repeat this exercise for every horizon i, and we may obtain an estimate for the moments $M_{ij}^{(k)}$ for each jth scenario. Note that least-squares regression is performed once across all scenarios for each horizon.

3.5. Opening Balance and Haircut

The initial margin at $t = 0$ requires special care. First, for both non-parametric and parametric approaches from Sections 3.3 and 3.4, we exclude the cash payment during the initial margin horizon. However, as cash payment amount $C_j(0, d)$ during the first initial margin horizon is in general scenario dependent, a consequence of this feature is that a verbatim implementation of either Sections 3.3 or 3.4 unrealistically implies that the unadjusted initial margin values $\mathrm{IM}_j(0)$ at $t = 0$ are scenario-dependent. Instead, for $t = 0$, Simple VaR method from Section 3.2 is used.

Next, as the initial margin is typically calculated according to the ISDA SIMM model rather than Simple VaR method, the difference between the opening balance of initial margin, denoted IM_0 and the $t = 0$ unadjusted initial margin value calculated with Simple VaR method, $\mathrm{IM}_j(0)$, can be material. We adjust the simulated initial margin to match the opening balance. The opening balance-adjusted initial margin $\widetilde{\mathrm{IM}}_j(t_i)$, $t_i > 0$, is given as

$$\widetilde{\mathrm{IM}}_j(t_i) = \left(\frac{\mathrm{IM}_0}{\mathrm{IM}_j(t_0)}\right)\mathrm{IM}_j(t_i). \tag{26}$$

More sophisticated fitting of the initial margin profile has been proposed (e.g., in [1] and [3]), but the multiplier is adopted for simplicity. Further, as there can be a significant amount of model risk in the approximation methods, in the context of credit risk management and regulatory capital calculation, we introduce an optional model haircut $\mathcal{H}$ to the simulated initial margin,

$$\widehat{\mathrm{IM}}_j(t) = \begin{cases} \mathrm{IM}_0, & t = 0 \\ (1-\mathcal{H})\,\widetilde{\mathrm{IM}}_j(t) & t > 0. \end{cases} \tag{27}$$

In general, under such a context, underestimating the amount of initial margin leads to a more conservative estimate of credit exposure. Such an assumption may not be as suitable in other contexts, such as managing collateral funding or liquidity risks.

3.6. Cornish-Fisher Expansion

It is possible to extend beyond the local normality assumption in Equation (5). For example, by estimating the first four moments $M_{ij}^{(1)}$, $M_{ij}^{(2)}$, $M_{ij}^{(3)}$ and $M_{ij}^{(4)}$, as well as the conditional skewness $S_j(t_i, t_{i+1})$ and excess kurtosis $K_j(t_i, t_{i+1})$

$$S_j(t_i, t_{i+1}) = \frac{1}{\sigma_j^3(t_i, t_{i+1})}\left(M_{ij}^{(3)} - 3M_{ij}^{(1)}\sigma_j^2(t_i, t_{i+1}) - \left(M_{ij}^{(1)}\right)^3\right), \tag{28}$$

$$K_j(t_i, t_{i+1}) = \frac{1}{\sigma_j^4(t_i, t_{i+1})}\times \left(M_{ij}^{(4)} - 4M_{ij}^{(1)}M_{ij}^{(3)} + 6\left(M_{ij}^{(1)}\right)^2\sigma_j^2(t_i, t_{i+1}) + 3\left(M_{ij}^{(1)}\right)^4\right) - 3, \tag{29}$$

we may utilize the Cornish-Fisher expansion ([16]) to estimate the portfolio PnL quantile

$$\text{Quantile}_j^{(q)}(t_i, t_{i+1}) = M_{ij}^{(1)} + \sigma_j(t_i, t_{i+1}) w_{ij}(z), \tag{30}$$

where $z = \Phi^{-1}(q)$, and

$$w_{ij}(z) = z + [S_j(t_i, t_{i+1}) h_1(z)] + \left[K_j(t_i, t_{i+1}) h_2(z) + S_j^2(t_i, t_{i+1}) h_{11}(z)\right], \tag{31}$$

$$h_1(z) = \frac{1}{6}\left(z^2 - 1\right), \tag{32}$$

$$h_2(z) = \frac{1}{24}\left(z^3 - 3z\right), \tag{33}$$

$$h_{11}(z) = -\frac{1}{36}\left(2z^3 - 5z\right). \tag{34}$$

As the local normality assumption typically underestimates the portfolio VaR and initial margin, under the context of credit risk management and regulatory capital calculation, we continue to use the local normality assumption. However, the Cornish-Fisher expansion may be more suitable in other contexts, such as funding and liquidity management.

3.7. Testing Approach

In the following sections, we will discuss how to test the proposed models presented in this section. There are two categories of testing approaches proposed in this paper. The first is to assess the appropriateness of its in-simulation consistency. Essentially, our approximation models forecasting future IM values by attempting to estimate forward VaR values from simulated PnLs and simulated portfolio values. This is discussed in Section 4.3. The second is to assess the appropriateness of our simulated IM distribution with respect to its historical distribution. This historical backtesting assessment is elaborated in Section 5.

4. Exception Counting and Validation Results

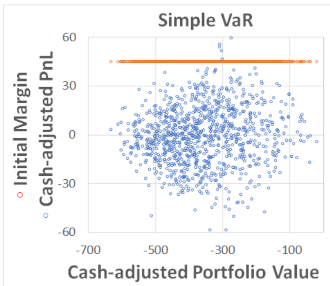
This section discusses possible approaches to assess the appropriateness of the in-simulation self-consistency of an initial simulation model. Comparing simulated distributions with respect to historical patterns will be left to Section 5. Section 4.1 outlines the benchmark portfolio we use to perform our tests. Section 4.2 explores the relationship between simulated portfolio value and its regression-based initial margin (or VaR). Section 4.3 describes our exception counting schemes and results.

4.1. The Benchmark Portfolio

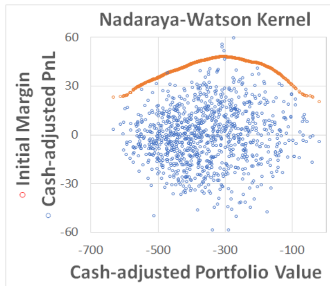
All tests presented below are based on a benchmark portfolio derived from a counterparty subject to IM regulation, which contains over 100 trades, most of which are interest rate swaps and interest rate swaptions, as well as some cross currency swaps. The currencies are denominated mainly by EUR,USD and GBP. The remaining maturities of the portfolio are well distributed between 1-year and 20-year horizons; the average remaining maturity of the portfolio is 5 Years. It is assumed in the tests that the portfolio will keep the same remaining maturity and the same moneyness over the test period. Note that many interesting single-trades results and regression analysis are available in [3]. In this paper, our contribution is to present possible self-consistency tests and historical backtesting approaches of a given benchmark portfolio that are largely agnostic towards the specific initial margin simulation methodology used, as well as to present example results for the initial margin simulation methodologies discussed.

