A short presentation on Viscoelasticity and Creep

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Outline

1. Introduction
   - Viscoelasticity
     - First order viscoelasticity
     - Governing differential equation
     - Standard Linear Solid model(Zener model)
     - Burgers model
     - Generalized Maxwell model
   - Creep
   - Stress Relaxation

Introduction

- Linear (and nonlinear) viscoelasticity
  - Time dependence
    - Creep
    - Stress Relaxation
  - Frequency Dependence

First order viscoelasticity

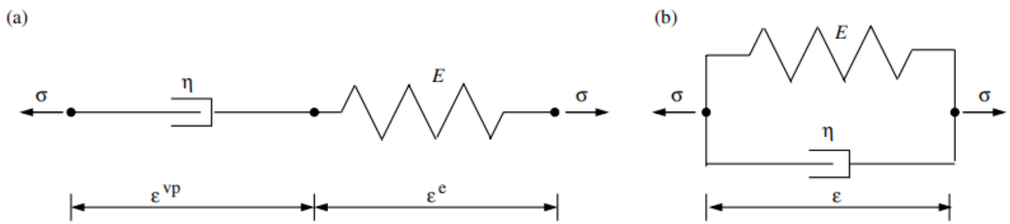


Figure 1: Maxwell model(a) and Kelvin model(b)

In the model, there's a "dashpot," also referred to as a "damper." The force provided by the damper is directly proportional to the velocity of deformation. When described using stress and strain, suppose its damping is η

$$\sigma = \dot{\varepsilon}_V \cdot \eta$$

where V represents viscosity and E represents elasticity.

Governing differential equation

In series, the stress on the left and right parts is equal, and the total strain is the sum of both:

$$\sigma = \sigma_E = \sigma_V, \varepsilon = \varepsilon_E + \varepsilon_V$$

Therefore, for the Maxwell model, its governing differential equation is:

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta}$$

In parallel, the strain in the upper and lower parts is equal, and the stress is the sum of both:

$$\sigma = \sigma_E + \sigma_V, \varepsilon = \varepsilon_E = \varepsilon_V$$

Therefore, for the Kelvin model, its governing differential equation is:

$$\sigma = E\varepsilon + \eta\dot{\varepsilon}$$

Standard Linear Solid model(Zener model)

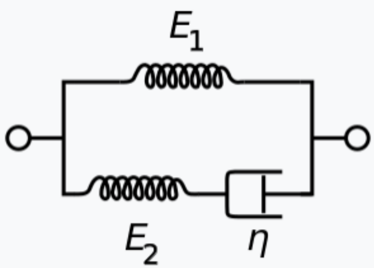
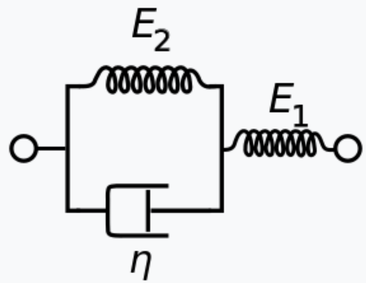
Maxwell representation 麦克斯韦表示	Kelvin representation 开尔文表示
	
$\sigma + \frac{\eta}{E_2}\dot{\sigma} = E_1\varepsilon + \frac{\eta(E_1 + E_2)}{E_2}\dot{\varepsilon}$	$\sigma + \frac{\eta}{E_1 + E_2}\dot{\sigma} = \frac{E_1 E_2}{E_1 + E_2}\varepsilon + \frac{E_1\eta}{E_1 + E_2}\dot{\varepsilon}$

Figure 2: Two representations for viscoelasticity

Deduce for Kelvin Standard Linear Solid model

For LHS:

$$\sigma = E_2\varepsilon_2 + \eta\dot{\varepsilon}_2 \tag{1}$$

For RHS:

$$\sigma = E_1\varepsilon_1, \quad \dot{\sigma} = E_1\dot{\varepsilon}_1 \tag{2}$$

Combine equations (1) and (2) together:

$$\varepsilon = \varepsilon_1 + \varepsilon_2 = \frac{1}{E_1}\sigma + \frac{1}{E_2}\sigma - \frac{\eta}{E_2}\dot{\varepsilon}_2 \tag{3}$$

$$\varepsilon + \frac{\eta}{E_2}\dot{\varepsilon} = \left(\frac{1}{E_1} + \frac{1}{E_2}\right)\sigma + \frac{\eta}{E_1E_2}\dot{\sigma} \tag{4}$$

Finally we end up with:

$$\sigma + \frac{\eta}{E_1 + E_2}\dot{\sigma} = \frac{E_1E_2}{E_1 + E_2}\varepsilon + \frac{E_1\eta}{E_1 + E_2}\dot{\varepsilon} \tag{5}$$

Burgers model

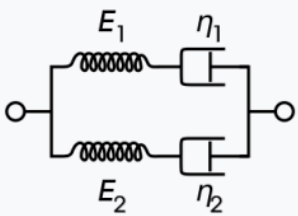
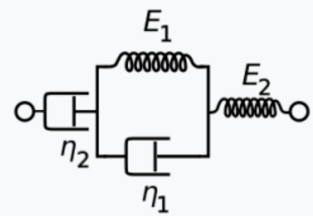
Maxwell representation 麦克斯韦表示	Kelvin representation 开尔文表示
	
