Assignment 4

Submission through Moodle is due by Sunday, November 3rd at 23:55

Total points: 45

1. [20 Points] For each of the following languages, give a context-free grramar (CFG).

   (a) (5 Points) $L_a = \{a^n b^m : m, n \geq 0 \text{ and } 2n \leq m \leq 3n\}$

   (b) (5 Points) $L_b = \{a^n b^m c^k : k = 2m + n\}$

   (c) (5 Points) $L_c = \{a^n b^m c^k : n = m \text{ or } m \leq k\}$

   (d) (5 Points) $L_d = \{w \in \{a, b\}^* : w \neq xx, \text{ for any } x \in \{a, b\}^*\}$.

2. [10 Points] Consider the language $L = \{a^{n+1} b^n : n \geq 0\}$.

   (a) (5 points) Describe in English the complement $\overline{L}$ of L. Your description should specify the types of strings that are in $\overline{L}$. That is, it is not acceptable to say $\overline{L}$ includes every string over $\{a, b\}$ that is not in L, which is obviously true.

   (b) (5 points) Give a CFG for $\overline{L}$.

3. [15 Points] Let G be the following CFG in which S is the start variable:

   $S \rightarrow AB \mid aB$
   $A \rightarrow aab \mid \lambda$
   $B \rightarrow bbA$

   (a) (5 Points) Using the procedure discussed in the class, convert G into an equivalent grammar in Chomsky Normal Form (CNF).

   (b) (5 Points) Find an equivalent grammar to G in Greibach Normal Form (GNF).

   (c) (5 Points) Suppose we modify the original grammar G as follows: *remove* the λ-production $A \rightarrow \lambda$ and instead *add* the unit producion $A \rightarrow A$. Let us call the resulting grammar as G'. Convert G' into CNF, and simplify, if possible. Also describe in English the language $L(G')$.