STAT 710 Third Exam
8:25am-9:15am, April 14, 2011

Please show all your work for full credits.

1. Let $X_1, ..., X_n$ be i.i.d. observations having the Lebesgue p.d.f.

$$f_\theta(x) = \frac{1}{2}(1-\theta^2)e^{\theta x - |x|},$$

where $\theta \in (-1, 1)$ is an unknown parameter. Consider the hypotheses $H_0 : \theta \leq 0$ versus $H_1 : \theta > 0$.

   (a) (3 points) Show that there is a statistic Y and a constant c such that the UMP test of size $\alpha \in (0, \frac{1}{2})$ rejects H_0 if and only if $Y > c$. (You don't need to obtain the value of c, but provide a formula to compute c.)

   (b) (2 points) Obtain the constant c in part (a) when $n = 1$.

   (c) (3 points) Obtain $c = c_n$ such that $\lim_{n\to\infty} P_0(Y > c_n) = \alpha$, where P_0 is the probability under $\theta = 0$ and Y is in part (a).

2. Let $X_1, ..., X_n$ be i.i.d. observations having the Lebesgue p.d.f.

$$f_\theta(x) = \frac{1}{2}(1-\theta^2)e^{\theta x - |x|},$$

and $Y_1, ..., Y_n$ be i.i.d. observations having the Lebesgue p.d.f.

$$f_\varphi(y) = \frac{1}{2}(1-\varphi^2)e^{\varphi y - |y|},$$

where $\theta \in (-1, 1)$ and $\varphi \in (-1, 1)$ are unknown parameters. Suppose that $X_1, ..., X_n$ and $Y_1, ..., Y_n$ are independent. Consider the hypotheses $H_0 : \theta = \varphi$ versus $H_1 : \theta \neq \varphi$.

   (a) (3 points) Find two statistics Y and U such that the UMPU rejects H_0 if and only if $Y < c_1(U)$ or $Y > c_2(U)$, where $c_1(U)$ and $c_2(U)$ are functions satisfying some constraints. (Give the constraints but do not need to simplify them.)

   (b) (3 points) Derive the form of the likelihood ratio test statistic.

   (c) (3 points) Derive the form of Wald's test statistic.

   (d) (3 points) Derive the form of Rao's score test statistic.