

COMP 335: Introduction to Theoretical Computer Science

Assignment 5

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1. [10 Points] Show that the following grammar G , where S is the starting variable, is ambiguous.

Grammar:

$$S \rightarrow AB \mid aaaB$$

$$A \rightarrow a \mid Aa$$

$$B \rightarrow b$$

Proof of Ambiguity:

First we decompose the grammar so that productions only have 1 result.

$$S \rightarrow AB$$

$$S \rightarrow aaaB$$

$$A \rightarrow a$$

$$A \rightarrow Aa$$

$$B \rightarrow b$$

Then we show that there are at least different 2 leftmost derivations that result in the same string.

$$1) S \Rightarrow^2 aaaB \Rightarrow^5 aaab$$

$$2) S \Rightarrow^1 AB \Rightarrow^4 AaB \Rightarrow^4 AaaB \Rightarrow^3 aaaB \Rightarrow^5 aaab$$

2. [10 Points] A context-free grammar $G = (V, T, S, P)$ is said to be a simple grammar or s-grammar if all its productions are of the form $A \rightarrow ax$, where $A \in V$, $a \in T$, $x \in V^*$, and any pair (A, a) occurs at most once in P .

Find an s-grammar for $L = \{a^n b^{2n} : n \geq 1\}$.

Answer:

$$S \rightarrow aXB$$

$$X \rightarrow aXB_2 \mid b$$

$$B_2 \rightarrow bB$$

$$B \rightarrow b$$

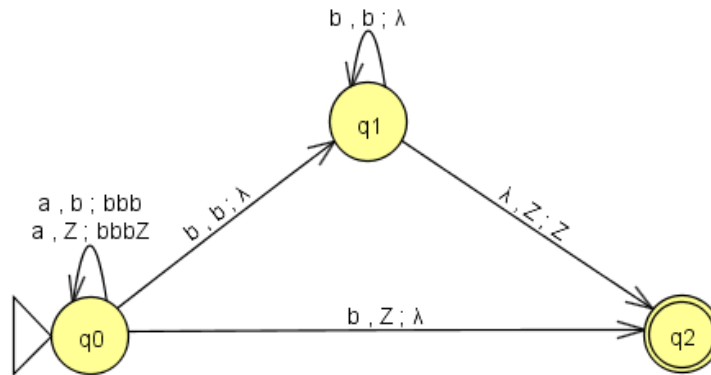
3. [10 Points] Give an NPDA with 2 states that accepts $L = \{a^n b^{n+1} : n \geq 0\}$

Answer: Note: The start symbol in the stack is Z .

4. [20 Points] For each of the following CFLs, give a "direct" design for an NPDA. That is, it is not acceptable to first find a CFG and then convert it into an NPDA.

Note: The start symbol in the stack is Z .

(a) $L_1 = \{a^n b^{2n+1} : n \geq 0\}$



(b) $L_2 = \{w \in \{a, b\}^* : n_a(w) \leq 3n_b(w)\}$

5. [20 Points] Show that the following CFLs are deterministic.

(a) $L_1 = \{(ab)^n b (ba)^n : n \geq 0\} \cup \{(ab)^n b : n \geq 0\}$

(b) $L_2 = \{w \in \{a, b\}^* : n_a(w) \neq n_b(w)\}$