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A Sequential Study of Migration and Job Search

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This paper designs a multiarmed bandit (MAB) sequential model for the analysis of the migration–job search process. The implications either are compatible with well-known migration behavior or, when novel, are also plausible. For example, regions with large wage variability attract migrants, and regions with large nonpecuniary returns increase both in migration and out migration. A major advantage of this approach is the relative ease with which martingale estimators can be derived from the martingale structure of the model. These martingale methods are exemplified for the return migration phenomenon.

> No one can deliver his labour in a market in which he himself is not present . . . and the unwillingness to quit home, and to leave old associations, including perhaps some loved cottage and burial-ground, will often turn the scale against a proposal to seek better wages in a new place. [Marshall 1948, p. 567]

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I. Introduction

The mobility of resources among alternative projects is the essence of economics. In a fluctuating economy with imperfect information, resources are constantly in motion, seeking out their best use. In capitalist economies, prices guide the mobility decisions of both the firm and the employee. Firms move to those industries and locations that promise the greatest discounted expected returns, while the individual's combined choice of education, occupation, region, and job is a sequential process yielding the highest discounted expected utility.

The economics of information describes how individuals design experiments to extract the most information about an uncertain environment. When the experiments are sequential, there is a tension between a procedure that tests a wide range of alternatives, thereby acquiring considerable information before acting, and a procedure that acts after learning about only a few alternatives. Both the discount rate and the time horizon play key roles in resolving this conflict between the costs and the benefits of experimentation.[1] Berry and Fristedt (1985, p. 5) observe, "it may be wise to sacrifice some potential early payoff for the prospect of gaining information that will allow for more informed choices later. This aspect prompted Whittle (1982, p. 210) to claim that a bandit problem 'embodies in essential form a conflict evident in all human action.' The 'information versus immediate payoff' question makes the general problem difficult; the issue is seldom . . . clear."

This paper analyzes the migration–job search decision.[2] It is an important topic in the economics of information. We develop a sequential decision model that combines the multiarmed bandit (MAB) methodology[3] with the theory of belated information.

[1] For a thorough discussion of this important issue, see Bather (1981), Kelly (1981), and the monograph by Berry and Fristedt (1985). This monograph is especially valuable in that it contains a general measure theoretic framework for studying bandit problems and an extensive bibliography of the bandit literature with brief and cogent descriptions of each paper. For example, we are reminded that Thompson (1933) "is the paper that started it all."

[2] Greenwood (1975) surveys the early migration literature. More recent research is contained in Rothenberg (1977), Bartel (1979), Herzog and Schlottmann (1983), Linneman and Graves (1983), Goss and Schoening (1984), Pissarides and McMaster (1984), Herzog, Hofler, and Schlottmann (1985), and Isserman et al. (1985). Mortensen (1984) and Lippman and McCall (1976, in press) survey the search literature.

[3] A formal definition will be given shortly. For now, think of a MAB as a slot machine with n arms, $n \geq 2$, where winning on arm i has an unknown probability P_i, $i = 1, \ldots, n$. The problem is to choose the "best" arm. Discussions of this problem are in Whittle (1981) and Kumar (1985). The MAB problem is a member of the class of Bayesian adaptive control problems. In theory, its solution can be obtained via dynamic programming. However, Gittins and Jones (1974)

The paper begins with a brief review of the MAB method that covers both its attractiveness for the study of economic processes and its current limitations. Modifying the MAB by melding it with the belated information model[4] is the prelude to the paper's major goal—the economic analysis of migration and job search behavior. While the literature on each of these topics is enormous, there is neither a sequential model that addresses the migration decision nor a dynamic model that explicitly considers the sequential decision making associated with both job search and migration.

Several of the model's implications are compatible with well-known migration behavior, whereas others are novel but plausible. These include the following. (1) Cities with large nonpecuniary returns increase both in migration and out migration. (2) Remigration is a decreasing function of information known prior to migration. (3) The substantial return migration that is observed in many empirical studies is consistent with our model. That is, migrants who possess inferior information about a new location are likely to return to locations they have visited previously, especially those where they have accumulated large amounts of specific human capital. In his discussion of labor mobility in the Middle Ages, Marshall (1948, p. 186) observes, "No doubt the 'wander years' were chiefly those of youth, and after these were over the wanderer was likely to settle down in the place in which he was born." Finally, David (1974) observed that regions with large wage variability would be attractive to migrants. This conjecture has found empirical support and is implied by our MAB model.

Section III contains a generalized bandit model of migration with wage rate at location i being a random variable. An application of this methodology to the illegal alien problem is developed in Section IV.

One of the major advantages of this approach is the relative ease with which econometric tests are derived from the MAB model. As we will see, the MAB is a martingale stochastic process and implicitly contains these econometric estimators. Furthermore, Aalen (1978), Bremaud (1981), and others showed that these estimators are also martingales and hence possess many appealing asymptotic properties. Because of the importance

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uncovered a deep and simple structure that decomposed the n-dimensional dynamic program into N one-dimensional dynamic programs. For an alternative approach to the MAB problem, see Lai and Robbins (1985), which generalizes the original Robbins model (1952).

[4] The major building blocks are Salop (1973), Gittins (1979), Wilde (1979), Whittle (1980, 1982–83), Roberts and Weitzman (1980), Lippman and McCall (1981, in press), and McCall and McCall (1981). The most important "building block" was Lippman's distinction between good news (1) and bad news (−1).

of the topic, Section V shows how the return migration phenomenon can be estimated using martingale methods.[5]

The concluding section mentions several empirical applications of this methodology that are currently under way and indicates an application to location theory.[6]

II. The Sequential Migration Model

The elements of the MAB method are presented first. Then the belated information model of job search is briefly reviewed as it is merged with the MAB. These methods are then applied to job search and migration.

