Math 518: HW 2 due Friday, September 5, 2014.

Background knowledge: For the basics of topology, linear algebra, and calculus which are assumed in these problems, see Appendices A, B, and C of Lee's *Introduction to Smooth Manifolds*.

Homework policy: Late homework will not be accepted; however, your lowest two homework grades will be dropped so you are effectively allowed two infinitely late assignments. Collaboration on homework is permitted, nay encouraged, but you must write up your solutions individually and understand them completely.

1. Let X be the set of all points $(x, y) \in \mathbb{R}^2$ where $y = \pm 1$, and let M be the quotient of X by the equivalence relation $(x, -1) \sim (x, 1)$ for $x > 0$.

   (a) Show that M is locally Euclidean but is not a Hausdorff topological space.

   (b) While (a) implies that M is not a topological manifold as defined in class, are there still coordinate charts forming a smooth atlas on M?

2. Exercise 7 of Chapter 1 of Lee, which is on page 30.

3. Exercise 8 of Chapter 1 of Lee, which is on page 31.

4. Use the chain rule to show that if $\phi\colon \mathbb{R}^m \to \mathbb{R}^n$ is a diffeomorphism, then $m = n$. Use this to prove that if smooth manifolds M and N are diffeomorphic then $\dim M = \dim N$.

5. Consider the 2-dimensional torus T in $\mathbb{R}^3$ shown below, where the inner radius is 2 and the outer radius is 4 and hence each vertical circle has radius 1. Alternatively, T is the surface of revolution given by spinning the circle $(x-3)^2 + z^2 = 1$ about the z-axis. Give an explicit smooth atlas of charts for T exhibiting it as a 2-manifold.