PHYS 266 – University Physics II Laboratory

Experiment 1: Electric Fields

Overview

We begin a two-week study of electric fields where you will first deduce the strength and shape of an electric field near a set of conductors by making experimental measurements and next deduce the same from simulations. You will measure or simulate the electric potential and use these results to infer information about the electric field.

Definition of Electric Field

An electric field is said to exist at a point in space if a small test charge, placed at that point, experiences a force. The electric field $\vec{E}$ is a vector that is defined as the ratio of the force $\vec{F}$ experienced by the test charge divided by the magnitude of the test charge q. The direction of the electric field is the direction of the force when a **positive** test charge is used. Mathematically this can be expressed as

$$\vec{E} = \frac{\vec{F}}{q}$$

This basic definition of electric fields is interesting from a theoretical point of view but not particularly useful from an experimental perspective. When you actually place a test charge at a location it will influence the field at that location, unless the charge is very small. We will describe an alternative, more indirect, approach to measuring and calculating electric fields below.

Relationship Between Electric Field and Electric Potential

It is conceptually easy to place a small test charge in an electric field, and find the force on it, though not practical to do so. The electric field must be measured by other indirect means, such as measuring the electric potential and using relationships between the electric potential and the electric field to find the electric field strength. The electric potential can be easily measured with a high impedance voltmeter.

The electric potential is best defined in terms of <u>potential difference</u>, since the potential at any point may be set arbitrarily to zero. A potential difference exists between two points in an electric field when work is required to move a charge from one point to the other. The electric potential difference ΔV between these two points then is defined as the work W, required to move a small positive test charge q from one point to the other, divided by the magnitude of the test charge, i.e.,

$$\Delta V = \frac{W}{q}$$

The work, and hence the potential difference between the two points, is independent of the path that is taken between the two points. Using the basic definition of work,

$$W = \vec{F} \cdot \vec{d}$$

a relationship between the electric potential difference and the electric field strength can be found. Keep in mind that the dot product depends upon the angle between the force and the displacement. It takes zero work to move in a direction perpendicular to the force and the work is maximum when parallel to the force. Using this definition of work we can write the expression for the potential difference between two points described by the displacement $\vec{d}$ as

$$\Delta V = \frac{W}{q} = \frac{\vec{F} \cdot \vec{d}}{q} = \vec{E} \cdot \vec{d}$$

Again, remember that the dot product depends on the angle between the electric field and the displacement. If the displacement is perpendicular to the electric field, then the potential difference will be zero. When the displacement $\vec{d}$ is in the same direction as $\vec{E}$ then $\vec{E} \cdot \vec{d}$ reduces to Ed and the potential difference is given by

$$\Delta V = Ed.$$

So, when we pick a small displacement d in the direction of the electric field we can find the magnitude of the electric field from

$$E = \frac{\Delta V}{d}$$

If a test charge were moved in a direction perpendicular to the electric field, no work would be done in doing so. When no work is done, there is no change in potential, and the charge moves along what is called an equipotential line. Therefore, ***electric field lines and equipotential lines are perpendicular***. The potential undergoes no change in a direction perpendicular to the electric field and undergoes a maximum change in a direction parallel to the field.

Finding Electric Field from Electric Potential

Your goals over the next two weeks will be to find the electric field by first finding the electric potential. In Experiment 1A you will measure the potential and in Experiment 1B you will compute it using a computational approach.

Once the potential is known, the ***direction of the electric field is parallel and opposite to the direction in which the electric potential increases the most***. In calculus, an operator called the gradient and symbolized by ∇ (called del operator) is introduced to signify changes that must be made in a direction in which the change is greatest. With this notation the electric field may be written in terms of calculus so that

$$\vec{E} = -\nabla V = -\lim_{\Delta s \to 0} \frac{\Delta V}{\Delta s} \hat{s},$$

where ΔV is the change in potential, Δs is the distance over which this change take place, and $\hat{s}$ is the unit vector in the direction over which the change in potential is the greatest. The unit vector $\hat{s}$ will be perpendicular to the equipotential lines. The electric field is the *negative* gradient of the potential because the electric field is in the *opposite* direction to the direction in which the potential is increasing. The symbol $\lim_{\Delta s \to 0}$ means that Δs must be very small.

The mks units of electric field are Newton/Coulomb (N/C) which are equivalent to volts/meter (V/m). More commonly, though, the electric field strength is measured in units of volts/cm (V/cm), a hybrid of cgs and mks units.

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1A: Electric Field from Measurements of Electric Potential

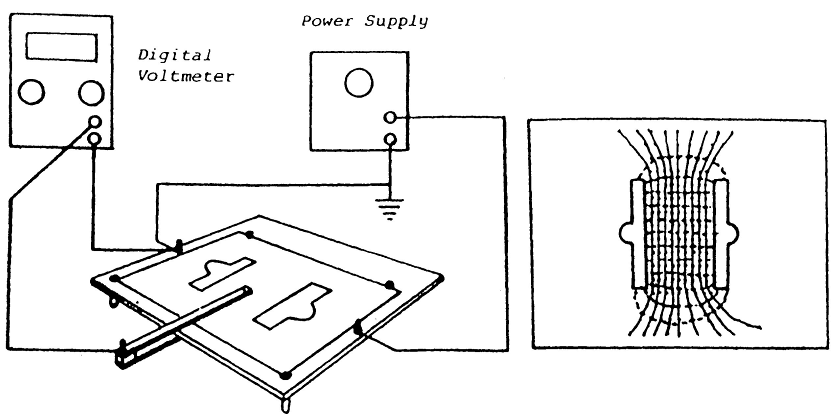
Objectives

The objectives of this experiment are:

- to learn how to measure the electric potential,
- to study the relationships between the electric potential and electric field strength,
- to learn how to predict the electric field direction, and
- to learn how to compute the electric field strength.

Procedure

The Cenco-Overbeck apparatus is used to map out equipotential lines from which the electric field can be derived. An electric field will be produced in a resistive medium (conducting paper) by the application of a voltage (emf) to two conducting electrodes that are attached to the resistive medium. The resistive medium is a conducting paper that has been impregnated with carbon and the conducting electrodes have been made by painting the paper with silver conducting paint. The source of the emf will be a DC power supply.



Points at the same potential lie on a line called an equipotential line. Once the equipotential lines have been mapped, the electric field lines, which are perpendicular to the equipotential lines, may be sketched. The strength of the electric field at any point is found by measuring the potential difference between adjacent equipotential lines and then dividing it by the distance between them, when the distance is measured along the electric field line (perpendicular to the two equipotential lines).

The number of plates that are examined depends on your instructor. It is generally accepted that 2 plates are examined each class.

1. <u>Measure the Equipotential Lines</u>

1.1. Examine the apparatus, and make sure that it is connected as shown in the above figure. The apparatus consists of a board on which the conducting paper with an electrode configuration may be attached to the bottom. The top side of the board provides for the placement of a sheet of graph paper and has guide pins for the positioning of a template that matches the position of the electrodes on the bottom of the board. The top portion of the board has a terminal on each side that is connected to the two conducting electrodes. A "U" shaped probe provides a mechanism by which the potential may be measured by contacting the conducting paper on the bottom. The corresponding position may then be recorded on the paper on the top. Check the bottom of your apparatus for the electrode configuration

1.2. Turn the digital voltmeter on to the 20 DCV range, or if using a multimeter set it to the 20 DCV range.

1.3. Obtain a sheet of blank paper and attach it to the top of the board. The feet of the board are spring loaded so that if you push down on each corner of the board, a rubber pad will rise up so each corner of the graph paper may be inserted and held down when the spring is allowed to return to its normal position. Sometimes the rubber pads don't hold well so it is advisable to use a piece of masking tape to keep the paper from moving.

1.4. Place the clear plastic template over the positioning pins and trace the outline of the parallel electrodes. Use a colored fiber tipped pen, so that it stands out, or shade the electrodes with pencil or ink. Remove the template.

1.5. Place the "U" shaped probe so that it is between the electrodes near the center of the board. When readings are being made with the probe, it should be squeezed together slightly so good contact occurs between the probe and the conducting paper. Be sure the probe's indicating circle and contact point are always directly opposite.

1.6. Move the probe to the right electrode and adjust the power supply so that the electrode's potential is 10 volts. Also note that the potential is the same at every point on the electrode.

1.7. Move the probe to your left and then to your right and note how the voltage changes.

1.8. Move the probe toward and then away from you and note how the voltage changes.

1.9. Note the directions in which the potential increases and decreases the most and the least. Record the observations.

1.10. Move the probe to the left electrode and note that the potential there is zero. Also note that the potential is zero at every point on the conducting electrode. This results from arbitrarily choosing the zero point by connecting the ground side of the voltmeter to this electrode. Ground potential is customarily zero.

