STAT 710 Third Exam
8:50am-9:40am, April 20, 2012

Please show all your work for full credits.

1. Let $X_1, ..., X_n$ be i.i.d. observations having the Lebesgue p.d.f.

$$\theta f(x) + (1-\theta)g(x),$$

where $\theta \in [0,1]$ is an unknown parameter and f and g are two different known Lebesgue p.d.f.'s. Consider the hypotheses $H_0 : \theta = 0$ versus $H_1 : \theta \neq 0$.

   (a) (2 points) Derive the form of Wald's test statistic.

   (b) (2 points) Derive the form of Rao's score test statistic.

   (c) (2 points) Derive the form of the likelihood ratio test statistic.

   (d) (2 points) Let R_n be the test statistic in part (b). Show directly (without using a theorem) that $R_n \to_d \chi^2_1$, the chi-square distribution with 1 degree of freedom.

2. Let X be a non-negative integer valued random observation with

$$P_\theta(X = x) = f_\theta(x),$$

where θ is an unknown parameter, $\theta \geq \theta_0$, θ_0 is a fixed constant, $0 < \theta_0 < 1$,

$$f_\theta(x) = \frac{e^{-\theta}\theta^x}{x!} I_{\{0,1,2,...\}}(x)$$

when $\theta > \theta_0$, and

$$f_\theta(x) = \frac{\theta_0^x}{1+\theta_0} I_{\{0,1\}}(x)$$

when $\theta = \theta_0$. Consider the hypotheses $H_0 : \theta = \theta_0$ versus $H_1 : \theta > \theta_0$.

   (a) (2 points) Show that the family of the distributions of X has monotone likelihood ratio in X.

   (b) (2 points) Derive a UMP test of size $\alpha = \theta_0/(1+\theta_0)$.

   (c) (2 points) Obtain the power function of the UMP test in (b).

   (d) (2 points) Derive a UMP test of size $\alpha < \theta_0/(1+\theta_0)$.

   (e) (2 points) Derive a UMP test of size $\alpha > \theta_0/(1+\theta_0)$.

   (f) (2 points) Obtain the likelihood ratio $\lambda(X)$.