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SCHC 501: Section H01
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# SCHC 501 Homework 5

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**Problem 1** Translate the following sentences into statement logic. Use lower case letters for atomic statements and give the "key" to the translation, i.e., say which atomic statements are the correspondents of which English sentences. (In some cases you may want to use a syntactically different version of the English sentence.)

(a) Either John is in that room or Mary is, and possibly they both are.

(b) The fire was set by an arsonist, or there was an accidental explosion in the boiler room.

(c) When it rains, it pours.

(d) Sam wants a dog, but Alice prefers cats.

(e) If Steve comes home late and has not had any supper, we will reheat the stew.

(f) Clarence is well educated only if he can read Chuvash.

(g) Marsha won't go out with John unless he shaves off his beard and stops drinking.

(h) The stock market advances when public confidence in the economy is rising and only then.

(i) A necessary but perhaps not sufficient condition for negotiations to commence is for Barataria to cease all acts of aggression against Titipu.

In each case say what, if anything, has been "lost in translation"; that is, what semantic properties of the English sentence are not represented in the logical formula.

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For each part, we give the statement logic followed by a key. If not specified, nothing significant has been lost in translation.

**(a)** $j \vee m$

- $j$: John is in that room.
- $m$: Mary is in that room.

**(b)** $a \vee e$: The sentence is intended to be an exclusive or, but the default in logic is inclusive.

- $a$: The fire was set by an arsonist.
- $e$: There was an accidental explosion in the boiler room.

**(c)** $r \to p$

- $r$: It rains.
- $p$: It pours.

**(d)** $d \,\&\, c$: The "but" in English implies that there is a difference, but this is lost in the statement logic.

- $d$: Sam wants a dog.
- $c$: Alice prefers cats.

**(e)** $(\ell \,\&\, h) \to s$

- $\ell$: Steve comes home late.
- $h$: Steve has not had any supper.
- $s$: We will reheat the stew.

**(f)** $e \to c$

- $e$: Clarence is well educated.
- $c$: Clarence can read Chuvash.

**(g)** $m \to (b \,\&\, d)$

- $m$: Marsha will go out with John.
- $b$: John shaves his beard.
- $d$: John stops drinking.

**(h)** $s \leftrightarrow c$

- $s$: The stock market advances.
- $c$: Public confidence in the economy is rising.

**(i)** $n \to a$

- $n$: Negotiations commence.
- $a$: Barataria ceases all acts of aggression against Titipu.

**Problem 2** The following sentences contain various sorts of ellipsis, so that some connectives appear not to be connecting whole statements[1]. Reformulate them so that the connectives connect statements (using different connectives if necessary) and translate into symbolic notation as above.

(a) John and Bill are going to the movies, but not Tom.

(b) Susan doesn't like squash or turnips.

(c) If neither Peter nor Fred is going to the party, then neither will I.

(d) If Mary hasn't gotten lost or had an accident, she will be here in five minutes.

(e) A bear or a wolf frightened the boys.

(f) A party or a softball game would have amused the children.

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(a) We can rewrite this as "John and Bill, but not Tom, are going to the movies," and use the statement logic $j \,\&\, b \,\&\, \sim t$ with key:

- $j$: John is going to the movies.

[1] I'm not sure I understand the question, honestly...

- $b$: Bill is going to the movies.
- $t$: Tom is going to the movies.

(b) We can rewrite this as "Susan likes neither squash nor turnips," and use the statement logic $\sim (s \vee t)$ with key:

- $s$: Susan likes squash.
- $t$: Susan likes turnips.

(c) We can rewrite this as "I will not go to the party if neither Peter nor Fred go", and use the statement logic $(\sim (p \vee f)) \to \sim m$ with key:

- $p$: Peter goes to the party.
- $f$: Fred goes to the party.
- $m$: I go to the party.

(d) We can rewrite this as "Mary will be here in five minutes if she has neither gotten lost nor had an accident," and use the statement logic $(\sim (\ell \vee a)) \to f$ with key:

- $\ell$: Mary has gotten lost.
- $a$: Mary had an accident.
- $f$: Mary will be here in five minutes.

(e) We can rewrite this as "The boys were frightened by either a bear or by a wolf," and use the statement logic $b \vee w$ with key:

- $b$: A bear frightened the boys.
- $w$: A wolf frightened the boys.

(f) We can rewrite this as "The children would have been amused either by a party or by a softball game," and use the statement logic $p \vee g$ with key:

- $p$: A party would have amused the children.
- $g$: A softball game would have amused the children.

**Problem 3** Let $p$, $q$, and $r$ be true and let $s$ be false. Find the truth values of the following statements.

(a) $((p \,\&\, q) \,\&\, s)$

(b) $(p \,\&\, (q \,\&\, s))$

(c) $p \to s$

(d) $s \to p$

(e) $((p \,\&\, q) \leftrightarrow (r \,\&\, \sim s))$

(f) $(p \to (q \leftrightarrow (r \to s)))$

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**(a)** We have $((p \,\&\, q) \,\&\, s) = ((\mathbf{T} \,\&\, \mathbf{T}) \,\&\, \mathbf{F}) \equiv \mathbf{T} \,\&\, \mathbf{F} \equiv \mathbf{F}$.

