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STAT 509: Section 003
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August 25, 2022

STAT 509 Homework 1a

Problem 1 Solve the following integral:

$$\int_0^\infty \lambda^2 x e^{-\lambda x} dx$$

Solution.

$$\begin{aligned}
\int_0^\infty \lambda^2 x e^{\lambda x} dx &= \lim_{b\to\infty} \int_0^b \lambda^2 x e^{-\lambda x} dx \\
&= -\lambda \lim_{b\to\infty} \int_0^b -\lambda x e^{-\lambda x} dx \\
&= -\lambda \lim_{b\to\infty} \left[e^{-\lambda x} x - \int e^{-\lambda x} dx \right]_0^b && \text{(by parts)} \\
&= -\lambda \lim_{b\to\infty} \left[e^{-\lambda x} x + \frac{e^{-\lambda x}}{\lambda} \right]_0^b \\
&= \lim_{b\to\infty} \left[-\lambda e^{-\lambda x} x - e^{-\lambda x} \right]_0^b \\
&= \lim_{b\to\infty} \left[-\frac{\lambda x + 1}{e^{\lambda x}} \right]_0^b \\
&= \lim_{b\to\infty} \left[-\frac{\lambda b + 1}{e^{\lambda b}} + \frac{\lambda(0) + 1}{e^{\lambda(0)}} \right] \\
&= 1 - \lim_{b\to\infty} \left[\frac{\lambda b + 1}{e^{\lambda b}} \right] \\
&= 1 - \lim_{b\to\infty} \left[\frac{\lambda}{\lambda e^{\lambda b}} \right] && \text{(L'Hôpital's rule)} \\
&= 1 && \text{(denominator grows without bound, so the limit is 0)}
\end{aligned}$$

Problem 2 Solve the following integral:

$$\int_0^t \frac{2x}{100} e^{-(x/10)^2} dx$$

Solution.

$$\begin{aligned}
\int_0^t \frac{2x}{100} e^{-(x/10)^2} dx &= -\int_0^t -\frac{2x}{100} e^{-x^2/100} dx \\
&= -\left[e^{-x^2/100} \right]_0^t \\
&= -e^{-t^2/100} + e^{-0^2/100} t = 1 - e^{-t^2/100}
\end{aligned}$$