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MATH 300: Section H01
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MATH 300 Homework Set 1

Problem 1

(a) **Not Proposition:** This is not declarative but a question.

(b) **Proposition:** This is a declarative statement, and while it may be difficult to determine the truth value in 2021, it definitely has either the value true or false.

(c) **Proposition:** Like (b), this is declarative, and is a historical fact so it either has the value true or false but not both.

(d) **Proposition:** This is declarative and has the value of false, but not true.

(e) **Not Proposition:** This is a propositional expression, but without a value assigned to x it is not a proposition because it does not make sense to compare the value an unassigned variable to a real number.

(f) **Proposition:** This is a declarative sentence, and while we do not yet know which value it takes on, it will be definitively either true or false after 2025.

(g) **Proposition:** This is a declarative sentence that is false but not true.

(h) **Not Proposition:** This is imperative rather than declarative, and thus cannot have a truth value.

(i) **Proposition:** Like (f), this is a declarative sentence that we do not yet know the truth value of yet but will when the time period ends. Technically, this is a propositional expression, but it is assumed that the starting point is whenever the sentence is said.

(j) **Proposition:** This is a declarative sentence that is true but not false.

(k) **Not Proposition:** This is a declarative sentence, but it is not a proposition because it can be neither true nor false. If you assume it is true, it says that it must be false, which leads to a contradiction. If you assume it is false, this would mean that it must be true, which also leads to a contradiction. Thus, it can hold neither truth value and cannot be a proposition.

(l) **Not Proposition:** If you assume the statement is true, it says that the statement is not one of the false statements in the homework set, so it is true there is no contradiction. If you assume the sentence is false, this means that the statement is one of the false statements in the homework set, so there is also no contradiction. The statement can be both true and false, and thus cannot be a proposition.

Problem 2

(a)

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

(b)

P	Q	R	$P \wedge Q$	$P \wedge R$	$(P \wedge Q) \vee (P \wedge R)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	F	F	F
F	F	F	F	F	F

(c)

P	Q	$\neg Q$	$Q \vee \neg Q$	$P \wedge (Q \vee \neg Q)$
T	T	F	T	T
T	F	T	T	T
F	T	F	T	F
F	F	T	T	F

(d)

P	Q	$P \wedge Q$	$\neg(P \wedge Q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

(e)

P	Q	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

(f)

P	Q	R	$\neg Q$	$P \vee \neg Q$	$(P \vee \neg Q) \wedge R$
T	T	T	F	T	T
T	T	F	F	T	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	T	T	T
F	F	F	T	T	F

Problem 3

(a) **Equivalent:** By the idempotent laws.

(b) **Not Equivalent:** They take different truth values when Q is false.

(c) **Not Equivalent:** They take different truth values when P and Q have different truth values from each other.

(d) **Equivalent:** By the associative laws.

(e) **Equivalent:** By De Morgan's laws.

(f) **Not Equivalent:** They take different truth values when P is true and Q and R are false.

Problem 4

(a) $A \oplus \neg B$ —— **True** —— A is true and $\neg B$ is false, so exactly one is true.

(b) $\neg C \wedge \neg A$ —— **False** —— $\neg A$ is false, so the conjunction is false.

(c) $A \wedge \neg C$ —— **True** —— Both A and $\neg C$ are true, so the conjunction is true.

(d) $\neg A \wedge B \wedge \neg C$ —— **False** —— $\neg A$ is false, so the conjunction is false.

Problem 5

P is not a proposition. First, assume P is true. Then, Q is true, which means that P must be false, so a contradiction ensues. Then, assume P is false. Then, Q is false, which means that P must be true, so a contradiction ensues. Thus, P can be neither true nor false, and as a result cannot be a proposition.

Problem 6

(a) **Neither:** The proposition is true when both P and Q are true, but false when they take on different truth values.

(b) **Tautology:** $P \wedge \neg P$ is false by the negation laws, so negating this leads to a tautology.

(c) **Tautology:** Let $C = (A \wedge B) \vee (A \wedge \neg B) \vee (\neg A \wedge B) \vee (\neg A \wedge \neg B)$. C always evaluates to true as shown below, so it is a tautology.

A	B	$\neg A$	$\neg B$	$A \wedge B$	$A \wedge \neg B$	$\neg A \wedge B$	$\neg A \wedge \neg B$	C
T	T	F	F	T	F	F	F	T
T	F	F	T	F	T	F	F	T
F	T	T	F	F	F	T	F	T
F	F	T	T	F	F	F	T	T

(d) **Neither:** The second part of the conjunction depends on R while the first does not, so the proposition can be neither a tautology nor a contradiction.

(e) **Neither:** The proposition is true when P is true and R and Q are false, but it is false when all three variables are false. Thus, it can be neither a tautology nor a contradiction.

Problem 7

(a) **Tautology:** Both A and $\neg B$ must be true, so the conjunction is true and a tautology.

(b) **Tautology:** $\neg A$ is false, so the conjunction must be false. Negating the conjunction leads to true and a tautology.

Problem 8

(a) We will win neither the first game nor the second one.

(b) 641,371 is not a composite integer.

(c) $x > y$ and $m^2 > 1$.

(d) Sue will not choose yoghurt or will choose ice cream.

(e) n is odd or n is a multiple of 5.

(f) None of the natural numbers x, y, z are prime.

(g) The function f does not have positive first or second derivatives at x_0.

