Homework #1
SOLUTION

Section 1.1

1pt for each problem and sub-problem

10 (d)(e)(f)

d) If the votes have been counted, then the election is decided.

e) If the votes have not been counted, then the election is not decided.

f) If the election is not decided, then the votes have not been counted.

12 (c)(e)

c) If you miss the final exam, then you do not pass the course.

e) It is either the case that if you have the flu then you do not pass the course or the case that if you miss the final exam then you do not pass the course (or both, it is understood).

22 (a)(b)(c)(d)(e)(g)

a) The necessary condition is the conclusion: If you get promoted, then you wash the boss's car.

b) If the winds are from the south, then there will be a spring thaw.

c) The sufficient condition is the hypothesis: If you bought the computer less than a year ago, then the warranty is good.

d) If Willy cheats, then he gets caught.

e) The "only if" condition is the conclusion: If you access the website, then you must pay a subscription fee.

g) If Carol is on a boat, then she gets seasick.

Section 1.2

4

The condition stated here is that if you use the network, then either you pay the fee or you are a subscriber. Therefore the proposition in symbols is $w \to (d \vee s)$.

16 all

a) If the explorer (a woman, so that our pronouns will not get confused here—the cannibals will be male) encounters a truth-teller, then he will honestly answer "no" to her question. If she encounters a liar, then the honest answer to her question is "yes," so he will lie and answer "no." Thus everybody will answer "no" to the question, and the explorer will have no way to determine which type of cannibal she is speaking to.

b) There are several possible correct answers. One is the following question: "If I were to ask you if you always told the truth, would you say that you did?" Then if the cannibal is a truth teller, he will answer yes (truthfully), while if he is a liar, then, since in fact he would have said that he did tell the truth if questioned, he will now lie and answer no.

36 all

Note that Diana's statement is merely that she didn't do it.

a) John did it. There are four cases to consider. If Alice is the sole truth-teller, then Carlos did it; but this means that John is telling the truth, a contradiction. If John is the sole truth-teller, then Diana must be lying, so she did it, but then Carlos is telling the truth, a contradiction. If Carlos is the sole truth-teller, then Diana did it, but that makes John truthful, again a contradiction. So the only possibility is that Diana is the sole truth-teller. This means that John is lying when he denied it, so he did it. Note that in this case both Alice and Carlos are indeed lying.

b) Again there are four cases to consider. Since Carlos and Diana are making contradictory statements, the liar must be one of them (we could have used this approach in part (a) as well). Therefore Alice is telling the truth, so Carlos did it. Note that John and Diana are telling the truth as well here, and it is Carlos who is lying.

Section 1.3

10

We construct a truth table for each conditional statement and note that the relevant column contains only T's. For part (a) we have the following table.

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \to q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

For part (b) we have the following table. We omit the columns showing $p \to q$ and $q \to r$ so that the table will fit on the page.

p	q	r	$(p \to q) \to (q \to r)$	$q \to r$	$[(p \to q) \to (q \to r)] \to (p \to r)$
T	T	T	T	T	T
T	T	F	F	T	T
T	F	T	T	T	F
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	F	T	F
F	F	T	T	T	F
F	F	F	T	T	T

For part (c) we have the following table.

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

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It is easy to see from the definitions of the logical operations involved here that each of these propositions is true in the cases in which p and q have the same truth value, and false in the cases in which p and q have opposite truth values. Therefore the two propositions are logically equivalent.