4.2. Relationship Between Portfolio Value and VaR

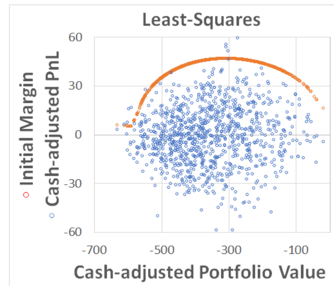
For any of the simulation methods proposed in Section 3, the objective is to infer the scenario-wise initial margin (or VaR) from the simulated cash-adjusted portfolio value and simulated PnLs. Hence, one preliminary test is to examine the relationship between the simulated unadjusted initial margin and the cash-adjusted portfolio value and PnLs. Figure 1 shows the comparison between the simulated unadjusted initial margin against the scatter plot of the cash-adjusted PV vs PnL at the one year simulation horizon for the benchmark portfolio, across the three methods proposed in Section 3. Intuitively, we expect the unadjusted initial margin to act as an envelope to the cash-adjusted portfolio value vs PnL scatter plot, such that only a small number of the simulated cash-adjusted PnLs exceed it. By construction, the unadjusted initial margin produced by Simple VaR method of Section 3.2 is invariant with respect to the simulated cash-adjusted portfolio value. Both the Nadaraya-Watson kernel method of Section 3.3 and least-squares parametric approach of Section 3.4 appear to perform respectably, but as least-squares approach is constrained by the parametric form imposed, it appears to be less flexible. In particular, as we impose a quadratic form for the variance of the portfolio cash-adjusted PnL, the simulated initial margin has a functional form of a square root of a quadratic function of the cash-adjusted portfolio value. For the non-parametric Nadaraya-Watson kernel, there is no particular a priori imposed functional form, and the the value of the simulated initial margin of a given cash-adjusted portfolio value depends on the simulated cash-adjusted PnLs of nearby cash-adjusted portfolio values.



(a) Simple VaR



(b) Nadaraya-Watson



(c) Least-Squares

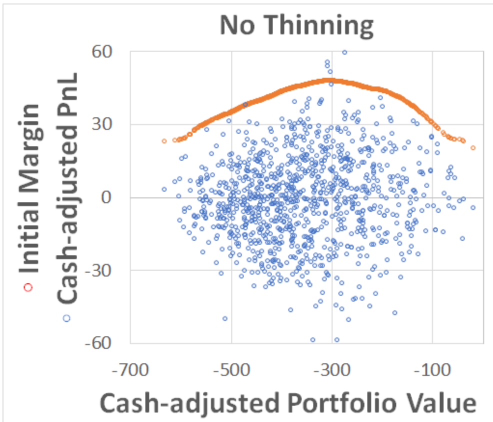
Figure 1: A comparison of scatter plots of portfolio value vs simulated initial margin and PnL for initial margin models. The simulation horizon is 1-yr, with 1000 simulated scenarios. All figures in millions.

As discussed in Section 3.3, a challenge of employing the non-parametric kernel approach is that performance can be a concern. In such instances, thinning provides a practical approach to speed up the calculation. A good demonstration of the comparison of accuracy with the introduction of thinning can also be seen from the scatter plots. Figure 2 demonstrates this. In the figure, we compare the results where we perform kernel regression for every scenario, against the results where we perform kernel regression for only 50 out of 1000 scenarios. We see that the results change very little in this example.

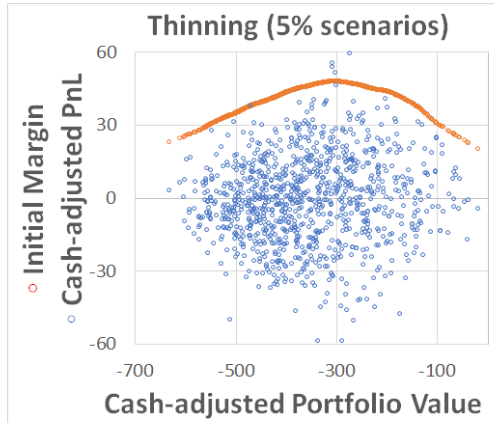
4.3. Exception Counting Schemes

Inspired by market risk management practices [17], one of the simplest ways of validating an initial margin model, focused specifically on the unadjusted initial margins defined in Equations (2) and (3), is via *exception counting*. The scheme described below focuses specifically on received initial margin, although posted initial margin can be validated analogously. Concretely, an *exception* occurs when $I_{ij} = 1$, where

$$I_{ij} = \begin{cases} 1, & \Delta\bar{V}_j(t_i) > \text{Quantile}_j^{(99\%)}(t_i) \\ 0, & \Delta\bar{V}_j(t_i) \le \text{Quantile}_j^{(99\%)}(t_i). \end{cases} \tag{35}$$



(a) Nadaraya-Watson Kernel, without thinning



(b) Nadaraya-Watson Kernel, using only 5% of scenarios

Figure 2: A comparison between the simulation results via the Nadaraya-Watson kernel, with or without thinning. The left graph shows the result where we calculate the unadjusted initial margin for each of the 1000 scenarios. The right only for 50 of the 1000 scenarios, with the rest by interpolation.

If the Monte Carlo simulation and the initial margin simulation are self-consistent, then the statistics of I_{ij} should have certain patterns. For simplicity, we assume that the initial margin horizons do not overlap, or that $t_i + d \leq t_{i+1} \, \forall \, i = 1, ..., H-1$. There are at least two possible testing schemes. The first is to count exception *across scenarios*, where for any horizon index i, the realized average exception rate r_i should be roughly

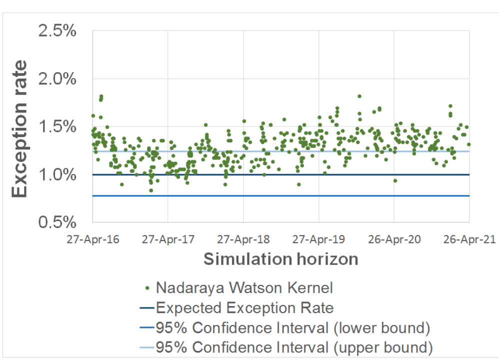
$$r_i \equiv \frac{1}{N} \sum_{j=1}^{N} I_{ij} \approx 1\%. \tag{36}$$

