Propagating errors in the adiabatic expansion lab

In the first lab, we have a relatively simple set of measurements. For each gas, we really only made two pressure measurements: P_1 and P_3. P_2 was determined based on the atmospheric records. ***The numbers I use here as experimental errors are not to be taken literally – they are hypothetical! You should use your own values and errors in determining the correct values.***

In this case, our overall function is to calculate the heat capacity ratio, γ:

$$\gamma = \frac{\ln\left(P_1 / P_2 \right)}{\ln\left(P_1 / P_3 \right)} \tag{1}$$

Since our function is of three variables, we need three terms in our propagation:

$$\Delta^2(\gamma) = \left(\frac{\partial \gamma}{\partial P_1} \right)^2_{P_2,P_3} \Delta^2(P_1) + \left(\frac{\partial \gamma}{\partial P_2} \right)^2_{P_1,P_3} \Delta^2(P_2) + \left(\frac{\partial \gamma}{\partial P_3} \right)^2_{P_1,P_2} \Delta^2(P_3) \tag{2}$$

In Eq. 2, the values $\Delta^2(P_i)$ are the square of the uncertainties. For example, if you found your value of P_1 to be 17.3±0.7 psi, you would use

$$\Delta^2(P_i) = 0.7^2 \text{ psi}^2 = 0.49 \text{ psi}^2$$

But… Since you measured the pressures P_1 and P_3 relative to P_2, you will have to use the same process to first approximate the uncertainty in the measured values. Assuming you had the raw data shown in the table below (P_1 and P_3 are in gauge pressure here), we can calculate each of our uncertainties.

P_1	1.68±0.09 psi
P_2	14.71±0.35 psi
P_3	0.46±0.09 psi

For example, we can see that the value of P_1 is 1.68 psi +14.71 psi = 16.39 psi. To get the uncertainty in the value (ΔP_1), we calculate the partial derivatives and scale accordingly:

$$\Delta^2\left(P_{1,abs}\right)=\left(\frac{\partial P_{1,gauge}}{\partial P_1}\right)^2_{P_2,P_3}\Delta^2\left(P_{1,gauge}\right)+\left(\frac{\partial P_{1,gauge}}{\partial P_2}\right)^2_{P_1,P_3}\Delta^2\left(P_2\right) \tag{2}$$
$$=(1)^2(0.09\,psi)^2+(1)^2(0.35\,psi)^2=0.1306\,psi^2$$

Meaning that the uncertainty in P_1 that we should use in our gamma calculations is ±0.36 psi (and the same for ΔP_3).

Next, we need to calculate the partial derivative of γ with respect to each pressure. Once you have all of your partial derivatives in terms of the variables P_1, P_2, and P_3, you can insert the ***measured*** values of P_1, P_2, and P_3 into the expression for each. As an example, I calculated (and you need to check, because I did it rather quickly) that

$$\left(\frac{\partial\gamma}{\partial P_1}\right)_{P_2,P_3}=\frac{\ln\left(\frac{P_1}{P_3}\right)-\ln\left(\frac{P_1}{P_2}\right)}{P_1\left(\ln\left(\frac{P_1}{P_3}\right)\right)^2};\left(\frac{\partial\gamma}{\partial P_1}\right)^2_{P_2,P_3}=\left(\frac{\ln\left(\frac{P_1}{P_3}\right)-_1\ln\left(\frac{P_1}{P_2}\right)}{P_1\left(\ln\left(\frac{P_1}{P_3}\right)\right)^2}\right)^2 \tag{3}$$

So that

$$\left(\frac{\partial\gamma}{\partial P_1}\right)^2_{P_2,P_3}\Delta^2 P_1=\left(\frac{\ln\left(\frac{16.39\,psi}{15.17\,psi}\right)-\ln\left(\frac{16.39\,psi}{14.71\,psi}\right)}{16.39\,psi\left(\ln\left(\frac{16.39\,psi}{15.17\,psi}\right)\right)^2}\right)^2(0.36\,psi)^2=0.0128\,psi^2 \tag{4}$$

Do this for each term (P_1 is the hardest derivative), add them up, and you have the square of the uncertainty in your overall measurement. This calculation needs to be shown in your calculations section, but in your text, it is sufficient to say that you used propagation of errors to determine that γ = 1.39±0.38 (or whatever your number is). It seems like a lot of work for a single number, but it must be done! If everyone were fastidious about this, papers would be a lot more reliable.