COMP 335: Introduction to Theoretical Computer Science

Assignment 1

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1. [15 Points] For each of the following statements write if the statement is TRUE or FALSE. If the statement is TRUE then provide a proof. If the statement is FALSE then provide a counter-example.

   (a) For every language L we have $L^2 \subseteq L^3$

   Answer:

   Proof:

   (b) For every two languages L_1 and L_2 we have $(L_1 \cup L_2)^* \subseteq (L_1 L_2)^*$

   Answer:

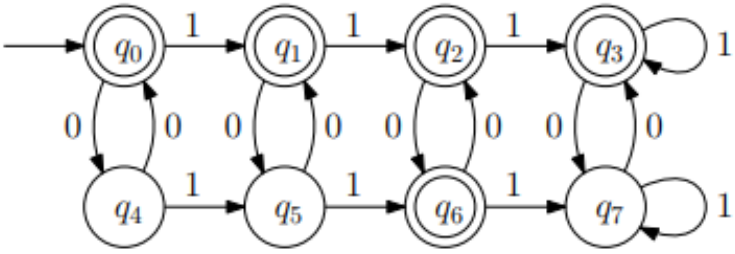
   Proof:

   (c) Let L_1 and L_2 be two languages such that $\lambda \in L_1 \cap L_2$. Then it holds that $(L_1 L_2)^* = (L_2 L_1)^*$

   Answer:

   Proof:

2. [10 Points] The following is a transition diagram for a DFA over the alphabet $\Sigma = \{0, 1\}$. Answer the following questions about this automaton:



(a) What is the start state? What is the set of accept states?

Answer:

Start State = q_0

Accept States = $\{q_0, q_1, q_2, q_3, q_6\}$

(b) What is the sequence of states the DFA goes through on input 101100?

Answer:

$(q_0, q_1, q_5, q_6, q_7, q_3, q_7)$

(c) Does the machine accept every string w that contains exactly two 1s? Why or why not?

Answer:

(d) Does the machine reject every string w that has odd number of 0s? Why or why not?

Answer:

(e) Describe the language accepted by the machine using the set builder notation.

Answer:

3. [30 Points] For each of the following languages, give a DFA that accepts it.

(a) $\{ba^n b^m : n \geq 3, m \geq 2\}$

Answer:

(b) $\{w \in \{a, b\}^* :$ every maximal substring w consisting entirely of symbols a is of length exactly $3\}$

Answer:

(c) $\{w \in \{a, b\}^* : w$ does not contain bab as a substring$\}$

Answer:

(d) $\{w \in \{a, b\}^* : w$ begins with bb and $n_b(w) \bmod 3 = 0\}$

Answer:

(e) $\{a^m b^n : mn > 4\}$

Answer:

(f) $\{vwv^R : v, m \in \{a, b\}^*$ and $\mid v \mid = 2\}$

Answer: