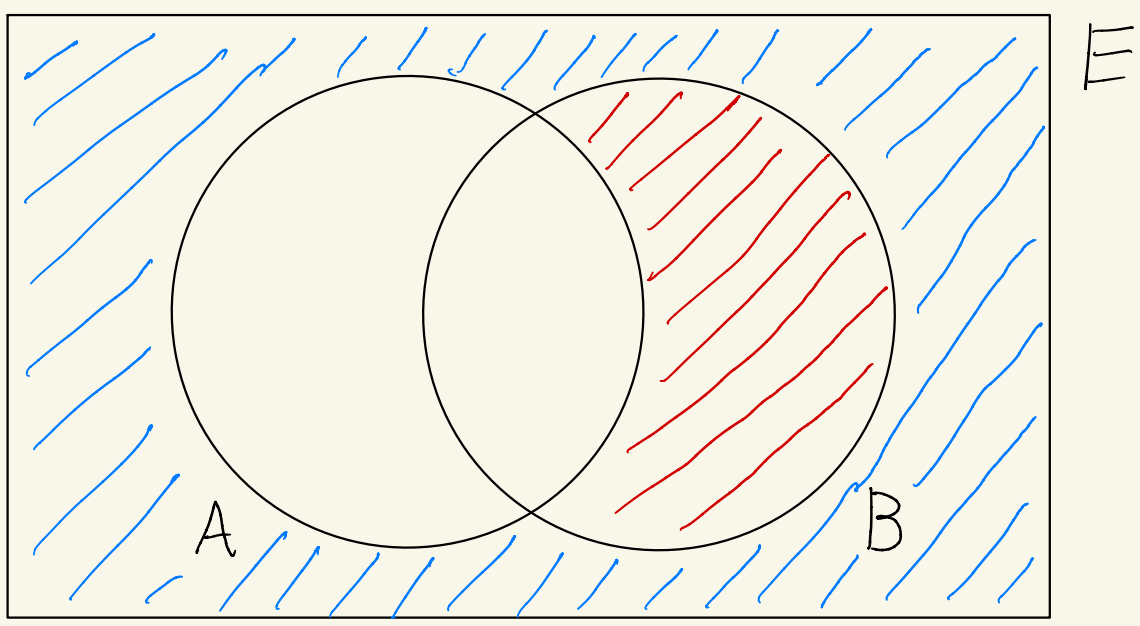
SCHC 501 - Homework 14

Nathan Bickel

1. Explain and illustrate in a Venn diagram with two sets A and B in a domain E what the combined effects of the four universal conditions on quantifiers are.

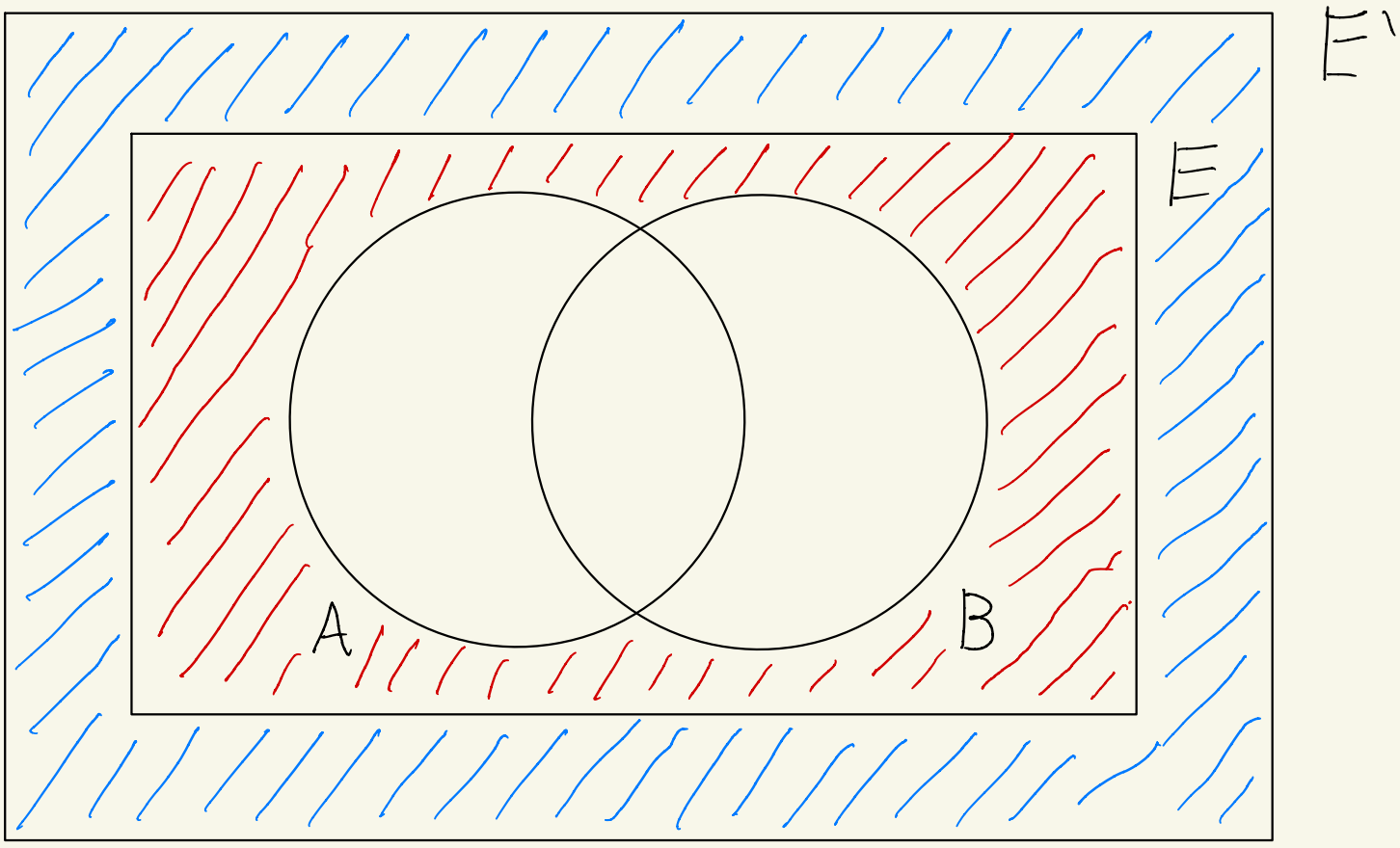
- Let D be a determiner in a model $M = \langle E, [\![\ ]\!] \rangle$. Then D is <u>conservative</u> iff for all $A, B \subseteq E$, we have $D_E AB \Leftrightarrow D_E A(A \cap B)$.



Thus, any element in one shaded area can move to the other shaded area without affecting $D_E AB$. In other words, B−A can be any subset of E−A and $D_E AB$ will not change in truth value.

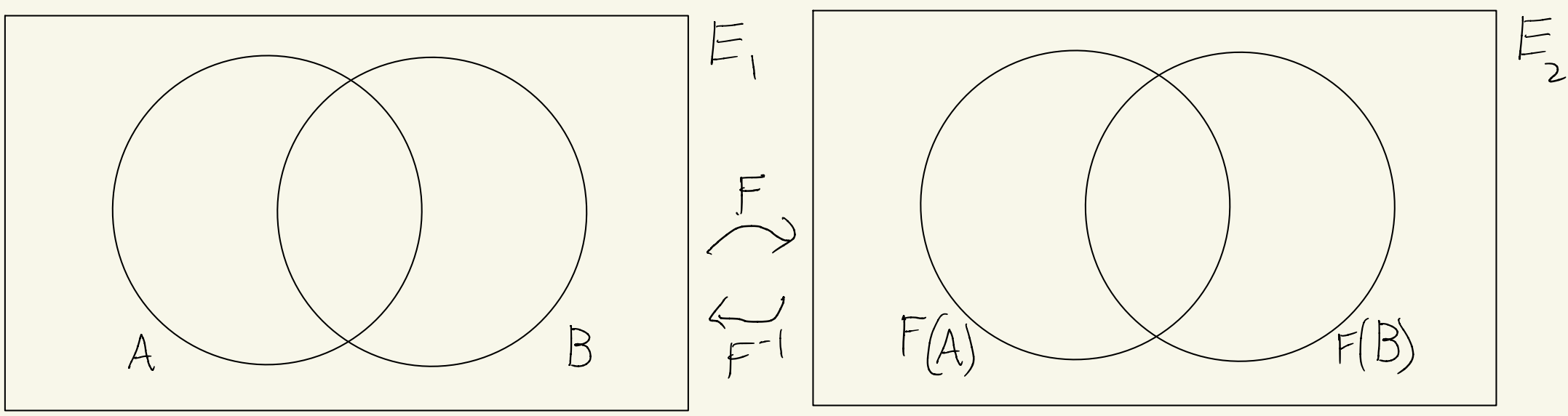
- Let D be a determiner in a model $M = \langle E', [\![\ ]\!] \rangle$. Then D is <u>extensional</u> iff for all $E \subseteq E'$ and for all $A, B \subseteq E$, we have $D_E AB \Leftrightarrow D_{E'} AB$.

