

COMP 335: Introduction to Theoretical Computer Science

Assignment 1

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1. [15 Points] For each of the following statements write if the statement is TRUE or FALSE. If the statement is TRUE then provide a proof. If the statement is FALSE then provide a counter-example.

(a) For every language L we have $L^2 \subseteq L^3$

Answer: FALSE

Counter Example:

$$\Sigma = \{a, b\}$$

$$L = \{a, ab\}$$

$$L^2 = LL = \{aa, aab, aba, abab\}$$

$$L^3 = L^2L = \{aaa, aaaba, aaba, aabab, abaa, abaab, ababa, ababab\}$$

$$\{aa, aab, aba, abab\} \not\subseteq \{aaa, aaaba, aaba, aabab, abaa, abaab, ababa, ababab\}$$

$$\therefore L^2 \not\subseteq L^3$$

(b) For every two languages L_1 and L_2 we have $(L_1 \cup L_2)^* \subseteq (L_1L_2)^*$

Answer:

Proof:

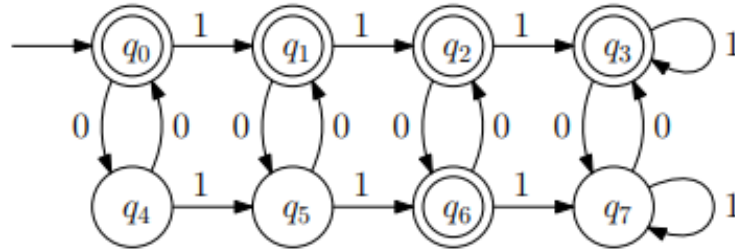
(c) Let L_1 and L_2 be two languages such that $\lambda \in L_1 \cap L_2$. Then it holds that

$$(L_1L_2)^* = (L_2L_1)^*$$

Answer:

Proof:

2. [10 Points] The following is a transition diagram for a DFA over the alphabet $\Sigma = \{0, 1\}$. Answer the following questions about this automaton:



- (a) What is the start state? What is the set of accept states?

Answer:

Start State = q_0

Accept States = $\{q_0, q_1, q_2, q_3, q_6\}$

- (b) What is the sequence of states the DFA goes through on input 101100?

Answer: $(q_0, q_1, q_5, q_6, q_7, q_3, q_7)$

- (c) Does the machine accept every string w that contains exactly two 1s? Why or why not?

Answer: $L = \{0^*10^*10^*\}$

Yes. Since the string always contains exactly two 1s, the machine traverses to one of these states $\{q_2, q_6\}$. Since $q_2, q_6 \in F$ and only 0 appear in the string after consuming both 1s, the machine can only transition between these acceptance states.

- (d) Does the machine reject every string w that has odd number of 0s? Why or why not?

Answer: $L = \{w \in \Sigma^* : \text{Where } n_0(w) \bmod 2 \neq 0\}$

No. Since there's an odd number of 0s, the any string will always terminate in the following states $\{q_4, q_5, q_6, q_7\}$. However, not all of these states are acceptance states (only $q_6 \in F$). Therefore, depending on how many 1s are present in the string, we may or may not reach an acceptance state.

Counter Example: $w = 01$, Final State = q_5

(e) Describe the language accepted by the machine using the set builder notation.

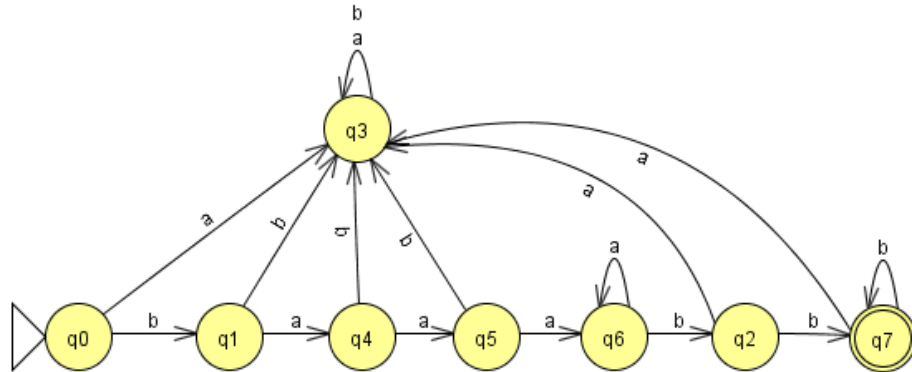
Answer:

$$L = \{w \in \Sigma^* : \text{Where } n_0(w) \bmod 2 = 0\} \cup \{w \in \Sigma^+ : \text{Where } n_0(w) \bmod 2 \neq 0 \wedge n_1(w) = 2\} \quad (1)$$

3. [30 Points] For each of the following languages, give a DFA that accepts it.