The MAB Model[7]

There are N bandits. The state of bandit i at time t is denoted by $x_i(t)$ ($i = 1, \ldots, N$; $t = 1, 2, \ldots$), where $x_i(t)$ can be thought of, in the finite state space case, as a (random) function from t to the set $\{1, 2, \ldots, L\}$, which enumerates the states.[8] The decision maker must play one bandit during each time period. Suppose bandit i is chosen at time t. The decision maker receives a bounded reward (which may be negative) of amount $R_i[x_i(t)]$, and the bandit moves to a new state, say, $x_i(t + 1)$, where the motion is determined by a stationary Markov chain with transition matrix $P^{r,s}$, $r, s = 1, 2, \ldots, L$.[9] The $N - 1$ bandits that were not chosen remain fixed at their initial position, $x_j(t) = x_j(t + 1)$, $j \neq i$. The goal of the decision maker is to sequence the play of the bandits to maximize the discounted expected rewards, $E \sum_{t=1}^{\infty} \beta^t R_j[x_j(t)]$, where $\beta \equiv 1/(1 + r)$, and r is the appropriate interest rate, $r \geq 0$.

To illuminate the structure of the MAB problems, we present a simple example. Suppose a prospector is considering N sites to mine. He has enough equipment, labor, and so on to explore only one site per period.[10]

[5] The martingale's role in the Gittins methodology is most apparent in Karatzas (1984). His proof used martingale arguments, the diffusion process is a martingale, and the estimator of the diffusion's semigroup is a martingale.

[6] There have been several applications of the bandit methodology to economic problems. These include Rothschild (1974), Schmalansee (1975), Viscusi (1979), Roberts and Weitzman (1980), McCall and McCall (1981), and Miller (1984).

[7] There are several expository pieces on the MAB problem. These include Whittle (1980, 1982–83), Ross (1983), and Kumar and Varaiya (1986).

[8] The state of a bandit may, alternatively, be $x_i(t)$, x_i, or x, depending on the situation in which it is used.

[9] When discussing the transition matrices of bandit processes, it is convenient to use the values of $x_i(t)$, namely, $\{1, 2, \ldots, L\}$.

[10] This assumption is important. If two or more mines could be worked at once, the natural extension of Gittins's result will not be valid. For an interesting example of this, see Ross (1983).

For the moment, let us fix arbitrarily a period as 1 month.[11] Also, for simplicity assume that if gold is found at the ith site the prospector receives a prize, P_i, each period from that point on. However, continued receipt requires that he continues to expend all his resources. If no gold is found at site i, then an expense, C_i, is incurred. Next, assume that for each site he has an initial probability of "success," q_i, where we define this as the probability of striking gold next period given that the mine contains gold. The probability of "no gold" at site i is r_i. At the outset, the probability that gold will be found next period is $(1 - r_i)q_i = s_i$. Each period of failure causes a revision of s_i downward according to Bayes's rule. The statistic s_i contains all the information about site i and constitutes the state space.[12] More generally, any set of variables that are sufficient statistics for a particular bandit may be taken to represent the state space.[13] The generality of the MAB comes from the fact that P_i, C_i, and s_i can vary across sites.

The objective of the prospector is to maximize his expected discounted return from prospecting the set of N mines (β is the discount rate). Thus each period he receives either $-C_i$ or P_i if site i is worked and gold is or is not discovered, respectively. Using the above notation,

$$R_i[x_i(t)] = \begin{cases} -C_i & \text{if gold is not found at } i, \\ P_i & \text{if gold is found at } i. \end{cases}$$

The only limitation imposed by the bandit model is that both P_i and C_i are bounded above. The independence of bandits assumption and the assumption that only the state of the bandit being played can change are closely related in this (as well as most other) applications. In this example it specifically implies that, when you are prospecting site i, you gain no information about any other site $j \neq i$. In particular, you gain no knowledge of the probability of success of the jth project. If sites were located on the same mountain or very close to each other, the MAB model may not be appropriate.

Gittins and Jones (1974) solved the general MAB problem by devising an index that could be calculated for each bandit. The optimal sequence of play was to follow the simple rule, Always play that bandit with the largest index. The index for the ith bandit at time t is given by

[11] Fixed periods is not a binding restriction for the Gittins result. Periods could have random lengths. These are called "variable stage" bandits.

[12] This example, therefore, has a continuous state space on [0, 1].

[13] In a more complicated model one could gain information on P_i as well. Thus the state space may be multidimensional.

$$\xi_i[x_i(t)] = \max_{T>1} \frac{E\{\sum_{t=1}^{T-1} \beta^t R_i[x_i(t)] | x_i(1) = x_i\}}{E[\sum_{t=1}^{T-1} \beta^t | x_i(1) = x_i]}, \tag{1}$$

where the maximization is with respect to the set of stopping times $T > 1$.[14] This ratio was called the Gittins index by Whittle (1980).

The major contribution of Gittins and Jones was to collapse an extremely complicated n-dimensional problem into N relatively simple one-dimensional problems.

The Gittins index possesses some noteworthy qualities. (i) The index assigned to a particular project depends only on the state of that project. (ii) The Gittins index can be interpreted as the average discounted reward per discounted time over a stopping time T that is chosen to maximize this average. Hence the Gittins procedure is an optimal stopping rule. (iii) The optimal index is simply an amount such that the decision maker is indifferent between retiring and pursuing the best of the N opportunities while maintaining the retirement option. The MAB problem is different from other stopping rule problems in that (*a*) unplayed bandits are motionless; (*b*) unplayed bandits neither add nor subtract from the total return; and (*c*) the motion of the chosen bandits is Markovian.[15]

To show that the MAB problem can be formulated as a dynamic program, let $V(x_1, \ldots, x_N)$ be defined by

$$\max_{u \in (1,2,\ldots,N)} \{R_u[x_u(t)] + \sum_{s=1}^{L_u} P^{x_u,s} V(x_1, \cdot, x_{u-1}, s, x_{u+1}, \ldots, x_N)\},$$

where V is the unique solution to the dynamic program, $U = \{1, 2, \ldots, N\}$ is the control set, and u_t is the control variable at time t. Clearly, $V(x_1, \ldots, x_N)$ is the value function of a dynamic program. Furthermore,

[14] A stopping time is a random variable T satisfying

$$\{T \leq t\} \in H_t \tag{i}$$

for all t. Roughly speaking, H_t is the history of the process from zero until t. The event $\{T \leq t_0\}$ means that the process is stopped at or before t_0. Clearly, we want this event to belong to the process history, and this is implied by (i).

