Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy

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Macro Reading Group
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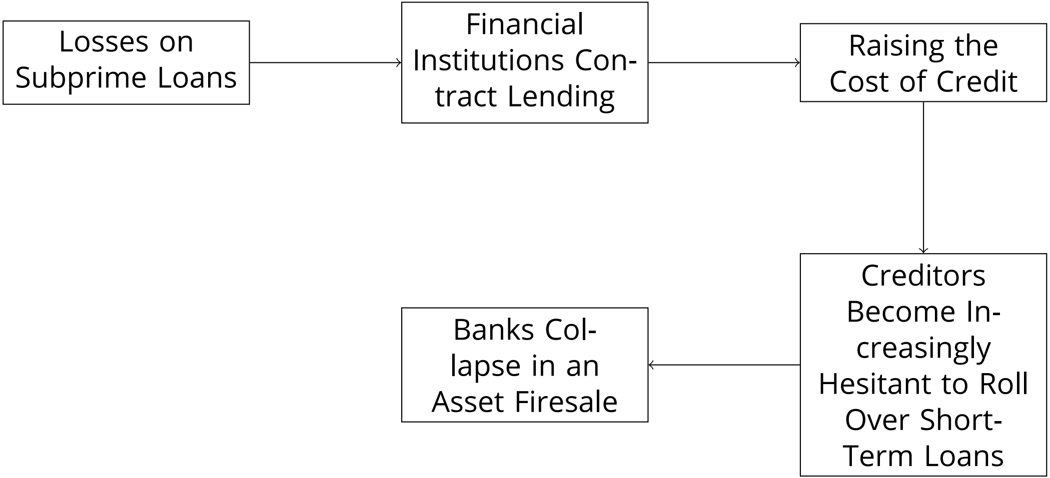
Roadmap

Great Recession Shadow Banks



Relevant Literature

There are three key strands of literature present in this paper:

- Macroeconomic Models with a Banking Sector and Liqudity Risk (Gertler and Karadi (2011), Gertler and Kiyotaki (2011))
- Bank Runs (Diamond and Dybvig (1983), Cole and Kehoe (2000))
- Macroeconomic Models with Bank Runs (Ennis and Kiester (2003), Martin, Skeie, and von Thadden (2014))

Relationship to BGG

Recall the notion of a financial accelerator in **Bernanke, Gertler and Gilchrist (1996)**:

Definition

A financial accelerator is a mechanism by which shocks to the economy are propagated and amplified through credit markets

We will add to this financial accelerator by including a notion of bank runs, which further propagates economic shocks.

Roadmap

Environment

- **Agents**: Households and Bankers, both of measure 1
- **Time**: Discrete, $t = 0, 1, 2, ...$
- **Goods**: A nondurable good and **capital**, a durable asset
  - Capital does not depreciate and is fixed in total supply
  - Capital is held by both banks and households, with $K_t^b + K_t^h = 1$
- **Shocks**: Z_t is the shock to capital productivity, it is **aggregate**

Capital Evolution

The capital stock evolves differently depending on if it is held by **households** or **banks**:

Definition

For banks, capital stock evolves as

$$K_t^b \to \begin{cases} Z_{t+1} K_t^b \text{ output} \\ K_t^b \text{ capital} \end{cases} \tag{1}$$

Definition

For households, capital stock evolves as

$$\begin{cases} K_t^h \text{ capital} \\ f(K_t^h) \text{ goods} \end{cases} \to \begin{cases} Z_{t+1} K_t^h \text{ output} \\ K_t^h \text{ capital} \end{cases} \tag{2}$$

Why do bank runs occur?

Bank runs in this model are not classic bank runs, as in Diamond-Dybvig, but rather are generated by banks' inability to cover short term liabilities with long-term imperfectly liquid assets.

There are two key features of the model that lead to bank runs:

- Financing constraints on banks
- Inefficiencies in the home storage of capital

We will be highlighting these more in the coming sections of the presentation, as households and banks handle runs differently.

