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THE MEASUREMENT OF RESISTANCE OF UNKNOWN WIRES USING THE WHEATSTONE BRIDGE APPARATUS

Abstract:

Using Ohm's law, the resistance of a meter wire is calculated. Using the same wire, a Wheatstone bridge is constructed with a standard resistance panel to calculate the resistance of 5 unknown wires. The resistivities of the unknown wires are derived and compared to literature values of the same material. The results are found to be similar: $\rho(\mathrm{Cu}) = 1.7 \times 10^{-8}\Omega\mathrm{m}$ where our value agrees within two standard deviations of $(1.65 \pm 0.62) \times 10^{-8}\Omega\mathrm{m}$.

Introduction:

In this lab, we will apply ohm's law to find the resistance of a wire. We will use this wire to calculate the resistance and other properties of 5 coils. We cannot simply use a digital multimeter for this lab because the resistance of conductors like copper is too low for precise measurement.

A Wheatstone apparatus consists of a galvanometer and 4 resistors, two of known resistance R_1 and R2, one of well-defined variable resistance R_s, and one of unknown resistance R_x. The two known resistors in our apparatus turn out to be a bisection of the same meter wire whose resistance was calculated. Due to the mathematics of the Wheatstone bridge, only the ratio R_1 to R_2 is required, not absolute resistance. So, due to constant resistance density in the wire, the ratio of lengths of the wire bisections is sufficient. R_S is simply a decade resistor box that allows us to create any resistance between 0.1Ω and 1111.0Ω in 0.1Ω increments. R_X is a coil of wire with known material, length and thickness.

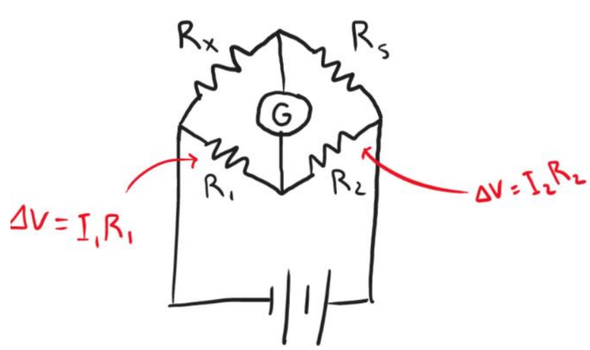

FIGURE 1: WHEATSTONE BRIDGE DIAGRAM

When making a measurement with the bridge, we want the current across the bridge to be zero. When this is the case, the voltage across R_1 and R_X is equal, and the voltage across R_2 and R_S is equal. This allows equate the ratios of resistance:

$$\frac{R1}{R2} = \frac{Rx}{Rs}$$

Once the resistance of a coil is calculated, we need to compare our results to literature values. Resistance of a wire is increased when the length of the wire increases. When the cross-sectional area of the wire decreases, the resistance increases. This means we must use a property of a conductor which normalizes both variables:

$$\rho = \frac{\mathrm{RA}}{\mathrm{L}}$$

Resistivity is defined for a wire as resistance times cross-sectional area over length. These values are tabulated for different materials which allows us to compare to the literature.

Experimental Procedure:

The resistance of a meter wire is determined by measuring the voltage across the wire and measuring the current through the circuit. The power supply is adjusted so that the ammeter measures five values between 0 and 80 mA. At each current, the voltage is measured and a plot of voltage over current is generated. The slope is calculated and recorded with uncertainty as the resistance of the meter wire. Next the thickness of the wire is measured at five different points using a micrometer. The values are averaged to obtain the diameter of the meter wire. The resistivity can be calculated now.

In the second half of the experiment, the Wheatstone bridge is connected according to figure 1. The entire length of the wire is R1 and R2, the node between the two is a contactor that you can move around that is connected to the galvanometer. Select some Rs around .3 Ohms, move the system voltage to around .3V and move the contactor around to try and make I=0. The most accurate measurements are made when the contactor is close to the middle position so incrementally increase Rs by .1 Ohms until you can place the contactor near the middle while minimizing the magnitude of the galvanometer. Now increase the voltage to 1.5V to get a more accurate measurement of current and flick the contactor by tiny amounts to truly zero in. Now Measure the length of the wire to the left of the contactor as L1 and to the right as L2, note Rs and move to the next coil.