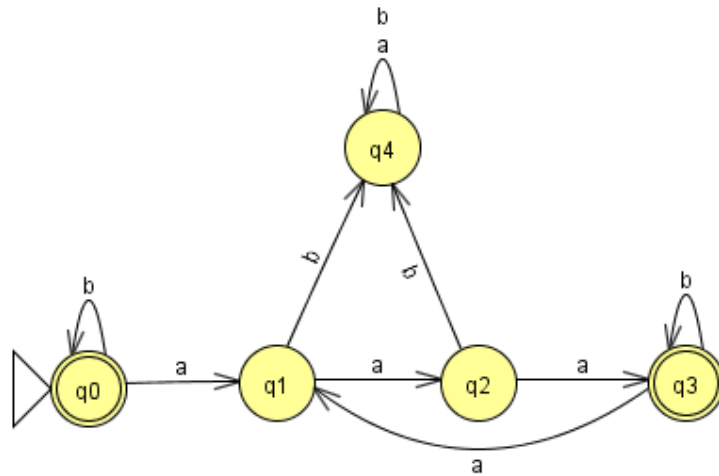
(a) $\{ba^n b^m : n \geq 3, m \geq 2\}$

Answer:



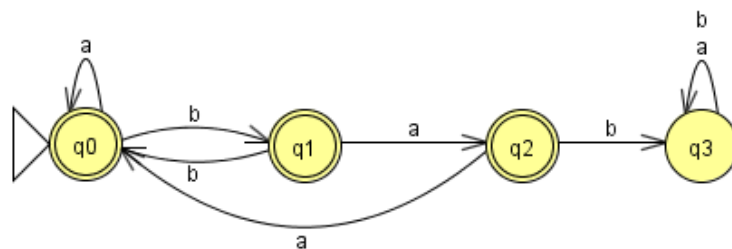
(b) $\{w \in \{a, b\}^* : \text{every maximal substring } w \text{ consisting entirely of symbols } a \text{ is of length exactly } 3\}$

Answer:



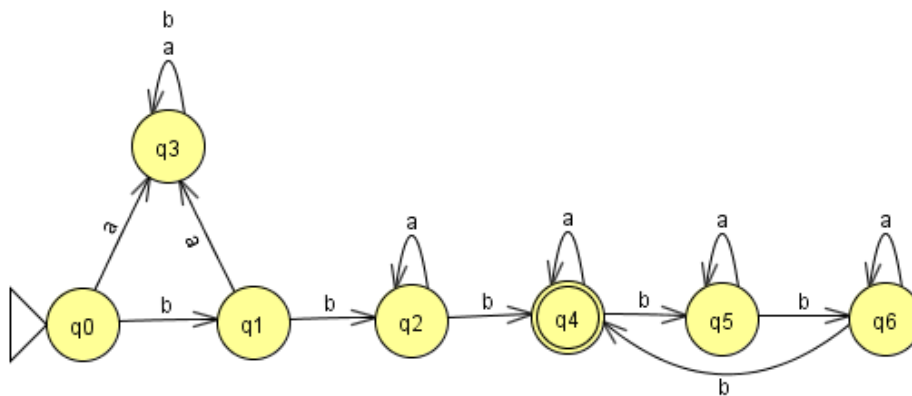
(c) $\{w \in \{a, b\}^* : w \text{ does not contain } bab \text{ as a substring}\}$

Answer:



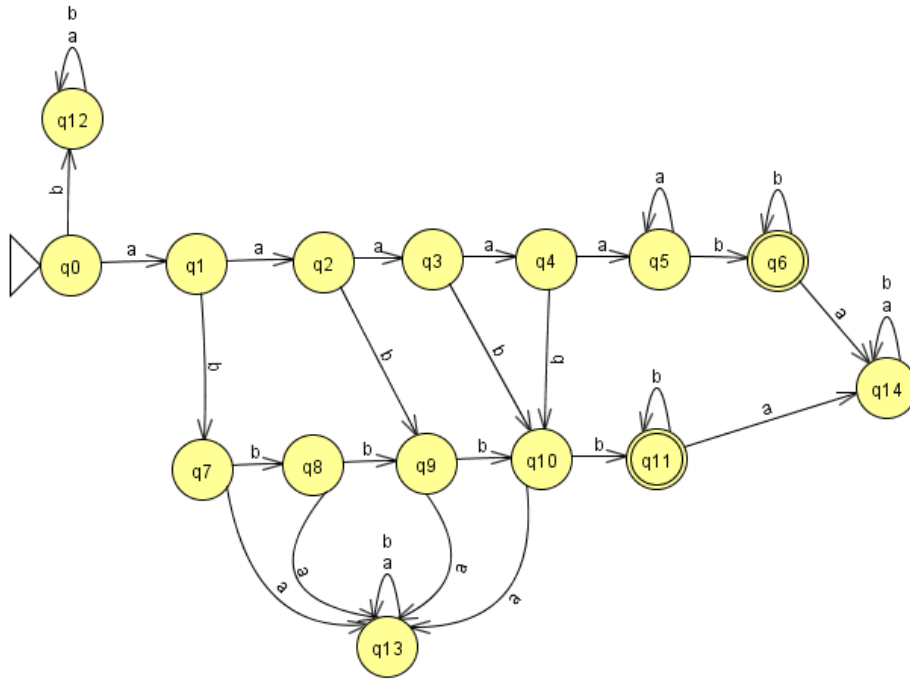
(d) $\{w \in \{a, b\}^* : w \text{ begins with } bb \text{ and } n_b(w) \bmod 3 = 0\}$

Answer:



(e) $\{a^m b^n : mn > 4\}$

Answer:



(f) $\{vww^R : v, w \in \{a, b\}^* \text{ and } |v| = 2\}$

Answer:

