Estimation of Future Initial Margins in a Multi-Curve Interest Rate Framework*

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Abstract

We propose an approach for the dynamical estimation of initial margins. We determine initial margins at future points in time by computing a risk measure of the modelled price increment over a margin period of risk. As an example, we produce the initial margin process for interest rate swap clearing where we assume that the swap price process is driven by a two-factor multi-curve interest rate model that exhibits good calibration properties. The obtained initial margin dynamics incorporate "forward-looking" information present in swaptions market data to which the swap price model is calibrated. We compare the model-generated initial margin process to initial margin data provided by clearing houses and propose adjustments to reduce the observed gap. In doing so, we in effect calibrate the initial margin process to additional market information possibly present in historical market data but not captured in the swaptions market. The margin valuation adjustment (MVA) process is obtained by an application of the risk-neutral valuation formula where the initial margin process is taken as the underlying instrument. We conclude with answers to questions we have received from the financial industry.

*Non-technical summary of a manuscript in preparation by the same authors. The authors welcome comments and suggestions. Corresponding author: a.macrina@ucl.ac.uk.

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1 Introduction

One of the effects of the recent financial crisis has been the regulatory efforts to try to reduce the systematic credit risk in the inter-dealer market. A path towards achieving this goal has been to advocate for vanilla instruments to be cleared through central counterparties (CCPs). In September 2016, bilateral margin for transactions between two main users become mandatory; the same rules will be progressively extended to a bigger cohort of derivative users between September 2016 and September 2019. The result of these new policies is that the margin paradigm becomes the standard for derivatives trading, not the exception. The fundamental concepts relating to margining are summarised in Section 2.

In central clearing and in standard Credit Support Annex (CSA) setups, the variation margin (VM) is paid in cash that accrues at an overnight rate. This feature has two effects: (a) In a coherent framework, the valuation of the underlying derivatives are to be performed by applying the nowadays standard *collateral discounting*, see e.g. Bielecki & Rutkowski [1], Henrard [3], Johannes & Sundaresan [4] and Pallavicini *et al.* [7]. This is the way that main clearing houses and derivatives dealers value portfolios—at least for the major currencies. This is the approach we also take. (b) The first order currency exposure of the trade is covered. The currency exposure of the portfolio is the same as the currency exposure of the VM; the counterparties are not exposed to different currency risks in case of default. To the first order, the currency risk of portfolios can be ignored; this is the approach we take here.

The cash VM does not offer protection against interest rate movements of the derivatives portfolios in case of default. The purpose of the initial margin (IM) is to make *ex ante* provisions for such a risk during the margin period of risk (MPR) and thus to mitigate an adverse impact on profits. The IM is usually deposited in cash or in (sovereign) bonds. The cash deposit accrues at the overnight rate (minus a spread) and the bonds require a haircut. The requirement to deposit an important sum in cash or bonds for a long period of time is a costly constraint for the depositing party. The BCBS estimated[1] in its quantitative impact study in 2013 that the IM requirement for bilateral margin could reach between 0.7 and 1.7 trillion EUR. The IM provisions are ring-fenced and may not be used by the involved (counter)parties. This long-term requirement of depositing a typically large amount that changes on a daily basis, is the centre of our analysis. We focus on interest rate instruments with the interest rate swaps (IRS) being our workhorse. The main questions are: How could the dynamics of the IM value be modelled? Can efficient dynamic models be proposed? Can we apply the obtained IM dynamics to estimate the long-term cost of the margin requirements?

In order to propose answers to such questions, we develop a method based on the explicit description of the IM process in a multi-curve interest rate setup. Indeed, another effect of the crisis has been a wider spread between different reference interest rates. The basis swaps have non-trivial spreads, and as such the overnight-indexed and LIBOR-indexed swaps need to be priced in a multi-curve setup. Similarly, the cost of funding

[1]See Appendix C in `http://www.bis.org/publ/bcbs242.pdf`

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of major derivatives dealers now differ from the overnight fixing and the LIBOR fixings. The funding costs tend to vary among different dealers, also. Thus, we take into account the impact of this new financial environment and develop a method for dynamical IM computations in a multi-curve interest rate setup, see e.g. Henrard [3], Crépey *et al.* [2] and Mercurio [6].

The present note is a non-technical summary of some first results. The full mathematical background to this research work is the focus of a forthcoming paper.

2 Margins

The standard method for margin computation makes a distinction between *variation margin* (VM) and *initial margin* (IM). The variation margin is the exchange of the portfolio present value between (counter)parties on a regular basis—usually daily. The credit risk embedded in the transaction decreases to the extend that today's value is covered. The initial margin targets the future. "Initial margin" is a misnomer in the sense that the margin is not fixed at the inception of the financial contract and neither does the initial margin remain unchanged during the contract's lifetime. On the contrary: the IM is computed and exchanged on a regular basis, like the variation margin. It is an "initial" margin only in the sense that it is an estimation of the potential changes in the value of the contract, which is computed in advance before these actually occur. The IM is then later adapted to the actual changes in line with the variation margin. The changes referred to are variations of, say, the portfolio value between the last VM payment and the time the trade counterparty has been able to replace a defaulted portfolio. The *a priori* estimation of this time period is called *margin period of risk* (MPR). This period is set by regulation to five business days for central clearing and to ten business days for bilateral clearing. The changes to be covered by the IM are the changes arising from any market movements. The purpose of the IM is to cover the potential future exposure of the instruments over the MPR and thus to reduce the credit risk between the trade counterparties.

The computation of the IM is done in practice by applying different methods. The main approach is to use Value-at-Risk (VaR) or Expected Shortfall (ES) as the risk measure for the potential exposure of the MPR. The risk is usually measured disregarding a potential default by a member, even if the IM amount will be used only in case of default. CCPs usually compute the risk measure based on historical data and on volatility rescaling, which is used to take into account volatility clustering, i.e. the fact that some periods are more volatile than others. Each clearing house may apply a different risk measure. LCH.Clearnet calculates the IM by the "Portfolio Approach to Interest Rate Scenarios" (PAIRS). It is described as an expected shortfall[2] approach mainly based on ten years of OTC data, along with a certain number of corrections or "add-ons". EUREX utilises a method called PRISMA based on 3 years of recent historical data and some stress periods. The method is VaR at 99%.[3] CME applies a VaR approach with a

[2] To a 99.7% confidence level

[3] The VaR is obtained by computing VaR several times on subsamples at a lower percentile (95%) and

99.7%. Even if the general method considered by each of the clearing houses is publicly known, the details of the methodologies and the data utilised are not. Each day, clearing houses provide a report of the VM and IM to their members; the numbers provided are the figures for the previous end-of-day to be paid on the next day. The replication of the clearing house methodology for the payment of the daily margin is not required by the members. The replication of the static daily IM can nevertheless be useful to the member for margin attribution, clearing client servicing, or what-if analyses of portfolio compositions.

