In-Class Midterm Exam

Name:_____

Instructions:

1. This exam is open book. You may use textbooks, notebooks, class notes, and a calculator.
2. Laptops, tablets, phones and other devices with wireless capability must be turned off and put away.
3. Do all your work in the spaces provided. If you need additional space, use the back of the preceding page, indicating *clearly* that you have done so.
4. To get full credit, you must show your work. Partial credit will be awarded.
5. Some partial computations have been provided on some questions. You may find some *but not necessarily all* of these computations useful. You may assume that these computations are correct.
6. Do not dwell too long on any one question. Answer as many questions as you can.
7. Note that some questions have multiple parts. For some questions, these parts are independent, and so you can work on part (b) or (c) separately from part (a).

For grader's use:

Question	Possible Points	Score
1	35	
2	35	
3	30	
Total	100	

1. An experiment was conducted to determine how the yield of a chemical reaction was related to three factors: Temperature, pH, and Agitation Rate. For convenience we will call these T, P, and R. Temperature was set at either 140 C or 160 C, pH was set at either 7.5 or 8.5, and Agitation Rate was set at either 25 or 35 rpm. The yield (Y) was measured as the percent of the theoretical maximum. The experiment was done as follows. On a given day, one of the combinations of T, P, and R was selected at random, the reaction was run using those settings, and Y was measured. The entire experiment took 16 days, and each combination was used twice. The data appear below. For each combination of T, P, and R, the two values of Y are listed. Also listed is the sample variance of those two values.

Temperature	140	140	140	140	160	160	160	160
pH	7.5	7.5	8.5	8.5	7.5	7.5	8.5	8.5
Rate	25	35	25	35	25	35	25	35
Y	60, 65	61, 60	52, 58	83, 81	58, 58	61, 63	55, 52	75, 78
s^2	12.5	0.5	18	2	0	2	4.5	4.5

   (a) (15 points) Perform a formal test to determine whether there is evidence of a three-way interaction between Temperature, pH, and Agitation Rate.

(b) The data have been repeated below:

Temperature	140	140	140	140	160	160	160	160
pH	7.5	7.5	8.5	8.5	7.5	7.5	8.5	8.5
Rate	25	35	25	35	25	35	25	35
Y	60, 65	61, 60	52, 58	83, 81	58, 58	61, 63	55, 52	75, 78
s^2	12.5	0.5	18	2	0	2	4.5	4.5

(10 points) Regardless of your answer to part (a), consider the possibility of a two-way interaction between pH and Agitation Rate. Make a suitable plot to examine the possible presence of such an interaction. (No formal analysis is necessary.)

(c) The data have been repeated below:

Temperature	140	140	140	140	160	160	160	160
pH	7.5	7.5	8.5	8.5	7.5	7.5	8.5	8.5
Rate	25	35	25	35	25	35	25	35
Y	60, 65	61, 60	52, 58	83, 81	58, 58	61, 63	55, 52	75, 78
s^2	12.5	0.5	18	2	0	2	4.5	4.5

(10 points) Continue with part (b). Suppose you wanted to compare each pair of points in your plot by using an LSD comparison. Determine the LSD in such a case and determine which pairs of points in the plot are significantly different. (You may assume that the overall F test is significant.)

2. An experiment was conducted to study the effect of three chemical treatments intended to reduce the chance of fire for wood exposed to high heat. The chemical treatments were: Aluminum hydroxide, huntite and hydromagnesite combined, and calcium silicate. In addition, the samples of wood being tested were finished in one of two different ways: either rough finish, or smooth finish.

   To perform the experiment, 5 logs were sampled from a forest of spruce trees. Each log was cut into 6 boards. At random, each of the 6 boards was assigned to one of the 6 treatment combinations (chemical and type of finish) listed below:

   sa Smooth finish and treatment with aluminum hydroxide.

   shh Smooth finish and treatment with huntite and hydromagnesite.

   sc Smooth finish and treatment with calcium silicate.

   ra Rough finish and treatment with aluminum hydroxide.

   rhh Rough finish and treatment with huntite and hydromagnesite.

   rc Rough finish and treatment with calcium silicate.

   Two days after treatment, each board was exposed to a source of high heat, and the time was measured before the board caught fire.

