CHEM 452 HW 1 Hand-in Solutions

1.22 Use the hint: $x = \frac{h\nu}{k_B T}$

When you substitute:

① Change any ν to x by using $\nu = \frac{k_B T}{h} x$

② Change the integration variable from $d\nu$ to dx:

$$\frac{dx}{d\nu} = \frac{h}{k_B T} \Rightarrow d\nu = \frac{k_B T}{h} dx$$

Thus: $\frac{E_{total}(T)}{V} = \int_0^\infty \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{h\nu/k_BT}-1} d\nu$ becomes

$$= \int_0^\infty \left[\frac{8\pi h}{c^3}\left(\frac{k_BT}{h}\right)^3 x^3\right]\left[\frac{1}{e^x-1}\right]\left[\frac{k_BT}{h}\right]dx$$

now, remove constants:

$$= \frac{8\pi k_B^4 T^4}{c^3 h^3} \underbrace{\int_0^\infty \frac{x^3}{e^x-1} dx}$$

this is given:

$$= \left(\frac{8\pi k_B^4 T^4}{c^3 h^3}\right)\left(\frac{\pi^4}{15}\right)$$

T^4 dependence!

@ 1100 : 0.00111 J/m^3

@ 6000 : 0.981 J/m^3

P2.14 Test if $\hat{O}\psi = R\psi$, where ψ is the function, $\hat{O}$ is the operator, and R is the eigenvalue if it exists.

a) $\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta \frac{d}{d\theta}\cos\theta\right) = \frac{1}{\sin\theta}\frac{d}{d\theta}(\sin\theta)(\sin\theta)$

$= \frac{1}{\sin\theta}\left[-2\sin\theta\cos\theta\right] = -2\cos\theta$ $\boxed{R = -2}$ $\boxed{\text{YES}}$

b) $\left(\frac{d^2}{dx^2} e^{-2ix^2}\right) + \left(16x^2 e^{-2ix^2}\right)$

$= \left(\frac{d}{dx} 4ixe^{-2ix^2}\right) + 16x^2 e^{-2ix^2}$

$= \left(-4ie^{-2ix^2} - 16x^2 e^{-2ix^2}\right) + 16x^2 e^{-2ix^2}$

$= -4ie^{-2ix^2}$

$R = \boxed{-4i}$

must use chain rule...

$\frac{df(u)}{dx} = \frac{df(u)}{du}\frac{du}{dx}$

here, $f(u) = e^{-2ix^2}$, so $u = -2ix^2$

$\frac{du}{dx} = -4ix$

$\frac{df(u)}{du} = e^u$

here, had to chain rule again

c) $\left(\frac{d^2}{dx^2}\cos x \sin x\right) - 2\cos x \sin x$

$= \frac{d}{dx}\left(-\sin^2 x + \cos^2 x\right) - 2\cos x \sin x$

$= -2\sin x\cos x - 2\sin x\cos x - 2\cos x\sin x$

$= -6\sin x\cos x$ $\boxed{\text{yes}}$ $\boxed{-6}$