Members are also interested in estimating the amount to be paid as initial margin at a future date. They use this estimation in several related analyses, the most likely being the margin valuation adjustment (MVA), i.e. the present value of the IM cost associated with a trade (portfolio) over its lifetime. The cost is mainly composed of the funding rate minus the interest received on the IM amount posted. The current valuation of the future cost linked to the IM pledge is known as MVA.

3 Initial margin process

Given its definition that we shall produce shortly, the value of the IM at a future date will depend on the dynamics of the LIBOR and OIS curves up to that date. To estimate the future risk exposures and thus to develop a consistent dynamical model for the IM process, we consider a stochastic model for the dynamics of asset prices. The model may include information on current market prices and produce probabilistic trajectories of how the prices might evolve in the future. We then compute the IM on the basis of asset price processes whereby the emphasis is on the margin computation given the probabilistic nature of financial risk exposures. Hence, instead of keeping track of observed quantiles as would be the case if one were to replicate a CCP methodology, we produce IM values by applying a risk measure on the possible future stochastic evolution of interest rates. (We focus on interest rates since the type of financial asset we have in mind is an interest rate swap and associated swaptions.) As a consequence, the resulting IM values no longer rely exclusively on historical data, or in other words on observations of past prices or past events in financial markets. In fact part of the idea here is to incorporate the market's view on future interest rate levels by calibrating to interest rate option prices. Nevertheless, the proposed dynamical model for the IM may be calibrated to historical (market) data or may take into account current IM values computed by a clearing house.

To be able to apply such an approach while valuing speed of computations, we rely on rather parsimonious but nonetheless flexible interest rate models. By choosing a flexible *factor model* for interest rates and functional *proxies for the risk measures* utilised by the clearing houses, we are able to develop an IM process that can be expressed explicitly in terms of the underlying uncertainty factors. We obtain the stochastic dynamics of the IM without relying on scenario simulations, thus reducing the complexity of the calculation and guaranteeing, by construction, that the dynamics of the initial margin are consistent

rescaling using a factor from a Student-t distribution. The VaR is also complemented by a correlation breakdown adjustment and floored by a stress period VaR.

with the underlying interest rate model. The idea behind the method we propose for the IM computation of a swap is simple: We consider a specific interest rate model for LIBOR in a multi-curve setup, which gives rise to the stochastic evolution of the swap price process Sw_t. We provide a brief description of the model and suggest calibration schemes in the following sections. The IM process is then obtained by computing the expected shortfall (ES), or another risk measure, of a swap price increment adjusted for the coupon payment C_T at time $T \in [t, t+\delta)$ given the (market) information $\mathcal{F}_t$ available at time t. The time interval $[t, t+\delta]$ is the MPR. The mathematical formula, with which we define the IM process, is given as follows:

$$IM_t = \mathrm{ES}\left[Sw_t - \frac{h_{t+\delta}}{h_t} Sw_{t+\delta} - \frac{h_T}{h_t} C_T \,\middle|\, \mathcal{F}_t \right], \tag{1}$$

where h_t is a discount process. For instance, but not necessarily, we may think of h_t as being the OIS discount factor associated with the LIBOR multi-curve model. We may compute the expected shortfall under different probability measures, although, it is usually calculated under the (subjective) historical measure. The choice of the underlying pricing model determines the dynamical features of the IM process and its tractability. We consider the rational multi-curve models proposed by Crépey *et al.* [2] (see Section 5 below for a brief account), which provide a good compromise between model flexibility, tractability and good calibration properties. The IM process may then be calibrated to market data, such as options, and existing IM data. Indeed, information on swaption prices, for example, allow the model to include the market view as to where rates might be at a future date. The approach proposed in this paper has the advantages of (i) being able to accommodate various risk measures, e.g. VaR and expected shortfall, (ii) the MPR can be selected arbitrarily, and (iii) model calibration can be performed under a risk-neutral and under the statistical measure. As shown in the next section, the numerical efficiency is rather satisfactory. At this stage the modelling setup can be applied to single-currency portfolios. The extension towards including multi-currency portfolios could be the objective of forthcoming research. The application of this methodology to a portfolio of swaps and FRAs is straightforward: it suffices to view Sw_t and C_T in Equation (1) as, respectively, the t-value and the cash flow at T of the whole portfolio. Moreover, this valuation scheme can also be applied to more general interest rate portfolios by considering, in line with common industry practice, their representation in terms of a set of hedging swaps and forward rate agreements.

4 Numerical implementation

To illustrate the proposed method, let us consider the dynamics of the IM process for a pseudo-portfolio consisting only of a single forward starting swap. In this way, we isolate effects related to the fixing of the LIBOR and simplify the interpretation of the obtained results. We emphasise here that the additional complexity of considering a portfolio of swaps, instead of only one instrument, is almost negligible. This is due to the closed-form expression available for swap prices within the pricing approach considered in this work.

Before we move on, we pause briefly to clarify the usage of probability measures in our setup. The IM formula (1) has as its argument increments of a price process (swaps) of which model needs to be calibrated. The price model Sw_t is calibrated to option data (volatilities implied from swaptions) which is risk-neutral data. Once the parameters of the price model Sw_t have been calibrated to option data, we no longer need to work with a risk-neutral measure, and the price model is ready for the application to the IM computation. Now, whenever exposure risk management is the focus, one ought to make risk measurements under the statistical probability measure $\mathbb{P}$. This means that the considered probability measure in formula (1) should be $\mathbb{P}$. Depending on the price models utilised for Sw_t, the computation of the risk measure may be easier to perform under a $\mathbb{P}$-equivalent probability measure. This is our case, and we first work with an equivalent measure associated with a particular numeraire that arises in connection with the rational multi-curve models. For details on this equivalent numeraire measure $\mathbb{M}$, we refer to Crépey *et al.* [2] and Macrina [5]. Hence, we analyse the result obtained for IM_t under the equivalent measure, and then ask the question of how the computation needs to be adjusted to obtain the initial margins in line with risk exposures under $\mathbb{P}$ (or a kind of proxy measure $\mathbb{P}^*$ interpreted as the subjective probability measure of a clearing house). The so-called *rational multi-curve interest rate models* we choose to utilise are briefly introduced in Section 5 and are developed in Crépey *et al.* [2]. The model calibration for the application to the IM computation within this modelling framework is presented in Section 6. The numerical results are produced based on market data in EUR as of 27 July 2015. We consider the case of a swap starting in five years time with a tenor of five years, a fixed rate of 1% and a notional of one million. After calibrating the model to swaption data, we obtain the dynamics for the LIBOR process and the price process for the considered swap as shown in Figure 1. The model calibrated to swaption prices features a positive drift for the LIBOR process (with six-month tenor) and a quite stable drift between the OIS forward rate and the LIBOR processes. The shape of the swap price process appears to be in line with what is expected, with clear jumps appearing at the coupon payment dates.

Once the swap price process underlying the IM formula (1) is calibrated to swaption implied volatilities, the IM can be computed with expected shortfall taken at the 99.7% level under the numeraire measure associated with the process h_t. Figure 2 shows the mean and deviations of the resulting IM process. We draw attention to the significant difference between the lower and upper deciles of the IM.

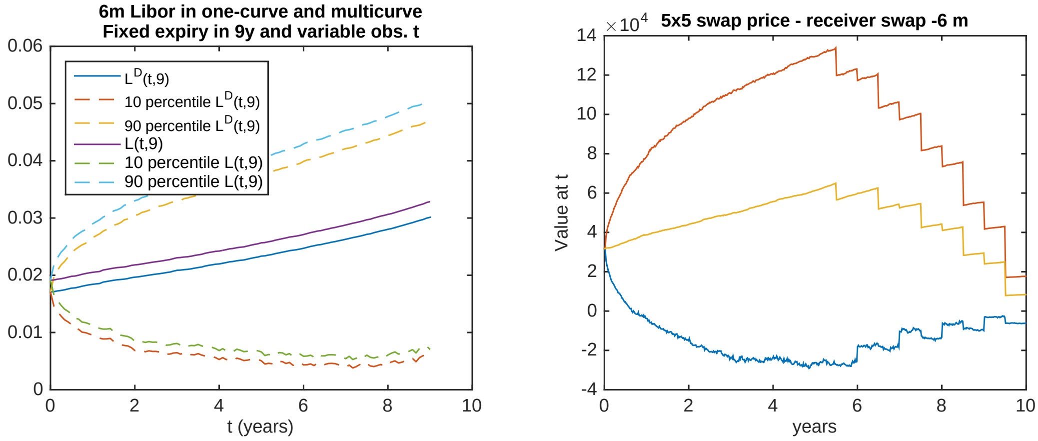


Figure 1: Left: Dynamics of the LIBOR process (two-factor model). The notation $L(t;9)$ is explained in Section 5. Right: Price process of the 5Y×5Y swap (mean in yellow colour, upper and lower deciles in red and blue colours, respectively).

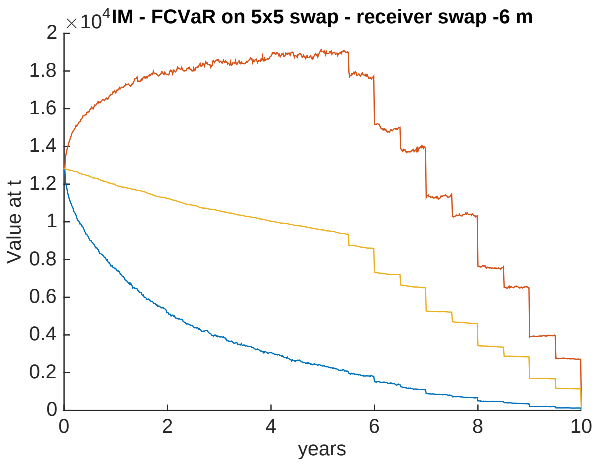


Figure 2: IM process under the numeraire measure associated with the process h_t (mean in yellow colour, upper and lower deciles in red and blue colour).

We benchmark the modelled IM by comparing it to the IM obtained by applying the valuation method of two clearing houses, LCH and CME. We report these reference values in Table 1. We see that the IM at $t = 0$ computed by the clearing house methods is higher than the value at $t = 0$ we obtain with formula (1). We show this discrepancy in Figure 3 where we plot the P&L distribution over one MPR computed on the basis of (i) the LCH method and (ii) the IM process (1) at $t = 0$. Also in Figure 3, we show the ratios between VaR and ES based on these two distributions for levels of risk above

75%. The modelled IM process (1) appears to underestimate the tail behaviour present in the LCH database.

CCP	PV	IM
LCH	33,841€	23,449€
CME	34,496€	28,154€

Table 1: Present value and IM calculated using the methodology and information of clearing houses, for the 5Y×5Y swap.

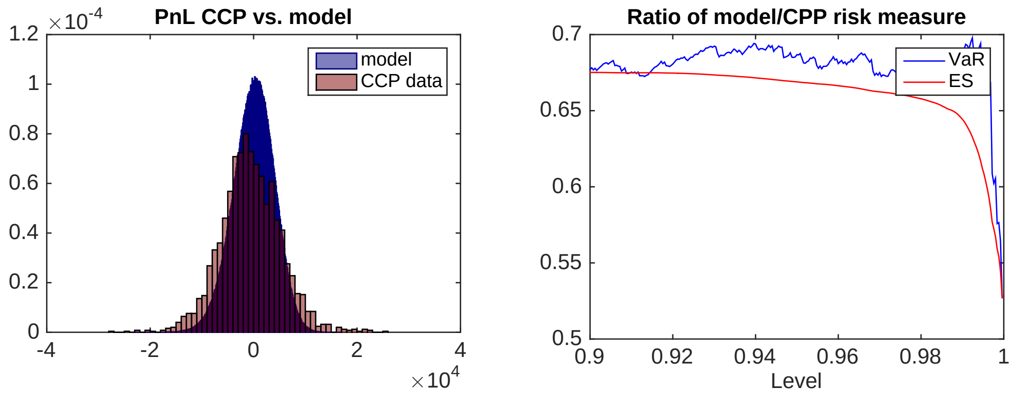


Figure 3: Left: P&L for a one-week evolution (MPR) based on LCH data vs. model-generated data under the numeraire measure. Right: Comparison for VaR and ES as computed by (i) LCH and (ii) IM formula under the numeraire measure.