1.11. Move the probe around until you find a point at one volt potential. Mark that point on the graph paper (draw a small circle using the hole at the end of the probe) and repeat this procedure many times until you can construct with certainty the one-volt equipotential line. Don't be afraid to go around behind the electrodes to measure the potential and find points. Connect the points with a smooth solid line.

1.12. Repeat the previous step for the 3-, 5-, 7-, and 9-volt equipotential lines.

2. ***Find the Electric Field***

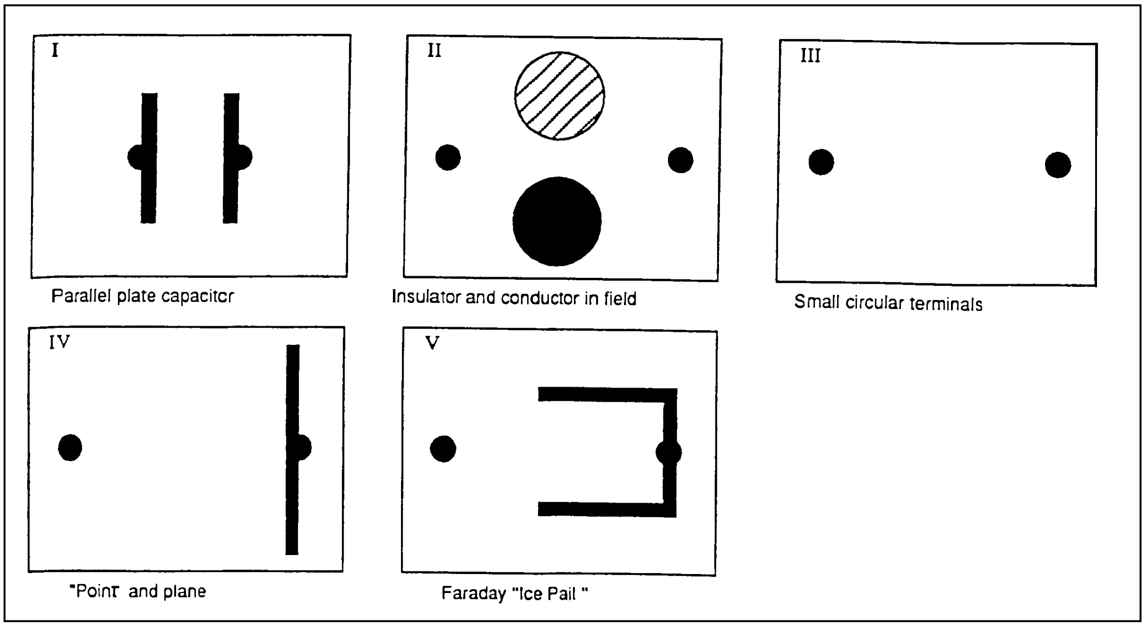
2.1. Draw continuous dashed lines perpendicularly to the equipotential lines and indicate the direction of the electric field on the dashed lines.

2.2. Have an instructor designate points on your graph at which you are to compute the magnitude of the electric field strength. Label them A, B, etc.

2.3. Compute the electric field strength at the indicated point by moving a little distance (less than 0.5 cm) to the left along the electric field line and a little distance to the right along the electric field line. Record the electric potential at each of these adjacent points, and find their potential difference, ΔV. Measure the distance between the two points to find Δs. Compute the electric field strength by dividing ΔV by Δs.

2.4. Repeat the entire process for other electrode geometries required by your instructor.

This figure shows the various plates of electrode geometries that are available. Which plates you will use depend on your instructor.



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1B: Electric Field from Simulations of Electric Potential

Objective

The objective of this activity is to understand how to use a computer to solve a boundary-value problem to determine the potential in a region when the potential is known on some given boundary. You will learn a technique called the Method of Relaxation to solve this problem.

The Method of Relaxation

A Geometrical Approach

In the first part of the Electric Fields experiment, you used a power supply to place a potential difference between two conductors and used a voltmeter to map the *equipotential lines* in the region between the conductors. Since the electric field lines are perpendicular to the equipotential lines, you were able to draw a map the electric field. In this week's experiment, you are going to use a computer to simulate the results of this experiment and explore how you can predict electric potential, and therefore electric field lines, for any geometry. This technique, called the *Method of Relaxation*, is described below and in a video at http://physics.wku.edu/up/phys266/e1/1b/.

