CSE15 Quiz 3 Name: _______Solution_______________________

Note: 6 questions, maximum 13 points, show all steps (as appropriate) to get full credit

1. [2 points] Find the prime factorization of each of these integers.

   a) 39

      $39 = 3 \cdot 13$

   b) 81

      $81 = 3^4$

2. [2 points] What is the greatest common divisor of the following pair of integers

   $3^7 \cdot 5^3 \cdot 7^3$ and $2^{11} \cdot 3^5 \cdot 5^9$

   $3^5 \cdot 5^3$

   (Note: you can leave your answer in factored form.)

3. [3 points] Use mathematical induction to prove that

$$1^2 + 3^2 + 5^2 + \cdots + (2n+1)^2 = (n+1)(2n+1)(2n+3)/3$$

   whenever *n* is a nonnegative integer.

   Let P (n) be $1^2 + 3^2 + 5^2 + \cdots + (2n+1)^2 = (n+1)(2n+1)(2n+3)/3$.

   Basis step: P (0) is true because $1^2 = 1 = (0+1)(2\cdot0+1)(2\cdot0+3)/3$.

   Inductive step: Assume that P (k) is true.

   Then $1^2 + 3^2 + \cdots + (2k+1)^2 + [2(k+1)+1]^2 = (k+1)(2k+1)(2k+3)/3 + (2k+3)^2$

   $= (2k+3)[(k+1)(2k+1)/3 + (2k+3)]$

   $= (2k+3)(2k^2+9k+10)/3 = (2k+3)(2k+5)(k+2)/3$

   $= [(k+1)+1][2(k+1)+1][2(k+1)+3]/3.$

4. [2 points] Find $f(2)$, $f(3)$, $f(4)$, and $f(5)$ if f is defined recursively by

$$f(0) = -1$$
$$f(1) = 2$$
and for $\mathrm{n} = 1, 2, \ldots$
$$f(n+1) = f(n) + 3f(n-1)$$

$$f(2) = -1$$
$$f(3) = 5$$
$$f(4) = 2$$
$$f(5) = 17$$