Name: ______________________________

Show sufficient work to make *very clear* your method of solution.

1. (a) (4 points) Show that if A and B are mutually exclusive and $P(A), P(B) > 0$, then A and B can not be independent.
   (b) (8 points) Let A and B be events with $0 < P(A) < 1$ and $0 < P(B) < 1$. Suppose that $P(A|B) > P(A|B^c)$, where the superscript c denotes complement. Show that $P(B|A) > P(B)$.
   (c) (8 points) Let A, B, C, and D be four events. Suppose that if all B, C, and D occur, then A occurs. Show that $P(A) \geq P(B) + P(C) + P(D) - 2$. (*Hint:* $(\cap_i A_i)^c = \cup_i A_i^c$).
2. Let $Z_1, Z_2, ..., Z_p$ be independent Normal(0, 1) random variables.
   (a) (5 points) Find the joint PDF for $(Z_1 + Z_2, Z_2 + Z_3)$. Name the distribution and specify all parameters.
   (b) (5 points) Find $P(\max(Z_1 + Z_2, Z_2 + Z_3) < 0)$.
   (c) (5 points) Find the PDF for the random variable $X = Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2$.
   (d) (5 points) Find $\mathrm{Corr}(Z_1, Z_1^4 + Z_2)$. Are Z_1 and $Z_1^4 + Z_2$ independent?
3. Consider a continuous random variable X with PDF $f(x) = cx^2.(1 - x)$, for $0 < x < 1$ and 0 otherwise. Further let $Y|X = x \sim \mathrm{Bernoulli}(x)$.
   (a) (5 points) Find c and the marginal PDF for Y.
   (b) (5 points) Find the conditional PDF $X|Y = y$.
   (c) (10 points) Find $\mathrm{Corr}(X, Y)$.
4. Suppose that X is a random variable with PDF $f_\theta(x) = c(\theta)x^{\theta-1}.(1 - x)$ for $0 < x < 1$ (0 otherwise) and $\theta > 0$.
   (a) (5 points) Find $c(\theta)$.
   (b) (5 points) Show that X belong to a natural parameter exponential family and state the base measure and sufficient statistic.
   (c) (5 points) Find $E[\log X]$.
   (d) (5 points) Find the MGF for $Y = \log X$.
5. Let $X_1, X_2, ..., X_p$ be mutually independent random variables with distribution $X_i \sim \mathrm{Exponential}(\lambda)$.
   (a) (5 points) Find the MGF for $Y = \sum_{i=1}^{p} a_i.X_i$ where $a_i > 0$ for all i. Clearly specify any constraints on t.
   (b) (5 points) Find the CDF for for $Y = \min(X_1, X_2, ..., X_p)$. Name the distribution.
   (c) (5 points) Find a g such that $g(X_i) \sim \mathrm{Uniform}(0, 1)$.
   (d) (5 points) Find the joint PDF for $(Y_1, Y_2) = (\frac{X_1}{X_2}, X_2)$. Hence or otherwise, find the PDF for $R = \frac{X_1}{X_2}$.