The previous experiment where you measured potential probably has given you some intuition in how the voltages establish themselves in space. Look at the figure below… The right-most square has been set to a 10 V potential compared to the leftmost square. What potential difference do you expect between

0	?	10

the leftmost square and the middle square? Even if you had not performed the previous experiment, your intuition probably tells you that the square marked with a ? would be 5V.

Now let's consider a different case that is slightly more complex. You should be able to see that this problem boils down to recognizing that the first case has been replicated in a different orientation. Thus, the answer to ? is 5 V again.

	0	
0	?	10
	10	

Therefore, the potential of a given square is the average of the potentials in the squares that immediately surround the square in question.

For the first case, we can write:

? = ½ (0 + 10)
? = 5

For the second case, we can write:

? = ¼ (0+0+10+10)
? = 5

And now, in general, for any 2-dimensional geometry, to find a potential, $V_{x,y}$ at a point (x, y):

$$V_{x,y} = \frac{1}{4}\left(V_{x-1,y} + V_{x+1,y} + V_{x,y-1} + V_{x,y+1}\right)$$

A More Rigorous Approach

We can show that the electrostatic potential at any point in space must be the average of the potential at all surrounding points in a more rigorous manner using calculus. First, recall that electrostatic potential, $V(x, y)$, is a scalar function that is equal to the negative gradient of the electric field. Mathematically, we write this as

$$\vec{E} = -\nabla V,$$

or equivalently

$$E_x = -\frac{\partial V}{\partial x} \text{ and } E_y = -\frac{\partial V}{\partial y}.$$

This means that <u>the electric field is in the *opposite* direction to the direction of the greatest increase of the potential</u>. Equivalently, the electric field will always be *perpendicular* to the lines of constant potential, the equipotential lines you measured last week.

Later you will learn that the electric field depends on the charge distribution ρ according to Gauss' Law which can be written in differential form as

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}.$$

But in the space between the electrodes the charge density is zero, so the right-hand side of the above equation is zero and the left-hand side becomes

$$\nabla \cdot \vec{E} = \nabla \cdot (-\nabla V) = \nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0.$$

This equation, $\nabla^2 V = 0$, called Laplace's equation, implies that if the function $V(x, y)$ is concave up in one direction it must be concave down in the other. To solve Laplace's equation we need to approximate the second derivative for non-infinitesimal dx and dy. So, the first derivative can be written as a central difference for a small change in position of δ as

$$\frac{\partial V}{\partial x} \approx \frac{V\left(x + \frac{1}{2}\delta\right) - V\left(x - \frac{1}{2}\delta\right)}{\delta}.$$

For the second derivative you replace V with its derivative and after a little work you obtain

$$\frac{\partial^2 V}{\partial x^2} \approx \frac{V(x+\delta) + V(x-\delta) - 2V(x)}{\delta^2}.$$

Using a similar result for the second derivative with respect to y, and then simplifying leads to

$$V(x, y) = \frac{1}{4}[V(x+\delta, y) + V(x-\delta, y) + V(x, y+\delta) + V(x, y-\delta)].$$

So again, we see that potential at any point in our grid is the average of the potential at four nearest points (left, right, above, and below).

If the calculus and algebra in this section is a little tough for you at this point just focus on the main takeaway, which is that the <u>potential at any point can be found by taking the average value at surrounding points</u>.

Implementation of the Relaxation Method

In order to use this approach, the following requirements must be met:

1. The geometry in question must be divided into equal spaces.
2. The potentials at the boundaries must be known.

The second requirement is not hard to meet. We know where in the geometry that we apply voltage and we know in the geometry where we have grounded the surface. From our study of potentials and fields we also know that the electric potential goes to zero when radial position, r, becomes very far from any charges.

In order to get a precise solution for the potentials, the geometry in question must be divided into small spaces. How small or how large depends on the precision desired as well as the processing speed of the computer. It is true that one could perform this calculation by hand, but a computer is designed for this type of calculation.

As an example, let's consider another case where an 8 x 8 set of cells has different potentials placed on its boundaries. Three sides of this 8x8 grid have been grounded (zero potential) and the fourth side has been set to 10 V. The interior cells, which we do not really know the values for yet, are initially all set to zero volts.

