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SCHC 501 - Homework Ch. 13

1a. Give the derivation tree for the following statements of SL according to the syntax given in (13-1) and (13-2).

i. $((\sim p \,\&\, q) \vee p)$

ii. $(\sim(p \,\&\, q) \vee p)$

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iii. $\sim((p \& q) \vee p)$

b. Give the compositional semantic interpretation of the three trees of (1a), assuming that both atomic statements are true.

i. $((\sim p \& q) \vee p)$

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ii. $(\sim(p \& q) \vee p)$

iii. $\sim((p \& q) \vee p)$

2. Assume the alternative syntax for SL, which has, besides the category for atomic statements, two other categories for connectives: Neg for negation which combines with only one statement at a time, and Conn for all other four connectives which

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combine with two statements at a time.

a. Write the two new syntactic rules for this syntax of SL.

(i) If f is a connective in Neg and ϕ is a wff, then $f(\phi)$ is a wff.

(ii) If f is a connective in Conn and ϕ and ψ are wffs, then $f(\phi, \psi)$ is a wff.

b. Give the derivation tree for the three statements given under 1a.

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i. $((\sim p \,\&\, q) \vee p)$

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ii. $(\sim(p \& q) \vee p)$

iii. $\sim((p \& q) \vee p)$

To give SL with this syntax a compositional semantics we first have to interpret the connectives according to their truth values as functions taking either one or a pair of truth values, which is/are the values of their constitutive statement(s), and assigning a truth value. For instance, conjunction is interpreted by the following function:

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$\langle 1, 1 \rangle \to 1$

$\langle 1, 0 \rangle \to 0$

$\langle 0, 1 \rangle \to 0$

$\langle 0, 0 \rangle \to 0$

c. Give the interpretations of the other connectives of SL as such functions.

$\sim$:	$\vee$:	$\to$:	$\leftrightarrow$:
$1 \to 0$	$\langle 1, 1 \rangle \to 1$	$\langle 1, 1 \rangle \to 1$	$\langle 1, 1 \rangle \to 1$
$0 \to 1$	$\langle 1, 0 \rangle \to 1$	$\langle 1, 0 \rangle \to 1$	$\langle 1, 0 \rangle \to 0$
	$\langle 0, 1 \rangle \to 1$	$\langle 0, 1 \rangle \to 0$	$\langle 0, 1 \rangle \to 0$
	$\langle 0, 0 \rangle \to 0$	$\langle 0, 0 \rangle \to 1$	$\langle 0, 0 \rangle \to 1$

d. Write the two compositional semantic rules for SL with this syntax. Hint: Use the convention of $[\![P]\!]$ standing for the semantic interpretation of P.

(i) If f is a connective in Neg and ϕ is a wff, then $[\![f(\phi)]\!] = f([\![\phi]\!])$.

(ii) If f is a connective in Conn and ϕ and ψ are wffs, then $[\![f(\phi, \psi)]\!] = f([\![\phi]\!], [\![\psi]\!])$.

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e. Show how each of the statements of (1a) is interpreted compositionally according to the three trees you gave in (2b).

i. $((\sim p \,\&\, q) \vee p)$

ii. $(\sim(p \,\&\, q) \vee p)$

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iii. $\sim((p \& q) \vee p)$

6. Assume the following constants and variables:

Type	Variables	Constants
e	x, y, z	j, m
$\langle e,t\rangle$	P, Q	walks, man
$\langle e,\langle e,t\rangle\rangle$	R	loves
$\langle\langle e,t\rangle,t\rangle$	P	someone, everyone

Check whether the following expressions are well-formed expressions of TL. If so, specify their type. Draw derivation trees for at least three of the well-formed ones.

a. $\lambda x\ \text{loves}(m)(x)$: Well-formed, type $\langle e,t\rangle$

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λx loves(m)(x), <e,t>

x, e

loves(m)(x), <e,t>

loves(m), <e,t>

x, e

loves, <e,<e,t>>

m, e

b. λR R(m)(j): well-formed, type t

λR R(m)(j), t

R, <e,<e,t>>

R(m)(j), t

R(m), <e,t>

j, e

R, <e,<e,t>>

m, e

c. Someone(walks): Well-formed, type t

Someone(walks), t

someone, <<e,t>,t>

walks, <e,t>

d. λP P(j): well-formed, type <<e,t>,t>

e. λx someone(x): Not well-formed, someone does not take argument of type e

f. λx(P(x) & Q(x)): Well-formed, type <e,t>

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g. $\lambda P(P(j) \& P(m))$: well-formed, type $\langle\langle e,t\rangle,t\rangle$

h. $(\lambda P\, P(j))(walk(s))$: well-formed, type t

i. $\lambda x\, \exists y\, loves(x)(y)$; well-formed, type $\langle e,t\rangle$

j. $\lambda P(\forall P(P(P) \leftrightarrow P(j)))$; well-formed, type $\langle\langle\langle e,t\rangle,t\rangle,t\rangle$

7. Using the same variable-conventions as above, and the constants and semantic interpretation of TLA given in 13.2.3, specify for each of the following the type and a description of the semantic value for M and g as given in (13-36).

Optional abbreviations: Plus(a,b) for Plus(b)(a)
gr(a,b) for gr(b)(a)

a. $(\lambda x\, gr(x,5))$: Type $\langle e,t\rangle$

The value is the set of numbers greater than 5.

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b. $(\lambda x\,(odd(x) \,\&\, gr(x,5)))(8)$: Type t

8 is not odd, so the value is false.

c. $(\lambda x\ Plus(x,3))(4)$: Type e

The value is 7.

d. $(\lambda x\,(\lambda y\ gr(y,x)))(5)(8)$: Type t

5 is the entity at which $(\lambda x\,(\lambda y\ gr(y,x)))$ is being evaluated, so this is equivalent to $(\lambda y\ gr(y,5))(8)$. This is equivalent to $gr(8,5)$, so the value is true.

e. $(\lambda x\,(\lambda y\ gr(y,x))(5))(8)$: Type t

5 is the entity at which $(\lambda y\ gr(y,x))$ is being evaluated, so this is equivalent to $(\lambda x\ gr(5,x))(8)$. This is equivalent to $gr(5,8)$, so the value is false.

f. $\lambda x\ \exists y\,(Plus(y,y) = x)$: Type $\langle e,t\rangle$

The value is the set of natural numbers x such that $x = y + y$ for some natural number y. This is exactly the set of even natural numbers.

g. $(\lambda P\ P(8))(\lambda x\ gr(x,5))$: Type t

This is equivalent to $(\lambda x\ gr(x,5))(8)$, which is equivalent to $gr(8,5)$. Thus, the value is true.

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