[15] Varaiya, Walrand, and Buyukkoc (1985) observe that *a* and *b* are the real determinants of the index optimality, whereas *c* merely simplifies the calculation of the index and is not essential. Indeed, it "led researchers to adopt a DP framework, thereby obscuring the problem's simple structure." They also maintain that the MAB problem is "perhaps the simplest nontrivial problem in stochastic control for which a relatively complete analysis is available" (p. 426).

if there is a $u = g(x_1, \ldots, x_N)$ such that a maximum is achieved, then g is both an optimal and a stationary policy.

A hammer-and-tongs dynamic programming analysis of this Markov decision process is exceedingly difficult. But by imposing *a–c* we can decompose the problem into N simpler subproblems. The solution to each of these subproblems constitutes the Gittins index for that bandit, and again the optimal policy is to work the bandit at time t that has the largest index. The subproblem for the ith bandit is a simple bandit problem in which the only two "bandits" that can be chosen are the ith bandit or a retirement option that yields m. Thus, for the ith subproblem, the value function in state $x_i(t)$ is

$$V_i[x_i(t), m] = \max\{R_i[x_i(t)] + \beta \sum_{s=1}^{L} P^{x_{i,s}} V_i(s, m); m\}.$$

The Gittins index of the ith bandit, $Z_i^{x_i}$ (or simply Z_i^x), is then given by that value of m that yields indifference between continuing and stopping (retiring). Hence

$$Z_i^x = R_i(x) + \beta \sum_{s=1}^{L} P^{x,s} V_i(s, Z_i^x). \tag{2}$$

The optimal policy is to choose the ith project at time t if[16]

$$Z_i^x = \max_{1 \le j \le N} \{Z_j^x\}.$$

There are two alternative ways of formulating the Gittins index. Which is more useful depends on the particular problem. The next subsection contains an example in which the "forward induction" approach proves insightful, whereas the analysis of the optimal job search–migration policy applies the dynamic programming approach.

Early in the 1960s, Manne (1960) and Derman (1962) showed that Markov decision processes could be solved using linear programming. After Gittins and others unveiled the structure of a solvable MAB, Varaiya et al. (1985) showed that much of it could be recast as a purely deterministic problem, and, indeed, Chen and Katehakis (1986) demonstrated that it could also be solved using linear programming. Varaiya et al. designed a recursive method for estimating the solution, whereas

[16] For a simple optimality proof using the dynamic programming approach, see Whittle (1982–83).

Dupac and Herkenrath (1982) developed a stochastic approximation method for accomplishing the same task.[17]

The Sequential Search Model and the Gittins "Forward Induction" Index

To exemplify the Gittins strategy we apply this forward induction method to the familiar sequential search model.

The forward induction rule maximizes the ratio of average return to average time with respect to an optimally chosen stopping time. More specifically, at each stopping time the decision maker chooses the length of time until the next stop to maximize the expected return per unit of discounted time. Thus the optimal policy is defined by a sequence of steps proceeding forward in time. Forward policies are frequently easier to obtain than backward induction policies. However, they are not optimal for all Markov decision processes. One of Gittins's major achievements was to prove that, for a restricted class of Markov decision processes, the family of alternative bandit processes, a policy is optimal if and only if it coincides almost always with a forward induction policy. The distinctive feature of this class of alternative bandits is that only one of the n bandits composing the class can be turned on at any specified time; the remaining $n - 1$ bandits must be off.

Consider the choice problem confronting the decision maker when there is but one job and retirement is the only alternative activity. Let the job be in state i, and suppose the lump payment from retirement, S, is exactly equal to the critical number ξ_i. In this circumstance the decision maker is indifferent between working and retiring. The key point implicit in this indifference assertion is the assumption that if work is chosen it continues until an optimally chosen stopping time. For an arbitrary stopping time T,

$$\xi_i \geq E(\text{discounted return earned by } T) + \xi_i E(\beta^T).$$

That is, the return from retirement, ξ_i, exceeds the return from continued work for all those stopping times T that are not optimal. Equality is achieved in the above inequality only when the stopping time is optimal. Hence

$$\xi_i = \max_{T:T>0} \frac{E(\text{discounted return earned by } T)}{1 - E(\beta^T)}.$$

[17] There is an interesting coincidence here in that Robbins (1952) was the first to formulate both stochastic approximation and the experimental design that led to the MAB problem. His recent solution to another version of the MAB problem is contained in Lai and Robbins (1985).

Multiplying both sides by $1 - \beta$ gives

$$(1-\beta)\xi_i = \max_{T:T>0} \frac{E(\text{discounted return earned by } T)}{E(1+\beta+\beta^2+\ldots+\beta^{T-1})}$$
$$= \max_{T:T>0} \frac{E(\text{discounted return earned by } T)}{E(\text{discounted time prior to } T)}.$$

Using this derivation of the Gittins index leads to an alternative formulation of an optimal policy for N jobs. Calculate the Gittins index for each job by evaluating the ratio of expected discounted earnings to time T to the expected discounted time prior to T at the optimal T and dividing by $1 - \beta$. Then choose that job with the largest index. Work in this job if its index exceeds S; otherwise, retire.

