Math 560 Homework 2 Fall 2021

Due: Wednesday, September 8

1. **Building NFAs**

   Draw NFAs for the following languages, with the specified number of states. You may assume the alphabet is always $\{0, 1\}$.

   (a) The language $\{w \mid w \text{ ends in } 00\}$, 3 states.

   (b) The language $\{w \mid w \text{ contains an even number of 0s or exactly two 1s}\}$, 6 states.

   (c) The language $0^*1^*0^+$, 3 states.

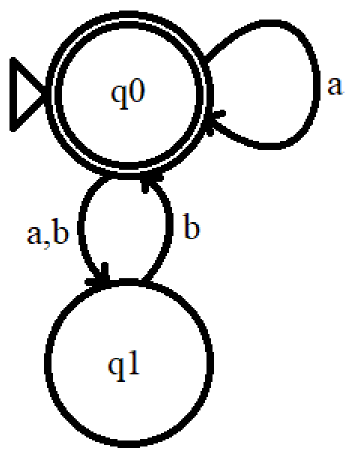
   (d) The language $1^*(001^+)^*$, 3 states.

   (e) The language 0^*, 1 state.

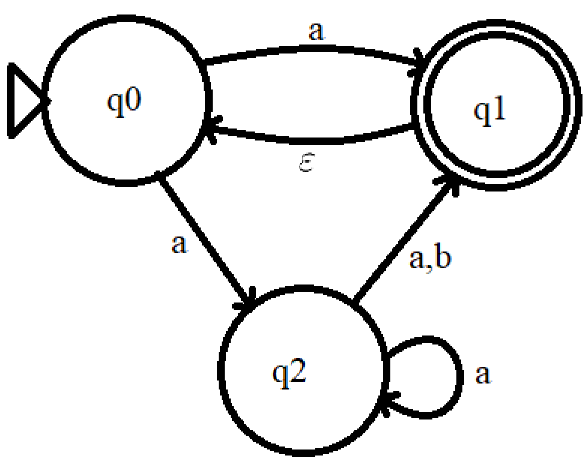
2. **Subset Construction**

   Turn the following NFAs into DFAs using the subset construction. It should be clear from your drawing of a DFA how the subset construction was applied.

   (a)



   (b)



3. **Reversing a Language and Building a Binary Adder**

   If A is a language, we define its reverse, $\mathit{reverse}(A)$, as the language of all strings in A written in reverse, i.e. $\mathit{reverse}(A) = \{w^{\mathcal{R}} \mid w \in A\}$. For example, if the string '00110' is in language A, then the string '01100' is in $\mathit{reverse}(A)$.

   (a) Prove that the set of regular languages is closed under the reverse operation, i.e., prove that if language A is regular, then $\mathit{reverse}(A)$ is also regular. (Hint: if A is regular, then there is an NFA M_1 that recognizes it. How would you change that NFA into a new machine M_2 that recognizes the reverse?)

   (b) Define the alphabet

   $$\Sigma_3 := \left\{ \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix} \right\}.$$

   The alphabet Σ_3 contains eight size-3 columns of 0s and 1s. A string of symbols in Σ_3 thus builds threes rows of 0s and 1s. Consider each row to be a binary number and define the language

   $$B = \{w \in \Sigma_3 \mid \text{the bottom row of } w \text{ is the sum of the top two rows}\}.$$

   For example,

   $$011 + 001 = 100 \quad \text{but} \quad 01 + 00 \neq 11,$$

   so

   $$\begin{bmatrix}0\\0\\1\end{bmatrix} \begin{bmatrix}1\\0\\0\end{bmatrix} \begin{bmatrix}1\\1\\0\end{bmatrix} \in B \quad \text{but} \quad \begin{bmatrix}0\\0\\1\end{bmatrix} \begin{bmatrix}1\\0\\1\end{bmatrix} \notin B.$$

   Show that B is regular. (Hint: show that $\mathit{reverse}(B)$ is regular, and by the previous problem conclude that $\mathit{reverse}(\mathit{reverse}(B)) = B$ is also regular.)

4. **Intersection and Complement**

   (Hint: DFAs are easier to use for the following proofs.)

   (a) Prove that the set of regular languages is closed under the complement operation, i.e., prove that if language A is regular, then its complement $\bar{A}$ is also regular.

   (b) Prove that the set of regular languages is closed under the intersection operation, i.e., prove that if languages A and B are regular, then $A \cap B$ is also regular.