CSE 15
Discrete Mathematics

Lecture 18 – Mathematical Induction (2)

Announcements

- HW #8
  - Due **5pm** 11/21 (Wed).

- Reading assignment
  - Ch.6.1 – 6.3 of textbook

Strong Induction
(Ch. 5.2)

- Strong Induction
- Example Proofs using Strong Induction

Strong Induction

- *Strong Induction*: To prove that $P(n)$ is true for all positive integers n, where $P(n)$ is a propositional function, complete two steps:
  - *Basis Step*: Verify that the proposition $P(1)$ is true.
  - *Inductive Step*: Show the conditional statement $[P(1) \land P(2) \land \cdots \land P(k)] \rightarrow P(k + 1)$ holds for all positive integers k.

Proof Using Strong Induction

Example: Prove that every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps.

Solution: Let *P*(*n*) be the proposition that postage of *n* cents can be formed using 4-cent and 5-cent stamps.

- BASIS STEP: *P*(12), *P*(13), *P*(14), and *P*(15) hold.
  - *P*(12) uses three 4-cent stamps.
  - *P*(13) uses two 4-cent stamps and one 5-cent stamp.
  - *P*(14) uses one 4-cent stamp and two 5-cent stamps.
  - *P*(15) uses three 5-cent stamps.

Proof Using Strong Induction

- INDUCTIVE STEP: The inductive hypothesis states that $P(j)$ holds for $12 \le j \le k$, where $k \ge 15$. Assuming the inductive hypothesis, it can be shown that $P(k + 1)$ holds.
- Using the inductive hypothesis, $P(k - 3)$ holds since $k - 3 \geq 12$. To form postage of $k + 1$ cents, add a 4-cent stamp to the postage for $k - 3$ cents.

Hence, $P(n)$ holds for all $n \geq 12$. ◀