Further, with confidence level p, the average exception rate r_i should fall within the confidence interval

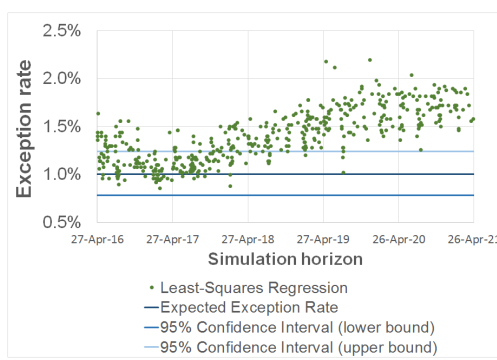
$$\left[\frac{B^{-1}(N, 1\%, 1-p)}{N}, \frac{B^{-1}(N, 1\%, p)}{N} \right], \tag{37}$$

where $B^{-1}(N, q, x)$ is the inverse cumulative distribution function of the binomial distribution with N trials, success probability q for each trial, at quantile level x. In words, observing the exception rate of a given horizon across scenarios not only give an indication of the self-consistency of the simulated initial margin (or VaR) at the specific horizon, but tracking the exception rates across simulation horizons also provides insights into the stability and reasonableness of initial margin simulation over longer simulation horizons.

Figure 3 demonstrates the results of simulated exception rates of our benchmark portfolio, using both the Nadaraya-Watson non-parametric kernel of Section 3.3 and the least-squares parametric regression of Section 3.4. The Monte-Carlo simulation runs for $N = 5000$ scenarios for 5 years, and we compare the simulated exception rates against the expected exception rate of 1%, as well as the 95% confidence interval. While the Nadaraya-Watson kernel appears to be more stable over time, both methods display the same tendency that the exception rate is slightly higher than ideally expected. One source for this tendency is due to the local normality assumption, which typically leads to an underestimation of the initial margin. The local normality assumption may be improved, for example, by using the Cornish-Fisher expansion discussed in Section 3.6 instead. A comparison between the local normality assumption and the Cornish-Fisher expansion, using the Nadaraya-Watson kernel, is illustrated in Figure 4, where the Cornish-Fisher expansion exhibits a much smaller bias towards underestimation of initial margin. However, in the context of calculating credit exposure, an underestimation of initial margin can generally be tolerated or even desired as it leads to a conservative exposure and capital estimate. Note that, by comparison, Simple VaR method of Section 3.2 is a special case where by construction, the exception rate is always 1% for all horizons.

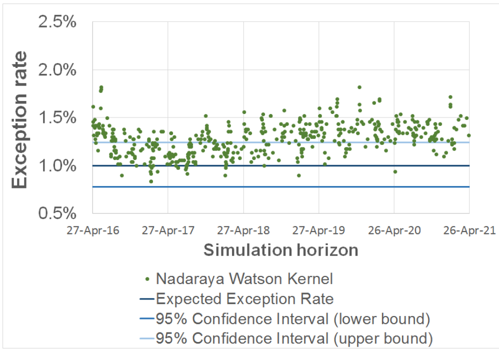


(a) Nadaraya-Watson Kernel

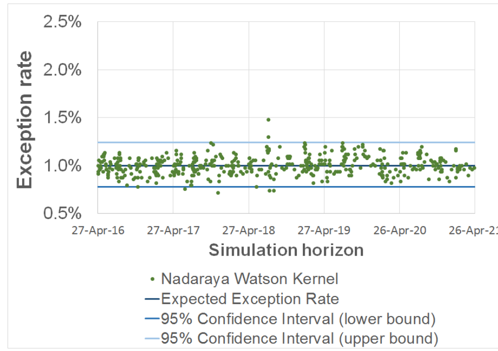


(b) Least-Squares Regression

Figure 3: A comparison of exception rate across scenarios for initial margin models



(a) Local Normality



(b) Cornish-Fisher Expansion

Figure 4: A comparison of exception rate across scenarios, where the portfolio distribution is either obtained from the first two moments and via the local normality assumption (left), or from the first four moments and via the Cornish-Fisher expansion (right). In both examples, the moments are estimated via the Nadaraya-Watson kernel.

Another possible exception counting scheme is to count exceptions *through time*. We first collect the exception count E_j for scenario j through all horizons

$$E_j = \sum_{i=1}^{H} I_{ij}. \tag{38}$$

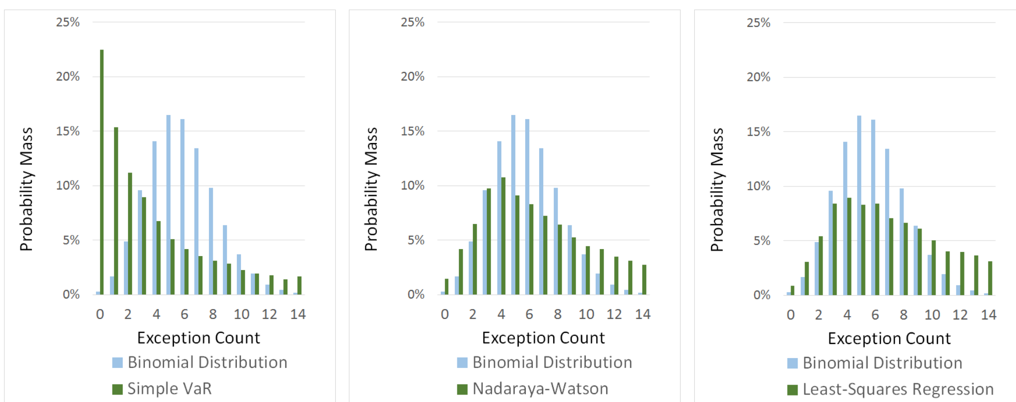
Assuming that exceptions occur randomly at 1% rate, with an ideal simulation method, E_j should be distributed according to a binomial distribution

$$\mathbb{P}\left(E_j = n\right) = b\left(H, 1\%, n\right), \tag{39}$$

where $b(H, q, n)$ is the probability mass function for a binomial distribution with H trials, success probability q for each trial, at n successes. With N total Monte Carlo scenarios, we can compare the simulated histogram of E_j and compare it with the probability mass function of the binomial distribution. In words, while counting exceptions across scenarios provides an indication for the self-consistency of the IM simulation on a given horizon, counting exceptions through simulation time provides further insights into the statistical reasonableness and self-consistency of the initial margin simulation along a set of given Monte Carlo paths.