Results and Data:

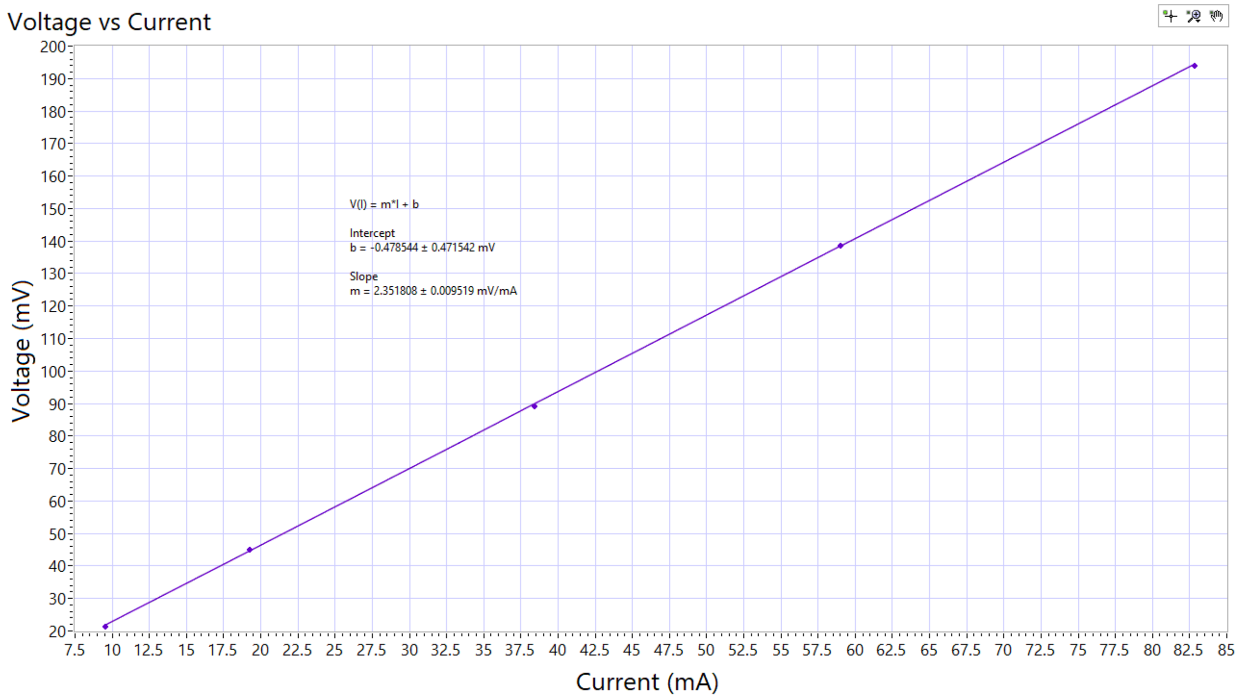

FIGURE 2: PLOT OF VOLTAGE OVER CURRENT FOR THE METER WIRE.

The slope of this graph is the average value for V/I which is resistance by Ohm's law. Our value for the resistance of the meter wire is calculated to be (2.352+-0.009)Ωm.

The calculation of our meter wire was done using student-t distribution. Using the formulas

$$s^2 = \frac{1}{n-1}\sum(x_i - \bar{x})^2$$

for variance s of a set x with n elements and for critical value t:

$$\bar{x} \pm \frac{ts}{\sqrt{n}}$$

With 95% confidence interval and 4 degrees of freedom the critical value of a two tailed distribution is 2.776. Therefore, for our data:

diameter (mm)
0.5023
0.5025
0.5026
0.2016
0.5026

$$\overline{x} = (0.5023 \pm 0.0005)mm$$
$$s = 0.000421$$

So, our 95% confidence interval is $(0.5023 \pm 0.0005)mm$

The following calculation gives the area of the wire.

