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MATH 300: Section H01
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February 8, 2022

# MATH 300 Homework 4

### Problem 1

(a) Simplification

(b) Disjunctive Syllogism

(c) Modus Ponens

(d) Addition

(e) Hypothetical Syllogism

### Problem 2

$R$: It rains.

$F$: It is foggy.

$S$: The sailing race is held.

$L$: The lifesaving demonstration continues.

$T$: The trophy is awarded.

We claim the following is a valid argument:

$$\begin{array}{ll} & (\neg R \vee \neg F) \Rightarrow (S \wedge L) \\ & S \Rightarrow T \\ & \neg T \\ \hline \therefore & R \end{array}$$

By modus tollens:

$$\begin{array}{ll} & S \Rightarrow T \\ & \neg T \\ \hline \therefore & \neg S \end{array}$$

By addition (and De Morgan's law):

$$\begin{array}{ll} & \neg S \\ \hline \therefore & (\neg S \vee \neg L) \equiv \neg(S \wedge L) \end{array}$$

By modus tollens (and De Morgan's law):

$$\begin{array}{ll} & \neg(S \wedge L) \\ & (\neg R \vee \neg F) \Rightarrow (S \wedge L) \\ \hline \therefore & \neg(\neg R \vee \neg F) \equiv (R \wedge F) \end{array}$$

By simplification:

$$\begin{array}{ll} & R \wedge F \\ \hline \therefore & R \end{array}$$

## Problem 3

Let the universe consist of all things.

$M(x)$: $x$ is a man.

$I(x)$: $x$ is not an island.

$$\begin{array}{ll} & (\forall x)(M(x) \Rightarrow I(x)) \\ & \neg I(\text{Manhattan}) \\ \hline \therefore & \neg M(\text{Manhattan}) \end{array}$$

This is valid by universal modus tollens.

## Problem 4

(a)

i. Since you did not use the whirlpool, you are not sore by modus tollens.

ii. Since you are not sore, you did not play hockey yesterday by modus tollens.

(b)

i. Since it was not sunny on Tuesday, it was either partly sunny or you did not work by disjunctive syllogism.

ii. Since it was not partly sunny on Friday, if you worked on Friday, then it was sunny by disjunctive syllogism.

(c)

i. Since all insects have six legs, and dragonflies are insects, dragonflies have six legs by modus ponens.

ii. Since all insects have six legs, and spiders do not have six legs, spiders are not insects by modus tollens.

iii. Since spiders eat dragonflies, and dragonflies are insects, there are some insects that spiders eat by existensial generalization.

(d)

i. Since every student has an internet account, and Homer does not, Homer is not a student by modus tollens.

(e)

i. Since tofu is healthy, and all foods that are healthy do not taste good, tofu does not taste good by modus ponens.

(f)

i. Since you are dreaming or hallucinating, and you are not dreaming, you are hallucinating by disjunctive syllogism.

ii. Since you are hallucinating, you see elephants running down the road by modus ponens.

## Problem 5

(a) **Correct**: By modus tollens.

(b) **Incorrect**: It is not specified that the only fun cars to drive are convertibles, so other cars could be included with convertibles as being fun to drive. Isaac could have one of those cars, and thus his car would be fun to drive without being a convertible.

(c) **Incorrect**: It is not specified that Quincy likes only action movies, so he could like other kinds of movies along with action movies. *Eight Men Out* could be one of those movies, so it would be a movie that Quincy likes without being an action movie.

(d) **Correct**: By universal instantiation.

**Problem 6**

(a) **Incorrect**: This is not an iff statement, so it says nothing about what happens if $x$ is irrational. The premise is true (if you cannot write $x^2$ as a quotient of two integers, you certainly won't be able to do so for $x$), but the conclusion is false (take $x = \sqrt{2}$, for example).

(b) **Correct:** By universal instantiation.

**Problem 7**

(a) Since everyone who owns a red convertible has gotten at least one speeding ticket, Linda (who owns a red convertible) has gotten a speeding ticket by universal instantiation. Since Linda is in the class, someone in the class has gotten a speeding ticket by existential generalization.

(b) Let the universe consist of the five roommates. Since it is true for all of the roommates that they have taken discrete mathematics, and it is true that any person (and thus any roommate) can take a class in algorithms if they have taken discrete, any of the roommates can take a class in algorithms by modus ponens.

(c) Let the universe be movies produced by John Sayles. Since he made a movie about coal miners, there is a movie about coal miners in the universe. Since all movies in the universe are wonderful, then by universal instantiation, there is a wonderful movie about coal miners.

(d) Since there exists an $x$ in the class that has been to France, and all $x$ who go to France go to the Louvre, then by universal modus ponens $x$ has gone to the Louvre. Since $x$ is in the class, there is someone in the class who has gone to the Louvre by existensial generalization.

**Problem 8**

Assuming the universe of x and y consists of Max and at least one other person, $S(\text{Max}, \text{Max})$ does not follow from $(\exists x)S(x, \text{Max})$ because it only stipulates that someone must satisfy that condition and not Max in particular.

**Problem 9**

Let the universe consist of all people.

$H(x)$: $x$ is Hungarian.

$P(x)$: $x$ likes paprika.

(a)

$$\begin{array}{ll} & (\forall x)(H(x) \Rightarrow P(x)) \\ & H(\text{Laszlo}) \\ \hline \therefore & P(\text{Laszlo}) \end{array}$$

This is valid and uses universal modus ponens.

(b)

$$
\begin{array}{ll}
 & (\forall x)(H(x) \Rightarrow P(x)) \\
 & \neg H(\text{Joe}) \\
\hline
\therefore & \neg P(\text{Joe})
\end{array}
$$

This is not valid. It is not specified that only Hungarians like paprika, so we can say nothing about Joe or other non-Hungarians' paprika-related preferences.

(c)

$$
\begin{array}{ll}
 & (\forall x)(H(x) \Rightarrow P(x)) \\
 & P(\text{Laszlo}) \\
\hline
\therefore & H(\text{Laszlo})
\end{array}
$$

This is not valid. It is not specified that only Hungarians like paprika, so just because someone likes paprika doesn't necessarily mean they are Hungarian.

(d)

$$
\begin{array}{ll}
 & (\forall x)(H(x) \Rightarrow P(x)) \\
 & \neg P(\text{Laszlo}) \\
\hline
\therefore & \neg H(\text{Laszlo})
\end{array}
$$

This is valid and uses modus tollens.

**Problem 10**

An implication and its contrapositive are logically equivalent, and an inverse and its contrapositive (the converse) are also logically equivalent. Thus, Bob and Diana are both definitely incorrect, because there's no way to have an implication be true without its contrapositive be true as well, and no way for the converse to be true without the inverse also being true.

Ann is right if none of the statements are true. For example, the implication "if it is day, then it is night" yields the other three sentences "if it is not night, then it is not day", "if it is not day, then it is not night", and "if it is night, then it is day", all of which are of course false.

Cecil is right if either the implication/contrapositive or the inverse/converse (but not both) are true. For example, the implication "if it is 2020, then there is a global disease pandemic" yields the other three sentences "if there is not a global disease pandemic, then it is not 2020", "if it is not 2020, then there is not a global disease pandemic", and "if there is a global disease pandemic, then it is 2020". The first two statements are true (unless you believe in *alternative facts*), and the last two are false because there are other years in which there are global disease pandemics (2021, 2022, etc...).

Edith is right if both the implication/contrapositive and inverse/converse are true. For example, "if a chess piece moves in an L, then it is a knight" yields the other three sentences "if a chess piece is not a knight, then it does not move in an L", "if a chess piece does not move in a knight, then it is not a knight", and "if a chess piece is a knight, then it moves in an L". Thus, all four sentences are true (the knight is the one and only piece that moves in an "L" pattern).

**Problem 11**

(a) **No**: The only case in which this is guaranteed to be true is if the universe has exactly one member. While $(\exists x)P(x)$ only says there is at least one member that satisfies $P(x)$, $(\forall x)P(x)$ says that every member does, so there can be a part of the universe that doesn't satisfy $P(x)$.

(b) **Yes**: Since the universe has at least one member, if all the members satisfies $P(x)$, then it is certainly true that at least one member will satisfy $P(x)$.

(c) **Yes**: The truth value is always false. There is no member of the universe, so there cannot possibly be a member that satisfies any predicate.

(d) **Yes**: The truth value is always true. The truth set of $P(x)$ is the entire universe. In other words, there is nothing in the universe that doesn't satisfy $P(x)$ (or that does satisfy $P(x)$.