Figure 5 compares the simulated exception counts E_j for the benchmark portfolio for the three different approaches outlined in Sections 3.2-3.4. The

Monte Carlo simulation runs for $N = 5000$ scenarios and $H = 585$ non-overlapping two-week periods. The simulated histograms of exception counts are compared against the ideal binomial distribution. For Simple VaR, the test results show that along many Monte Carlo paths, lower-than-expected exception counts are observed. Along these Monte Carlo paths, initial margin may be overestimated and credit exposure may be underestimated. While this does not directly indicate that expected exposure itself is underestimated, as expected exposure depends on the compensatory nature of Monte Carlo paths that are both overestimated and underestimated, this is nonetheless less than ideal. In contrast, the behaviors of the Nadaraya-Watson non-parametric kernel regression and least-squares parametric regression are more reasonable, with the Nadaraya-Watson kernel behaving slightly better in the example. Both these methods exhibit a tendency to producing higher-than-expected exception counts, which in the context of credit exposure calculation leads to a conservative exposure estimate.



(a) Simple VaR (b) Nadaraya-Watson (c) Least-Squares

Figure 5: A comparison of exception counts through time for initial margin models

These exception counting schemes, while simple, can be adopted by many forward-IM or forward-VaR simulation schemes. Further, as the schemes only involve intermediate simulation results required for the initial margin simulation methods discussed in Section 3, such as cash-adjusted portfolio PnLs, they add little to overall simulation effort.

5. Historical Backtesting of Initial Margin Simulation Models

In Section 4, we have seen some evidence of in-simulation consistency for the IM distribution as defined by the regression model. Exception counting schemes have proven useful for that purpose. In this section, we estimate how our approximation model compares against historical backtesting. Section 5.1 provides further background and motivation to our backtesting approach. Section 5.2 provides some details about the data prepared for this testing. Section 5.3 compares the historical spot initial margin value as calculated by the ISDA SIMM model against our approximation model. Section 5.4 outlines a backtesting procedure that follows the methodology outlined in [18, 19]. We present the backtesting results of the IM simulation model itself in Section 5.5, and the backtesting result of the portfolio value collateralized with both initial and variational margins in Section 5.6.

5.1. Background and Motivation

As pointed out by [1], depending on the context and the purpose of model usage, it may or may not be important to have an accurate assessment of the distribution of the simulated IM values against historically realized values. However, under the context of counterparty credit risk management and calculation of regulatory capital, it is often deemed best practice to be able to demonstrate appropriate historical backtesting results ([19, 20]), either internally or externally to the regulators ([21]). To achieve this, a historical backtesting procedure is introduced to assess the performance of the proposed model.

Regarding the target of backtesting against which we assess the appropriateness of our IM model, there are at least two viable options: the historical ISDA SIMM values, or the historical 10-day 99% stressed VaR values, which is the confidence level specified by the regulatory principle ([4]). Here we choose to backtest against the ISDA SIMM values, since in practice the market standard is to calculate the collateral amount with respect to the ISDA SIMM model. Also note that the ISDA working group is establishing a backtesting process ([22]) for the ISDA SIMM model which should track the performance of that model against the 10 day 99% stressed VaR.

Further, for illustrative purposes, the backtesting results are done with respect to the Nadaraya-Watson kernel approach of Section 3.3. However, the backtesting approach described here should apply to a broad class of initial margin simulation models. Further, the backtesting results are generated with the model haircut $\mathcal{H}$ introduced in Section 3.5 set to zero. Within the context of credit risk management and regulatory capital calculation, with the historical backtesting results as a guide, the model haircut $\mathcal{H}$ can be appropriately set to achieve the desired level of conservatism ([1]).

5.2. Data Preparation and Building a Historical SIMM Series

To support historical backtesting, the first task is to prepare and define the historical data to be used. This broadly includes three aspects:

- historical market data
- historical portfolios
- historical initial margin values

For the backtesting study, we select a five-year historical backtesting window from 2011 to 2016. We prepare two Scenarios of backtesting data: the Real Market Scenario and a Hypothetical Market Scenario. We describe their preparation approach and rationales below.

5.2.1. Real Market Scenario

This Scenario uses actual historically-realized market data. However, what this market data contains needs further specification. As we need to assess the quality of forecasted IM values using our approximation models against historical IM values, the historical data not only need to include historical market rates (e.g., interest rates, FX rates, etc), but also historical model parameters for the simulation models of the market rates. We use the historical model parameters calibrated to the stress period during the backtesting window for this study.

In addition to historical market data, we also need to prescribe a series of historical portfolios for the backtesting window. We use the benchmark portfolio described in Section 4.1, and with its trade maturities and moneyness adjusted throughout the backtesting window such that the average portfolio maturities and moneyness remain constant through the backtesting window.

The last element of the data preparation is the historical IM values. This presents a practical challenge, as bilateral IM does not come into effect until September 2016 at the earliest. Instead, we take the historical market data and historical benchmark portfolios described above, and compute a series of *would-be* historical portfolio sensitivities and historical IM values via the ISDA SIMM model through the backtesting window.

5.2.2. Hypothetical Market Scenario

Broadly speaking, a successful IM simulation consistent with historical backtesting requires three elements:

1. A set of forecasting models and their model parameters for the market rates (e.g., interest rates, FX rates, etc) consistent with historical backtesting.
2. A set of trade valuation models consistent with the calculation of the spot IM calculation model (e.g., valuation models used to generate portfolio sensitivities for the ISDA SIMM model).
3. Given a set of market rates paths (historically realized or simulated), an IM forecasting model consistent with the behavior of the spot IM calculation model (e.g., the ISDA SIMM model).

Within our simulation and testing framework, we use same set of valuation models for both forecasting initial margin via approximations, as well as generating portfolio sensitivities for the spot ISDA SIMM calculation and preparation of historical IM values, so element 2 above should not be a significant contributor to our backtesting result. The focus of Section 3 is to define a suitable model for element 3. However, even with a suitable IM forecasting model, the simulated IM values can still be inconsistent with historical backtesting due to poor forecasting of market rates of element 1. Hence, it is important to prepare testing data to be able to separate the effects of element 1 and 3.

To this end, we prepare a different set of backtesting data separate from the Real Market Scenario above. We fix the market rates simulation model parameters constant throughout the five-year backtesting window, and take the historical market rates at the beginning of the backtesting window (i.e., 2011) as the initial point. We then forward project a series of daily market rates for the entire backtesting window, and use the simulated market rates as a substitute for actual historical market data. We then take the same series of portfolio as for the Real Market Scenario, and apply the same approach to construct the *would-be* historical ISDA SIMM IM values.