$$\pi r^2 = 3.141593\left(\frac{(0.5023 \pm 0.0005)\mathrm{mm}(1\mathrm{m}/1000\mathrm{mm})}{2}\right)^2 = (1.982 \pm 0.004) \times 10^{-6}\ \mathrm{m}^2$$

Using the definition of resistivity we can also calculate:

$$\rho = \frac{\mathrm{RA}}{\mathrm{L}} = \frac{(2.352 \pm 0.009)\Omega(1.982 \pm 0.004) \times 10^{-6}\ \mathrm{m}^2}{1.00 \pm 0.01\mathrm{m}} = (4.66 \pm 0.07) \times 10^{-6}\Omega\mathrm{m}$$

For the second half of the experiment, the data L1, L2 and Rs was collected. The ratio in the table is presented to make calculations more straightforward and serve as a visual aid. Note that measurements are most accurate when the ratio is close to one. We can calculate Rx for example:

$$\mathrm{R_x} = \mathrm{R_s}\frac{\mathrm{L_1}}{\mathrm{L_2}} = 0.5\Omega * 1.064 = 0.5320\Omega$$

Coil No.	L1 (cm)	L2 (cm)	L1/L2	Rs (Ω)	Rx (Ω)
1	51.55	48.45	1.064	0.5	0.5320
2	49.50	50.50	0.9802	1.9	1.862
3	57.20	42.80	1.336	1.0	1.336
4	49.20	50.15	0.9940	4.2	4.175
5	49.55	50.45	0.9822	9.5	9.330

To eliminate any discrepancy due to uneven resistance density on the wire: R1 and R2 are flipped:

Coil No.	L1 (cm)	L2 (cm)	L1/L2	Rs (Ω)	Rx (Ω)
1	55.00	45.60	0.818	0.5	0.4091
2	49.55	50.45	1.0180	2.0	2.036
3	49.38	50.62	1.025	1.2	1.230
4	49.20	50.80	1.0320	4.2	4.336
5	50.90	49.10	0.9646	9.5	9.164

For our last trial we have:

Coil No.	L1 (cm)	L2 (cm)	L1/L2	Rs (Ω)	Rx (Ω)
1	42.00	58.00	1.381	0.6	0.8286
2	50.55	49.45	0.9782	2.1	2.054
3	51.80	48.20	0.9305	1.3	1.210
4	49.45	50.55	1.022	4.3	4.396
5	52.63	47.37	0.9000	10.0	9.000

Each coil has a specific cross-sectional area and length specified below. We can calculate resistivity for each now. It is relevant to consider the first four coils consist of copper and the last is nickel silver alloy: thus, the larger resistivity.

Coil No.	R (Ω)	L (m)	Cross-sectional area ($10^{-8}m^2$)	Resistivity (10^{-8} Ωm)
1	0.59±.12	10±.5	32±2	1.8±.6
2	1.98±.06	10±.5	7±1	1.3±.3
3	1.26±.04	20±.5	32±2	2.0±.2
4	4.30±.07	20±.5	7±1	1.5±.3
5	9.16±.10	10±.5	32±2	29±4

Compared to the literature values: the first five coils are all copper and the last is a nickel silver alloy. The resistivity of both are $1.7*10^{-8}$ Ωm and $33*10^{-8}$ Ωm respectively. Using the two standard error criterion, all of our calculated resistivities agree with the expected values of the metals used for the experiment.

Conclusion:

Using the Wheatstone bridge, the resistance of 5 different wires with various lengths, areas and materials was calculated. Since resistivity is a property of a substance, we should expect the resistivity of all copper wires to match that of the literature value of $1.7*10^{-8}$ Ωm. We find each of our measurements of resistivity to agree with the literature value within two standard errors. The same result is found for the nickel silver alloy wire with literature resistivity of $33*10^{-8}$ Ωm and measured $(29±4)*10^{-8}$ Ωm.

Although our values agree, we were unable to make a precise measurement for the resistivity of copper. During the process of zeroing the current on the Wheatstone bridge, we found that the conductor would sometimes introduce random error most likely due to a poor contact being made to the wire.