Concordia University

Dept. of Computer Science and Software Engineering

COMP 335 – Introduction to Theoretical Computer Science

Fall 2024

Assignment 3

Submission through Moodle is due by Sunday, October 20th at 23:55

1. [20 Points] For each of the following languages over $\Sigma = \{a, b\}$, write a regular grammar and then convert it into an equivalent NFA using the procedure described in class.

   (a) (10 Points) $L(r)$ where $r = ((a + b)(a + b))^*b + a((a + b)(a + b))^*$; and

   (b) (10 Points) $\{w \in \{a, b\}^* : w \text{ ends in } a \text{ and } |w| \equiv 1 \pmod 3\}$.

2. [25 Points] Fix an alphabet Σ. For any string w with $|w| \geq 2$, let $skip(w)$ be the string obtained by removing the first two symbols of w. Define two operators on languages:

$$f_1(L) = \{w \in \Sigma^* : skip(w) \in L\}, \text{ and}$$
$$f_2(L) = \{skip(w) \in \Sigma^* : w \in L\}$$

   (a) (5 Points) Consider $L' = L(bba^*)$ over the alphabet $\Sigma = \{a, b\}$. Write a regular expression representing $f_1(L')$. Write another regular expression representing $f_2(L')$.

   (b) (10 Points) Claim: for every regular language L the language $f_1(L)$ is regular. Clearly state whether the claim is TRUE or FALSE, and then prove your answer.

   (c) (10 Points) Claim: for every regular language L the language $f_2(L)$ is regular. Clearly state whether the claim is TRUE or FALSE, and then prove your answer.

3. [20 Points] For each of the following languages, use the Pumping Lemma and/or closure properties of regular languages to show that the language is not regular.

   (a) (10 Points) $L_1 = \{0^k 1^\ell : k \geq \ell^4 \geq 0\}$.

   (b) (10 Points) $L_2 = \{a^n : n \text{ is not a perfect cube}\}$.