

1. (a) At first, we have an input string S , a front index is equal to 0, and a back index is equal to 1.

$\text{recurValid}(S, \text{front}, \text{back}) :$

if back is equal to the length of S :

return $\text{dict}(S[\text{front} : \text{back}])$

else if $\text{dict}(S[\text{front} : \text{back}])$ is True :

return $\text{recurValid}(S, \text{front}, \text{back} + 1)$ or

$\text{recurValid}(S, \text{back}, \text{back} + 1)$

else $\text{dict}(S[\text{front} : \text{back}])$ is False :

return $\text{recurValid}(S, \text{front}, \text{back} + 1)$

(b) $dpValid(S)$:

n is the length of string S

construct a dp table with n rows and $n+1$ columns with None filled initially

for every element j in range $(1, ~~length~~ + 1)$:

$dp[0][j] = dict(string[0:j])$

for every element z in range $(1, n)$:

for every element j in range $(z+1, n+1)$:

if $dict(string[z:j])$ is true :

$dp[z][j] = dp[z-1][j]$ or $dp[z-1][z]$

else $dict(string~~[z:j]~~[z:j])$ is false :

$dp[z][j] = dp[z-1][j]$

return $dp[n-1][n]$

(c) We first have an empty solution array
Then we look into our dp table and set index $j = n$
initially (the dp table has n rows and $n+1$ columns)

while $j > 0$:

look at the j th column
for every element z in range $(0, n)$:

if $dp[z][j]$ is True:

append z into our solution array, stop search
and jump to the z th column ($j = z$) and
start search again

if all grid points in this column is False, stop the
whole process and indicate that input string S is
an invalid text document

If we successfully arrive at the first column (0th), we
can say that input string S is a valid text document

Reverse our solution array and assume its element:

$[a_1, a_2, \dots, a_n]$ and $a_1 = 0$ is inevitable

Now we can reconstruct our String S by dividing it into
several valid words (remember n is the length of S):
 $S[a_1:a_2], S[a_2:a_3], \dots, S[a_{n-1}:a_n], S[a_n:n]$

(d) Since front < back is required in our algorithm, we did not
fill a triangle area in the dp table where front \geq back
So the running time: $n \times (n+1) - \frac{1}{2} \times n \times n$
 $= \frac{1}{2}n^2 + n$

$\in O(n^2)$ (n is the length of
string S)