Roadmap

Households in Bank Runs

Definition

$$R_{t+1} \begin{cases} \bar{R}_{t+1} \text{ if no bank run} \\ x_{t+1}\bar{R}_{t+1} \text{ if bank run} \end{cases} \tag{3}$$

Household Problem

Definition

The household problem can be expressed as the following:

$$E_t\left[\sum_{i=0}^{\infty}\beta^i \ln(C^h_{t+i})\right]$$
$$\text{subject to } C^h_t + D_t + Q_t K^h_t + f(K^h_t) = Z_t W^h + R_t D_{t-1} + (Z_t + Q_t)K^h_{t-1} \quad (4)$$

Importantly, R_t is the return on bank deposits in the **no run environment**, so we are assuming that households do not anticipate bank runs. This assumption is relaxed in later sections.

First Order Conditions for Households

- Deposits (Household-Bank Interaction): $E_t(\Lambda_{t,t+1})R_{t+1} = 1$ where $\Lambda_{t,t+1} = \beta^i \frac{C_t^h}{C_{t+i}^h}$
- Direct Capital Holdings of Households: $E_t(\Lambda_{t,t+1} R_{t+1}^h) = 1$ with $R_{t+1}^h = \frac{Z_{t+1}+Q_{t+1}}{Q_t+f'(K_t^h)}$

As $f'(K_t^h)$ is increasing in capital held by the household, which means in the limiting case of a bank run, the price of capital drops sharply.

Roadmap

Timing of Bank Decisions

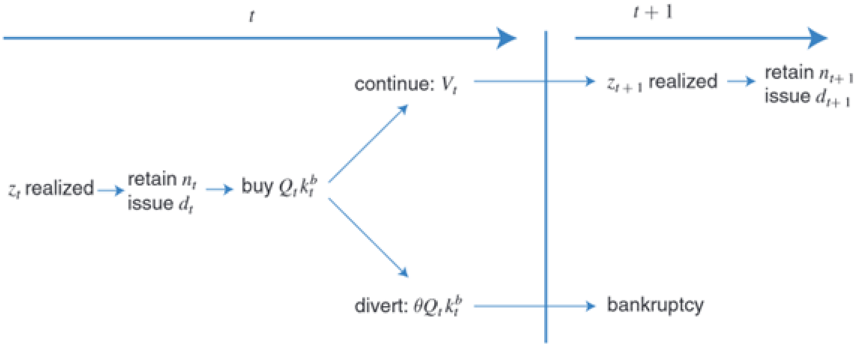


Figure 1. Timing

Figure: Timing of Bank Decisions

Banker Problem

Definition

The banker problem can be expressed as the following:

$$V_t = E_t \left[\sum_{i=1}^{\infty} \beta^i (1-\sigma)\sigma^{i-1} c_{t+i}^b \right] \tag{5}$$

- Bankers only consume in the last period of existence
- Bankers are risk neutral
- Bankers are finitely lived, dying with probability $1-\sigma$ each period (TV Condition)

Banker Net Worth Evolution

Bankers have evolving net worth based on their survival time:

- Bankers Born in Period t: $n_t = w^b$, net worth is equal to initial endowment
- Bankers Surviving from Period $t-1$: $n_t = (Z_t + Q_t)k^b_{t-1} - R_{t-1}D_{t-1}$, net worth is equal to return on capital from the previous period minus deposits
- Bankers Dying in Period t: $c^b_t = n_t$, bankers consume all their net wealth in the period they die

Deposit Constraint

Bankers finance their asset holdings with deposits and equity, so $Q_t k^b_t = d_t + n_t$

Moral Hazard Problem

Banks need a reason to limit the amount of issued deposits

Definition

Given the assumption that bankers can take $\theta Q_t k_t^b$ out of the bank without the investors knowing, we can write the **incentive constraint** for bankers as:

$$\theta Q_t k_t^b \leq V_t \tag{6}$$

If this is not satisfied, households will not loan to firms as they have incentive to cheat.

Restating the Banker Problem

Armed with all of this new information, we can restate the banker problem as:

Banker Problem

$$
\begin{aligned}
&V_t = E_t\left[\beta(1-\sigma)n_{t+1} + \beta\sigma V_{t+1}\right] \\
&\text{subject to } \theta Q_t k_t^b \leq V_t \\
&\text{and } Q_t k_t^b = d_t + n_t \\
&\text{and } n_t = (Z_t + Q_t)k_{t-1}^b - R_t D_{t-1}
\end{aligned}
$$

Constraint Math

This is all extra math to generate a reframing of the Bank problem that we will see in the following slides.

