1. Consider the matrix $A = \begin{pmatrix} 3 & 1 & -5 & 0 & 5 \\ 2 & 1 & -3 & 1 & 7 \\ 1 & 1 & -1 & 1 & 6 \end{pmatrix}$ which is row equivalent to $B = \begin{pmatrix} 1 & 0 & -2 & 0 & 1 \\ 0 & 1 & 1 & 1 & 5 \\ 0 & 0 & 0 & 1 & 3 \end{pmatrix}$.

(a) What is the relationship between the solution sets to $\mathcal{LS}(A)$ and $\mathcal{LS}(B)$? **(1 point)**

(b) Find a matrix C that is row equivalent to A and is in reduced row echelon form. Label any row operations you preform. **(2 points)**

(c) Parameterize all solutions to the linear system $\mathcal{LS}(A)$. **(3 points)**

(d) Find the dimension of the rowspace $\mathcal{R}(A)$ of A. **(2 points)**

2. Let W be a subset of a vector space V. We say that W is *awesome* when both:

(i) The subset W is nonempty.

(ii) For all w_1 and w_2 in W and $a \in \mathbb{R}$, the linear combination $aw_1 + w_2$ is also in W.

Prove that if W is a subspace then it is awesome. **(6 points)**

3. Let S in $\mathrm{Mat}_{2\times 2}(\mathbb{R})$ be the subspace of symmetric matrices, i.e., those whose transpose A^t is equal to A.

(a) Give an explicit basis for S, carefully justifying your answer. **(6 points)**

(b) What is the dimension of S and why? **(2 points)**

4. Suppose $\{w_1, w_2\}$ are linearly independent vectors in a vector space V. For v in V, prove that if $\{v, w_1, w_2\}$ is linearly dependent then v is in $\text{span}(w_1, w_2)$. **(6 points)**

5. Suppose V is a vector space where $\dim V = 2$.

(a) For any subspace $W \subset V$, what are the possibilities for $\dim W$? **(2 points)**

(b) Suppose $W_1 \subset W_2 \subset W_3 \subset W_4$ are all subspaces of V. Prove that at least two of W_1, W_2, W_3 and W_4 are the same. Hint: Use part (a). **(5 points)**

6. Circle true or false as appropriate; you do **not** need to provide any justification. **(1 point each)**

(a) If $A \in \mathrm{Mat}_{3\times 3}(\mathbb{R})$ is row equivalent to a matrix in RREF that has no zero rows then the linear system $\mathcal{LS}(A)$ is consistent. $\boxed{\text{true} \quad \text{false}}$

(b) If $\{u, v, w\}$ is a basis for $\mathbb{R}^3$ then $\{u - v, v - w, w - u\}$ is also a basis for $\mathbb{R}^3$. $\boxed{\text{true} \quad \text{false}}$

(c) The set $\left\{(x, y, z) \in \mathbb{R}^3 \;\middle|\; x + y + z = 0\right\}$ is a subspace of $\mathbb{R}^3$. $\boxed{\text{true} \quad \text{false}}$

(d) The elements $f(t) = \sin^2(t)$, $g(t) = \cos^2(t)$ and $h(t) = 1$ are linearly independent in $\mathcal{F}(\mathbb{R}, \mathbb{R})$. $\boxed{\text{true} \quad \text{false}}$

(e) Suppose a linear system with 4 variables and 6 equations has $(1, 2, 0, 1)$ and $(3, 0, 1, 5)$ as solutions. Then the total number of solutions to this system is finite. $\boxed{\text{true} \quad \text{false}}$

(f) The set $\{a \sin(t) + b \cos(t) \mid a, b \in \mathbb{R}\}$ is a subspace of $\mathcal{F}(\mathbb{R}, \mathbb{R})$. $\boxed{\text{true} \quad \text{false}}$

(g) With the terminology of Problem 2, if a subset W of a vector space V is awesome then it is a subspace. $\boxed{\text{true} \quad \text{false}}$