STAT 709 Third Exam
8:25am-9:15pm, Nov 23, 2010

Please show all your work for full credits.

1. Let $(X_1, Y_1), ..., (X_n, Y_n)$ be i.i.d. from the bivariate normal distribution with the Lebesgue p.d.f.

$$f(x,y) = \frac{1}{2\pi\sigma^2\sqrt{1-\rho^2}} \exp\left\{ -\frac{x^2}{2\sigma^2(1-\rho^2)} + \frac{\rho xy}{\sigma^2(1-\rho^2)} - \frac{y^2}{2\sigma^2(1-\rho^2)} \right\},$$

where $\sigma^2 > 0$ and $\rho \in (-1, 1)$.

(a) (2 points) When both σ^2 and ρ are unknown, obtain the UMVUE of σ^2.

(b) (2 points) Show that the variance of the UMVUE in (a) attains the Cramér-Rao lower bound.

(c) (2 points) When σ^2 is unknown and ρ is known, obtain the UMVUE of σ^2.

(d) (2 points) When σ^2 is unknown and ρ is known, obtain the Fisher information about σ^2 contained in $(X_1, Y_1), ..., (X_n, Y_n)$ and the variance of the UMVUE in (c).

(e) (4 points) When both σ^2 and ρ are unknown, obtain the UMVUE of $(1+\rho)/(1-\rho)$.

2. Let $X_1, ..., X_n$ be i.i.d. with the Lebesgue p.d.f.

$$f(x) = \frac{1}{2\theta} \exp\left\{ -|x-\mu|/\theta \right\},$$

where $\mu \in \mathcal{R}$ and $\theta > 0$.

(a) (2 points) When $\mu = 0$ and θ is unknown, show that both $T_n = n^{-1}\sum_{i=1}^n |X_i|$ and $T_n' = \sqrt{(2n)^{-1}\sum_{i=1}^n X_i^2}$ are consistent estimators of θ.

(b) (4 points) In (a), obtain the asymptotic relative efficiency of T_n w.r.t. T_n'.

(c) (2 points) When both μ and θ are unknown, show that $n^{-1}\sum_{i=1}^n |X_i - \bar{X}|$ is a consistent estimator of θ, where $\bar{X}$ is the sample mean.