While the IM distribution is ultimately driven by both the accuracy of the IM approximation methods and the market rates simulation models, the objective of creating a Hypothetical Market Scenario[7] separate from the Real

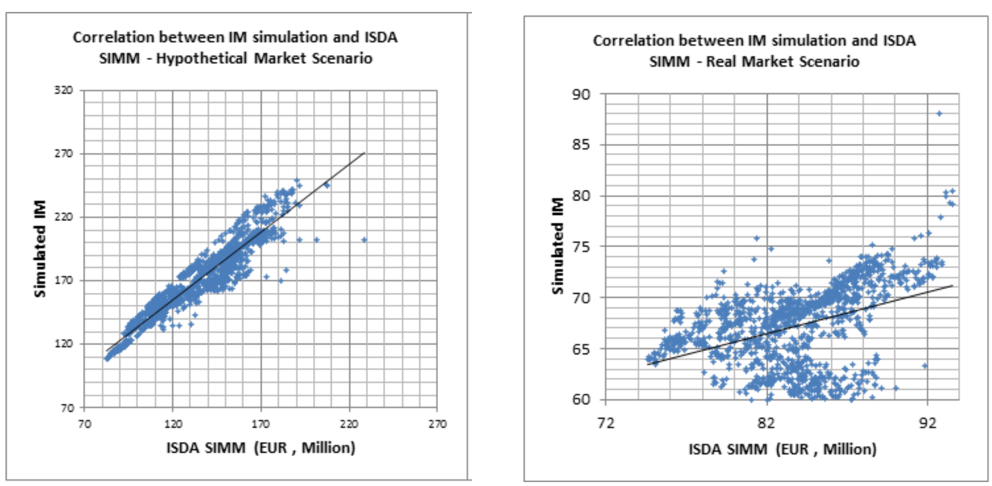
[7] In fact, we can create multiple versions of Hypothetical Market Scenarios with the same set of model parameters, to further study the statistical properties.

Market Scenario is that a comparison between the backtesting results of the two Scenarios highlights the impact of simulation models used for market rates and the effects of elements 1 and 3.

5.3. Comparing Spot Initial Margins by the ISDA SIMM model vs the Approximation Model

The simplest historical backtesting is to assess the model performance at $t = 0$. More specifically, in this section, we compare the correlation between the spot initial margin calculated by the ISDA SIMM model against the spot initial margin calculated by our approximation model without adjustment to the opening balance provided by the ISDA SIMM model. The purpose of this test is to investigate whether the proposed initial margin model can capture the same market dynamics as the ISDA SIMM model in a changing market scenario.

The calculation is performed on daily steps over the five year market scenario. The tests are performed for both the Hypothetical Market Scenario and the Real Market Scenario. The results, scatter plots of initial margins by the ISDA SIMM model against the approximation model over the five year window, are summarized in Figure 6.



(a) Hypothetical Market Scenario

(b) Real Market Scenario

Figure 6: Correlation between IM with the ISDA SIMM model and the Approximation Model

The *left part* of Figure 6 show the result for the Hypothetical Market

Scenario. The result demonstrates a strong positive correlation between the spot IM given by the approximation model and the ISDA SIMM model over time. In this Scenario, the same set of market rates model parameters is used for both generating the historical market rates and simulating the initial margin. It also demonstrates the desired strong correlation between the proposed approximatio model and the ISDA SIMM model for the calculation of the spot initial margin. The same tests have been performed in additional Hypothetical Market Scenarios, and all tests show the same pattern. By all these tests, we conclude the proposed IM simulation model is as sensitive as the ISDA SIMM model respect to the market movement.

As described in Section 3, the proposed initial margin simulation model takes the simulated portfolio values at the forward time horizon as input, so that the forecast quality of the initial margin simulation model partially depends on the appropriateness of the market rates simulation models. To demonstrate this, we repeat the above test using the Real Market Scenario, where the market rates simulation models do not have the same dynamics as the historical market rates. As shown by the *right part* of Figure 6, the correlation is not as significant[8]. This study emphasizes the importance of anchoring the spot initial margin to the opening IM balance calculated by the ISDA SIMM model, as described in section 3.5.

5.4. The Backtesting Procedure

The next step is to assess the forecasting quality of our approximation model. We describe our backtesting approach in this section, and present our results in Section 5.5. We follow the approach of [18, 19, 20]: given a backtesting window $[t_{\text{start}}, t_{\text{end}}]$ (i.e., 2011-2016) and a forecasting horizon Δ_f (e.g., $\Delta_f = 2$ weeks) for which we would like to backtest, we proceed as follows:

1. The backtesting window $[t_{\text{start}}, t_{\text{end}}]$ is divided into M multiple congruent backtesting periods $[t_i, t_{i+1}]$, where $t_1 = t_{\text{start}}$, $t_{i+1} = t_i + \Delta_f$, and M is the largest index such that $t_{M+1} \leq t_{\text{end}}$. Note that for simplicity, by construction the backtesting periods are non-overlapping.

[8]Note that in our testing data, the same set of historical portfolio data was used in both the Real Market Scenario and the Hypothetical Market Scenario. This resulted in different portfolio states and moneyness, and different range of initial margin values observed.

2. For each backtesting period $[t_i, t_{i+1}]$, conditional on the market data and historical SIMM IM value at $t = t_i$, simulate N IM values for $t = t_{i+1}$. These N simulated IM values form a cumulative distribution, and we can observe where the historical SIMM IM value at $t = t_{i+1}$ falls in the simulated cumulative distribution. More specifically, the historical SIMM IM value implies a backtesting quantile $q_i \in [0, 1]$ with respect to the cumulative distribution of the simulated IM values.
3. The outcome of the above exercise is a collection $\{q_i\}_{i=1}^{M}$ from which one can construct the cumulative distribution function associated with the backtesting quantiles of the model:

$$F_{\mathrm{Q}}(x) = \frac{1}{M} \sum_{i=1}^{M} I_{\{q_i \leq x\}}, \tag{40}$$

   where I is the indicator function. The simulated IM distribution would be a good fit to the dynamics of the historical IM if F_Q is close to the standard uniform distribution U.
4. Further, we can define a sensible *distance* metric D that measures how closely $F_{\mathrm{Q}}(x)$ matches the standard uniform distribution $U(x)$. Example distance metrics include the Kolmogorov-Smirnov statistic, the Anderson-Darling statistic, or the Cramer-von Mises statistic ([18, 19]). Given a distance metric, let the distance between $F_{\mathrm{Q}}(x)$ and $U(x)$ be D^{Model}.
5. In practice, the smaller the D^{Model}, the better the simulated IM matches the historical distribution. However, even with an ideal IM simulation model, the distance metric D^{Model} will not be zero. Hence, we benchmark D^{Model} by the following: repeat the above procedure K times, but instead of deriving $\{q_i\}_{i=1}^{M}$ by comparing historical IM values with model simulated cumulative distributions, derive $\{q_i^{\mathrm{ideal}}\}_{i=1}^{M}$ by comparing model simulated IM values with model simulated cumulative distributions[9]. Each repetition yields a distance value D^{ideal}, and K repetitions yield a collection of $\{D_j^{\mathrm{ideal}}\}_{j=1}^{K}$. This collection represents a simulated distribution of D values when the simulation model behaves appropriately with respect to the historical distribution. We then can assess the reasonableness of D^{Model} against the histogram of $\{D_j^{\mathrm{ideal}}\}_{j=1}^{K}$. Details and testing results are given in Section 5.5.