The forward induction procedure can also be applied to the elementary search model. Letting ξ be the reservation wage, the inequality

$$\xi \geq E(\text{discounted return till } T) + E(\beta^T)E(X|X \geq \xi_T)$$

holds strictly for all nonoptimal stopping rules T and holds as an equality when T is set equal to the optimal stopping rule T^*. That is,

$$\xi = \max_{T:T>0} E(\text{discounted return till } T) + E(\beta^T)E(X|X \geq \xi_T)$$
$$= E(\text{discounted returns till } T^*) + E(\beta^{T^*})E(X|X \geq \xi),$$

where $\xi_{T^*} \equiv \xi$. Now set $\beta = 1$ to give

$$\xi = \frac{-c}{1-F(\xi)} + \int_{\xi}^{\infty} x dF(x)/1 - F(\xi),$$

which can be rearranged to yield the familiar H function,

$$c = \int_{\xi}^{\infty} (x-\xi) dF(x) \equiv H(\xi).$$

When there are several distributions F_i to search from the c_i the corresponding cost of search, $i = 1, 2, \ldots, N$, the optimal procedure is to calculate ξ_i for each and search that distribution with the maximal ξ_i, say, $\bar{\xi} = \max_i \xi_i$. This special case of the Gittins procedure has been proposed before (see Lippman and McCall 1976). We now know the conditions under which this rule is valid—namely, all other ξ_i's must remain fixed while the searcher samples the best distribution. When the environment is stationary and the searcher knows the costs, c_i, and distributions, F_i, $i = 1, \ldots, N$, there is nothing to learn, and he never deviates from searching the market with reservation wage $\bar{\xi}$.

An alternative interpretation of this model is to assume that F_i represents the subjective offer distribution that the searcher attaches to firm i. The searcher orders the firms by their reservation wages, ξ_i, $i = 1, 2, \ldots, N$. Thus the first firm visited has reservation wage $\bar{\xi}$. Let $\bar{\bar{\xi}}$ be the second-highest reservation wage, and suppose a nonrecallable offer $\bar{w}$ is received from the first firm. If $\bar{w} \geq \bar{\bar{\xi}}$, the searcher accepts employment at the first firm; if $\bar{w} < \bar{\bar{\xi}}$, he solicits an offer from the second firm. He proceeds in this fashion until employment is achieved. Salop (1973) has termed this process "systematic search."

A Bandit Model of Migration

There are N cites available for location, and the wages associated with each city, w_i, $i = 1, 2, \ldots, N$ are known. For each city there is a moving cost, k_i, $i = 1, \ldots, N$, that is incurred once, the first time the city is occupied. This assumption is somewhat unrealistic in that traveling costs—when relocation to a former search city is concerned—are assumed to be zero. Our model stresses the costs of learning attributes of a location. Finding desirable stores, restaurants, parks, and so on could consume considerable time and expense, as can the formation of a new social network. We postulate that these costs greatly outweigh moving costs, especially early in a career when accumulated physical capital is negligible. Another way to view the situation is that one incurs costs to learn city attributes, which in turn influence subsequent migration decisions. Finally, it is assumed that city attributes are stable over time, again implying that location costs are incurred only once for each city.[18]

After moving and working 1 period, city attributes are learned, and a decision is made whether to quit and move elsewhere or to remain. For simplicity we assume that a priori the migrant views city i's attributes as a simple random variable, $\alpha_i V$, $\alpha_i > 0$ and $P(V = 1) = P(V = -1) = 1/2$. Since the model assumes that the worker is an expected income maximizer—and so risk neutral—this assumption is not restricting. The objective is to design a test policy that maximizes expected discounted returns.[19]

[18] On a more mathematical level, the zero cost assumption of relocation to a formerly inhabited city is needed to ensure that the Gittins solution is valid. One of the major assumptions needed for a solution is project independence. The number of cities need not be fixed at N but may be increasing. This arm-acquiring generalization is contained in Whittle (1981).

[19] This is the essential feature of the belated information model. The details may be found in Lippman and McCall (1981).

In MAB terminology there are only three possible states,

$$x_i(t) = \begin{cases} 0, \\ 1, \\ 2, \end{cases}$$

at the outset; $t = 0$; and all projects are in state 0. If project i is worked initially at time t^*, then $x_i(t^*) = 0$, one receives $w_i - k_i$, and the project moves to $x_i(t^* + 1)$, where

$$x_i(t^* + 1) = \begin{cases} 1, & p = ½, \\ 2, & p = ½. \end{cases}$$

These are both absorbing states in the sense that if project i is worked again the state does not change. Earnings are either $w_i + \alpha_i$ or $w_i - \alpha_i$, according to whether $x_i(t^* + 1) = 1$ or 2, respectively.

From (2) the Gittins index of project i in state 0 satisfies the following equation:

$$Z_i^0 = w_i - k_i + \frac{1}{2}\beta\{\max[Z_i^0, V_i(1, Z_i^0)] + \max[Z_i^0, V_i(2, Z_i^0)]\}.$$

A result due to Gittins (1979) simplifies its calculation. If a project is deteriorating in the sense that

$$V_i[x_i(t), m] \geq V_i[x_i(t+1), m] \ \forall \ x_i(t+1),$$

then the Gittins index $Z_i^{x_i(t)} = R_i(x_i)/(1 - \beta)$. Using this result gives the following lemma.

LEMMA 1. (*a*) If $k_i \geq \alpha_i/(1 - \beta)$, the Gittins index is given by

$$Z_i^0 = w_i - k_i + \beta w_i/(1 - \beta).$$

(*b*) If $k_i < \alpha_i/(1 - \beta)$, the Gittins index is given by

$$Z_i^0 = [w_i - k_i + 1/2\beta/(1 - \beta)(w_i + \alpha_i)]/(1 - 1/2\beta).$$

PROOF. See the Appendix.

As anticipated, the location decision depends not only on wages and moving costs but—when α_i is "large" relative to k_i—also on city attributes. In addition a city's "attractiveness" increases in α_i; that is, Z_i^0 is increasing in α_i. For example, suppose all cities are identical except for city attributes. That is, $k_i = k_j$, and $w_i = w_j \ \forall \ i \neq j$, whereas $\alpha_i \neq \alpha_j$ $\forall \ i \neq j$, and assume that $k_i < \alpha_i/(1 - \beta) \ \forall \ i$. Then the optimal policy is to order the cities from highest to lowest α_i and to test each city until either one gets a positive α_i or all cities have been tested. The reasoning is clear: one prefers to "win" a large α as soon as possible. It may seem that the cost of "losing" is higher for riskier cities, but the cost is the same for all cities, namely, the cost of moving to the next city.

