STAT 710
First Exam, Feb 12, 2016

Please show all your work for full credits.

1. Consider the estimation of an unknown $\theta \in \mathcal{R}$ under the squared error loss. Let $\hat{\theta}$ be an unbiased estimator of θ and $R_{\hat{\theta}}(\theta)$ be the risk of $\hat{\theta}$. Assume that $\sup_\theta R_{\hat{\theta}}(\theta) < \infty$.
   (a) (2 points) Show that the estimator $\hat{\theta} + c$ is not minimax, where $c \neq 0$ is a known constant.
   (b) (2 points) Show that the estimator $c\hat{\theta}$ is not minimax, where $c \neq 1$ is a known constant.

2. Let $X_1, ..., X_n$ be i.i.d. with Lebesgue p.d.f.
$$f_\theta(x) = \theta(1+\theta)x(1-x)^{\theta-1}, \quad 0 < x < 1,$$
where $\theta > 0$ is unknown.
   (a) (3 points) Obtain the likelihood $\ell(\theta)$ and show that the likelihood equation
$$\frac{d \log \ell(\theta)}{d\theta} = 0$$
   has a unique solution and it must be the MLE.
   (b) (2 points) Obtain an explicit form of the MLE in (a).
   (c) (2 points) Suppose that $\theta \geq 1$. Obtain the MLE of θ and prove that it is MLE.

3. Let X and Y be two independent observations from the double exponential distribution with Lebesgue p.d.f. $f_\theta(x) = \frac{1}{2}e^{-|x-\theta|}$, where $\theta \in \mathcal{R}$ is unknown.
   (a) (3 points) Based on the two observations X and Y, obtain the likelihood function of θ and show that $(X+Y)/2$ is an MLE of θ. In this problem, do we have a unique MLE?
   (b) (3 points) Show that $(X+Y)/2$ cannot be a Bayes estimator of θ under the squared error loss.
   (c) (3 points) When $X < Y$, obtain an explicit form of the generalized Bayes estimator of θ under the squared error loss with respect to the improper prior = Lebesgue measure.