[9]A shortcut is to simply draw q_i^{ideal} from a uniform distribution.

5.5. Backtesting Results and Analysis

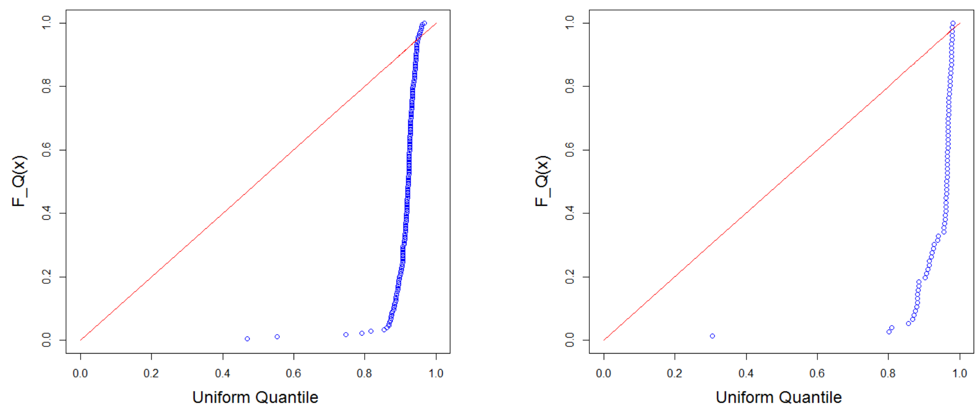
In this section, we follow the procedure outlined in Section 5.4 and compare $F_{\mathrm{Q}}(x)$ against $U(x)$. Figure 7 shows the quantile-quantile plots of $F_{\mathrm{Q}}(x)$ against $U(x)$ for forecasting horizons $\Delta_f = 1$w and 2w. As illustrated, the forecasting quality of the approximation model is quite poor compared to historical distribution of IM values. However, under the context of regulatory capital calculation in counterparty credit risk, it is often sufficient to demonstrate that the risk measures calculated are conservative. In this context, it is sufficient to demonstrate that the simulated IM distribution in some sense underestimates the received collateral[10] amount with respect to the historical IM distribution, as such underestimation leads to a conservative estimate of credit exposure and capital amount.

One possible characterisation of such underestimation is to demonstrate first-order stochastic dominance[11]. In Figure 7 the cumulative distribution of percentiles $F_Q(x)$ is in general largely inferior to the standard uniform distribution $U(x)$ except towards the highest percentiles. This means that the hypothesis of first-order stochastic dominance of SIMM over the IM Model is generally reasonable. While our approximation IM model is typically not accurate with respect to historical backtesting, it is typically quite conservative within the context of credit risk management and capital calculation. Note that the simulated distribution can be made even more conservative by using an appropriate model haircut as introduced in Section 3.5.

Another approach to assess the appropriateness of the model is with respect to a given definition of the D-metric. This approach to backtesting, an application of the Probability Integral Transform framework to the distribution of simulated IM, is also discussed in [1, 18, 19]. In this paper, we provide explicit analysis and results for the conservatism of the IM distribution in this section, and extend the approach to analyze collateralized portfolio value

[10]Here we assume that the posted initial margin will not generate counterparty credit risk, because according to final regulatory technical standards about margin reform [23], the initial margin shall be segregated from proprietary assets of the collecting counterparty on the books and records of a third party holder or custodian, or by other similar binding arrangements. This ensures that initial margin should be immediately available to the bank if the counterparty defaults.

[11]A random variable A first-order stochastically dominates another random variable B if $F_A(x) \leq F_B(x)$ for all x and $F_A(x) < F_B(x)$ for some x, with F_A and F_B the cumulative distributions of the random variables A and B respectively [24].



(a) 1w Horizon

(b) 2w Horizon

Figure 7: Quantile distribution F_Q (blue dots) against standard uniform distribution U (red line) using the Real Market Scenario. We have F_Q mostly inferior to U except towards the upper-right edge of the plot.

in Section 5.6. There are many appropriate choices, but for simplicity, we demonstrate using the Kolmogorov-Smirnov statistic [25], where the distance D between the distribution of percentile $F_{\mathrm{Q}}(x)$ and the standard uniform $U(x)$ is defined as:

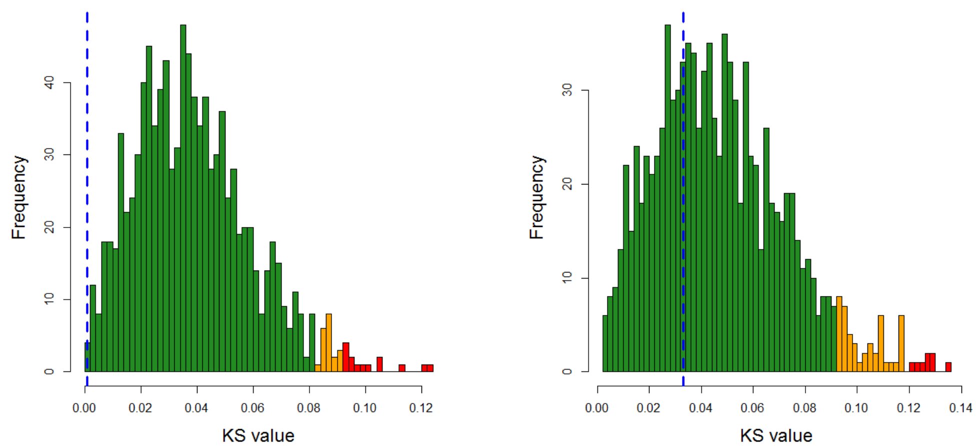
$$D = \sup_{x\in[0,1]} |F_{\mathrm{Q}}(x) - U(x)| \tag{41}$$

This distance metric is *two-sided* in the sense that what we test is how much the model distribution fits the data observed. As we noticed already in Figure 7, we already know that our simulated IM distribution is not accurate with respect to the historical IM distribution. Instead, we aim to demonstrate that our simulated IM distribution sufficiently underestimates that of the historical IM distribution. To this end, we instead define a *one-sided* metric D^+:

$$D^+ = \sup_{x\in[0,1]} (F_{\mathrm{Q}}(x) - U(x)) \tag{42}$$

to test whether the IM amounts simulated are consistently underestimated in comparison to historical SIMM amounts. This idea of one-sided metric is also discussed in [19].

