STAT 709 First Exam
8:25am-9:15am, Sept 28, 2010

Please show all your work for full credits.

1. Let $X_1, ..., X_k$ be random variables defined on a probability space.

   (a) (6 points) Let $X = (X_1, ..., X_k)$. Show that

   $$\sigma(X) = \sigma\left(\bigcup_{j=1}^{k} \sigma(X_j)\right).$$

   (b) (2 points) Let $Y = X_1 + \cdots + X_k$. Assume that $E|X_j| < \infty$ for all j. Find an explicit form of a function of Y for

   $$\sum_{j=1}^{k} E(X_j|Y).$$

   (You need to give reasons for your formula.)

   (c) (2 points) Let $Y = X_1 + \cdots + X_k$. Assume that $X_j = X_j^2$ for all j. Find an explicit form of a function of Y for

   $$\sum_{i=1}^{k} \sum_{j=i+1}^{k} E(X_i X_j|Y).$$

   (You need to give reasons for your formula.)

2. Let U, V, and Z be independent random variables defined on a probability space. Assume that f, g, and h are the Lebesgue p.d.f.'s of U, V, and Z, respectively.

   (a) (2 points) Derive the joint Lebesgue p.d.f. of (X, Y, Z), where $X = Z + U$ and $Y = Z + V$.

   (b) (4 points) Show that, for any $A \in \sigma(X)$,

   $$P(A|Y, Z) = P(A|Z) \text{ a.s.}$$

   (c) (4 points) Show that, for any Borel function h with $E|h(X)| < \infty$,

   $$E[h(X)|Y, Z] = E[h(X)|Z] \text{ a.s.}$$