Suppose for M cities $\alpha_i/(1-\beta) < k_i$, $M < N$. Then the optimal policy would be to test the first $N - M$ cities, when ordered from highest to lowest α_i, and continue testing until one "wins." If the first $N - M$ cities turn out badly, then remain at the last one—the expected gain from migration to the next-best city is less than expected cost.

This result seems contrary to the finding that people tend to move to well-known places. But here improved information about city i not only decreases α_i but also raises w_i; thus the net effect is an increase in Z_i^0.

This simple example may also explain why cities with large in migration also have large out migration. City i may possess a large α_i. This induces a large in migration. But after experimenting, many migrants, approximately half, will be disappointed. Thus a large value of α_i increases both in migration and out migration.

On the other hand, a city may be attractive mainly because of a high wage (small α_i). This corresponds to the case of good information prior to a migration decision. In this case, if information subsequent to the move turns out poorly, it is less likely that remigration will take place. The Gittins index declines less in city i, and it is likely to remain best. This provides a theoretical basis for the empirical evidence that remigration is a decreasing function of information known prior to migration (see Yezer and Thurston 1976; Allen 1979; and DaVanzo 1983).

Finally, it is clear that return migration is often the optimal policy. Suppose an individual is located originally in city i, his hometown, and $Z_i^0 = w_i + \alpha_i/(1-\beta)$.[20] Suppose there is one city j with w_j and α_j both sufficiently large that migration is optimal. Let all other cities $k \neq i, j$, be inferior to i. After migration, a large negative α_j may cause the index to decline enough that return migration takes place even though w_j is larger than w_i.

III. A General Bandit Model of Migration

The model is the same as before except that the wage w_i^ℓ at location i is a random variable with subjective probability distribution q_i^ℓ, $\ell = 1, 2, \ldots, L$, $\sum_{\ell-1}^{L} q_i^\ell = 1$. The individual pays a cost c_i—perhaps to an employment agency—to observe the true wage in city i. This wage

[20] We may also view this example in terms of an arm-acquiring MAB process (see Whittle 1981). The hometown is where one "settles," i.e., is, receives a positive α, with projects added according to a Poisson process. The Gittins procedure remains optimal when city j is considered as an additional opportunity.

may be considered his best wage offer obtained from a search process in which an amount c_i is allocated to the search process. All individual job information is obtained prior to a move decision—there is no belated information subsequent to a move with respect to the job. This implies no subsequent intracity job search. If he stays premanently in the city, he will work at his initial job. This simplifying assumption is needed to satisfy the Gittins solvability constraints.

Assume a potential migrant searches other cities before making a decision to move. For example, suppose a migrant moves to city i and finds that he dislikes the city after working there 1 period ($V = -1$). It may be optimal to search city j and discover his true w_j there. If w_j^ℓ is high, given α_j and k_j, he may move to j. If, on the other hand, w_j^ℓ is relatively low, then he may search the next most attractive city or, if the potential benefits of that search do not outweigh the costs c, return to work premanently in city i.

In terms of our MAB model, city i is in state 0 at the outset. If worked, $-c_i$ is earned, and the project moves to one of L states, $\ell = 1, 2, \ldots, L$. If the project is worked once again, $w_i^\ell - k_i$ is earned, and the project moves to one of two absorbing states with equal probability—as in the simple migration model—that we call 2ℓ or 3ℓ. Thus in total the project has $3L + 1$ states. Now, if ever worked again, project i yields either $w_i^\ell + \alpha_i$ or $w_i^\ell - \alpha_i$, depending on whether $x_i(t^*) = 2\ell$ or 3ℓ, respectively. Therefore the Gittins index for the ith job in state 0 satisfies

$$\begin{aligned} Z_i^0 = -c_i + \beta \sum_{\ell=1}^{L} q_i^\ell \max\Bigl(Z_i^0, w_i^\ell - k_i + \frac{1}{2}\beta\,\{\max[Z_i^0, V(2\ell, Z_i^0)] \\ + \max[Z_i^0, V(3\ell, Z_i^0)]\}\Bigr). \end{aligned} \tag{3}$$

From (3) it is clear that Z_i^0 is increasing in α_i. If city i is searched, that is, $Z_i^0 \geq Z_j^{x_j}\ \forall\ j \neq$ i, then the city will be tested—migration will take place—if and only if $Z_\ell^i \geq Z_j^{x_j}\ \forall\ j \neq i$. Let $Z^* = \max_{j\neq i} Z_j^{x_j}$. Then we obtain the following.

THEOREM 1. If the ith city is searched, there is a reservation wage ξ_i for the ith city such that migration to the ith city occurs if and only if $w_i^\ell \geq \xi_i$, where ξ_i is defined by (a)

$$\xi_i = Z^*(1-\beta) + k_i - \frac{\beta(k_i + \alpha_i)}{(2-\beta)},$$

if $\alpha_i/(1 - \beta) \geq k_i$, and ($b$)

$$\xi_i = (1-\beta)(Z^* + k^i),$$

if $\alpha_i/(1 - \beta) < k_i$.

PROOF. (*a*) From lemma 1,

$$Z_i^\ell = \left[w_i^\ell - k_i + \frac{1}{2}\beta(w_i^\ell + \alpha_i)/(1-\beta)\right] \Big/ (1 - 1/2\beta).$$

Consequently, the job is tested—migration occurs—if and only if

$$Z^* < \left[w_i^\ell - k_i + \frac{1}{2}\beta(w_i^\ell + \alpha_i)/(1-\beta)\right] \Big/ (1 - 1/2\beta),$$

which implies

$$Z^* = \left[\xi_i - k_i + \frac{1}{2}\beta(\xi_i + \alpha_i)/(1-\beta)\right] \Big/ (1 - 1/2\beta).$$

(*b*) Now,

$$Z_i^\ell = w_i^\ell - k_i + \frac{\beta w_i}{1-\beta}.$$

Arguing as in (*a*), we obtain

$$Z^* = \xi_i - k_i + \frac{\beta \xi_i}{10\beta}.$$

Q.E.D.

