

COMP 335: Introduction to Theoretical Computer Science

Assignment 6

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1. [30 Points] Classify the following languages into one of the three categories:

- (a) regular
- (b) context-free but not regular
- (c) not context-free

Prove your answer. Note that in order to show that a language is context-free but not regular, you need to prove both that it is context-free and also that it is not regular.

- (a) $L_1 = \{a^i b^j c^k \mid k = i \times j \text{ and } 0 < i < 10 < j\}$
- (b) $L_2 = \{xyz \mid x, y, z \in \{a, b\}^* \text{ and } n_a(x) = n_b(z)\}$
- (c) $L_3 = \{wuw^R \mid w, u \in \{a, b\}^* \text{ and } |w| = |u|\}$

2. [10 Points] Give a Turing machine for $L = \{a\} \cdot \{a, b\}^+$ that does not halt on rejection.

3. [20 Points] Give a Turing machine for each of the following languages.

(a) $L_1 = \{a^n b^m c^k \mid m \geq n, k \geq 1\}$

(b) $L_2 = \{xy \mid x \in \{a, b\}^+, y \in \{c\}^+ \text{ and } n_a(x) = n_c(y)\}$

4. [20 Points] Draw transition diagrams for Turing machines that compute the following functions. In each case, give a brief description in English of your solution strategy.

(a) $f(1^n) = 1^{2n}$

(b) $f(1^n) = 1^{n^2}$