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MATH 300: Section H01
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MATH 300 Homework 2

Problem 1

(a) If I remember to send you the address, then you sent me an e-mail message.

(b) If you were born in the United States, then you are a citizen of this country.

(c) If you keep your textbook, then it will be a useful reference in your future courses.

(d) If the Red Wings' goalie plays well, then they will win the Stanley Cup.

(e) If you get the job, then you had the best credentials.

(f) If there is a storm, then the beach erodes.

(g) If you log on to the server, then you have a valid password.

Problem 2

(a) I remember to send you the address and you did not send me an e-mail message.

(b) You were born in the United States and you are not a citizen of this country.

(c) You keep your textbook, and it is not a useful reference in your future courses.

(d) The Red Wings' goalie plays well, and they do not win the Stanley Cup.

(e) You get the job, and you do not have the best credentials.

(f) There is a storm, and the beach does not erode.

(g) You log on to the server, and you do not have a valid password.

Problem 3

(a)

p	$\neg p$	$p \Rightarrow \neg p$
T	F	F
F	T	T

(b)

p	$\neg p$	$p \Leftrightarrow \neg p$
T	F	F
F	T	F

(c)

p	q	$p \lor q$	$p \oplus (p \lor q)$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

(d)

p	q	$p \land q$	$p \lor q$	$(p \land q) \Rightarrow (p \lor q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

(e)

p	q	$\neg p$	$q \Rightarrow \neg p$	$p \Leftrightarrow q$	$(q \Rightarrow \neg p) \Leftrightarrow (p \Leftrightarrow q)$
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

(f)

p	q	$\neg q$	$p \Leftrightarrow q$	$p \Leftrightarrow \neg q$	$(p \Leftrightarrow q) \oplus (p \Leftrightarrow \neg q)$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

Problem 4

(a)

$$
\begin{aligned}
p \Leftrightarrow q &\equiv (\neg p \vee q) \wedge (\neg q \vee p) && \text{(definition of biconditional)} \\
&\equiv (\neg p \wedge (\neg q \vee p)) \vee (q \wedge (\neg q \vee p)) && \text{(distributive law)} \\
&\equiv ((\neg p \wedge \neg q) \vee (\neg p \wedge p)) \vee ((q \wedge \neg q) \vee (q \wedge p)) && \text{(distributive law)} \\
&\equiv ((\neg p \wedge \neg q) \vee \mathbf{F}) \vee (\mathbf{F} \vee (q \wedge p)) && \text{(negation law)} \\
&\equiv (\neg p \wedge \neg q) \vee (q \wedge p) && \text{(identity law)} \\
&\equiv (p \wedge q) \vee (\neg p \wedge \neg q) && \text{(commutative law)}
\end{aligned}
$$

(b)

$$
\begin{aligned}
(p \Rightarrow q) \wedge (p \Rightarrow r) &\equiv (\neg p \vee q) \wedge (\neg p \vee r) && \text{(definition of conditional)} \\
&\equiv \neg p \vee (q \wedge r) && \text{(distributive law)} \\
&\equiv p \Rightarrow (q \wedge r) && \text{(definition of conditional)}
\end{aligned}
$$

(c)

$$
\begin{aligned}
(p \vee q) \wedge (\neg p \vee r) \Rightarrow (q \vee r) &\equiv \neg((p \vee q) \wedge (\neg p \vee r)) \vee (q \vee r) && \text{(definition of conditional)} \\
&\equiv \neg(p \vee q) \vee \neg(\neg p \vee r) \vee (q \vee r) && \text{(de Morgan's law)} \\
&\equiv (\neg p \vee \neg q) \vee (p \wedge \neg r) \vee q \vee r && \text{(de Morgan's and associative laws)} \\
&\equiv ((\neg p \wedge \neg q) \vee q) \vee ((p \wedge \neg r) \vee r) && \text{(associative law)} \\
&\equiv ((\neg p \vee q) \wedge (\neg q \vee q)) \vee ((p \vee r) \wedge (\neg r \vee r)) && \text{(distributive law)} \\
&\equiv ((\neg p \vee q) \wedge \mathbf{T}) \vee ((p \vee r) \wedge \mathbf{T}) && \text{(negation law)} \\
&\equiv (\neg p \vee q) \vee (p \vee r) && \text{(identity law)} \\
&\equiv \neg p \vee p \vee q \vee r && \text{(associative/commutative law)} \\
&\equiv \mathbf{T} \vee q \vee r && \text{(negation law)} \\
&\equiv \mathbf{T} && \text{(domination law)}
\end{aligned}
$$

Problem 5

(a) The truth values of the events are the same, so all three statements are true.