We suspect that the observed discrepancy may be reduced if we consider the statistical measure $\mathbb{P}$ for the purpose of calculating the expected shortfall in (1). In fact, the IM calculated by both CCPs is based on data sets of historical information (about interest rate movements). Thus we consider re-weighting the quantiles obtained under the equivalent numeraire measure so as to diminishe the gap between the model-generated P&L and the P&L based on historical data the clearing house uses. Such an adjustment could be interpreted as a kind of measure change to the statistical measure $\mathbb{P}$. However, because we expect the historical database of different clearing houses to be incongruent and thus these to produce different IM values, we are reluctant to identify the adjusted quantiles as the $\mathbb{P}$-quantiles where $\mathbb{P}$ is the statistical probability measure. Instead, we introduce $\mathbb{P}^*$ for the subjective (perception of the $\mathbb{P}$ probability) measure that is implied by the historical data of a particular clearing house. Hence, we shall call the re-weighted quantiles, $\mathbb{P}^*$-quantiles, which are idiosyncratic and implied from the database provided by a particular clearing house.

Furthermore, in order to fit specific idiosyncrasies of a clearing house, we make use of a constant correction term to match the IM value at time $t = 0$. We show the updated comparison of the one-period histogram, ES-VaR ratios under the $\mathbb{P}^*$-measure, and the adjusted IM process in Figures 4 and 5(a), where one can observe a clear improvement. We notice that the change of measure spreads the probability mass in excess around the mean towards the tails. In Figure 5(a), we plot the associated IM process, where both, the risk exposure and the dynamics are computed by means of ES under the subjective measure $\mathbb{P}^*$. The initial value IM_0 is adjusted to match the initial margin computed by a clearing house at $t = 0$. For the calculation of MVA (see section 7), it will be advantageous to express the IM dynamics with respect to the $\mathbb{Q}$-measure. We show the corresponding plot in Figure 5(b). We emphasise that in Figure 5(b) we are not changing the measure for the calculation of the conditional expected shortfall (which is still $\mathbb{P}^*$); we do so only for the dynamics of the IM-process.

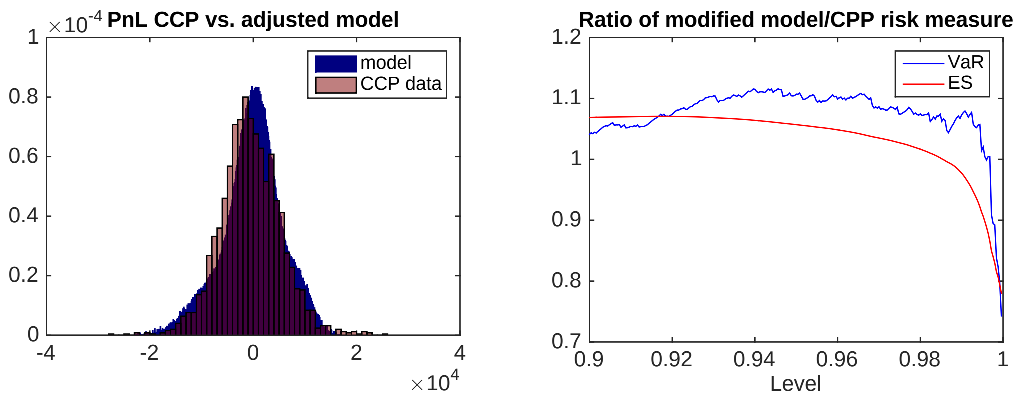
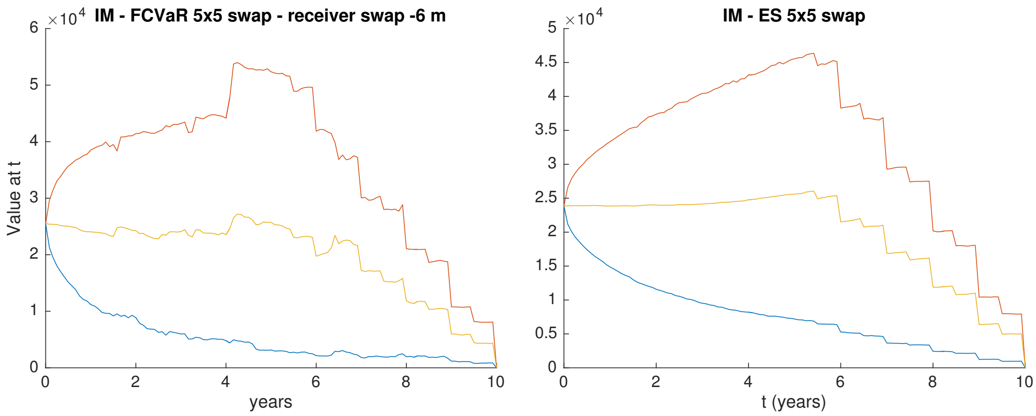


Figure 4: Left: P&L for a one-week evolution (MPR) based on LCH data vs. model-generated data under the subjective $\mathbb{P}^*$ measure. Right: Comparison for VaR and ES as computed by (i) LCH and (ii) IM formula under the $\mathbb{P}^*$ measure.

The type of model for the swap price process underlying the IM formula (1) which we have utilised to obtain the numerical analysis allows for mathematical simplifications and increased computational speed. As we shall briefly see in the following section, the example we consider for the IM process is based on a rational log-normal model.

The calculation of VaR and ES can be simplified by taking advantage of the stationarity of the multiplicative increments of the log-normal distribution and properties of the considered risk measures. As a result, the risk measures act only on functions of stationary terms, so that they can be computed before simulating the paths. This is particularly beneficial when a large number of points either in time or space are used for the simulation. In Table 2 we show the amounts of time necessary to compute the IM dynamics with our approach for some typical application cases. We compare with the values obtained when using a full Monte Carlo of Monte Carlo (MC^2) approach, as a proxy for the time that a direct full IM revaluation would imply. To provide further information on the relative costs of each step in the procedure, we also provide the time needed to compare the initial (pre-calculation) step and the evolution step.



(a) Adjusted IM under $\mathbb{P}^*$ (b) Adjusted IM under $\mathbb{Q}$

Figure 5: Adjusted IM process where ES is calculated under the $\mathbb{P}^*$ measure, under two different measures (Yellow: Mean. Red and Blue: Upper and lower deciles).

Paths	Periodicity (grid points)	MC^2	Refined Method		
			Initial	*Evolution*	*Total*
1250 ($\approx$ 5yrs history)	Weekly (520)	126.32	5.76	0.76	6.52
	Monthly (120)	28.52	5.73	0.20	5.93
2500 ($\approx$ 10yrs history)	Weekly (520)	423.37	5.73	0.92	6.65
	Monthly (120)	94.21	5.72	0.23	5.95

Table 2: CPU time in seconds for computing the IM process based on a 5Y×5Y swap until its expiry. The amounts of time refer to different numbers of Monte Carlo paths (for dynamics and risk measure calculation) and periodicity (or number of time steps in the simulation).

We emphasise that the pre-calculation step is valid as long as the model is not re-calibrated. If the IM process is computed for several portfolios or it is applied to estimate the marginal IM, then only the evolution part of the computation will require time. A full IM evaluation using the clearing house methodology would be even more time consuming than the MC^2 approach we present in Table 2, given that to replicate the methodology, a daily time sampling is required to update the historical scenarios.

5 Background: rational multi-curve interest rate models

In this section we take a glance at the modelling background of the IM process (1) and in particular at the model of the swap price process Sw_t in formula (1). The class of models we consider is the so-called rational multi-curve interest rate models proposed by Crépey *et al.* [2]. In the numerical analysis, we focus on a two-factor model with log-normally distributed factors under a given probability measure equivalent to the statistical measure. One factor impacts the general level of the base curve and another the spread between the different rate curves. This class of models can include more factors for both, the general dynamics of the base curve and the spread dynamics—see Macrina [5] for more material on asset pricing with rational models. For the purpose in this paper, it suffices to consider the two-factor lognormal model studied in detail by Crépey *et al.* [2]. Within this model specification, the price process P_{tT} of the OIS-discount bond with maturity T is given by

$$P_{tT} = \frac{P_{0T} + b_0(T)A_t^{(1)}}{P_{0t} + b_0(t)A_t^{(1)}}$$

for all $0 \leq t \leq T$. Here, P_{0T} denotes the initial term structure, $A_t^{(1)}$ is a martingale and $b_0(t)$ is a positive function satisfying certain properties and utilised to calibrate to market data (for details see Crépey *et al.* [2]). An important role is played by the discount process h_t in Equation (1), which in the present setup is defined by

$$h_t = P_{0t} + b_0(t)A_t^{(1)}.$$

The process h_t serves as the numeraire associated with a measure under which calculations simplify, and it is fixed once the distribution of the martingale $A_t^{(1)}$ is specified. For example, $A_t^{(1)}$ is here assumed lognormal and $b_0(t)$ a decreasing linear function. Earlier, we also mentioned the so-called *LIBOR process*. By $L(t; T_{i-1}, T_i)$ we denote the present value at time t of the LIBOR coupon that at time T_i pays the LIBOR fixed for the period $[T_{i-1}, T_i]$. As proposed by Crépey *et al.* [2], we consider the following model for the LIBOR process:

$$L(t; T_{i-1}, T_i) = \frac{L(0; T_{i-1}, T_i) + b_1(T_{i-1}, T_i)A_t^{(1)} + b_2(T_{i-1}, T_i)A_t^{(2)}}{P_{0t} + b_0(t)A_t^{(1)}},$$

where $A_t^{(2)}$ is a second martingale, and b_1 and b_2 are further degrees of freedom available for model calibration. The one-factor version of this model is obtained by setting $b_2 = 0$. In the present implementation, we have chosen to work with lognormal martingales. We take $A_t^{(1)}$ and $A_t^{(2)}$ to be defined by

$$A_t^{(i)} = \exp\left(a_i X_t^{(i)} - \tfrac{1}{2}a_i^2 t\right) - 1 \tag{2}$$

for positive constants a_i and where $X_t = (X_t^{(1)}, X_t^{(2)})$ is a two-dimensional Brownian motion with respect to a reference probability measure with constant correlation between its components. This two-factor model allows for closed-form price dynamics (or semi-closed form) of several of the most important financial contracts. For example the price at time t of a receiver swap with starting date $T_0 > t$, with coupon payment and fixing dates $T_1, \dots, T_n$, is given by

$$Sw_t = \frac{c_0 + c_1 A_t^{(1)} + c_2 A_t^{(2)}}{P_{0t} + b_0(t) A_t^{(1)}} \tag{3}$$

where

$$\begin{aligned}
c_0 &= \sum_{i=1}^{n} N_i \left[\alpha_i L(0; T_{i-1}, T_i) - \alpha_i' K P_{0T_i} \right], \\
c_1 &= \sum_{i=1}^{n} N_i \left[\alpha_i b_1(T_{i-1}, T_i) - \alpha_i' K b_0(T_i) \right], \\
c_2 &= \sum_{i=1}^{n} \alpha_i N_i b_2(T_{i-1}, T_i),
\end{aligned}$$

and where α_i (respectively α_i') represents the accrual factor of the payment at time T_i of the floating (respectively the fixed) leg. By analysing this quantity, we are able to obtain the (stochastic) dynamics of the IM exclusively in terms of the martingales $A_t^{(1)}$ and $A_t^{(2)}$.

6 Model calibration

In Section 3 we propose stochastic models for the IM dynamics, which is based on computing a risk measure (e.g. ES) of a price increment of an underlying financial instrument that is traded via a clearing house. In Section 5 we tell what class of model we apply to price an interest rate swap, which is the financial contract that we assume is being cleared. Given that one has selected a pricing model for the asset to be cleared, one needs to calibrate the pricing model (here, the swap price process) to relevant market data (for the swap these would typically be swaptions) in order to specify the parameters (degrees of freedom) of the selected pricing model. Next we give a brief account of how we analyse the behaviour of the rational multi-curve model we have selected given a set of option data.