$\sigma + \left(\frac{\eta_1}{E_1} + \frac{\eta_2}{E_2}\right)\dot{\sigma} + \frac{\eta_1\eta_2}{E_1E_2}\ddot{\sigma} = (\eta_1 + \eta_2)\,\dot{\varepsilon} + \frac{\eta_1\eta_2\,(E_1 + E_2)}{E_1E_2}\ddot{\varepsilon}$	$\sigma + \left(\frac{\eta_1}{E_1} + \frac{\eta_2}{E_1} + \frac{\eta_2}{E_2}\right)\dot{\sigma} + \frac{\eta_1\eta_2}{E_1E_2}\ddot{\sigma} = \eta_2\dot{\varepsilon} + \frac{\eta_1\eta_2}{E_1}\ddot{\varepsilon}$

Figure 3: Burgers model

Generalized Maxwell model

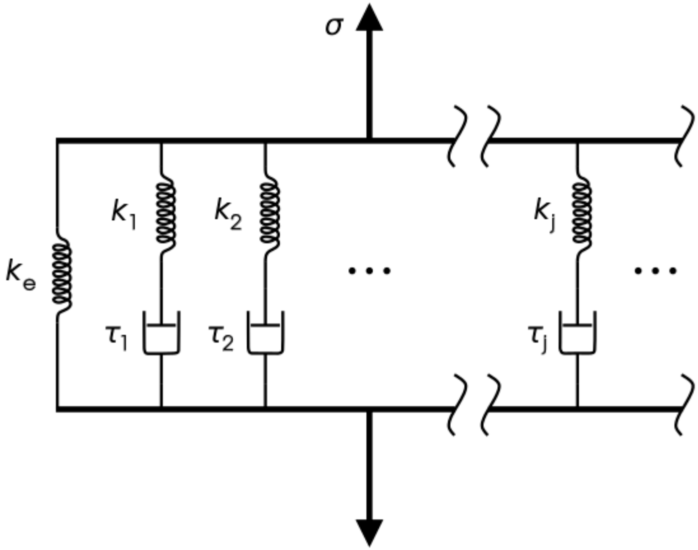


Figure 4: Generalized Maxwell model

Definition and mechanisms of Creep

Dislocation creep

At high stresses and temperatures, dislocations can climb and cross-slip, with vacancies diffusing along dislocation lines, leading to higher strain rates.

Diffusion creep(Nabarro-Herring Creep)

Under stress, the equilibrium concentration of vacancies differs across grain boundaries, causing vacancies to diffuse through the grain interior in a direction perpendicular to the tensile stress, resulting in deformation along the direction of stress.

Grain boundary creep(Coble Creep)

At lower temperatures and stresses, dislocation movement becomes difficult, and bulk diffusion is less effective. However, vacancies can diffuse along grain boundaries, facilitating creep.

Stress relaxation

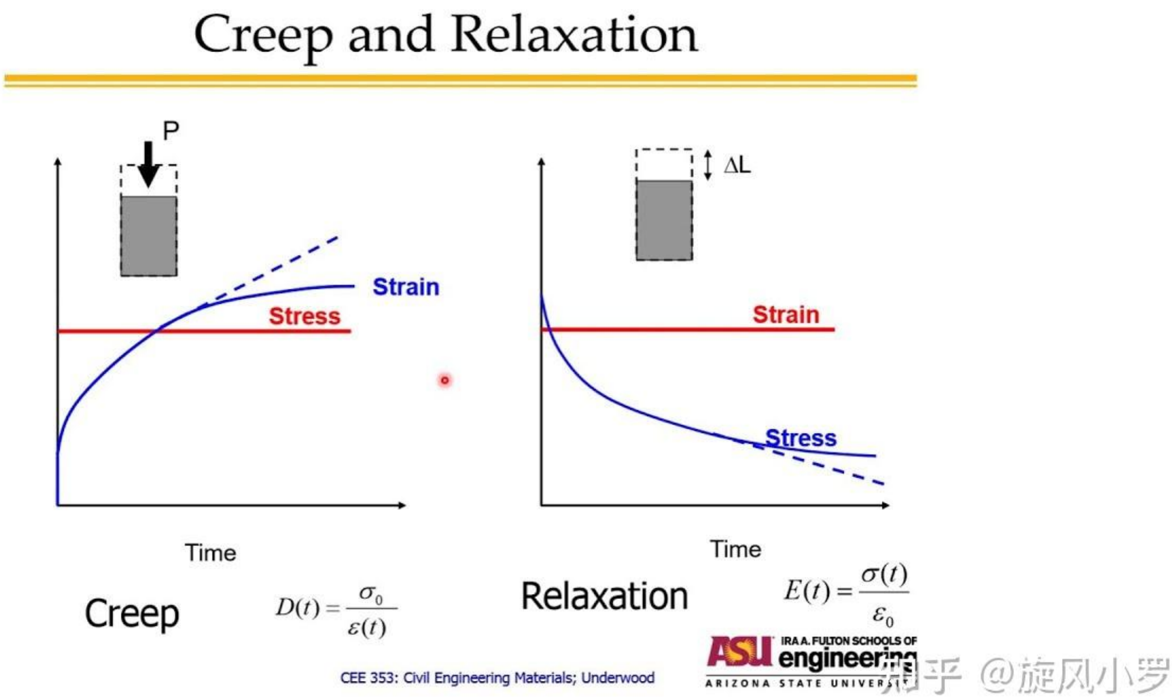


Figure 5: Creep and Relaxation

Comparasion of creep and stress relaxation

Creep

Creep occurs under **constant stress**, leading to ongoing material deformation over time. Initially, deformation may be quick, slowing down as the material adapts, potentially leading to irreversible changes if the stress persists.

Stress Relaxation

Stress relaxation happens with **constant strain**, where the initially induced stress in the material decreases over time due to internal adjustments or viscous flow, despite the strain remaining unchanged.