We use this one-sided Kolmogorov-Smirnov metric for our historical back-



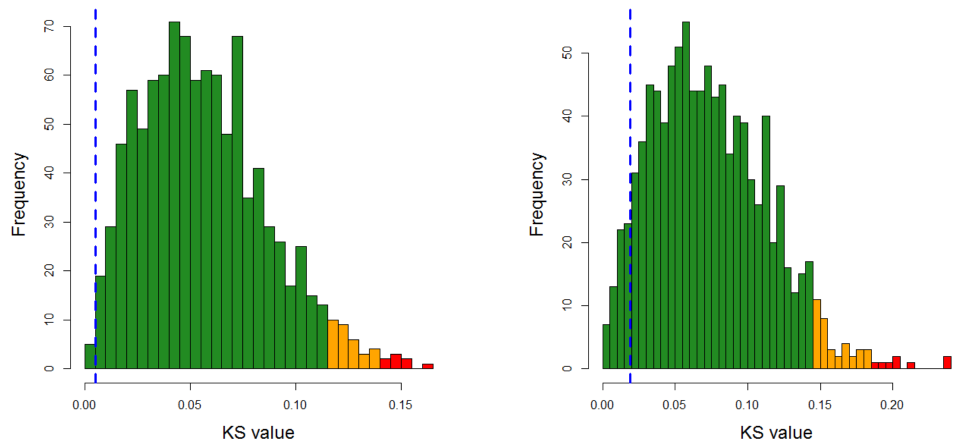
(a) Hypothetical Market Scenario (b) Real Market Scenario

Figure 8: Historical backtesting results with forecasting horizon of 1w, with the one-sided D^{+} metric("KS value"). The dashed blue line is D^{Model} and the histogram is based on $\{D_j^{\text{ideal}}\}_{j=1}^{K}$ values. With the Hypothetical Market Scenarios, $D^{\text{Model}} = 0.001$ (0.10 p-value) while with the Real Market Scenarios, $D^{\text{Model}} = 0.033$ (31.27 p-value).

testing test. Following the procedure outlined in Section 5.4, we plot the distance D^{Model} against the histogram of $\{D_j^{\text{ideal}}\}_{j=1}^{K}$ in Figures 8 and 9. The dashed blue line represents the D^{Model} value. Further, to define a confidence interval for the D^{Model} value, as well as to provide a *traffic-light* system for model risk monitoring, we use similar band coding colour suggested in [18]:

- Green: the band 0-95 percent of the first D^{ideal} values
- Amber: the band 95-99 percent of the first D^{ideal} values
- Red: the band 99-100 percent of the first D^{ideal} values

From Figures 8 and 9, we observe that the D^{Model} values are comfortably in the green band confidence interval in either the Hypothetical Market Scenario or the Real Market Scenario. This suggests that the simulated IM distribution sufficiently conservatively underestimates the historical IM distribution, with respect to the one-sided Kolmogorov-Smirnov metric. Further, as expected we observe that the quality of the market rates simulation



(a) Hypothetical Market Scenario (b) Real Market Scenario

Figure 9: Historical backtesting results with forecasting horizon of 2w, with the one-sided D^+ metric("KS value"). The dashed blue line is D^{Model} and the histogram is based on $\{D_j^{\text{ideal}}\}_{j=1}^K$ values. With the Hypothetical Market Scenarios, $D^{\text{Model}} = 0.005$ (0.40 p-value) while with the Real Market Scenarios, $D^{\text{Model}} = 0.019$ (5.50 p-value).

does play a role in the quality of the results of the IM simulation backtesting; in both figures, the Hypothetical Market Scenario performs better than the Real Market Scenario. Another contributing factor to the backtesting performance is the portfolio composition style. With respect to our benchmark portfolio, the backtesting performance appears reasonable. However, even though our backtesting framework is generic for a broad class of IM simulation models, the Nadaraya-Watson method assumes that the portfolio value itself serves as a good regressor for its risks. This assumption may need to be verified for different portfolio composition styles that a bank commonly encounters.

5.6. Effects of Portfolio Dynamics

Since the intent of the introduction of initial margin is to cover the potential increase in portfolio value over a $d = 10$-day horizon at a 99-percent confidence interval [4], to assess the appropriateness of an IM simulation method in the context of counterparty credit risk management and historical backtesting, it is prudent to also assess the the dynamics of IM jointly with the dynamics of portfolio value, especially that these two elements are in general not independent. It is possible to modify the backtesting approach discussed in Section 5.4 to explore this joint dynamics. More specifically, define the collateralized portfolio value as

$$\tilde{V}^d(t) = \mathrm{IM}^{\mathrm{received}}(t) + \mathrm{VM}(t) - V(t+d) + C(t, t+d), \tag{43}$$

where $\mathrm{VM}(t)$ is the value of variation margin at time t, and $V(t)$ is the value of the portfolio, and $C(t, t+d)$ is the net cash of the portfolio over time span $(t, t+d]$. Note that $\max(-\tilde{V}^d(t), 0)$ can be thought of as a simple model for calculating collateralized counterparty exposure[12], where the closeout period is assumed to match the initial margin horizon, and assuming that the counterparty is fully collateralized as of time t, the only closeout risk under consideration is the change in portfolio value over $[t, t+d]$. A discussion of more realistic models for calculating collateralized counterparty exposure is in [2].