(b) Yes. (i) and (iii) are equivalent, because they are each other's contrapositives.

(c)

i. Joe went to a movie but did not go out to dinner.

ii. Joe went out to dinner but did not go to a movie.

iii. Joe did not go out to dinner but did go to the movie.

(d)

i. Going out to dinner is necessary for going to a movie.

ii. Going to a movie is necessary for going out to dinner.

iii. Not going to the movie is necessary for not going out to dinner.

(e)

i. Going to a movie is sufficient for going out to dinner.

ii. Going out to dinner is sufficient for going to a movie.

iii. Not going out to dinner is sufficient for not going to the movie.

(f)

i. Joe went to a movie only if he went out to dinner.

ii. Joe went out to dinner only if he went to a movie.

iii. Joe did not go out to dinner only if he did not go to the movie.

Problem 6

(a)

p	q	$p \downarrow q$
T	T	F
T	F	F
F	T	F
F	F	T

(b)

p	$p \downarrow p$	$\neg p$
T	F	F
F	T	T

(c)

p	q	$p \downarrow q$	$(p \downarrow q) \downarrow (p \downarrow q)$	$p \vee q$
T	T	F	T	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	F

In both (b) and (c), the truth tables are equivalent, so the statements are equivalent.

(d) The collection of operators $\{\vee, \neg\}$ is also functionally complete because $\wedge$ can always be expressed using negation and de Morgan's laws. Since both $\neg$ and $\vee$ can be expressed using $\downarrow$, and $\{\vee, \neg\}$ is functionally complete, then $\{\downarrow\}$ must also be functionally complete.

Problem 7

(a) There is a student in my school that has visited North Dakota.

(b) Every student in my school has visited North Dakota.

(c) There is no student in my school that has visited North Dakota.

(d) There is a student in my school that has not visited North Dakota.

(e) Not every student in my school has visited North Dakota.

(f) Every student in my school has not visited North Dakota.

Problem 8

(a) Every animal hops if it is a rabbit.

(b) Every animal is a rabbit and hops.

(c) There is an animal that hops if it is a rabbit.

(d) There is an animal that is a rabbit and hops.

Problem 9

(a) $(\exists x)(C(x) \land D(x) \land F(x))$

(b) $(\forall x)(C(x) \lor D(x) \lor F(x))$

(c) $(\exists x)(C(x) \land \neg D(x) \land F(x))$

(d) $\neg(\exists x)(C(x) \land D(x) \land F(x))$

(e) $(\exists x)(\exists y)(\exists z)(C(x) \land D(y) \land F(z))$

Problem 10

(a) **True:** $1 > 0$

(b) **True:** $0 > -2$

(c) **False:** $2 \geq 2$

(d) **True:** Since $Q(0)$ is true, there is an x for which $Q(x)$ is true.

(e) **False:** Since $Q(1)$ is false, not all x make $Q(x)$ true.

(f) **True:** Since $Q(1)$ is false, there is an x for which $Q(x)$ is not true.

(g) **False:** Since $Q(0)$ is true, not all x make $Q(x)$ not true.

Problem 11

(a) **True:** If $x = \sqrt{2}$, then the proposition is true, so there does exist an x that satisfies it.

(b) **False:** Since the universe consists only of real numbers, there is no x for which $x^2 = -1$.

(c) **True:** Since x^2 takes a minimum at 0 for the real numbers, $x^2 + 2$ will always be at least 2 and thus always greater than 1.

(d) **False:** The expression is not true for $x = 0$.