Clearly, the results derived from the optimal policy in the simple model—return migration, correlation between high in migration and high out migration, and so on—extend to the more general model. However, in the general case a more complicated picture may emerge with periods of search occurring prior to migration, the reservation wage dictating the migration decision.

We conclude this section by stating the structure of the optimal migration policy. (1) Calculate the Gittins indices for the N cities. (2) Spend c_i and search the city with the largest index. If the best wage offer from this city w_i exceeds ξ_i, migrate there; observe i's pecuniary aspects; if $w_i < \xi_i$, search the city with the next highest Gittins index. (3) If the nonpecuniary aspects are positive α_i, stop; if they are negative α_i, test the N-1 Gittins indices to see if one of them exceeds your present return. If such an index exists, then work that project. This may involve searching a city, migrating to a new city, or relocating, depending on the project's state. If no such index exists, stop.

IV. International Migration

In this section we shall use a generalization of the Gittins result to analyze a special case of international migration—the illegal alien problem. In this extension projects may have variable stages.[21] The basic MAB model assumes that any project may be tried for a single period and then broken off. In some applications this is unrealistic. When one selects project i in state x_i, he is committed to continue that project for a time τ, which may be random. The Gittins result continues to apply with the index for project i in state x_i determined by the following optimal value function:

$$V_i(x_i, m) = \max[m, R_i(x_i) + E(\beta^t) \sum_{x'_i} P^{x_i, x'_i} V_i(x'_i, m)],$$

where $R_i(x_i)$ is now interpreted as the total expected reward from working the project one more time as opposed to 1 more period. The probability that a project begins in state x_i and moves to state x'_i at the end of the stage is denoted by P^{x_i, x'_i}.

We now analyze the decision confronting a potential illegal alien. We model this as a MAB process. This problem is much more complicated than the migration problem since capture probabilities must be considered both at the point of entry and at the workplace. The solution is simplified by applying the results from variable stage processes.

The individual has several opportunities in his or her home country plus the possibility of working—illegally—in a foreign country. For simplicity we will assume that there is only one such foreign opportunity.[22] Label this project F. The worker will illegally migrate to the foreign country if and only if $Z_F^{x_F} \geq \max_{i \neq F} (Z_i^{x_i})$.

To derive Z_F^0 explicitly, assume that there are two capture probabilities known with certainty by the worker (there is no Bayesian adjustment). These are the probability of capture on entry, P_F, and the probability of capture at the workplace, P'_F. Thus for each attempted entry there is an independent probability of capture, and once entry is successful there is an independent capture probability P'_F for each subsequent period of work. Also associated with each entry attempt is a cost, k_F, and after entry is achieved a wage of either $w + \alpha_F$ or $w_F - \alpha_F$ is received, each with probability ½.[23] We assume that return migration is not desirable.

[21] For a complete description, see Whittle (1982–83).

[22] Extending this to the case in which there are many foreign countries or many workplaces within a foreign country is accomplished easily.

[23] Alternatively, the wage may be known with certainty, but belated information concerning area characteristics is learned.

That is, once the alien successfully enters he prefers to stay regardless of the subsequent belated information and works until caught. Only when caught and returned to his home country may reentry be undesirable given the accumulated information.

Given the above structure, one can employ the results from variable project stage processes to simplify calculation of the index. That is, this problem is equivalent to the one that restricts the worker to try entry until successful and remain in the foreign country at the workplace until caught. This follows since the optimal policy in the unrestricted case entails the same behavior—the restrictions are not binding. Thus there are three stages, 0, 1, 2—entry with no information, entry with "good" information, and entry with "bad" information, respectively.

The optimal value function at stage 0 is

$$V_F(0,m) = \max\left\{m, -E\left(\sum_{n=0}^{\tau-1} \beta^n\right)k_F + E(\beta^\tau)E\left(\sum_{n=0}^{\sigma-1} \beta^n\right)w_F \right.$$
$$\left. + \frac{1}{2}E(\beta^\sigma)[V_F(1,m) + V_F(2,m)]\right\},$$

where τ is a geometric random variable representing the time of successful entry, and σ is the geometric random variable representing the time of capture at the workplace. Letting

$$E(\beta^\tau) \equiv \theta, \quad E(\beta^\sigma) \equiv \delta,$$

$$E\left(\sum_{n=0}^{\tau-1} \beta^n\right)k_F \equiv K, \quad E\left(\sum_{n=0}^{\sigma-1} \beta^n\right)w_F \equiv W,$$

$$E\left(\sum_{n=0}^{\sigma-1} \beta^n\right)(w_F + \alpha_F) \equiv W + A, \quad E\left(\sum_{n=0}^{\sigma-1} \beta^n\right)(w_F - \alpha_F) \equiv W - A,$$

we have

$$V_f(0,m) \equiv \max\left(m, -\mathrm{K} + \theta\left\{W + \frac{1}{2}\delta[V(1,m) + V(2,m)]\right\}\right),$$

where

$$V_f(1,m) = \max\{m; -K + \theta[(W+A) + \delta V(1,m)]\},$$

$$V_f(2,m) = \max\{m; -K + \theta[(W+A) + \delta V(2,m)]\}.$$

Thus

$$Z_F^0 = \frac{(-K+\theta W)\left(1-\delta+\frac{1}{2}\theta\delta\right)+\frac{1}{2}\theta^2\delta A}{(1-\delta)\left(1-\frac{1}{2}\theta\delta\right)}$$

Of course, government officials of the foreign country have some control over capture probabilities and entry costs—say, by increasing penalties for those who smuggle illegal aliens into the country. Thus to minimize "initial" entry attempts they would allocate their limited resources to minimize Z_F^0. But to minimize total attempts—given the assumptions of our model—it should explicitly be recognized that $Z_F^1 > Z_F^0 > Z_F^2$—reentrants are better informed—when optimally allocating resources.