Calibration to swaption prices. The liquid option prices available in the market are at the same time numerous and rather restrictive. There is for example an abundance of liquidly-traded swaptions with different expiries, tenors and strikes. But those liquid swaptions are restricted to a very limited type of swap: fixed versus LIBOR of a unique tenor. In EUR, fixed versus EURIBOR-6M are traded. In USD, fixed versus LIBOR-3M are traded. But other type of options, such as OIS swaptions or basis swaptions, are mostly non-existent. From the perspective of calibration, it means that the availability of option prices restricts the calibration (and the use) of model parameters. Furthermore, it may not be possible to distinguish, on the basis of market data, how much each term of a model contributes to the dynamics of a price process since model parameters may need to be specified in an *ad hoc* manner. For instance, since swaption prices depend on a mixture of the base curve dynamics and the LIBOR dynamics, they do not carry clear information about the market's view on the spread dynamics, which in the above rational multi-curve model are described by the functions b_1, b_2 and the constant a_1.

Leaving aside such calibration issues, we can still verify that the model is able to capture the general implied dynamics with a parsimonious choice of parameters. We calibrated a simplified version of the above model to 36 ATM swaptions and four OTM swaption smile curves where the swap price dynamics are influenced by only three parameters. In Figures 6 and 7, we plot the swaption market data and the model-generated prices for the case that the model is calibrated to the ATM and OTM swaptions.

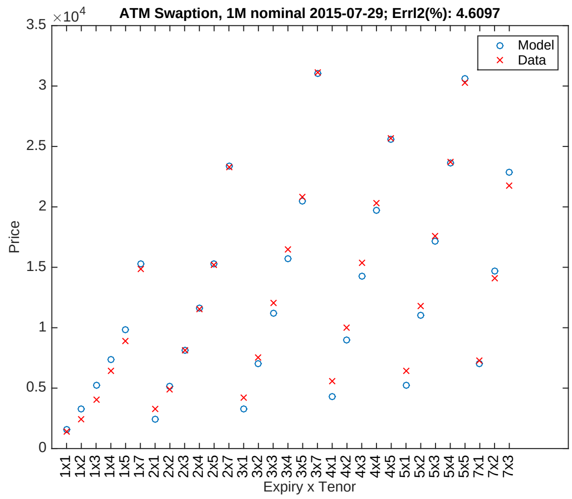


Figure 6: ATM swaption prices and prices obtained with a one-factor rational multi-curve model calibrated to ATM and OTM swaptions.

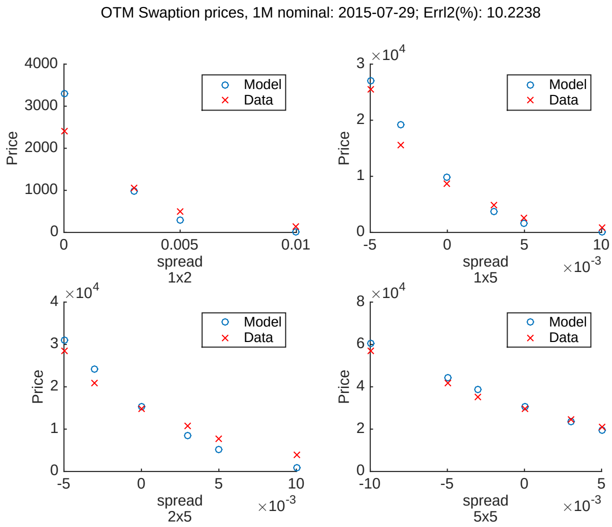


Figure 7: OTM swaption prices and prices obtained with a one-factor rational multi-curve model calibrated to ATM and OTM swaptions.

The plots show that a parsimonious parametrisation can fit the main market features in terms of volatility level and smile skewness. As mentioned above, there is no liquid market instrument that can provide information on the relation between curves in a multi-curve framework. This makes the task of calibrating the parameters of the second factor, which is associated with the spread dynamics, challenging. To get a sense of the difference in scale between the coefficients b_1 and b_2, we look at the spread between a LIBOR curve and the forward OIS curve. Figures 8 and 9 show the results for the calibrated two-factor model.

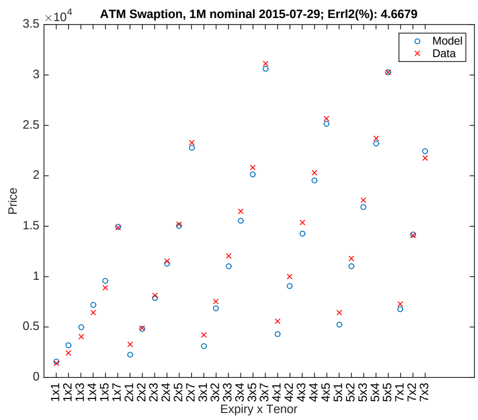


Figure 8: ATM swaption prices and prices obtained with a two-factor rational multi-curve model calibrated to ATM and OTM swaptions.

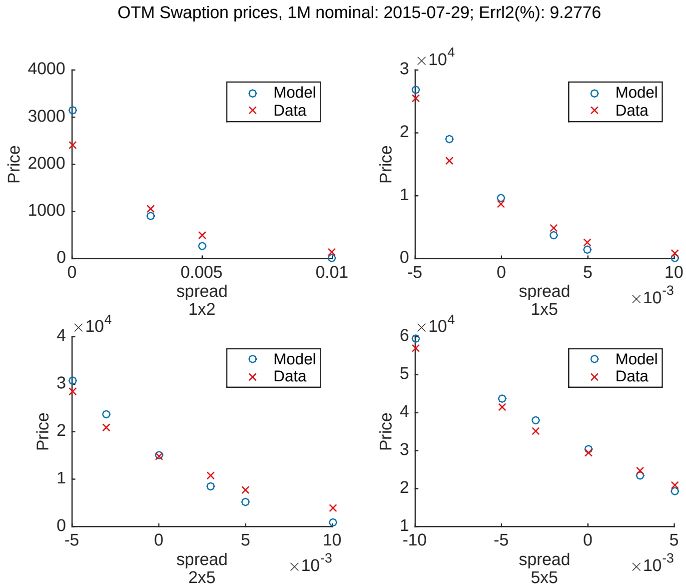


Figure 9: OTM swaption prices and prices obtained with a two-factor rational multi-curve model calibrated to ATM and OTM swaptions.