Made with Goodnotes



Thus, any element in one shaded area can move to the other shaded area without affecting $D_E AB$. In other words, $E-(A\cup B)$ can be any subset of $E'-(A\cup B)$ and $D_E AB$ will not change in truth value.

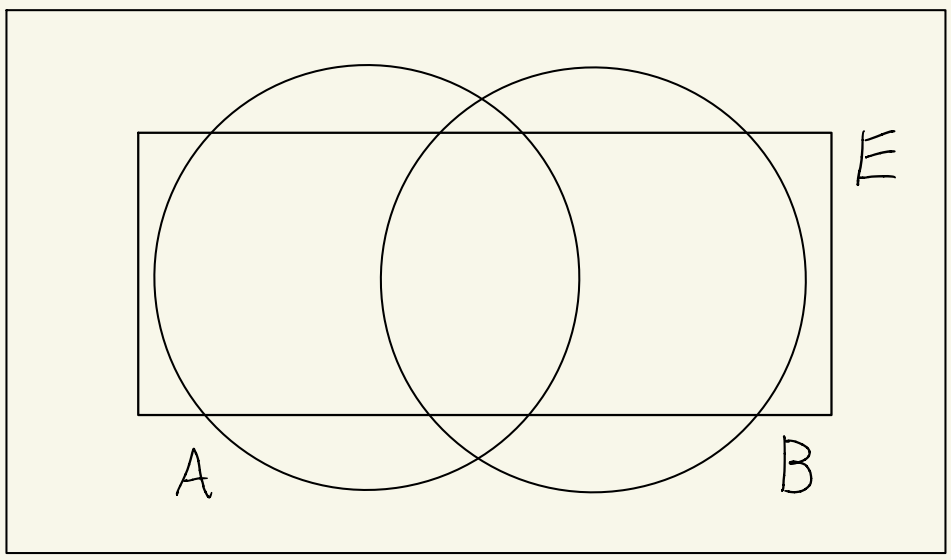
- Let D be a determiner in a model $M_1 = \langle E_1, [\![\ ]\!]\rangle$ and $M_2 = \langle E_2, [\![\ ]\!]\rangle$ a model. Then D is <u>quantitative</u> iff for all $A, B \subseteq E_1$ and all bijections $F: E_1 \to E_2$, we have $D_{E_1} AB \Leftrightarrow D_{E_2} F(A)F(B)$.

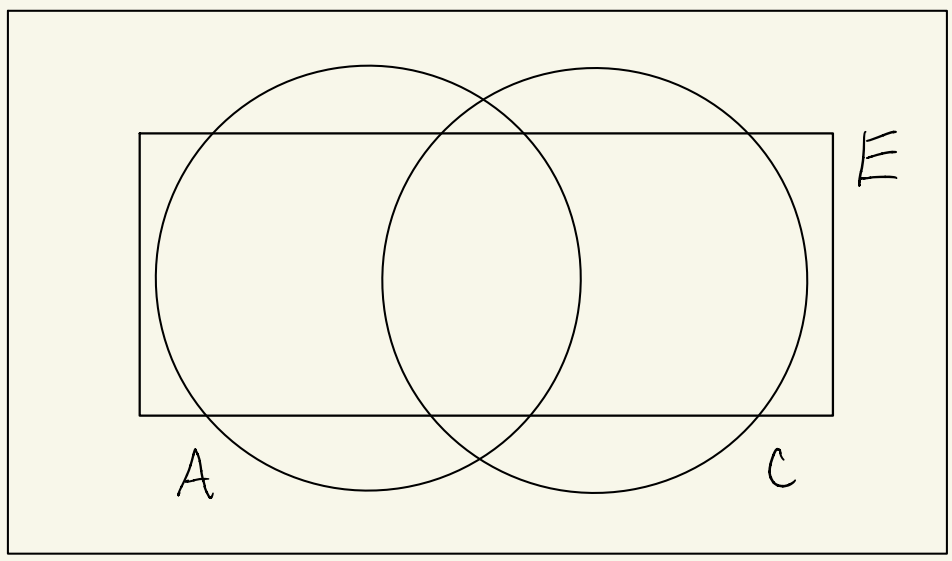


Thus, any two elements must be able to switch areas with each other in the diagram. Otherwise, there would be a bijection that would make $D_{E_1} AB \Leftrightarrow D_{E_2} F(A)F(B)$ false.

