COMP 335: Introduction to Theoretical Computer Science

Assignment 2

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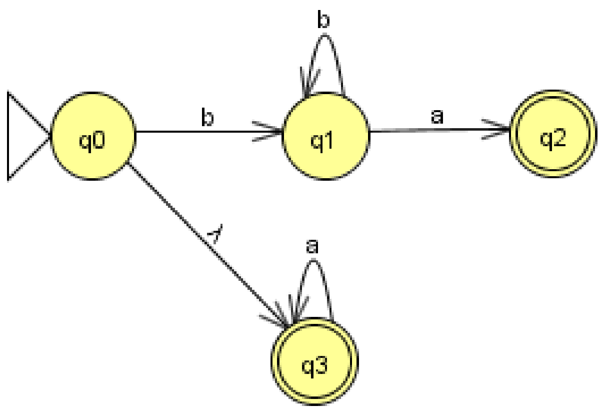
1. [20 Points] For each of the following languages over the alphabet $\Sigma = \{a, b\}$ give an NFA (as a transition diagram) with the specified number of states. *Hint*: try simplifying a DFA and/or use λ transitions.

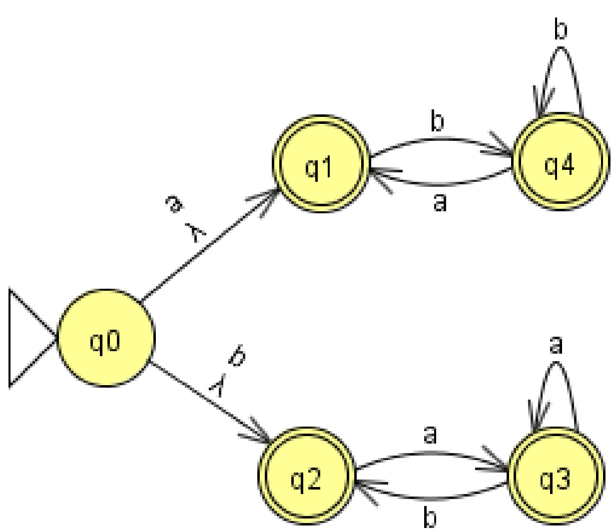
   (a) The language $\{a^n : n \geq 0\} \cup \{b^n a : n \geq 1\}$ with at most 4 states.

   Solution:



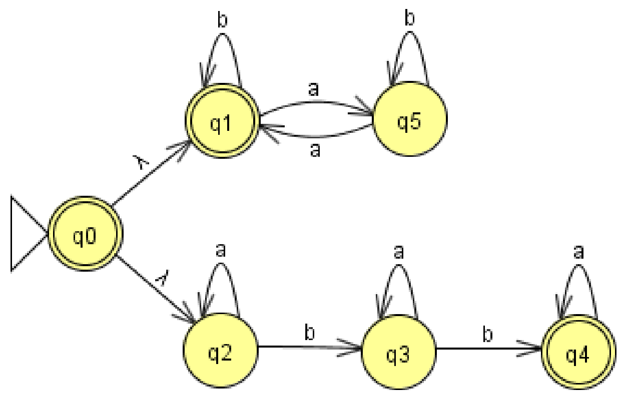
   (b) The language $\{w : w \text{ either has no consecutuve a's or no consecutive b's}\}$ with at most 5 states.

   Solution:



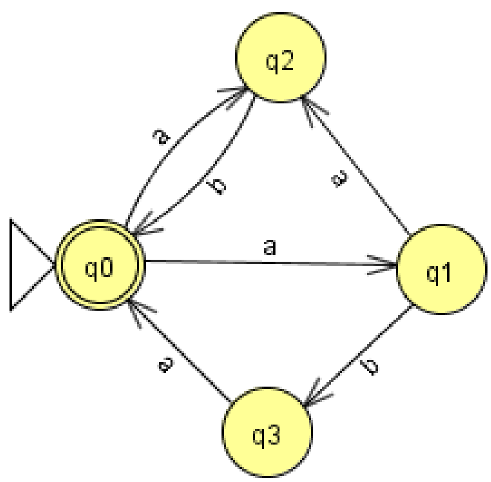
(c) The language $\{w : w \text{ contains an even number of a's or exactly two b's}\}$ with at most 6 states.

Solution:



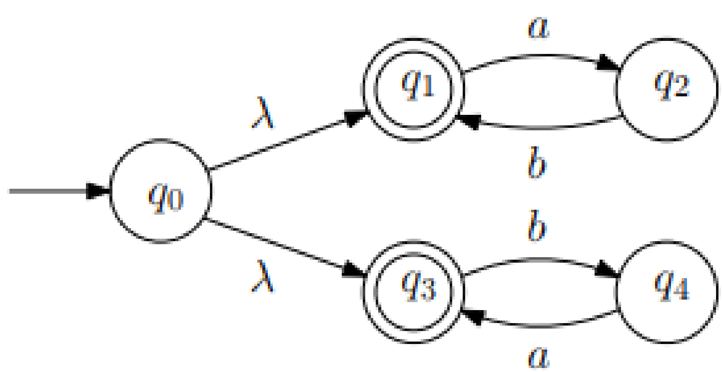
(d) The language $\{ab, aab, aba\}^*$ with at most 4 states.