For simplicity, further assume that at time t, $\mathrm{VM}(t) = V(t)$, then we may write

$$\tilde{V}^d(t) = \mathrm{IM}^{\mathrm{received}}(t) - \Delta\bar{V}(t), \tag{44}$$

where $\Delta\bar{V}(t)$ is the cash-adjusted portfolio profit-and-loss as defined in Equation (1). Then, given a particular backtesting period $[t_i, t_{i+1}]$, conditional on market condition at time t_i, instead of comparing the historical value of SIMM against N simulated values of IM at time t_{i+1}, we compare the historical value of collateralized portfolio value

$$\tilde{V}^d(t_{i+1}) = \mathrm{SIMM}^{\mathrm{received}}(t_{i+1}) - \Delta\bar{V}(t_{i+1}), \tag{45}$$

where $\mathrm{SIMM}^{\mathrm{received}}(t_{i+1})$ and $\Delta\bar{V}(t_{i+1})$ are the historical SIMM IM value and cash-adjusted portfolio PnL respectively, against N simulated values of

$$\tilde{V}^d_j(t) = \mathrm{IM}^{\mathrm{received}}_j(t) - \Delta\bar{V}_j(t), \tag{46}$$

[12]Note that assuming the counterparty is fully collateralized, since the initial margin is meant to provide protection for potential portfolio value increase with 99-percent confidence over horizon d, in most well-behaved situations $\tilde{V}^d(t)$ is positive, and in this simple exposure model, collateralized exposure exists only when $\tilde{V}_d(t) < 0$. One reason to analyze $\tilde{V}^d(t)$ instead of $\max(-\tilde{V}^d(t), 0)$ is because of the rarity of the occurence of negative values of $\tilde{V}^d(t)$. Another is that even for a positive value of $\tilde{V}^d(t)$, its magnitude provides an indication of the relative conservatism of the initial margin calculation methods.

where $j = 1, ..., N$, $\mathrm{IM}_j^{\text{received}}(t)$ and $\Delta\bar{V}_j(t)$ are the simulated received IM values and cash-adjusted portfolio profit-and-loss for the jth scenario conditional on market condition at time t_i. Based on this setup, we are able to repeat much of the analysis with respect to the collateralized portfolio value $\tilde{V}^d(t)$ instead of initial margin value.

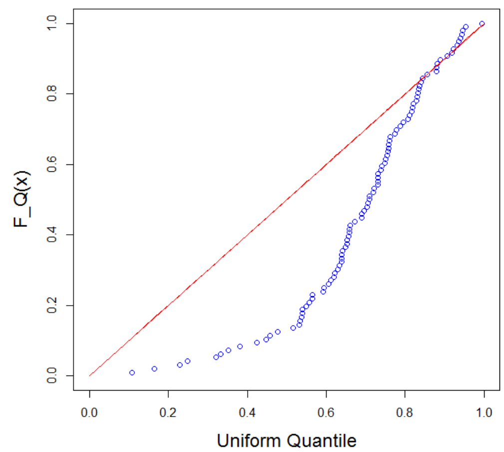


Figure 10: Quantile distribution F_Q (blue dots) against standard uniform distribution U (red line) using the Real Market Scenario. $F_Q(x)$ is derived from comparing simulated values of collateralized portfolio values against historically realized collateralized portfolio values instead of simulated IM values against historically realized SIMM IM values. We have F_Q mostly inferior to U except towards the upper-right edge of the plot.

To illustrate the above with an example, we choose the forecasting horizon $\Delta_f = 2\text{w}$, using the Real Market Scenario. We follow the procedure outlined in Section 5.4 and compare $F_Q(x)$ against $U(x)$. $F_Q(x)$ is derived from comparing simulated values of collateralized portfolio values against historically realized collateralized portfolio values instead of simulated IM values against historically realized SIMM IM values. Figure 10 shows the comparison. As the cumulative distribution of percentiles $F_Q(x)$ is in general largely inferior to the standard uniform distribution $U(x)$ except towards the highest percentiles. This indicates that it is generally reasonable to accept that the collateralized portfolio value calculated with SIMM exhibits first-order stochastic dominance over the collateralized portfolio value calculated with the IM approximation model[13].

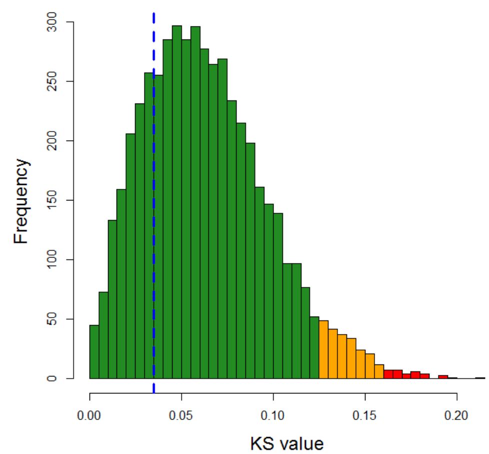


Figure 11: Historical backtesting results with forecasting horizon of 2w, with the one-sided D^+ metric("KS value"). The dashed blue line is D^{Model} and the histogram is based on $\{D_j^{\text{ideal}}\}_{j=1}^K$ values. With the Hypothetical Market Scenarios, $D^{\text{Model}} = 0.005$ (0.40 p-value) while with the Real Market Scenarios, $D^{\text{Model}} = 0.019$ (5.50 p-value).

The next step is to assess the appropriateness of the collateralized portfolio value simulation with respect to a reasonable D-metric, such as the one-sided Kolmogorov-Smirnov metric and the *traffic-light* system discussed in Sections 5.4 and 5.5. The assessment of the one-sided D-metric is provided in Figure 11. We observe that the D^{Model} value is comfortably in the green band confidence interval. This suggests that the collateralized portfolio value calculated with the IM model sufficiently conservatively under estimates the collateralized portfolio value calculated with the historical SIMM IM distri-

[13]If a random variable A first-order stochastically dominates another random variable B , given a non-increasing function f, $\mathbb{E}[f(B)] \geq \mathbb{E}[f(A)]$. With our simplified collateralized exposure model where the collateralized exposure is given as $\max(-\tilde{V}^d(t), 0)$, since $\max(-x, 0)$ is a non-increasing function in x, demonstrating first-order stochastic dominance ideally implies that expected exposure calculated with the IM model is larger, and therefore more conservative, than exposure calculated with the SIMM IM value. However, as the occurence of $\tilde{V}^d(t) < 0$ is rare, drawing such a statistical conclusion is perhaps too strong. It is more appropriate to conclude that based on our testing, our IM simulation model exhibits evidence of reasonable conservatism.

bution with respect to the one-sided Kolmogorov-Smirnov metric.

Note that while the notion of collateralized portfolio introduced here is quite simple, as it only captures the market risk of the portfolio itself during the closeout period, the backtesting framework discussed here is general enough to include additional effects, such as the market risk of the collateral assets, or the settlement risk of cash payments and trade flows [2], or the presence of multiple Credit Support Annexes (CSAs). We leave these further investigations as future work.

6. Conclusions

In this paper, we have proposed simple approximation approaches to forward simulate bilateral initial margin requirements. These approaches are not only practical to implement in realistic production environments, but are also conservative under the context of counterparty credit risk management and calculation of regulatory capital. The model is suitable for approximate forward initial margin in a context of calculating regulatory capital.

In conjunction, we have also proposed a series of testing approaches to assess the model appropriateness in terms of both model's in-simulation consistency and historical backtesting. Further, these testing approaches are not specific to our initial margin simulation methodology, and should be applicable to a broad class of initial margin simulation approaches.

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