   Some SAS output follows.

```
data fire;
input log finish$ chemical$ time;
datalines;
1 r a  15
2 r a  20
3 r a  25
4 r a  19
5 r a  30
1 r c  17
2 r c  14
3 r c  26
4 r c  31
5 r c  45
1 r hh 23
2 r hh 30
3 r hh 33
4 r hh 30
5 r hh 32
1 s a  19
2 s a  23
3 s a  25
4 s a  20
5 s a  22
1 s c  17
2 s c  15
3 s c  23
4 s c  37
5 s c  35
1 s hh 20
2 s hh 29
3 s hh 20
4 s hh 25
5 s hh 36
;
proc glm;
class log finish chemical ;
model time = finish|log|chemical;
```

   The (edited) results of the above code appear next.

```
                        Dependent Variable: time

                                   Sum of
Source                            Squares    Mean Square   F Value   Pr > F
Model                        1646.800000      56.786207        .      .
Error                           0.000000         .
Corrected Total              1646.800000

Source                       Type III SS
finish                        19.2000000
log                          753.8000000
log*finish                    58.4666667
chemical                     189.6000000
finish*chemical               16.8000000
log*chemical                 475.4000000
log*finish*chemical          133.5333333
```

(a) (10 points) Construct the appropriate ANOVA table for this experiment, indicating Source and df.

(b) (15 points) Test the null hypothesis that there is no main effect for `chemical`. (Only test this main effect; no other tests are required.)

(c) (10 points) The experiment was repeated with some changes. First, a total of 6 logs were used, all from a forest of fir trees. Each log was cut into two boards; one board was finished "smooth" and one was finished "rough." Also, all boards were treated with calcium silicate (the other chemical treatments were not used).

Two days after treatment, each board was cut into three pieces, and at random, each piece of a board was assigned to receive one of three temperature treatments: 300 C, 400 C, or 450 C. Again, the time was measured until the wood caught fire.

The investigators were particularly interested in whether there is an interaction between the finish (smooth or rough) and the temperature of the heat source. Of course, they are also interested in main effects.

Write down an ANOVA table for this experiment, indicating Source and df and indicate the degrees of freedom for testing the `finish` by `temperature` interaction.

3. (a) (15 points) An experiment was conducted to measure the concentration of lead in drinking water in Michigan. To do the study, 6 cities were sampled at random, and within each city, 3 houses were chosen at random. From each house, 1 water sample was taken. An Analysis of Variance was calculated, and it was found that SSCity = 392, and SSHouse = 460. Suppose I make a list of all the houses in Michigan that are in cities, and randomly choose two houses from the list. I then take one sample of water from each house. Estimate the variance of the mean concentration of the two samples.

   (Hinte: You'll need to make a reasonable assumption in order to proceed.)

(b) An experiment was conducted as a Randomized Complete Block Design with 8 blocks, and 5 treatments. Each treatment appeared exactly once in each block, and a single observation was taken for each combination of block and treatment.

i. (1 point) How many degrees of freedom are there for Treatments?

ii. (7 points) The 5 treatments in part (i) consisted of combinations of three factors: A, B, and C, each at two levels. The combinations used are listed below, using a standard notation:

Treatment:	1	2	3	4	5
Combination:	ab	a	b	1	c

Let SSTreatments represent the Sum of Squares for testing whether there is an overall difference among the 5 treatment groups (1) – (5). Let $\bar{y}_{i\cdot}$ represent the mean response for treatment i. Give contrast coefficients so that it is possible to partition SSTreatments into a set of contrast sums of squares, each with a single degree of freedom.
You should include contrasts to test the main effects for A and B and their interaction (when C is low), plus any additional contrasts to account for the degrees of freedom in SSTreatments.

You only need to give the contrast coefficients for each contrast; no other calculations are required.

iii. (7 points) Let μ_1 and μ_5 denote the mean response for treatments 1 and 5, respectively. The investigators would like to test the null hypothesis $H_0 : \mu_1 = 1.05\mu_5$ versus the two-sided alternative.

Consider the following italicized statement which is either True or False: *Because (+1) and (–1.05) do not add up to 0, the quantity* $1\mu_1 - 1.05\mu_5$ *is not a contrast, and therefore the hypothesis cannot be tested.*

Indicate whether the italicized statement is True or False and explain your answer.