Problem 9

(a) $(\neg(\neg P)) \vee ((\neg Q) \wedge (\neg S))$

(b) $(Q \wedge (\neg S)) \vee ((\neg(\neg(P)) \wedge Q)$

(c) $(P \wedge (\neg Q)) \vee ((\neg P) \wedge (\neg R)) \vee ((\neg P) \wedge S)$

(d) $\neg P \vee (Q \wedge (\neg(\neg(P)) \wedge Q) \vee R$

Problem 10

The last two columns have the same truth value at every line below, so they are equivalent.

A	B	$A \vee B$	$A \wedge B$	$\neg(A \wedge B)$	$(A \wedge B) \wedge \neg(A \wedge B)$	$A \oplus B$
T	T	T	T	F	F	F
T	F	T	F	T	T	T
F	T	T	F	T	T	T
F	F	F	F	T	F	F

Problem 11

(a) If you have the flu, you miss the final examination.

(b) You do not miss the final examination if and only if you pass the course.

(c) If you miss the final examination, you do not pass the course.

(d) You have the flu, you miss the final examination, or you pass the course.

(e) If you have the flu, you do not pass the course, or if you miss the final examination, you do not pass the course.

(f) You have the flu and you miss the final examination, or you do not miss the final examination and you pass the course.

Problem 12

(a) $r \wedge \neg q$

(b) $p \wedge q \wedge r$

(c) $p \Rightarrow r$

(d) $p \wedge \neg q \neg r$

(e) $(p \wedge q) \Rightarrow r$

(f) $r \Leftrightarrow (p \oplus q)$

Problem 13

(a) **True:** Both are true.

(b) **False:** The first is true and the second is false.

(c) **True:** It can never be true that it is winter at the same time as spring, summer, or fall, so the biconditional is true.

(d) **True:** Both are false.

(e) **True:** Both are true.

Problem 14

(a) If you get promoted, then you washed the boss's car. If you do not wash the boss's car, then you do not get promoted.

(b) If there are winds from the south, then there will be spring thaw. If there is not spring thaw, then there were not winds from the south.

(c) If you bought the computer less than a year ago, then the warranty is good. If the warranty is not good, then you bought the computer at least a year ago.

(d) If Willy cheats, then he gets caught. If Willy does not get caught, then he did not cheat.

(e) If you can access the website, then you paid a subscription fee. If you did not pay a subscription fee, then you cannot access the website.

(f) If you know the right people, then you will get elected. If you do not get elected, then you do not know the right people.

(g) If Carol is on a boat, she will get seasick. If Carol is not seasick, she is not on a boat.

Problem 15

(a) **Converse**: If I stay at home, then it snowed tonight.

Contrapositive: If I do not stay at home, then it did not snow tonight.

Inverse: If it does not snow tonight, then I will not stay at home.

(b) **Converse:** It is a sunny summer day whenever I go to the beach.

Contapositive: If I do not go to the beach, it is not a sunny summer day.

Inverse: I do not go to the beach whenever it is not a sunny summer day.

(c) **Converse:** If I sleep until noon, I stayed up late.

Contrapositive: If I do not sleep until noon, I did not stay up late.

Inverse: If I do not stay up late, I do not sleep until noon.

Problem 16

(a) The last two columns below have the same value at every line, so the propositions are logically equivalent.

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg p \wedge \neg q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$	$p \Leftrightarrow q$
T	T	F	F	T	F	T	T
T	F	F	T	F	F	F	F
F	T	T	F	F	F	F	F
F	F	T	T	F	T	T	T

(b) The last two columns below have the same value at every line, so the propositions are logically equivalent.

p	q	r	$p \Rightarrow q$	$p \Rightarrow r$	$q \wedge r$	$(p \Rightarrow q) \wedge (p \Rightarrow r)$	$p \Rightarrow (q \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	F	F	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

(c) Every line in the last column is true, so the proposition is a tautology.

p	q	r	$\neg p$	$p \vee q$	$\neg p \vee r$	$(p \vee q) \wedge (\neg p \vee r)$	$(q \vee r)$	$p \vee q) \wedge (\neg p \vee r) \Rightarrow q \vee r$
T	T	T	F	T	T	T	T	T
T	T	F	F	T	F	F	F	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T	T
F	F	T	T	F	T	F	T	T
F	F	F	T	F	T	F	F	T

Problem 17

(a) This is a list of propositions, because each sentence is declarative, and because the only way to assign truth values in a way that does not lead to contradictions is to say that "Exactly 399 of the sentences on this list are false" is true and that every other sentence is false. Not more than 1 sentence in this list can be true since they all state different numbers, and one of them must be true or else the 400^{th} statement would be false. The only way that exactly 1 statement can be true is that 399 of them are false, so this is the only valid truth assignment.

(b) This is a list of propositions because there is only one way to assign truth values consistently. If k sentences on the list are false, then the first k sentences on this will be true and the last $400 - k$ will be false. This is because if it is true that there are exactly k of something, then there must be at least $k - 1$ of it down to at least 1 of it, but there is not at least $k + 1$ of it. Thus, we need k sentences to be false, and we need them to be the last $400 - k$ sentences. Using the equation $k = 400 - k$ and solving for k yields $k = 200$, so the only valid assignment of truth values is for the first 200 statements to be true and the last 200 to be false.

(c) This is not a list of propositions because there is no way to assign truth values. We can use similar reasoning to part (b), but we need to replace 400 by 399. By the same logic, we need to solve k for $k = 399 - k$, which yields $k = 199.5$. Obviously, there is no way to have 199.5 of the sentences be false, so there is no way to assign truth values to the sentences in a consistent way.