V. The Return Migration Phenomenon: A Martingale Estimator[24]

To simplify the discussion assume that there are only two regions. At the beginning of our study, n_{12} is the total number of individuals in region 2 who moved from their home region, region 1. We wish to measure the strength of the return migration phenomenon.

Let $N_r(t)$ be the number of individuals in region 2 at $t = 0$ who return to region 1 in the time interval $[0, t]$. Each time an individual returns to region 1, N_r increases by one; hence N_r is called a counting process. As usual, we assume that two individuals cannot return simultaneously.

The intensity process is derived as follows. Let I_{dt} be a small time interval of length dt around time t. The history of the process up to an instant before t is denoted by H_{t-}, and the conditional probability that N_r jumps in I_{dt} given its history till just before t is denoted by $\lambda_r(t)$.

Let $dN_r(t)$ be the increment of N_r during I_{dt}, and let H_{t-} be the history up to but not including t. Then

$$\lambda_r(t)dt = \mathrm{pr}[dN_r(t) = 1|H_{t-}]. \tag{4}$$

Note that $H_s \subseteq H_t$, provided $s \leq t$. That is, the history (formally called the sigma algebra) is nondecreasing in t.

Since $dN_r(t)$ is a Bernoulli (0, 1) random variable,

$$E[dN_r(t)|H_{t-}] = \lambda_r(t)dt.$$

[24] There are several clear presentations of this relatively novel methodology. They include Bremaud (1981), Jacobsen (1982), Karr (1984, in press), and Andersen and Borgan (1985).

Now define a stochastic process, M_r, as having increments

$$dM_r(t) = dN_r(t) - \lambda_r(t)dt$$

over the interval I_{dt}, with $M_r(0) = 0$. Then

$$E[dM_r(t)|H_{t-}] = 0,$$

which implies that the return process

$$M_r(t) = N_r(t) - \int_0^t \lambda_r(s)ds$$

is a martingale.[25] The integral on the right-hand side, the integrated hazard rate, is called the compensator of the martingale process. It is extremely important in the development of martingale theory and martingale econometrics.

To be more complete let us assign a random variable $N_{ir}(t)$ to each of the n_{12} migrants in region 2 at $t = 0$ and derive the multivariate counting process $\underline{N}$.

For each of the n_{12} migrants who began in region 1 but were in region 2 at $t = 0$, say, the ith, we observe his mobility from $t = 0$ to $\bar{T}_i$, the time $\bar{T}_i$ is either the actual return time to region 1 or a censoring time. Define $d_i = 1$ if $\bar{T}_i$ is the actual return time and $d_i = 0$ if $\bar{T}_i$ is a censoring time, for the ith individual, $i = 1, \ldots, n_{12}$. Hence N_{ir} is zero before $\bar{T}_i$ and jumps to one at $\bar{T}_i$, provided $\bar{T}_i$ is the actual return time; otherwise, no jumps occur.

The corresponding intensity process is calculated as follows. If at any time t the individual has returned to region 1 or been censored, the conditional probability of a jump in I_{dt} is zero; if he has not been censored and has not returned to region 1 by t, the conditional probability is $\alpha_i(t)dt$, where $\alpha_i(t)$ is the "hazard rate"[26] for the actual return time of individual i. Thus letting[27]

[25] A sequence of random variables $Y_1, Y_2, \ldots,$ is a martingale if

$$E(|Y_i|) < \infty, \tag{ii}$$

and

$$E(Y_{n+1}|Y_1, \ldots, Y_n) = Y_n. \tag{iii}$$

[26] The hazard rate is the instantaneous return rate at t given that the individual has not returned by t. That is $\alpha(t)dt$ is the probability of a return in $(t, t + dt)$ given no return by t.

[27] The function $1(\cdot)$ is called the "indicator function." It equals one on the set $\{\bar{T}_i \geq t\}$ and zero elsewhere.

$$Y_i(t) = 1_i(\bar{T}_i \geq \text{t})$$

yields the conditional probability of a jump

$$P[dN(t) = 1 | H_{t-}] = \alpha_i(t) Y_i(t) dT. \tag{5}$$

By (4) and (5), the multivariate counting process $\underline{N} = (N_{1r}, \ldots, N_{nr})$ has intensity process $\underline{\lambda}$ with components

$$\lambda_i(t) = \alpha_i(t) Y_i(t), \quad i = 1, \ldots, n.$$

Assume $d_i(t) = d_r(t)$, for all i. Then, a nonparametric estimator of

$$A_r(t) = \int_0^t \alpha_r(s) ds$$

can be derived by noting that[28]

$$dN_{ir}(t) = \alpha_r(t) Y_r(t) + \text{“noise”},$$

so that

$$\int_0^t Y_r(s)^{-1} dN_r(s)$$

is the obvious estimator. When censoring is included, this estimator becomes

$$\hat{A}_r(t) = \int_0^t [J_r(s)/Y_r(s)] dN_r(s),$$

where $J_r(t) = 1[Y_r(t) > 0]$.

The estimator $\hat{A}_r(t)$ is called the Nelson-Aalen nonparametric estimator.[29]

VI. Conclusion

There are many problems in labor economics that may benefit from a MAB analysis. The allocation of new employees among the teams that constitute the firm is a promising application. International migration (legal and illegal, permanent and temporary) also has appeal. Indeed, the

[28] "Noise" is the fluctuation induced by the martingale stochastic process. Note also that $Y_r(t) = \sum_{i=1}^{n} Y_i(t)$.

[29] For a complete discussion, the reader should consult Andersen and Borgan (1985).