Calibration to IM values. The calibration of the model to swaption data allows us to incorporate information about the market's view on future interest rates movements. However, the IM process includes additional information that is not necessarily captured in the swaption market. For instance, as briefly mentioned above, the swaption market provides little information about the actual spread between OIS and LIBOR, and the lack of an OIS swaption market exacerbates such a shortcoming. This lack of information might explain the thin tails produced by the model-implied IM distribution. We address this issue by considering a subjective meaure $\mathbb{P}^*$ with respect to which the discrepancy between the modelled P&L distribution and the one produced by a reference clearing house is reduced. We choose a probability measure adjustment from a parametrised family of positive, integrable and normalised functions. The parameters are then obtained by a minimisation procedure so as to incorporate information about rarer events into the IM distribution. Furthermore, a constant factor is introduced to pinpoint the model-generated value of today's IM, i.e. IM_0, to the one published by each clearing house. This adjustment accounts for differences in the databases of each CCP and it also serves as a first-order approximation to include add-on corrections in the methodology of each clearing house. Such adjustments are integrated in the IM process that produces the model distribution shown in Figure 4.

7 Margin Valuation Adjustment (MVA)

Given that the initial margin formula (1) allows to estimate the initial margin at any future date, the cumulated cost associated with posting the initial margin over the lifetime of a cleared financial asset can also be calculated. This cumulated cost is what is usually referred to as the margin valuation adjustment (MVA). We consider initial margin payments during the time period $[t, \bar{T}]$ and ask what is the expected cumulated cost at time t, that is MVA_t, given the estimated IM payments over $[t, \bar{T}]$. The cost MVA_t can be calculated as if we computed the price at time t of a derivative on IM with maturity $\bar{T}$. That is,

$$\begin{aligned} MVA_t &= \frac{1}{D_t}\mathbb{E}^{\mathbb{Q}}\left[\int_t^{\bar{T}} D_u\, r_u^f\, IM_u\, \mathrm{d}u \,\middle|\, \mathcal{F}_t\right] \\ &= \frac{1}{D_t}\int_t^{\bar{T}} \mathbb{E}^{\mathbb{Q}}\left[D_u\, r_u^f\, IM_u \,|\, \mathcal{F}_t\right] \mathrm{d}u, \end{aligned} \tag{4}$$

where D_t is the OIS-discount factor (the numeraire associated with the risk-neutral measure $\mathbb{Q}$) and r_t^f is a funding cost, capturing the spread between the rate at which the clearing member borrows funds and the one it receives from the clearing house for the deposited IM. We make the assumption that the latter is the OIS rate[4].

Let us discuss now the borrowing rate. A clearing member has an estimate of the funding needs that it will have in the future. Its treasury usually satisfies such a liquidity requirement by securing a basket of funds with best-matching maturities. We make the simplifying assumption that the funds profile is kept fixed in time. Then, we propose to model the funding rate as a weighted average of LIBOR rates at different maturities plus

[4]In some CCPs, it is customary to pay OIS minus a deduction.

an idiosyncratic factor (that depends on the clearing member). That is,

$$r_t^f := \sum_{k=1}^{M} \gamma_k L\left(t, T_{i_k^*(t)}^{\Delta_k}, T_{i_k^*(t)+1}^{\Delta_k}\right) + A_t^{(3)} - r_t$$

where $\Delta_1, \ldots, \Delta_M \in \mathbb{R}_+$ are a set of maturities; $\gamma_1, \ldots, \gamma_M \in [0,1]$ with $\sum_{k=1}^{M} \gamma_k = 1$ is a set of weights specifying the composition of the clearing member's funding basket. Furthermore, $i_k^*(t) := \lfloor t\Delta_k^{-1} \rfloor$ and $T_i^{\Delta_k} = i\Delta_k$, for $i = 1, 2, \ldots$, are the dates at which each specific obligation arrives at its term. Following our simplifying assumption, we assume that these obligations are renewed at the new LIBOR rate. In addition, r_t is the OIS (short) rate in our approach, and $A_t^{(3)}$ is an idiosyncratic factor. We note that the MVA-process can attain negative values. The interpretation is that the clearing member has managed to fund their IM deposits at a lower rate than the one payed by the CCP on the deposited IM. This might occur, for example, there is a strong increase in rates, given the assumption that the clearing member fixes the funding rates periodically. Hence, negative values of the MVA should be understood as profit scenarios for the clearing member.

We show in Figure 10 a simulation for the MVA process under the $\mathbb{Q}$ measure, consistent with the IM estimation in Figure 2, when the clearing member rate is given in terms of six-month LIBOR, and the idiosyncratic factor $A_t^{(3)}$ is set to zero.

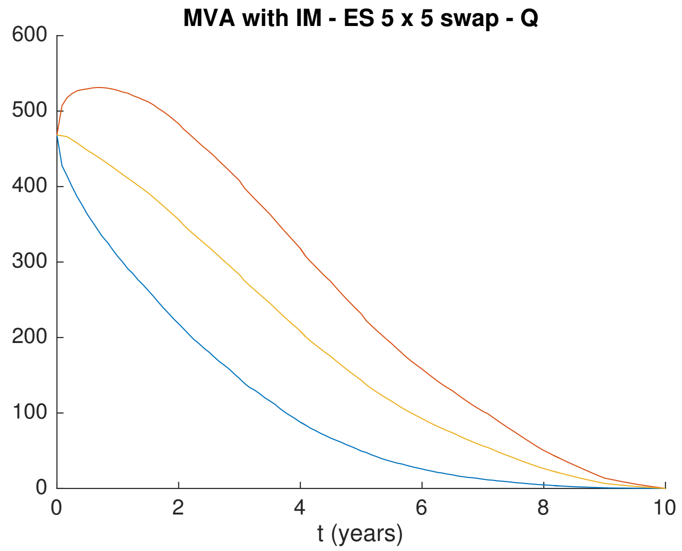


Figure 10: MVA process (Yellow: Mean. Red and Blue: Upper and lower deciles) under the $\mathbb{Q}$ measure.