0	0	0	0	0	0	10
0	0	0	0	0	0	10
0	0	0	0	0	0	10
0	0	0	0	0	0	10
0	0	0	0	0	0	10
0	0	0	0	0	0	10
0	0	0	0	0	0	10
0	0	0	0	0	0	10

Now let's use Equation 1 and calculate the values in the interior white cells by starting at the top left and going across each row one at a time. After 1 computation we obtain the following result.

0	0	0	0	0	0	10
0	0	0	0	0	2.5	10
0	0	0	0	0	3.125	10
0	0	0	0	0	3.2813	10
0	0	0	0	0	3.3203	10
0	0	0	0	0	3.3301	10
0	0	0	0	0	3.3325	10
0	0	0	0	0	0	10

N = 1 iteration

We see that most of the values are still zero except those cells nearest the 10 V potentials. Now we will calculate the averages in all cells again as shown below. Now more cells begin to have non-zero values.

0	0	0	0	0	0	10
0	0	0	0	0.625	3.4375	10
0	0	0	0	0.9375	4.4141	10
0	0	0	0	1.0547	4.6973	10
0	0	0	0	1.0938	4.7803	10
0	0	0	0	1.106	4.8047	10
0	0	0	0	1.1096	3.9786	10
0	0	0	0	0	0	10

N=2 iterations

The question is, *which is the correct solution?* The one that happened after one calculation or the solution after two calculations? The answer is neither. We must continue to do calculations (to *iterate*) until the solutions are stable. Now let's see what happens after 100 calculations or *iterations* (below left). Or perhaps 1000 iterations (below right).

0	0	0	0	0	0	10
0	0.3601	0.8051	1.4578	2.5637	4.7561	10
0	0.6363	1.4039	2.4634	4.0418	6.4611	10
0	0.7825	1.7122	2.9515	4.6799	7.0469	10
0	0.7827	1.7125	2.9518	4.6801	7.0471	10
0	0.6368	1.4047	2.4642	4.0424	6.4615	10
0	0.3606	0.8059	1.4586	2.5643	4.7565	10
0	0	0	0	0	0	10

N = 100 iterations

0	0	0	0	0	0	10
0	0.3601	0.8051	1.4578	2.5637	4.7561	10
0	0.6363	1.4039	2.4634	4.0418	6.4611	10
0	0.7825	1.7122	2.9515	4.6799	7.0469	10
0	0.7827	1.7125	2.9518	4.6801	7.0471	10
0	0.6368	1.4047	2.4642	4.0424	6.4615	10
0	0.3606	0.8059	1.4586	2.5643	4.7565	10
0	0	0	0	0	0	10

N=1000 iterations

We see that there is no difference in 100 iterations versus 1000 iterations. We say that the solution has *converged* to single solution. So, the correct solution is when the calculation has converged to a single set of values. In performing this type of calculation, it is easy to overestimate the number of iterations required to reach convergence. We can change the time required to reach convergence by changing the precision to which we perform the calculation. For example, if we had used two significant figures instead of 6 in the above example, the solution would have reached convergence much more quickly.

Procedure

You will use Microsoft Excel to calculate the potential for several electrode geometries. Some Microsoft Excel templates that depict the geometries of the conductor plates that you used last week with the Cenco-Overbeck apparatus are available at http://physics.wku.edu/up/phys266/. This site also includes a number of videos that help show how to use Microsoft Excel to implement the relaxation method to compute the potential as well as some that show how to best visualize the results.

Keep in mind that the computations you are performing are simply finding the potential at a given point by calculating the average value at all points that surround it.

1. ***Compute the potential as function of position for the parallel plate geometry.***

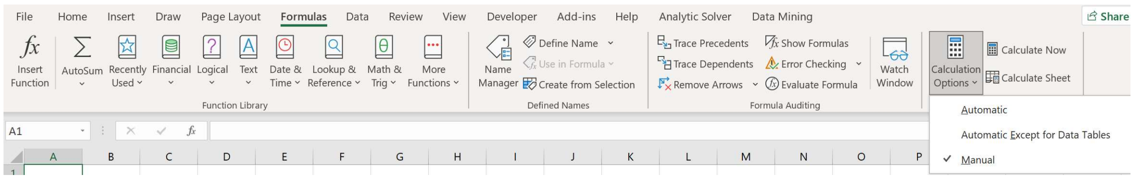
    1.1. Browse to http://physics.wku.edu/up/phys266/ and download the Microsoft Excel template file for the parallel-plate geometry. These templates have the conductors shaded with a light blue color, the boundaries of the region shaded with light gray, and the interior region with no shading or white in color.