**(b)** We have $(p \,\&\, (q \,\&\, s)) = (\mathbf{T} \,\&\, (\mathbf{T} \,\&\, \mathbf{F})) \equiv \mathbf{T} \,\&\, \mathbf{F} \equiv \mathbf{F}$.

**(c)** We have $p \to s = \mathbf{T} \to \mathbf{F} \equiv \mathbf{F}$.

**(d)** We have $s \to p = \mathbf{F} \to \mathbf{T} \equiv \mathbf{T}$.

**(e)** We have $((p \,\&\, q) \leftrightarrow (r \,\&\, \sim s)) = ((\mathbf{T} \,\&\, \mathbf{T}) \leftrightarrow (\mathbf{T} \,\&\, \sim \mathbf{F})) \equiv ((\mathbf{T} \,\&\, \mathbf{T}) \leftrightarrow (\mathbf{T} \,\&\, \mathbf{T})) \equiv \mathbf{T} \leftrightarrow \mathbf{T} \equiv \mathbf{T}$.

**(f)** We have $(p \to (q \leftrightarrow (r \to s))) = (\mathbf{T} \to (\mathbf{T} \leftrightarrow (\mathbf{T} \to \mathbf{F}))) \equiv (\mathbf{T} \to (\mathbf{T} \leftrightarrow \mathbf{F})) \equiv \mathbf{T} \to \mathbf{F} \equiv \mathbf{F}$.

**Problem 4** Construct truth tables for each of the following statements. Note whether any are logically equivalent.

(a) $(p \vee \sim q)$

(b) $\sim (\sim p \,\&\, q)$

(c) $((p \leftrightarrow q) \,\&\, p)$

(d) $((p \to (q \vee \sim r)) \,\&\, (p \to (q \vee \sim r)))$

(e) $(((p \to q) \to p) \to q)$

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We give each truth table. As can be observed from the truth tables, **(a)** and **(b)** are logically equivalent.

**(a)**

| $p$ | $q$ | $\sim q$ | $p \vee \sim q$ |
|---|---|---|---|
| $T$ | $T$ | $F$ | $T$ |
| $T$ | $F$ | $T$ | $T$ |
| $F$ | $T$ | $F$ | $F$ |
| $F$ | $F$ | $T$ | $T$ |

**(b)**

| $p$ | $q$ | $\sim p$ | $\sim p \,\&\, q$ | $\sim (\sim p \,\&\, q)$ |
|---|---|---|---|---|
| $T$ | $T$ | $F$ | $F$ | $T$ |
| $T$ | $F$ | $F$ | $F$ | $T$ |
| $F$ | $T$ | $T$ | $T$ | $F$ |
| $F$ | $F$ | $T$ | $F$ | $T$ |

**(c)**

| $p$ | $q$ | $p \leftrightarrow q$ | $(p \leftrightarrow q) \,\&\, p$ |
|---|---|---|---|
| $T$ | $T$ | $T$ | $T$ |
| $T$ | $F$ | $F$ | $F$ |
| $F$ | $T$ | $F$ | $F$ |
| $F$ | $F$ | $T$ | $F$ |

**(d)**

| $p$ | $q$ | $r$ | $\sim r$ | $q \vee \sim r$ | $p \to (q \vee \sim r)$ | $(p \to (q \vee \sim r))\,\&\,(p \to (q \vee \sim r))$ |
|---|---|---|---|---|---|---|
| $T$ | $T$ | $T$ | $F$ | $T$ | $T$ | $T$ |
| $T$ | $T$ | $F$ | $T$ | $T$ | $T$ | $T$ |
| $T$ | $F$ | $T$ | $F$ | $F$ | $F$ | $F$ |
| $T$ | $F$ | $F$ | $T$ | $T$ | $T$ | $T$ |
| $F$ | $T$ | $T$ | $F$ | $T$ | $T$ | $T$ |
| $F$ | $T$ | $F$ | $T$ | $T$ | $T$ | $T$ |
| $F$ | $F$ | $T$ | $F$ | $F$ | $T$ | $T$ |
| $F$ | $F$ | $F$ | $T$ | $T$ | $T$ | $T$ |

**(e)**

| $p$ | $q$ | $p \to q$ | $(p \to q) \to p$ | $((p \to q) \to p) \to q$ |
|---|---|---|---|---|
| $T$ | $T$ | $T$ | $T$ | $T$ |
| $T$ | $F$ | $F$ | $T$ | $F$ |
| $F$ | $T$ | $T$ | $F$ | $T$ |
| $F$ | $F$ | $T$ | $F$ | $T$ |