Name: ________________________________

Show sufficient work to make *very clear* your method of solution.

1. Let $X_1, ..., X_n$ be a random sample with a pdf

$$f(x) = \begin{cases} \theta\varphi^\theta x^{-(\theta+1)} & x > \varphi \\ 0 & x \le \varphi \end{cases}$$

where $\theta > 0$ and $\varphi > 0$ are fixed parameters.

   (a) Find the pdf of the 1st order statistic, $X_{(1)} = \min_i X_i$.

   (b) When φ is known and θ is unknown, show that the family of pdf's indexed by θ is an exponential family and find a sufficient and complete statistic T for θ.

   (c) When θ is known and φ is unknown, show that the minimum order statistic $X_{(1)}$ is a complete and sufficient statistic for φ.

   (d) Suppose that $\varphi = e^\theta$ is unknown. Show that the pair of statistics, T and $X_{(1)}$, is minimal sufficient for θ.

2. Let X_n be a random variable having the $Poisson(n)$ distribution, $n = 1, 2, ...$ In the following, the limiting process is with respect to $n \to \infty$.

   (a) Show that

$$\frac{X_n}{\sqrt{n}} - \sqrt{n} \text{ converges in distribution to } N(0,1)$$

   (b) For any positive integer k, show that

$$\frac{X_n^k}{n^{k-1/2}} - \sqrt{n} \text{ converges in distribution to } N(0,k^2)$$

   (c) Define

$$Y_n = \begin{cases} 1 & X_n = 0 \\ X_n & X_n > 0 \end{cases}$$

   Show that $Y_n - X_n$ converges in probability to 0.