COMP 335: Introduction to Theoretical Computer Science

Assignment 3

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1. [20 Points] For each of the following languages over $\Sigma = \{a, b\}$, write a regular grammar and then convert it into an equivalent NFA using the procedure described in class.

   (a) (10 Points) $L(r)$ where $r = ((a + b)(a + b))^* b + a((a + b)(a + b))^*$

   (b) (10 Points) $\{w \in \{a, b\}^* : w \text{ ends in } a \text{ and } \mid w \mid \equiv 1 \pmod 3\}$

2. [25 Points] Fix an alphabet Σ. For any string w with $\mid w \mid \geq 2$, let $skip(w)$ be the string obtained by removing the first two symbols of w. Define 2 operators on languages:

$$f_1(L) = \{w \in \Sigma^* : skip(w) \in L\}$$

$$f_2(L) = \{skip(w) \in \Sigma^* : w \in L\}$$

(a) (5 Points) Consider $L' = L(bba^*)$ over tha alphabet $\Sigma = \{a, b\}$. Write a regular expression representing $f_1(L')$. Write another regular expression representing $f_2(L')$.

(b) (10 Points) Claim: For every regular language L the language $f_1(L)$ is regular. Clearly state whether the claim is TRUE or FALSE, and then prove your answer.

Answer:

Proof:

(c) (10 Points) Claim: For every regular language L the language $f_2(L)$ is regular. Clearly state whether the claim is TRUE or FALSE, and then prove your answer.

Answer:

Proof:

3. [20 Points] For each of the following languages, use the Pumping Lemma and/or closure properties of regular languages to show that the language is not regular.

   (a) (10 Points) $L_1 = \{0^k 1^l : k \geq l^4 \geq 0\}$

   (b) (10 Points) $L_2 = \{a^n : n \text{ is not a perfect cube}\}$