Made with Goodnotes

• Let D be a determiner in a model $M = \langle E, I, D \rangle$. Then D is <u>variant</u> iff there exists $E' \supseteq E$ and $A, B, C \subseteq E'$ such that $D_{E'}AB$ and $\sim D_{E'}AC$.





Thus, we must be able to find A, B, C so that the left diagram makes D true and the right diagram makes D false.

2. Represent the following quantifiers and properties of quantifiers in the number-theoretic trees by describing the set of pairs which verify the quantifier by shading the appropriate area in the tree, and/or by describing the general pattern which the condition imposes on the tree.

<u>a. no men</u>

$|A - B| + |A \cap B| =$

0							0,0
1						1,0	0,1
2					2,0	1,1	0,2
3				3,0	2,1	1,2	0,3
4			4,0	3,1	2,2	1,3	0,4
5		5,0	4,1	3,2	2,3	1,4	0,5
6	6,0	5,1	4,2	3,3	2,4	1,5	0,6

Made with Goodnotes

b. not all men

$|A-B|+|A\cap B|=$

0				0,0			
1			1,0		0,1		
2		2,0		1,1		0,2	
3	3,0		2,1		1,2		0,3
4		4,0		3,1		2,2		1,3		0,4
5		5,0		4,1		3,2		2,3		1,4		0,5
6	6,0		5,1		4,2		3,3		2,4		1,5		0,6

c. neither man

$|A-B|+|A\cap B|=$

0				0,0			
1			1,0		0,1		
2		2,0		1,1		0,2	
3	3,0		2,1		1,2		0,3
4		4,0		3,1		2,2		1,3		0,4
5		5,0		4,1		3,2		2,3		1,4		0,5
6	6,0		5,1		4,2		3,3		2,4		1,5		0,6

d. at most two men

$|A-B|+|A\cap B|=$

0				0,0			
1			1,0		0,1		
2		2,0		1,1		0,2	
3	3,0		2,1		1,2		0,3
4		4,0		3,1		2,2		1,3		0,4
5		5,0		4,1		3,2		2,3		1,4		0,5
6	6,0		5,1		4,2		3,3		2,4		1,5		0,6

Made with Goodnotes

e. at least one man

$\lvert A-B\rvert+\lvert A\cap B\rvert=$							
0					0,0		
1				1,0		0,1	
2			2,0		1,1		0,2
3		3,0		2,1		1,2		0,3
4	4,0		3,1		2,2		1,3		0,4
5		5,0		4,1		3,2		2,3		1,4		0,5
6	6,0		5,1		4,2		3,3		2,4		1,5		0,6

f. exactly one man or exactly three men do not walk

$\lvert A-B\rvert+\lvert A\cap B\rvert=$							
0					0,0		
1				1,0		0,1	
2			2,0		1,1		0,2
3		3,0		2,1		1,2		0,3
4	4,0		3,1		2,2		1,3		0,4
5		5,0		4,1		3,2		2,3		1,4		0,5
6	6,0		5,1		4,2		3,3		2,4		1,5		0,6

g. Variation

Some, but not all, of the tree must be shaded.

h. Right monotone increasing

If a cell is shaded, everything to the right is shaded.

i. Symmetry

If a cell is shaded, then that row's left-most cell is shaded.

Made with Goodnotes

3. Determine the monotonicity properties of the following determiners:

- several: LMI, RMI
- at most 3: LMD, RMD
- none: LMD, RMD
- at least n: LMI, RMI
- some (singular): LMI, RMI
- these: RMI
- neither: LMI, LMD, RMD
- every: RMI, LMD
- all: RMI, LMD
- each: RMI, LMD
- infinitely many: LMI, RMI
- a finite number of: LMD, RMD
- most: RMI
- many: RMI

Made with Goodnotes