    1.2. Watch the video on setting up the relaxation method computation. You will enter formulas for each white cell in the interior of the region where the value of that cell is equal to the average of the four cells that surround it to the top, bottom, left, and right.

    1.3. Enter formulas in the gray boundary cells (excluding the corners) so that these cells equal the average value of the three cells surrounding it on the interior or boundary of the area. The formula in the corners should depend upon the two values that are adjacent to it on the two intersecting boundaries.

    1.4. Enter values of 0.0 or 10.0 in the appropriate blue cells for the conductors.

1.5. Having set up the formulas for each cell in your spreadsheet, it should be clear that if you consider two adjacent points in your spreadsheet that each one depends upon the other. The final exact values in your spreadsheet cannot all be computed at once because of this interdependence. This cyclic dependency is called recursion. Because of the recursion, the automatic computation that normally occurs in a spreadsheet has been disabled in these templates. If you select the Formulas ribbon along the menu bar and then select Calculation Options, you will notice that this has been set to Manual.



1.6. Now we must tell Excel to enable iterative calculations. To do so first select the Microsoft Office button at the top left of the window. Next select Excel Options. Select Formulas and then check the “Enable Iterative Calculations” checkbox. Set the “Maximum Iterations” value equal to 10. Put the “Maximum Change” value to 0.01. The Maximum Iterations value sets how many calculations happen after we press Shift-F9. The Maximum Change will stop the calculation if values do not change more than 0.01.

1.7. In order to make Excel start calculating, you must tell it to do so using the Calculate Now (F9) or Calculate Sheet (Shift-F9) buttons. Since we have multiple sheets, it is best to use the Calculate Sheet (Shift-F9) button to perform a calculation. Press Shift-F9. You will see the values of the cells in front of you change as they are calculated and re-calculated. You have already set the number of iterations to 10. So, this is the result of 10 iterations.

1.8. Continue to press Shift-F9 until the solution *converges*. Record the approximate number of iterations by counting each time you press Shift-F9.

2. ***Analyze the potential distribution and develop an understanding of the electric field using the potential.***

2.1. Overlay a color map on the potential grid by following the guidance shown in the videos or demonstrated by your instructor. You may also wish to create additional 3D models that show the potential as a function of position.

2.2. Either by hand or using the drawing tools in Excel, show the electric field lines as predicted by your computer model. Remember the electric field lines are perpendicular at all points the equipotential lines.

2.3. Save a copy of your spreadsheet output for your lab report.

2.4. Compare your computer model with the results of the “Electric Fields” experiment. In your lab write-up, comment on where the computer model succeeds and fails.

2.5. Complete this process for each electrode geometry required by your instructor.

Writing Your Report

You will write a report that summarizes your findings about the relationship between potential and electric fields both from your measurements last week and your simulation's this week. The body of the report should include elements from BOTH the 1A and 1B experiments, which each explore the same physics, but in different ways.

This lab report should, as always, include an introduction, experimental procedure, experimental results, and conclusions sections. However, in this course you should also write an abstract that provides a succinct summary of your report. This abstract should come at the beginning of the report, but it is most effective if you write it when you are done with the rest of the report. It should be brief, with only a few sentences, that provide high-level descriptions of what you did in the experiment and what main results you obtained. Please note that the abstract should not be an introduction and it should be able to stand on its own. As such, it will repeat the most important information that is described in more detail in your report.

Your report should compare the two methods (experimental and computational) for finding voltage (potential) and electric field (potential gradient). Specifically, you should address the following talking points in your narrative:

Using your measurements, computations, and understanding of the relationship between electrostatic potential and electric field:

- Describe the electric field (magnitude and direction) in the region between the parallel plates.
- Describe the magnitude of the electric field inside a conductor.
- Describe the direction of the electric field outside of and near the surface of a conductor.
- Compare the electric field geometry for the "point charge and a plate" case to the left (or right) half of the electric field geometry for the "two point charges" case.
- What do you consider the strengths and weaknesses of each method, and how well do they complement each other to increase your overall understanding of electrostatics?

Finally, remember that a lab report should "tell a story." Ideally, a student who had not yet done the lab could successfully complete it after reading your report. This does NOT mean that the report needs to include detailed verbiage from the lab manual, only that it should be complete and understandable to peers.