Problem 12

(a) $P(-2) \lor P(-1) \lor P(0) \lor P(1) \lor P(2)$

(b) $P(-2) \land P(-1) \land P(0) \land P(1) \land P(2)$

(c) $\neg P(-2) \lor \neg P(-1) \lor \neg P(0) \lor \neg P(1) \lor \neg P(2)$

(d) $\neg P(-2) \land \neg P(-1) \land \neg P(0) \land \neg P(1) \land \neg P(2)$

(e) $\neg(P(-2) \lor P(-1) \lor P(0) \lor P(1) \lor P(2))$

(f) $\neg(P(-2) \land P(-1) \land P(0) \land P(1) \land P(2))$

Problem 13

(a) $P(-5) \vee P(-3) \vee P(-1) \vee P(1) \vee P(3) \vee P(5)$

(b) $P(-5) \wedge P(-3) \wedge P(-1) \wedge P(1) \wedge P(3) \wedge P(5)$

(c) $P(-5) \wedge P(-3) \wedge P(-1) \wedge P(3) \wedge P(5)$

(d) $P(1) \vee P(3) \vee P(5)$

(e) $(\neg P(-5) \vee \neg P(-3) \vee \neg P(-1) \vee \neg P(1) \vee \neg P(3) \vee \neg P(5)) \wedge (P(-5) \wedge P(-3) \wedge P(-1))$

Problem 14

Let $S(x)$ be the statement "x is in the class". For parts (i), let the universe consist of all students s in the class. For parts (ii), let the universe consist of all people p.

(a) Let $P(x)$ be the statement "x has a cellular phone".

i. $(\forall s)P(s)$

ii. $(\forall p)(P(p) \wedge S(p))$

(b) Let $M(x)$ be the statement "x has seen a foreign movie".

i. $(\exists s)M(s)$

ii. $(\exists p)(M(p) \wedge S(p))$

(c) Let $W(x)$ be the statement "x can swim".

i. $(\exists s)\neg W(s)$

ii. $(\exists p)(\neg W(p) \wedge S(p))$

(d) Let $Q(x)$ be the statement "x can solve quadratic equations".

i. $(\forall s)Q(s)$

ii. $(\forall p)(Q(p) \wedge S(p))$

(e) Let $R(x)$ be the statement "x wants to be rich".

i. $(\exists s)\neg R(s)$

ii. $(\exists p)(\neg R(p) \wedge S(p))$

Problem 15

(a) Let $U(x)$ be the statement "x has visited Uzbekistan". The universe of discourse consists of all students s in the school.

i. $(\exists s)U(s)$

ii. $\neg(\forall s)\neg U(s)$

(b) Let $C(x)$ be the statement "x has studied calculus" and $P(x)$ be the statement "x has studied C++". The universe of discourse consists of all classmates c in the class.

i. $(\forall c)(C(c) \wedge P(c))$

ii. $\neg(\exists c)\neg(C(c) \wedge P(c))$

(c) Let $B(x)$ be the statement "x owns a bicycle" and $M(x)$ be the statement "x owns a motorcycle". The universe of discourse consists of all students s in the school.

i. $\neg(\exists s)(B(s) \wedge M(s))$

ii. $(\forall s)\neg(B(s) \wedge M(s))$

(d) Let $H(x)$ be the statement "x is happy". The universe of discourse consists of all students s in the school.

i. $(\exists s)\neg H(s)$

ii. $\neg(\forall s)H(s)$

(e) Let $T(x)$ be the statement "x was born in the twentieth century". The universe of discourse consists of all students s in the school.

i. $(\forall s)T(s)$

ii. $\neg(\exists s)\neg T(s)$

Problem 16

Let $P(x)$ be the statement "x is in the right place" and $C(x)$ be the statement "x is in excellent condition".

(a) Let the universe be all things t: $(\exists t)\neg P(t)$

(b) Let the universe be all tools t: $(\forall t)(P(t) \wedge C(t))$

(c) Let the universe be all things t: $\neg(\exists t)(P(t) \wedge C(t))$

(d) Let the universe be your tools t: $(\exists t)(\neg P(t) \wedge C(t))$

Problem 17

(a) There is a system that is open.

(b) Every system is malfunctioning or diagnostic.

(c) There is a system that is open or there is a system that is diagnostic.

(d) There is a system that is not available.

(e) Every system is not working.

Problem 18

(a) There is a real number x such that for all real y, $xy = y$.

(b) For every real, non-negative number x and real, negative number y, $x - y > 0$.

(c) For every real x and y, there exists a real number z such that $x = y + z$.

Problem 19

(a) There is a computer science class the school offers that a student at the school has taken.

(b) There is a student at the school that has taken every computer science class the school offers.

(c) Every student at the school has taken a computer science class the school offers.

(d) The school offers a computer science class that every student at the school has taken.

(e) Every computer science class the school offers has been taken by a student at the school.

(f) Every computer science class the school offers has been taken by every student at the school.

Problem 20

(a) Randy Goldberg enrolled in CS252.

(b) There is a student at the school that enrolled in Math 695.

(c) There is a class at the school that Carol enrolled in.

(d) There is a student in the school that enrolled in both Math 222 and CS252.

(e) At the school, there is a student x and a different student y such that if student x is enrolled in any class offered by the school, student y will be enrolled in the same class.

(f) There are two different students at the school that are enrolled in all the same classes offered by the school.

Problem 21

(a) $(\forall x)F(x, \text{Fred})$

(b) $(\forall y)F(\text{Evelyn}, y)$

(c) $(\forall x)(\exists y)F(x, y)$

(d) $\neg(\exists x)(\forall y)F(x, y)$

(e) $(\forall y)(\exists x)F(x, y)$

(f) $\neg(\exists x)(F(x, \text{Fred}) \wedge F(x, \text{Jerry}))$

(g) $(\exists y)(\exists z)((y \neq z) \wedge F(\text{Nancy}, y) \wedge F(\text{Nancy}, z)) \wedge \neg(\exists a)((y \neq a \neq z) \wedge F(\text{Nancy}, a))$

(h) $(\exists y)(\forall x)F(x, y) \wedge \neg(\exists z)(\forall x)((y \neq z) \wedge F(x, z))$

(i) $\neg(\exists x)F(x, x)$

(j) $(\exists x)(\exists y)((x \neq y) \wedge F(x, y)) \wedge \neg(\exists z)((x \neq z) \wedge (y \neq z) \wedge F(x, z))$

Problem 22

(a) There is a real number x such that for every real number y, $x + y = y$.

(b) For every real, non-negative number x and real, negative number y, $x - y > 0$.

(c) For every real, non-positive number x, there is a real, non-positive number y such that $x - y > 0$.

(d) For every real, non-zero number x and real, non-zero numbers y, $xy \neq 0$.

Problem 23

(a) **False:** $2 \neq 0$

(b) **True:** $2 = 2$

(c) **False:** (a) is a counterexample

(d) **False:** No matter what x is, the two sides of the equation will have a difference of 4

(e) **True:** (b) is an example

(f) **True:** For all x, $Q(x, 0)$ is true

(g) **True:** $Q(x, 0)$ is true for all x

(h) **False:** (d) is a counterexample

(i) **False:** Every false statement above is a counterexample

Problem 24

In parts (i), the universe of discourse consists of all integers. In parts (ii), the universe of discourse consists of all real numbers.

(a)

i.**True:** A squared integer will always equal another integer.

ii.**True:** A squared real number will always equal another real number.

(b)

i. **False:** Not all integers are perfect squares.

ii. **True:** All real numbers have a real square root.

(c)

i. **True:** $x = 0$ and a real number y will always satisfy this proposition.

ii. **True:** By the same reasoning.

(d)

i. **False:** The commutative property for addition has no exceptions.

ii. **False:** By the same reasoning.

(e)

i. **False:** This is only true for $x = 1$.

ii. **True:** y can be chosen as the reciprocal of x.

(f)

i. **False:** There is no x that satisfies $xy = 1$ for all y. If it is true for one value of y, it won't be for any others.

ii. **False:** By the same reasoning.

(g)

i. **True:** y can be chosen as $1 - x$.

ii. **True:** By the same reasoning.

(h)

i. **False:** This can be expressed as a system of equations, and the system has no solutions.

ii. **False:** By the same reasoning.

(i)

i. **False:** Take $x = 0$. To satisfy the first proposition, $y = 2$, but to satisfy the second, $y = -1$, so there is not a y that satisfies the system of equations for all x.

ii. **False:** By the same reasoning.

(j)

i. **False:** If x and y are not both even or both odd, $\frac{x+y}{2}$ will not be an integer.

ii. **True:** $\frac{x+y}{2}$ will always evaluate to a real number when x and y are both real.

Problem 25

(a) $(\forall z)(\forall y)(\forall x)\neg T(x, y, z)$

(b) $(\forall x)(\exists y)\neg P(x, y) \vee (\exists x)(\forall y)\neg Q(x, y)$

(c) $(\forall x)(\exists y)((Q(x, y) \wedge \neg Q(y, x)) \vee (Q(y, x) \wedge \neg Q(x, y)))$

(d) $(\exists y)(\exists x)(\exists z)(\neg T(x, y, z) \wedge \neg Q(x, y))$