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CSCE 355: Section 001
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CSCE 355 Homework 5

Problem 2 We proved that the regular languages are closed under (string) homomorphic images (this is also in the textbook). Is the same true for the context-free languages?

Yes. We can construct a CFG for the homomorphism φ by replacing every terminal symbol, let's call a, in the body of every production with $\varphi(a)$. This gives a valid CFG for the homomorphism.

Problem 3 Suppose the PDA $P = \big\{\{q,p\},\{0,1\},\{Z_0,X\},\delta,q,Z_0,\{p\}\big\}$ has the transition function

$$\delta(q,0,Z_0) = \{q, XZ_0\}.$$

Starting from the initial ID (q,w,Z_0), show all the reachable ID's when the input w is:

(b) 0011.

(c) 010.

(b) $(q,001,z_0) \vdash (q,011,xz_0) \vdash (q,11,xxz_0) \vdash (q,1,xxz_00 \vdash (q\varepsilon,xxz_0) \vdash (p,\varepsilon,xxz_0) \vdash (p,\varepsilon,xz_0) \vdash (p,\varepsilon,z_0)$ OR
$(q,001,z_0) \vdash (q,011,xz_0) \vdash (q,11,xxz_0) \vdash (p,11,xz_0) \vdash (p,1,xxz_0) \vdash (p,\varepsilon,z_0)$ OR
$(q,001,z_0) \vdash (q,011,xz_0) \vdash (q,11,xxz_0) \vdash (p,11,xz_0) \vdash (p,1,xxz_0) \vdash (p,1,xz_0) \vdash (p,1,z_0) \vdash (p,\varepsilon,\varepsilon)$

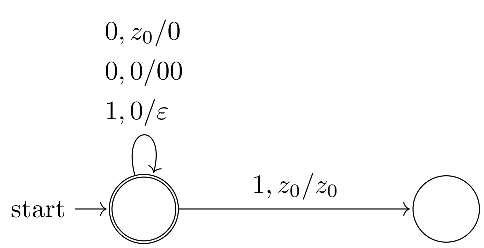
(c) $(q,010,z_0) \vdash (q,10,xz_0) \vdash (q,0,xz_0) \vdash (q,\varepsilon,xxz_0) \vdash (p,\varepsilon,xz_0) \vdash (p,\varepsilon,z_0)$ OR
$(q,010,z_0) \vdash (q,10,xz_0) \vdash (p,10,z_0) \vdash (p,0,\varepsilon)$

Problem 4 Design a PDA to accept each of the following languages. You may accept either by final state or by empty stack, whichever is more convenient.

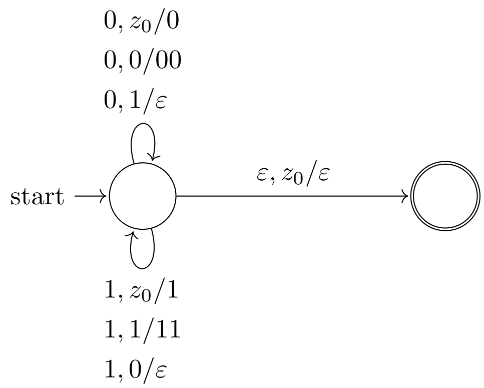
(b) The set of all strings of 0's and 1's such that no prefix has more 1's than 0's.

(c) The set of all strings of 0's and 1's with an equal number of 0's and 1's.

(a) Consider the PDA in the diagram below. Note $\Sigma = \{0,1\}$ and $\Gamma = \{0,z_0\}$, and the PDA accepts by final state.



(b) Consider the PDA in the diagram below. Note $\Sigma = \{0, 1\}$ and $\Gamma = \{0, 1, z_0\}$, and the PDA accepts by final state.



Problem 5 Convert the grammar

$$S \to aAA$$

$$A \to aS \mid bS \mid a$$

to a PDA that accepts the same language by empty stack.

The PDA is defined as follows: $P = \langle \{q\}, \{a, b\}, \{a, b, S, A\}, \delta, q, S, \emptyset \rangle$ with the transition function defined by

$$\delta(q, a, a) = \{(q, \varepsilon)\} \qquad \delta(q, b, b) = \{(q, \varepsilon)\}$$

$$\delta(q, \varepsilon, S) = \{(q, aAA)\} \qquad \delta(q, \varepsilon, A) = \{(q, aS), (q, bS), (q, a)\}$$

and this is all that defines the PDA.

Problem 6 Consider the 1-state restricted PDA $P = (\{q\}, \{0, 1\}, \{X, Z_0\}, \delta, q, Z_0)$, where δ is given by

$$\delta(q, 0, Z_0) = \{(q, \textbf{push } X)\} \qquad \delta(q, 1, X) = \{(q, \textbf{pop})\}$$

$$\delta(q, 0, X) = \{(q, \textbf{push } X)\} \qquad \delta(q, \varepsilon, Z_0) = \{(q, \textbf{pop})\}$$

Using either the method of the book or the method I described in class, convert P to an equivalent context-free grammar.

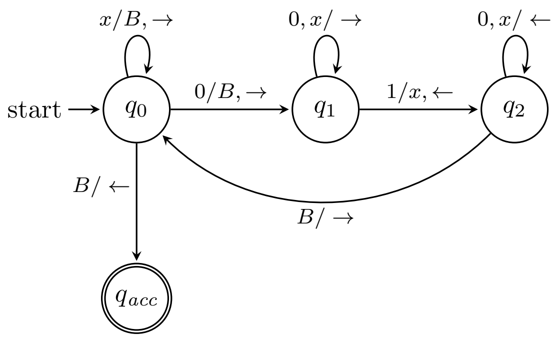
$S \to [qz_0q]$
$[qz_0q] \to \varepsilon$
$[qXq] \to 1$
$[qz_0q] \to 0[qXq][qz_0q]$
$[qxq] \to 0[qXq][qXq]$

Problem 7 Show that the language $L := \{ww \mid w \in \{0, 1\}^*\}$ is not pumpable (and hence not a CFL by the Pumping Lemma for CFLs).

Let $p > 0$, and consider $s := 0^p1^p0^{2p}1^p0^p$. Then $s \in L$ with $0^p1^p0^p$, and clearly $|s| \geq p$. Let u, v, w, x, y be strings such that $s = uvwxy$ with $|vwx| \leq p$ and $|vx| > 0$. Then, consider $i := 0$, which will yield

$uv^iwx^iy = uwy$. Then uwy will not be in L: since $|vwx| \le p$, vwx will only cover some string of 1s or 0s, possibly followed by a string of 0s or 1s. In any case, it is not possible that removing v and x will alter the same part of the two instances of $0^p1^p0^p$. Therefore, L is not CFL-pumpable, so L is not context-free. $\square$

Problem 8 Consider the standard, 1-tape Turing machine $M := \langle Q, \Sigma, \Gamma, \delta, q_0, B, F\rangle$ with input alphabet $\Sigma := \{0, 1\}$ and tape alphabet $\{0, 1, x, B\}$ (B is the blank symbol) given by the following transition diagram:



Give the complete computation path (sequence of IDs) of M on input "0101" (without the double quotes).

The computation path is

- $q_0 0101$
- $Bq_1 101$
- $q_2 Bx01$
- $Bq_0 x01$
- $BBq_0 01$
- $BBBq_1 1$
- $BBq_2 Bx$
- $BBBq_0 x$
- $BBBBq_0$
- $BBBq_{acc}B$.