Growth Rate of Net Worth

$$\frac{n_{t+1}}{n_t} = \frac{Z_{t+1} + Q_{t+1}}{Q_t} \frac{Q_t k_t^b}{n_t} - R_{t+1} \frac{d_t}{n_t}$$
$$= (R_{t+1}^b - R_{t+1})\phi_t + R_{t+1}$$

subject to $R_{t+1}^b = \frac{Z_{t+1}+Q_{t+1}}{Q_t}$ and $\phi_t = \frac{Q_t k_t^b}{n_t}$

R_{t+1}^b can be thought of as the realized rate of return on bank assets, and ϕ is the ratio of assets to net worth.

Tobin's Q Ratio

Reframing of Tobin's Q

$$\begin{aligned}\psi_t &= \max_{\phi_t} E_t\{\beta(1-\sigma+\sigma\psi_{t+1})\left[(R^b_{t+1}-R_{t+1})\phi_t + R_{t+1}\right]\} \\ &= \max_{\phi_t}\{\mu_t\phi_t + \nu_t\}\end{aligned}$$

subject to the incentive constraint $\theta\phi_t \leq \psi_t = \mu_t\phi_t + \nu_t$ where $\mu_t = E_t\left[\beta\Omega_{t+1}(R^b_{t+1} - R_{t+1})\right]$ (excess marginal value of assets over deposits) and $\nu_t = E_t\left[\beta\Omega_{t+1}\right]R_{t+1}$ (marginal cost of deposits) with $\Omega_{t+1} \equiv 1-\sigma+\sigma\psi_{t+1}$ (discount factor weighting).

Relationship Between Incentive Constraint and μ_t

Consider the following two environments, one where the incentive constraint exists and the other where it does not:

- **No Incentive Constraint**: Banks consume all excess return because they finance everything with deposits (infinite arbitrage), $R_t^b = R_t$ which then means $\mu_t = 0$. **This is a frictionless, no run environment.**
- **Incentive Constraint**: This means that $\mu > 0$, and banks will finance capital with their net worth after the incentive constraint binds, which is a textbook **financial accelerator**! If depositors choose not to roll over their deposits, this can be a bank run!

Aggregation (1)

Assets and Net Worth

$$Q_t K_t^b = \phi_t N_t$$

Evolution of Net Worth

$$N_t = \sigma \left[(Z_t + Q_t) K_{t-1}^b - R_t D_{t-1} \right] + W^B$$

Aggregation (2)

Total Output

$$Y_t = Z_t + Z_t W^h + W^b$$

Economy Budget Constraint

$$Y_t = f(K_t^h) + C_t^h + C_t^b$$

Roadmap

Households are the problem!

Households choose not to roll over their deposits if:

- If they percieve other households will not do the same
- If the forced liquidation makes the banks insolvent

Condition for Runs to Exist

For runs to exist, we need to have the following condition:

Condition for Runs

$$x_t = \frac{R^*_{t+1}}{R_{t+1}} = \frac{(Z_t + Q^*_t)K^b_{t-1}}{R_t D_{t-1}} < 1$$

$$x_t = \frac{R^{b*}_{t+1}}{R_{t+1}} \frac{\phi_{t-1}}{\phi_{t-1} - 1} < 1$$

with $R^{b*}_{t+1} = \frac{Z_t + Q^*_t}{Q_{t-1}}$ and ϕ_t equal to the leverage mutliple, $\frac{Q_t K^b_t}{N_t}$.

Run Threshold

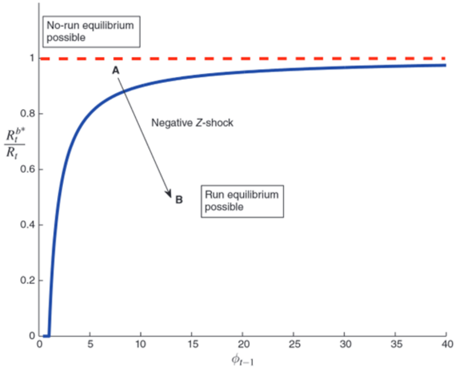


FIGURE 2. RUN THRESHOLD

Figure: Run Threshold

Bank Runs!

