

# Chapter 6

## Problems 5 and 6

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## Exercise 5

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For each of the following, use the “quick falsification” method to find an assignment of truth values to the atomic statements which makes the entire statement false.

(a)  $(p \vee q)$

(b)  $((p \vee q) \rightarrow (p \& q))$

(c)  $(\sim (\sim q \vee p) \vee (p \rightarrow q))$

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**(a)** Need  $(p \vee q) = 0$ :

$$p = 0, q = 0$$

**(b)** Need  $(p \vee q) = 1, (p \& q) = 0$ :

$$p = 1, q = 0$$

**(c)** Need  $\sim (\sim q \vee p) = 0, (p \rightarrow q) = 0$ :

$$p = 1, q = 0$$

## Exercise 5

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For each of the following, use the “quick falsification” method to find an assignment of truth values to the atomic statements which makes the entire statement false.

(d)  $((((p \rightarrow q) \rightarrow r) \rightarrow s) \rightarrow (p \rightarrow q))$

(e)  $((p \vee q) \& (r \& s)) \leftrightarrow (((p \& q) \& r) \& s)$

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**(d)** Need  $((p \rightarrow q) \rightarrow r) \rightarrow s = 1$ ,  $(p \rightarrow q) = 0$ , so need  $((0 \rightarrow r) \rightarrow s) = 1$ , so need  $(1 \rightarrow s) = 1$ :

$$p = 1, q = 0, r = 0, s = 1$$

**(e)** Need  $((p \vee q) \& (r \& s)) = 1$ ,  $((p \& q) \& r) \& s = 0$ , so need  $(p \vee q) = 1$ ,  $(r \& s) = 1$ ,  $(p \& q \& r \& s) = 0$ , so need  $(p \& q) = 0$ :

$$p = 0, q = 1, r = 1, s = 1.$$

## Exercise 6

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Let  $p$ ,  $q$ , and  $r$  be atomic statements. Which of the following are tautologies, contradictions or contingent statements?

- (a)  $(p \vee \sim p)$
  - (b)  $(p \vee q)$
  - (c)  $((p \& q) \rightarrow (p \vee r))$
  - (d)  $(\sim p \& \sim (p \rightarrow q))$
  - (e)  $((p \vee r) \rightarrow \sim p)$
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- (a)** Tautology: Clear
- (b)** Contingent: 1 when  $p = q = 1$  but 0 when  $p = q = 0$
- (c)** Tautology:  $(p \& q) \Rightarrow p \Rightarrow (p \vee r)$
- (d)** Contradiction:  $\sim (p \rightarrow q) \Rightarrow (p \& \sim q) \Rightarrow p$ , and  $(\sim p \& p)$  is a contradiction
- (e)** Contingent: 1 when  $p = r = 0$  but 0 when  $p = r = 1$