8 Questions and answers

A number of questions have reached us from the financial industry with regard to the approach presented in this paper. We think it may be useful to share our answers with anyone who might have similar questions. The following questions and answers are also available at: http://www.opengamma.com/blog

Q.1: *In the paper, several measures are mentioned. Can you describe what they represent? Which of those measures is actually used in the computations?*

A: There are two or three probability measures involved, depending on which perspective of the task one takes. There is a pricing measure, applied for option pricing and often called $\mathbb{Q}$. There is the historical or statistical measure, which represents the real-world (economic) probability measure, used for risk measurements/management, and which is often denoted by $\mathbb{P}$. Sometimes it may also be viewed as the subjective measure of the clearing houses. We make this distinction between the two interpretations for the $\mathbb{P}$-measure to take into account the fact that the data used by clearing houses might not be a perfect representation of the economic reality. Before we begin with the actual IM computations, we calibrate the (swap) price model to swaption data. For this task we apply the pricing dynamics of the price process. The fact that the IM process is based on the dynamics of the swap price, which is calibrated to option data, links our IM valuation to market views as to how the swap price might evolve in the future. However, calibration to option data only might not encode enough information in the IM dynamics. An information supplement is incorporated if we calibrate the IM process also to CCP data, which we assume is historical data. Therefore we need to consider the IM dynamics under the historical measure $\mathbb{P}$ so that we have consistent calibration to the clearing house data. Given that we have the explicit formulae for the measure changes among the considered probability measures, we are able to produce consistent IM dynamics and thus also consistent margin valuation adjustments. Since MVA can be viewed in principle as a replication pay-off dependent on the future IM payments, we may compute MVA as if it were a derivative. Because we have the means to compute the IM under the statistical $\mathbb{P}$-measure, while also having an explicit measure-change relation between the $\mathbb{P}$-measure and the risk-neutral measure $\mathbb{Q}$, we can compute the MVA under the pricing measure as customary when pricing derivatives. Our approach supports both views of a financial market—the risk-neutral one utilised for pricing and hedging, and the historical one applied for risk measurements as in the case of IM computations. It is this consistency between these two main tasks in financial practice that, in our view, is one of the valuable features of the proposed approach.

Q.2: *The paper makes use of the "rational multi-curve interest rate models". Why this class of models?*

A: When we started this research we had long discussions about the modelling of the interest rate market in a multi-curve framework. The first conclusion was that there is no standard "go-to" model. In the single-curve framework, we might have selected an HJM model. However we wanted a multi-curve interest rate setup with several different properties. The multi-curve models would need to give rise to term structures with realistic dynamics for the base curve and for the spread between OIS and LIBOR. The chosen modelling framework would need to produce parsimonious but nevertheless flexible models. It would also need to represent the market in a realistic way, thus calibrating well to term structures, option volatilities and skew (historical or implied smile). After reviewing several models, we found that the rational multi-curve models are satisfactory in this sense, and offer a desirable level of flexibility for calibration to market data (c.f. Section 6).

Q.3: *Can one calibrate your IM model to the IM as computed by the CCPs on the initial date?*

A: Yes, and we are continuing work in this direction to improve the relationship to the historical databases utilised by a CCP. As explained in Q.1, after calibrating the swap price model to swaption data, we calibrate the IM model by comparing the model-implied P&L with the P&L implied by data of a CCP. A small residual gap remains and this is one of the aspects we are focusing on in ongoing research work. However, we do obtain a perfect match for the starting value IM_0 by multiplying the model number by a constant factor so as to match the clearing house IM at time 0.

Q.4: *What back-testing could be done within this approach?*

A: We first need to clarify what we would like to validate through back-testing. One of the ultimate goals of our developments is to estimate MVA. It would be interesting to back-test the MVA computations through the analysis of the realised P&L of the hedging strategy proposed by the model. This would be a joint test of the IM dynamics we propose and the cost of the IM, which is beyond the scope of our current analysis. For such a validation, daily strategies and the residual P&L would be computed on a daily basis for the life of different instruments. The underlying market is the IRS market, where the maturities are relatively long. We would need to collect at least several years of CCP (or other IM model) data to achieve the comparison.

Another validation that could be done is the comparison between the estimated IM distribution of our approach and the actual distribution at a given time interval in the future, let us say one year. Estimating the model distribution today (in the historical measure) and waiting one year to compare to the realised IM will give us only one data point. To have a meaningful comparison set, we probably need at least 100 points, which would mean, if we want the different experiments to be independent, waiting 100 years. We can probably do with a shorter time interval and overlapping data, but in any case, it will require historical data, including CCP data, for long periods.

A third meaningful notion of back-testing in our context would be to consider IM figures computed by a clearing house, say between 2013 and 2014, calibrate the adjusted IM model to this data and then compute the IM values with the model and the realised swap curves for the period 2014-15. Then one compares the IM values required by the clearing house in the period 2014-15 with those obtained with the IM model.

Q.5: *Why should one apply your IM model if a perfect match with IM data by clearing houses cannot be obtained?*

A: First, as far as we know, current industry methods for the computation of IM do not allow to compute the IM at a future point in time and, in addition, for an arbitrary long margin period of risk. Not having a method for the calculation of future initial margin payments prevents one to be in a position to compute MVA. Secondly, we do not assume that one knows what the algorithm is with which a clearing house computes IM. In such a situation, we need to develop an "in-house" model for the computation of IM and then calibrate it to IM values posted in the past by clearing houses and any additional historical data one may view relevant for the IM computation. Thirdly, one could be of the opinion that the methodology of a clearing house needs innovation even if one knew exactly what the clearing house algorithm is. The proposed IM approach could be just viewed as a kind of dynamical extension of a generic clearing house method for IM computations.

So, returning to the question of "back-testing" in the answer to Q.4 above, we provide a framework to calculate IM that is essentially based on taking a risk measure of a price increment over a margin period of risk given the information available at the time the IM is computed. Since the goal is to calculate the IM at a future point in time, it is reasonable to calibrate our underlying price model to options, which provide "forward-looking" market information. Our view is that this is another valuable feature in our approach. It is of course conceivable that clearing houses do not adopt such a dynamic and "forward-looking" methodology to compute the IM, and thus one can expect that the clearing house IM and our IM values differ *a priori* by too much. One could say that this is the whole point of developing a new approach to IM if it were felt that clearing houses should innovate the way they compute the IM. However, our approach can accommodate an adjustment such that IM data used by clearing houses can be fed into our model in order for the model IM to be closer to the IM values of the clearing houses. The current proposed procedure is explained in Section 4.

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