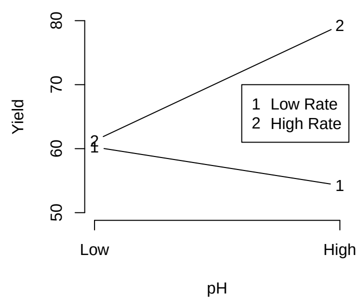
Stat 850 — Midterm Exam, Spring 17 — Partial Solutions

1. (a) We can use a T-test based on the 3-way interaction contrast. The pooled estimate of error variance is $s_p^2 = (12.5 + 0.5 + \cdots + 4.5)/8 = 5.5$. For each combination of factors, we calculate the average Y value, and get 62.5, 61.5, 55, 82, ... Finally, the contrast coefficients for the 3-way interaction can be written as $(1, -1, -1, 1, -1, 1, 1, -1)$. Together, this yields:

$$T = \frac{10}{\sqrt{5.5}\sqrt{\frac{8}{2}}} = 2.13$$

Comparing with a T_8 distribution we find that the p-value lies between 0.05 and 0.10, so there is weak evidence, at best, of a 3-way interaction.

(b) One way to draw the plot is:



The lines are far from parallel, so there appears to be evidence of a 2-way interaction between pH and Rate.

(c) Each mean in the figure has $n = 4$ and we use s_p from part (a) with 8 df. Thus

$$\text{LSD} = 2.306\sqrt{5.5 \times \frac{2}{4}} = 3.82$$

This yields a display like:

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54.25   60.25   61.25    79.25
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```

where means connected by a line are not significantly different.

2. (a) This is an RCBD where log is the block, and Finish and Chemical are treatment factors. The ANOVA table looks like:

Source	df
Block	4
Finish	1
Chemical	2
F×C	2
Error	20
Total	29

(b) We pool the SS from SAS for `log*finish` plus `log*chemical` plus `log*finish*chemical` (i.e. the block interactions) to get SSError = 667.4. Then

$$F = \frac{\text{MSC}}{\text{MSError}} = \frac{189.6/2}{667.4/20} = 2.84$$

Comparing to $F_{2,20}$ gives a p-value between 0.05 and 0.10, and so there is weak evidence of a main effect for chemical.

(c) This is a split-plot experiment where log is the block, finish is the whole plot treatment, and temperature is the subplot treatment. The ANOVA table looks like:

Source	df
log	5
Finish	1
WPError	5
Temp	2
F×T	2
SPError	20
Total	35

The test for F×T is an F test with 2 and 20 df.

3. (a) We can use a one-way random effects analysis where City is the "Treatment" and House is the "Error". The ANOVA table looks like:

Source	df	SS	MS
City	5	392	78.4
House	12	460	38.33
Total	17	852	

whence $\hat{\sigma}_H^2 = 38.33$ and $\hat{\sigma}_C^2 = (78.4 - 38.33)/3 = 13.36$.

Assume that the two newly sampled houses are in difference cities. Then a given observation (Y_{ij}) has variance $\sigma_C^2 + \sigma_H^2$, and the average of two observations will have variance $(\sigma_C^2 + \sigma_H^2)/2$, which we estimate to be $(38.33 + 13.36)/2 = 25.84$. (If you want to use notation like we did in class, this new sampling process has $k = 2$ and $n = 1$.)

4. (a) 4.

(b) Using the same order of treatment combinations as listed on the exam, possible contrast coefficients are

A	1	1	-1	-1	0
B	1	-1	1	-1	0
AB	1	-1	-1	1	0
C	1	1	1	1	-4

Note that "C" above is *not* the usual estimate of the main effect of C.

(c) The statement is False. It is true that $\sum c_i \neq 0$, and the quantity $\mu_1 - 1.05\mu_5$ is not a contrast. However, the mathematics of testing contrasts still holds, and we can still use a T-test:

$$T = \frac{\bar{y}_{1\cdot} - 1.05\bar{y}_{5\cdot}}{s\sqrt{1^2/8 + (1.05)^2/8}}$$

and we would compare this to T_{28}.

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Exam Summary

mean = 87, median = 89

Grade Distribution

   <80:  5
 80-84:  3
 85-89:  6
 90-94: 11
95-100:  3
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