German counterpart to the National Science Foundation has funded a 10-year study of guest-worker mobility. The analysis will be done at the University of Konstanz. Siegfried Berninghaus plans to use a MAB model similar to the one described here. These methods also are being applied to the movement of military personnel among training schools and jobs at the Rand Corporation. Finally, Alan Pitts (1986) is applying Markov process analysis (an innovative queuing model) to study labor turnover. He plans to test the model with martingale methods. His rich data set—a longitudinal file beginning in 1966—allows him to test a variety of hypotheses.

We also note that the analysis described here and the proposed econometric methods are pertinent to location theory.[30] Consider a firm that is planning to construct a plant in one of N locations. The crucial variables entering the firm's decisions are the availability of a trained labor force (or the ease with which one can be attracted), tax incentives provided by the local community, proximity of nonlabor inputs, and so on.[31] On the basis of all the available information, the index derived in Section III is calculated for each of the N possible sites. This index for site i is a measure of the profitability of the plant at site i, given the available information. Then the firm spends c_i dollars, intensively searches the site with the highest index, and receives a better estimate of profits. If this exceeds reservation profits, the firm begins building the plant. If not, the firm considers the location that now has the highest index. Suppose, though, that the reservation profits are exceeded. Then k_i dollars are spent, and building commences. During this initial phase of building, architectural plans are designed; ground, however, is not broken; and the firm learns the precise location and composition of the labor force and has personal contacts with city officials and other local firms. If this information is positive, it locates at i. If it is negative, it may render this location inferior to that location with the previously second highest Gittins index. The firm will then move to this superior site and acquire more information there. If, after experimenting with other sites, it discovers that site i is the best, it need not pay k_i dollars when it returns to i; that is, it starts almost where it left off.

Appendix

In this Appendix we prove lemma 1.

LEMMA 1. (*a*) If $k_i \geq \alpha_i/(1 - \beta)$, the Gittins index is given by

$$Z_i^0 = w_i - k_i + \beta w_i/1 - \beta.$$

[30] A recent survey has been completed by Stahl (1986). The article by Hotelling (1929) remains pertinent.

[31] For an empirical study of plant location, see Carlton (1983). The model presented here generalizes the paper by Pascal and McCall (1980).

(*b*) If $k_i < \alpha_i/(1 - \beta)$, the Gittins index is given by

$$Z_i^0 = \left[w_i - k_i + \frac{1}{2}\beta/(1-\beta)(w_i + \alpha_i)\right] \Big/ (1 - 1/2\beta).$$

Proof. (*a*) The first order of business is to calculate Z_i^1 and Z_i^2. But both state 1 and state 2 are absorbing. Thus applying Gittins result gives

$$Z_i^1 = \frac{w_i + \alpha_i}{1-\beta}, \quad Z_i^2 = \frac{w_i - \alpha_i}{1-\beta}.$$

Now suppose $Z_i^0 \geq (w_i + \alpha_i)/(1 - \beta)$. Since $v_i(x, m) = m$ for $m \geq Z_i^x$, we have

$$V_i(1, Z_i^0) = Z_i^0, \quad V_i(2, Z_i^0) = Z_i^0.$$

Hence

$$Z_i^0 = w_i + k_i + 1/2\beta 2 Z_i^0 = w_i - k_i + \beta Z_i^0$$

or

$$Z_i^0 = \frac{w_i - k_i}{1-\beta},$$

a contradiction since α_i, $k_i > 0$. Suppose

$$\frac{w_i + \alpha_i}{1-\beta} > Z_i^0 > \frac{w_i - \alpha_i}{1-\beta}.$$

Then

$$Z_i^0 = V(2, Z_i^0),$$

$$V(1, Z_i^0) = w_i + \alpha_i + \beta V(1, Z_i^0),$$

$$V(1, Z_i^0) = \frac{w_i + \alpha_i}{1-\beta}.$$

Thus

$$Z_i^0 = \left[w_i - k_i + \frac{\beta}{2(1-\beta)}(w_i + \alpha_i)\right] \Big/ \left(1 - \frac{1}{2}\beta\right)$$

or

$$k_i < \frac{\alpha_i}{(1-\beta)},$$

a contradiction. Hence

$$Z_{\mathrm{i}}^{\mathrm{o}} \leq \frac{w_{\mathrm{i}} - \alpha_{\mathrm{i}}}{1-\beta},$$

which implies

$$Z_i^{\mathrm{o}} = w_i - k_i + \frac{\beta w_i}{1-\beta}.$$

(*b*) From *a*, $Z_i^{\mathrm{o}} \leq (w_i + \alpha_i)/(1 - \beta)$. Suppose $Z_i^{\mathrm{o}} \leq (w_i - \alpha_i)/(1 - \beta)$; then

$$V(1, Z_i^{\mathrm{o}}) = w_i + \alpha_i + \beta V(1, Z_i^{\mathrm{o}}),$$

$$V(2, Z_i^{\mathrm{o}}) = w_i - \alpha_i + \beta V(2, Z_i^{\mathrm{o}}),$$

$$V(1, Z_i^{\mathrm{o}}) = w_i + \alpha_i/(1-\beta),$$

$$V(2, Z_i^{\mathrm{o}}) = w_i - \alpha_i/(1-\beta),$$

$$Z_i^{\mathrm{o}} = w_i - k_i + \frac{\beta w_i}{1-\beta}.$$

Hence

$$\frac{w_i - \alpha_i}{1-\beta} \geq w_i - k_i + \frac{\beta w_i}{1-\beta}$$

or

$$\frac{\alpha_i}{1-\beta} \leq k_i,$$

a contradiction. Thus

$$\frac{w_i + \alpha_i}{1-\beta} < Z_i^{\mathrm{o}} < \frac{w_i - \alpha_i}{1-\beta},$$

$$Z_i^{\mathrm{o}} = \left[w_i - k_i + \frac{\beta}{2(1-\beta)}(w_i + \alpha_i)\right] \Big/ \left(1 - \frac{1}{2}\beta\right).$$

Q.E.D.

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