A bank run in t works in the following way:

1. Households choose not to roll over their deposits en masse
2. Banks are forced to liquidate assets to pay off depositors, moving **all** capital to the household sector, $K_t^H = 1$

Banker Net Worth

$$N_{t+1} = W^b + \sigma(W^b)$$
$$N_{t+j} = W^b + \sigma((Z_{t+j} + Q_{t+j})K^b_{t+j-1} - R_{t+j}D_{t+j-1}) \; \forall i \geq 2$$

Liquidation Price

Through some algebra, we can derive the liquidation price of capital for the household:

Q^*

$$Q_t^* = E_t\left[\sum_{i=0}^{\infty} \Lambda_{t,t+i}(Z_{t+i} - \alpha K_{t+i}^h)\right] - \alpha \tag{7}$$

- Z_t manifests in the equation, so Q_t^* is vulnerable to fluctuations
- We notes that Q_t^* is **decreasing** in α
- Q_t^* is **increasing** in K_t^h! The longer the banks remain insolvent, the lower the price of the fire sale of capital.

Liquidity Shortage and Insolvency

If a bank run equilibrium exists:

- Banks are insolvent when $Q_t = Q_t^*$

If a bank run equilibrium does not exist:

- Banks are solvent when $Q_t = Q_t$

Asset price is based on liquidity which is effected by runs!

Roadmap

Impulse Response Functions - No Runs

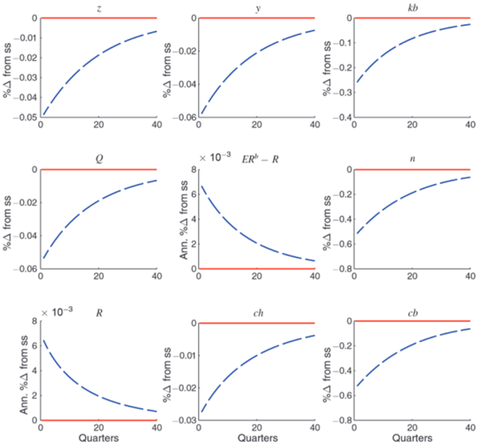


Figure 3. A Recession in the Baseline Model: No Bank Run Case

Figure: Impulse Response Functions

Impulse Response Functions - Runs

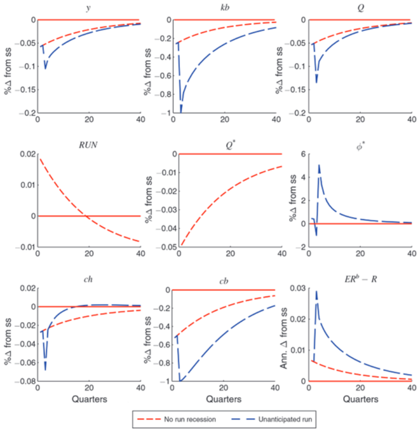


FIGURE 4. EX POST BANK RUN IN THE BASELINE MODEL

Figure: Impulse Response Functions

Anticipated Runs

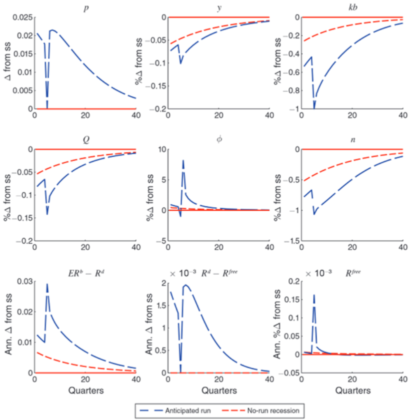


Figure 6. Recession with Positive Run Probability and Ex Post Run

Figure: Impulse Response Functions

Roadmap

What do we do?

The authors present problems, but they also present solutions:

- Deposit Insurance: Not possible in the model due to violation of the incentive constraint
- **Capital Requirements**
- **Lender of Last Resort**

Conclusion

We have generated a model that has a traditional financial accelerator **and** liquidity mismatch leading to instability in the banking sector!

Questions?