COMP 335: Introduction to Theoretical Computer Science

Assignment 5

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November 18, 2024
Fall 2024

1. [10 Points] Show that the following grammar G, where S is the starting variable, is ambiguous.

   Grammar:

   $S \to AB \mid aaaB$

   $A \to a \mid Aa$

   $B \to b$

   Proof of Ambiguity:

   First we decompose the grammar so that productions only have 1 result.

   $S \to AB$

   $S \to aaaB$

   $A \to a$

   $A \to Aa$

   $B \to b$

   Then we show that there are at least different 2 leftmost derivations that result in the same string.

   1) $S \Rightarrow^2 aaaB \Rightarrow^5 aaab$

   2) $S \Rightarrow^1 AB \Rightarrow^4 AaB \Rightarrow^4 AaaB \Rightarrow^3 aaaB \Rightarrow^5 aaab$

2. [10 Points] A context-free grammar $G = (V, T, S, P)$ is said to be a simple grammar or s-grammar if all its productions are of the form $A \to ax$, where $A \in V, a \in T, x \in V^*$, and any pair (A, a) occurs at most once in P.

   Find an s-grammar for $L = \{a^n b^{2n} : n \geq 1\}$.

   Answer:

   $S \to aXB$

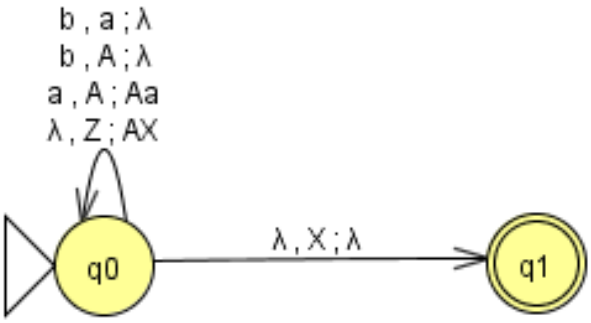
   $X \to aXB_2 \mid b$

   $B_2 \to bB$

   $B \to b$

3. [10 Points] Give an NPDA with 2 states that accepts $L = \{a^n b^{n+1} : n \geq 0\}$

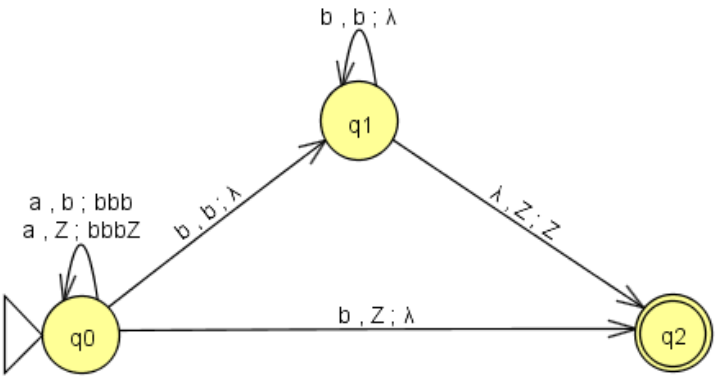
   Answer: Note: The start symbol in the stack is Z.



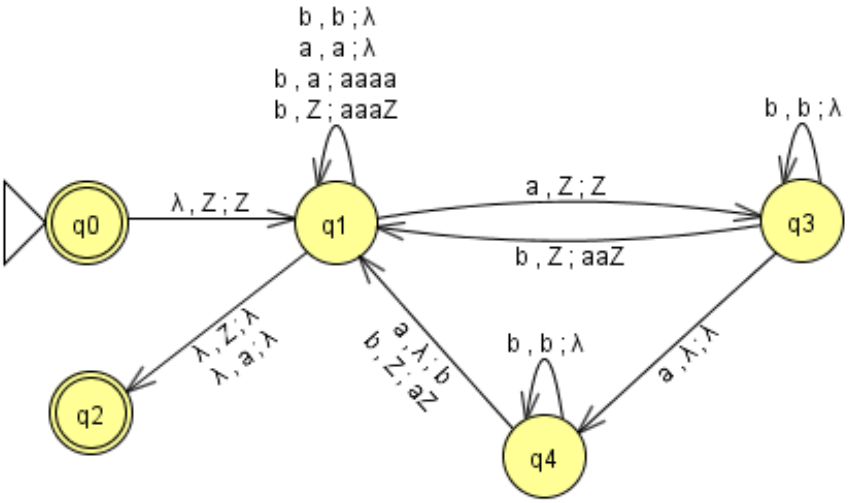
4. [20 Points] For each of the following CFLs, give a "direct" design for an NPDA. That is, it is not acceptable to first find a CFG and then convert it into an NPDA.

   Note: The start symbol in the stack is Z.

   (a) $L_1 = \{a^n b^{2n+1} : n \geq 0\}$

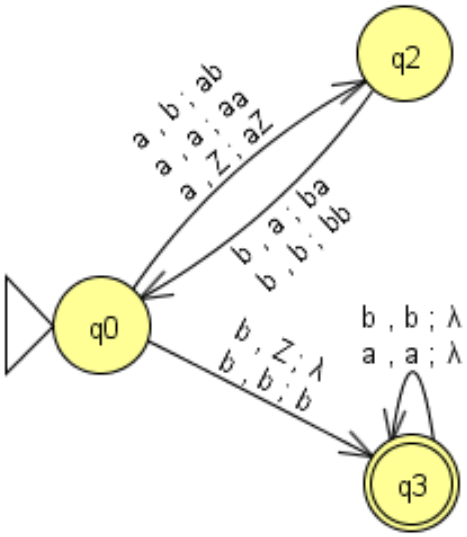


   (b) $L_2 = \{w \in \{a, b\}^* : n_a(w) \leq 3n_b(w)\}$



5. [20 Points] Show that the following CFLs are deterministic.

(a) $L_1 = \{(ab)^n b(ba)^n : n \geq 0\} \cup \{(ab)^n b : n \geq 0\}$



(b) $L_2 = \{w \in \{a, b\}^* : n_a(w) \neq n_b(w)\}$