Solution:

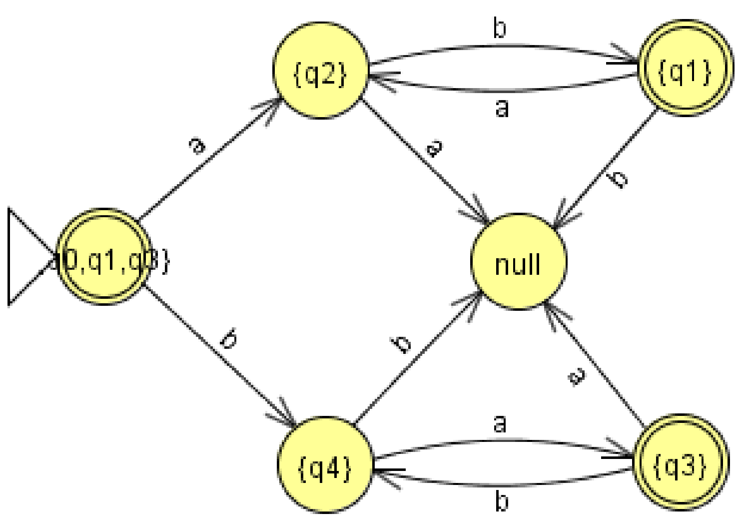


2. [20 Points] Let $\Sigma = \{a, b\}$. Convert each NFA below to a DFA using the subset construction. Draw the transition diagram of your DFA, label the states of your DFA by subset of states of the original NFA.

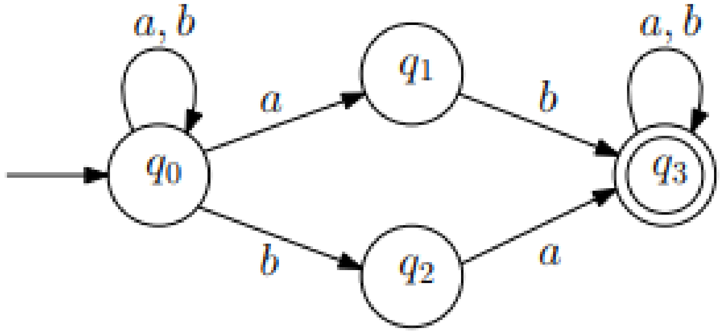
a)



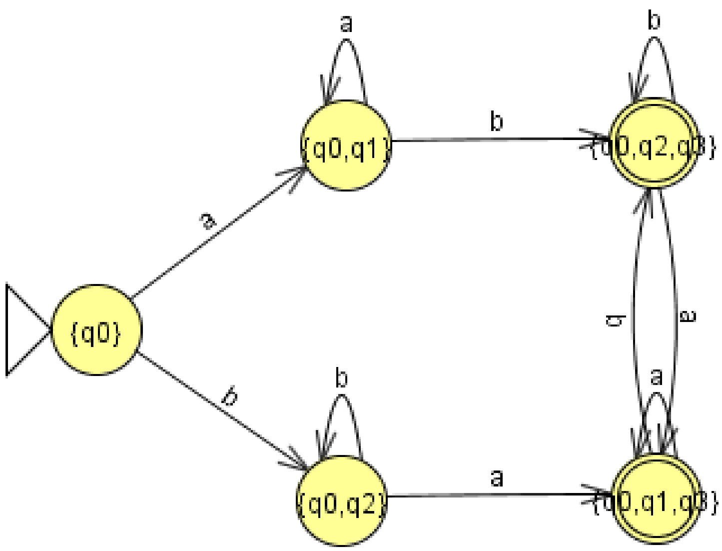
Solution: *Note:* States with "null" in them represent the empty set ($\emptyset$).



b)



Solution:



3. [20 Points] Find a regular expression for each of the following languages.

(a) $\{ba^n b^m : n \geq 3, m \geq 2\}$

Solution:

$$r = b(aaa)a^*(bb)b^*$$

(b) $\{w \in \{a, b\}^*$: every maximal substring of w consisting entirely of symbols a is of length exactly $3\}$

Solution:

$$r = b^* + (b^*(aaa)b^*)^*$$

(c) $\{w \in \{a, b\}^* : w$ does not contain bab as a substring$\}$

Solution:

$$r = a^*b^* + b^*a^* + (a^*b^*(aa)a^*b^*)^*$$

(d) $\{w \in \{a, b\}^* : w$ begins with bb and $n_b(w) \bmod 3 = 0\}$

Solution:

$$r = (bba^*ba^*)^*$$

$$r = bb(a^*(b)a^*)(a^*(b)a^*(b)a^*(b)a^*)^*$$