

Note: 4 questions on both sides, maximum 22 points, **show all steps (as appropriate) to get full credit.**

1. [4 points] Use a proof by cases to show that
 $\min(a, \min(b, c)) = \min(\min(a, b), c)$ whenever **$a, b,$** and **c** are real numbers.

There are three main cases, depending on which of the three numbers is smallest. If a is smallest (or tied for smallest), then clearly $a \leq \min(b, c)$, and so the left-hand side equals a . On the other hand, for the right-hand side we have $\min(a, c) = a$ as well. In the second case, b is smallest (or tied for smallest). The same reasoning shows us that the right-hand side equals b ; and the left-hand side is $\min(a, b) = b$ as well. In the final case, in which c is smallest (or tied for smallest), the left-hand side is $\min(a, c) = c$, whereas the right-hand side is clearly also c . Since one of the three has to be smallest we have taken care of all the cases.

2. [6 points] Let $A = \{0, 2, 4, 6, 8, 10\}$, $B = \{0, 1, 2, 3, 4, 5, 6\}$, and $C = \{4, 5, 6, 7, 8, 9, 10\}$. Find:

a. **$A \cap B \cap C$**

[4, 6]

b. **$(A \cup B) \cap C$**

[4, 5, 6, 8, 10]

3. [6 points] Find the power set of each of these sets, where ***a*** and ***b*** are distinct elements.

a. $\{a\}$

$\{\emptyset, \{a\}\}$

b. $\{a, b\}$

$\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

c. $\{\emptyset, \{\emptyset\}\}$

$\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

4. [6 points] Determine whether each of these functions is a **bijection** from ***R*** to ***R***.

a. $f(x) = -3x + 4$

This is a bijection since the inverse function is $f^{-1}(x) = (4 - x)/3$.

b. $f(x) = -3x^2 + 7$

b) This is not one-to-one since $f(17) = f(-17)$, for instance. It is also not onto, since the range is the interval $(-\infty, 7]$. For example, 42548 is not in the range.