# Homework 2

# Solution

**(0.5pt for each problem or sub-problem, total 20 points)**

**Section 1.4**

6 (d)(e)(f)

**d)** Some student in the school has not visited North Dakota. (Alternatively, there exists a student in the school who has not visited North Dakota.)
**e)** This is the negation of part **(b)**: It is not true that every student in the school has visited North Dakota. (Alternatively, not all students in the school have visited North Dakota.)
**f)** All students in the school have not visited North Dakota. (This is technically the correct answer, although common English usage takes this sentence to mean—incorrectly—the answer to part **(e)**. To be perfectly clear, one could say that every student in this school has failed to visit North Dakota, or simply that no student has visited North Dakota.)

8 (c)(d)

**c)** There exists an animal such that if it is a rabbit, then it hops. (Note that this is trivially true, satisfied, for example, by lions, so it is not the sort of thing one would say.)
**d)** There exists an animal that is a rabbit and hops. (Alternatively, some rabbits hop. Alternatively, some hopping animals are rabbits.)

10 (a)(c)(e)

**a)** We assume that this means that one student has all three animals: $\exists x(C(x) \wedge D(x) \wedge F(x))$.

**c)** $\exists x(C(x) \wedge F(x) \wedge \neg D(x))$

**e)** Here the owners of these pets can be different: $(\exists x\, C(x)) \wedge (\exists x\, D(x)) \wedge (\exists x\, F(x))$.

14 (a)(b)

**a)** Since $(-1)^3 = -1$, this is true.
**b)** Since $(\frac{1}{2})^4 < (\frac{1}{2})^2$, this is true.

24 (c)(d)

**c)** Let $S(x)$ be "$x$ can swim." Then we have $\exists x\, \neg S(x)$ the first way, or $\exists x(C(x) \wedge \neg S(x))$ the second way.
**d)** Let $Q(x)$ be "$x$ can solve quadratic equations." Then we have $\forall x\, Q(x)$ the first way, or $\forall x(C(x) \rightarrow Q(x))$ the second way.

28 (a)(b)

Let $R(x)$ be "$x$ is in the correct place"; let $E(x)$ be "$x$ is in excellent condition"; let $T(x)$ be "$x$ is a [or your] tool"; and let the domain of discourse be all things.

**a)** There exists something not in the correct place: $\exists x \neg R(x)$.

**b)** If something is a tool, then it is in the correct place place and in excellent condition: $\forall x (T(x) \rightarrow (R(x) \wedge E(x)))$.

34 (c)(d)

**c)** Let $S(x)$ be "$x$ can keep a secret," where the domain of discourse is people. The original statement is $\neg\exists x S(x)$, the negation is $\exists x S(x)$, "Some people can keep a secret."

**d)** Let $A(x)$ be "$x$ has a good attitude," where the domain of discourse is people in this class. The original statement is $\exists x \neg A(x)$, the negation is $\forall x A(x)$, "Everyone in this class has a good attitude."

40 (a)(b)

There are many ways to write these, depending on what we use for predicates.

**a)** Let $F(x)$ be "There is less than $x$ megabytes free on the hard disk," with the domain of discourse being positive numbers, and let $W(x)$ be "User $x$ is sent a warning message." Then we have $F(30) \rightarrow \forall x W(x)$.

**b)** Let $O(x)$ be "Directory $x$ can be opened," let $C(x)$ be "File $x$ can be closed," and let $E$ be the proposition "System errors have been detected." Then we have $E \rightarrow ((\forall x \neg O(x)) \wedge (\forall x \neg C(x)))$.

42 (c)(d)

**c)** Let $S(x, y)$ be "System $x$ is in state $y$." Recalling that "only if" indicates a necessary condition, we have $S(\text{firewall}, \text{diagnostic}) \rightarrow S(\text{proxy server}, \text{diagnostic})$.

**d)** Let $T(x)$ be "The throughput is at least $x$ kbps," where the domain of discourse is positive numbers, let $M(x, y)$ be "Resource $x$ is in mode $y$," and let $S(x, y)$ be "Router $x$ is in state $y$." Then we have $(T(100) \wedge \neg T(500) \wedge \neg M(\text{proxy server}, \text{diagnostic})) \rightarrow \exists x S(x, \text{normal})$.

**Section 1.5**

4 (b)(c)(d)

**b)** There is a student in your class who has taken every computer science course.

**c)** Every student in your class has taken at least one computer science course.

**d)** There is a computer science course that every student in your class has taken.

8 (a)(b)

**a)** $\exists x \exists y Q(x, y)$

**b)** This is the negation of part **(a)**, and so could be written either $\neg\exists x \exists y Q(x, y)$ or $\forall x \forall y \neg Q(x, y)$.

12 (f)(g)(i)

**f)** $\exists x \neg I(x)$ **g)** $\neg \forall x I(x)$ (same as **(f)**) **i)** $\exists x \forall y (x \neq y \leftrightarrow I(y))$

18 (a)(c)

**a)** $\forall f (H(f) \to \exists c A(c))$, where $A(x)$ means that console $x$ is accessible, and $H(x)$ means that fault condition $x$ is happening

**c)** $(\forall b \exists m D(m,b)) \leftrightarrow \exists p \neg C(p)$, where $D(x,y)$ means that mechanism $x$ can detect breach $y$, and $C(x)$ means that process $x$ has been compromised

24 (a)(b)

**a)** There exists an additive identity for the real numbers—a number that when added to every number does not change its value.

**b)** A nonnegative number minus a negative number is positive.

28 (c)(d)(e)

**c)** true (let $x = 0$)

**d)** false (the commutative law for addition always holds) **e)** true (let $y = 1/x$)

30 (a)(b)(c)

We need to use the transformations shown in Table 2 of Section 1.4, replacing $\neg\forall$ by $\exists\neg$, and replacing $\neg\exists$ by $\forall\neg$. In other words, we push all the negation symbols inside the quantifiers, changing the sense of the quantifiers as we do so, because of the equivalences in Table 2 of Section 1.4. In addition, we need to use De Morgan's laws (Section 1.3) to change the negation of a conjunction to the disjunction of the negations and to change the negation of a disjunction to the conjunction of the negations. We also use the fact that $\neg\neg p \equiv p$.
**a)** $\forall y \forall x \neg P(x,y)$ **b)** $\exists x \forall y \neg P(x,y)$ **c)** $\forall y (\neg Q(y) \vee \exists x R(x,y))$

36 (a)(b)

In each case we need to specify some predicates and identify the domain of discourse.
**a)** Let $L(x,y)$ mean that person $x$ has lost $y$ dollars playing the lottery. The original statement is then $\neg\exists x \exists y (y > 1000 \wedge L(x,y))$. Its negation of course is $\exists x \exists y (y > 1000 \wedge L(x,y))$; someone has lost more than \$1000 playing the lottery.
**b)** Let $C(x,y)$ mean that person $x$ has chatted with person $y$. The given statement is $\exists x \exists y (y \neq x \wedge \forall z (z \neq x \to (z = y \leftrightarrow C(x,z))))$. The negation is therefore $\forall x \forall y (y \neq x \to \exists z (z \neq x \wedge \neg(z = y \leftrightarrow C(x,z))))$. In English, everybody in this class has either chatted with no one else or has chatted with two or more others.